Lecture 23: AB Testing 1

CS109A Introduction to Data Science

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Announcements

HW 7 Clarifications:

- Don’t get tripped up on the notation (what *Z* represents).
- Reporting: do not multiply by 100 (leave in decimal form)
- Scoring: not just the leaderboard (because there is a ‘hidden’ test set)
- Kaggle submissions: be sure to accept the terms and then join the competition

HW 8: will be **short** and on solely on Ed. Very little coding.

Outline

- Causal Effects
- Experiments and *AB*-testing
- *t*-tests, binomial *z*-test, fisher exact test, oh my!
- Obama 2008

Association vs. Causation

In many of our methods (regression, for example) we often want to measure the association between two variables: the response, Y, and the predictor, X. For example, this association is modeled by a β coefficient in regression, or amount of increase in R^2 in a regression tree associated with a predictor, etc...

If β is *significantly different* from zero (or amount of R^2 is greater than by chance alone), then there is evidence that the response is associated with the predictor.

How can we determine if β is *significantly different* from zero in a model?

Association vs. Causation (cont.)

But what can we say about a *causal association*? That is, can we manipulate *X* in order to influence *Y*?

Not necessarily. Why not?

There is potential for confounding factors to be the driving force for the observed association.

Controlling for confounding

How can we fix this issue of confounding variables?

There are 2 main approaches:

1. Model all possible confounders by including them into the model (multiple regression, for example). Or use fancy methods ('causal methods') to account for the confounders.
2. An *experiment* can be performed where the scientist manipulates the levels of the predictor (now called the *treatment*) to see how this leads to changes in values of the response.

What are the advantages and disadvantages of each approach?

Controlling for confounding: advantages/disadvantages

1. Modeling the confounders

- Advantages: cheap
- Disadvantages: not all confounders may be measured.

2. Performing an experiment

- Advantages: confounders will be *balanced*, on average, across treatment groups
- Disadvantages: expensive, can be an artificial environment

Experiments and *AB*-testing

Completely Randomized Design

There are many ways to design an experiment, depending on the number of treatment types, number of treatment groups, how the treatment effect may vary across subgroups, etc...

The simplest type of experiment is called a Completely Randomized Design (CRD). If two treatments, call them treatment *A* and treatment *B*, are to be compared across *n* subjects, then *n*/2 subject are randomly assigned to each group.

- If n = 100, this is equivalent to putting all 100 names in a hat, and pulling 50 names out and assigning them to treatment *A*.

Experiments and *AB*-testing

In the world of Data Science, performing experiments to determine causation, like the completely randomized design, is called *AB*-testing.

AB-testing is often used in the tech industry to determine which form of website design (the treatment) leads to more ad clicks, purchases, etc... (the response). Or to determine the effect of a new app rollout (treatment) on revenue or usage (the response).

Assigning subject to treatments

In order to balance confounders, the subjects must be properly randomly assigned to the treatment groups, and sufficient enough sample sizes need to be used.

For a CRD with 2 treatment arms, how can this randomization be performed via a computer?

You can just sample $n/2$ numbers from the values 1, 2, ..., n without replacement and assign those individuals (in a list) to treatment group *A*, and the rest to treatments group *B*. This is equivalent to sorting the list of numbers, with the first half going to treatment *A* and the rest going to treatment *B*.

This is just like a 50-50 test-train split!

Beyond just A vs. B

How can an AB test be expanded to include more than two options? What if there are more than just one type of treatment?

The **multivariate experimental design** generalizes this approach. If there are two treatment types (font color, and website layout), then both treatments' effects can (and should) be tested simultaneously. Why?

In a **full factorial experimental** design, each and every combination of treatments are considered different treatment groups. Experiments online are cheap. Full factorial designs are often possible and feasible.

Better than AB…

I DON'T ALWAYS RUN A WEBSITE TEST
BUT WHEN I DO, IT'S ALWAYS MULTIVARIATE
made on imgur

t-tests, binomial *z*-test, fisher exact test, oh my!

Analyzing the results

Just like in statistical/machine learning, the analysis of results for any experiment depends on the form of the response variable (categorical vs. quantitative), but also depends on the design of the experiment.

For *AB*-testing (classically called a 2-arm CRD), this ends up just being a 2-group comparison procedure, and depends on the form of the response variable (aka, if *Y* is binary, categorical, or quantitative).

Analyzing the results (cont.)

For those of you who have taken Stat 100/101/102/104/111/139:

If the response is quantitative, what is the classical approach to determining if the means are different in 2 independent groups?

- a 2-sample *t*-test for means

If the proportions of successes are different in 2 independent groups?

- a 2-sample *z*-test for proportions

2-sample *t*-test

Formally, the 2-sample *t*-test for the mean difference between 2 treatment groups is:

$H_0: \mu_A = \mu_B$ vs. $\overset{\text{Ha}}{\cancel{H_0}}: \mu_A \neq \mu_B$

theoretical population means

$$t = \frac{\bar{Y}_A - \bar{Y}_B}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}}$$

empirical means

The *p*-value can then be calculated based on a $t_{\min(n_A, n_B)-1}$ distribution.

The assumptions for this test include (i) independent observations and (ii) normally distributed responses within each group (or sufficiently large sample size).

2-sample z-test for proportions

Formally, the 2-sample z test for the difference in proportions between 2 treatment groups is:

$H_0: p_A = p_B$ vs. $\underset{\text{Ha}}{\cancel{H_0}}: p_A \neq p_B$

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}_p(1-\hat{p}_p)\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}$$

where $\hat{p}_p = \frac{n_A\hat{p}_A + n_B\hat{p}_B}{n_A + n_B}$ is the overall ‘pooled’ proportion of successes.

The *p*-value can then be calculated based on a standard normal distribution.

t-test is more robust than z-test

Normal approximation to the binomial

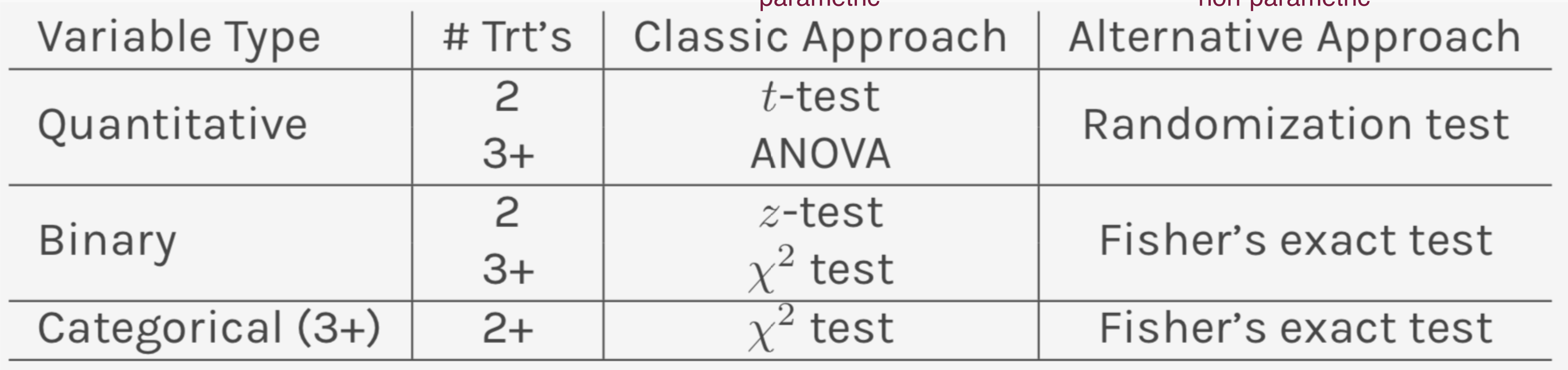
The use of the standard normal here is based on the fact that the binomial distribution can be approximated by a normal, which is reliable when $np \geq 10$ and $n(1 - p) \geq 10$.

What is a Binomial distribution? Why can it be approximated well with a Normal distribution?

Summary of analyses for CRD Experiments

parametric

non-parametric



Variable Type	# Trt's	Classic Approach	Alternative Approach
Quantitative	2	t-test	Randomization test
	3+	ANOVA	
Binary	2	z-test	Fisher's exact test
	3+	χ^2 test	
Categorical (3+)	2+	χ^2 test	Fisher's exact test

The classical approaches are typically *parametric*, based on some underlying distributional assumptions of the individual data, and work well for large *n* (or if those assumptions are actually true). The alternative approaches are *nonparameteric* in that there is no assumptions of an underlying distribution, but they have slightly less power if assumptions are true and may take more time & care to calculate.

Analyses for CRD Experiments in Python

- *t*-test:
  `scipy.stats.ttest_ind`
- proportion *z*-test:
  `statsmodels.stats.proportion.proportions_ztest`
- ANOVA *F*-test:
  `scipy.stats.f_oneway`
- χ^2 test for independence:
  `scipy.stats.chi2_contingency`
- Fisher's exact test:
  `scipy.stats.fisher_exact`
- Randomization test: ???

ANOVA procedure

The classic approach to compare 3+ means is through the Analysis of Variance procedure (aka, ANOVA).

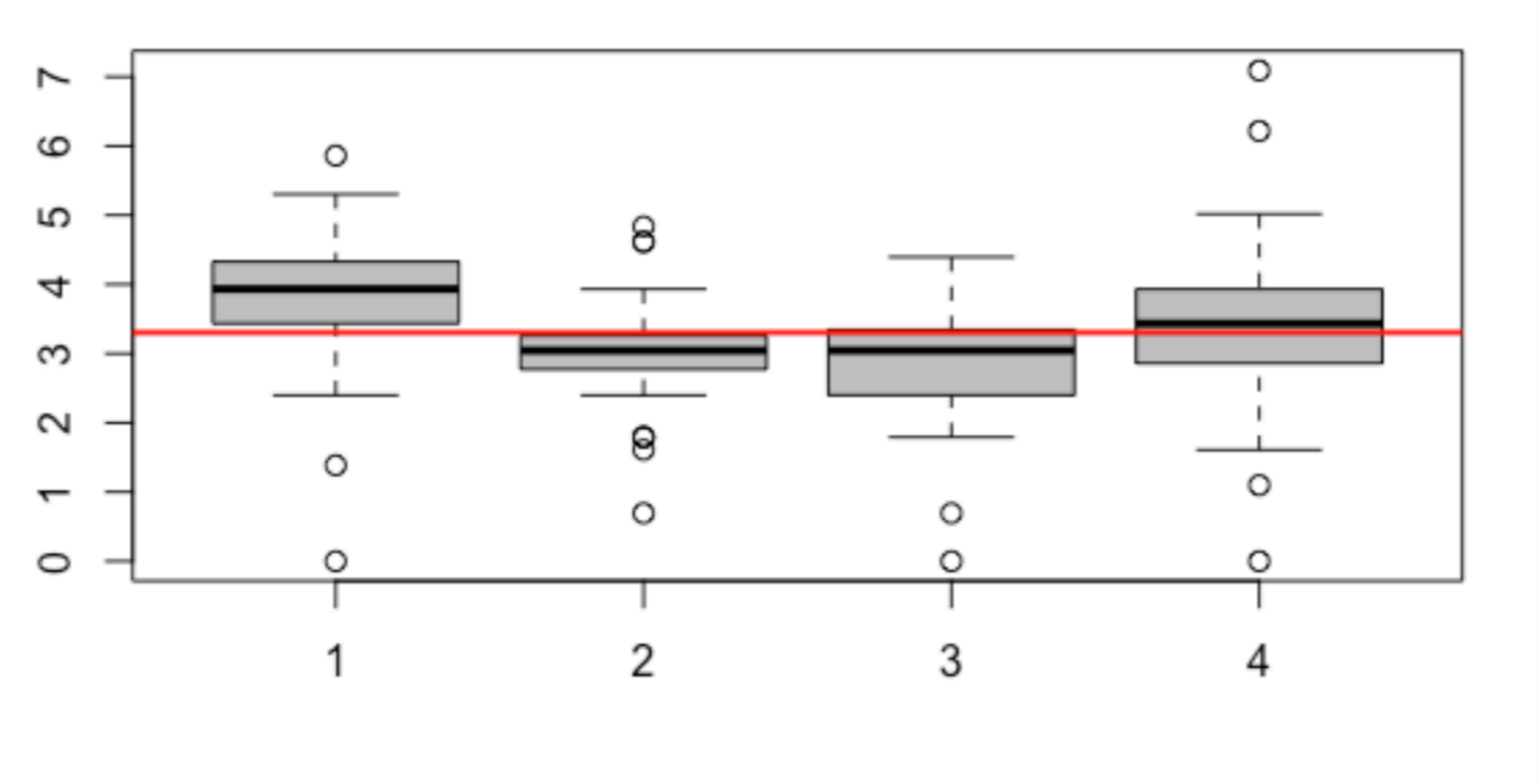
The ANOVA procedure's *F*-test is based on the decomposition of sums of squares in the response variable (which we have indirectly used before when calculating R^2).

$$SST = SSM + SSE$$

In this multi-group problem, it boils down to comparing how far the group means are from the overall grand mean (*SSM*) in comparison to how spread out the observations are from their respective group means (*SSE*).

A picture is worth a thousand words...

Boxplot to illustrate ANOVA



ANOVA *F*-test

Formally, the ANOVA F test for differences in means among 3+ groups can be calculated as follows:

H_0: the mean response is equal in all *K* treatment groups.
H_A: there is a difference in mean response somewhere among the treatment group.

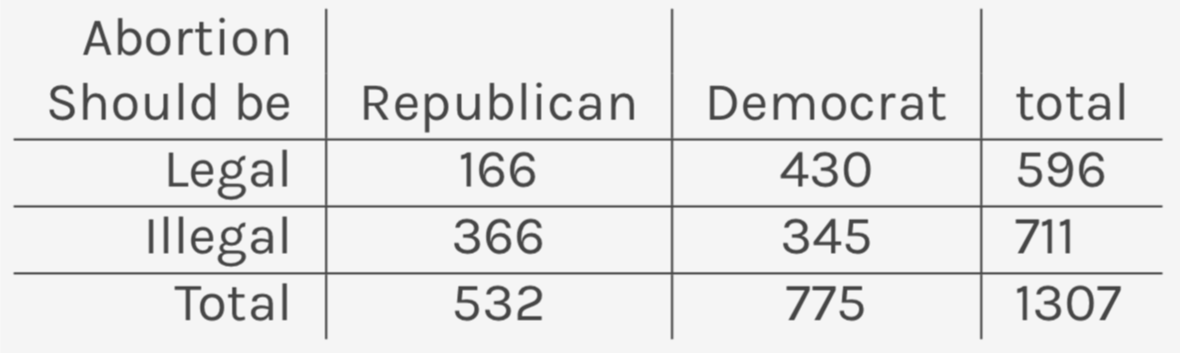
$$F = \frac{\sum_{k=1}^{K} \frac{n_k(\bar{Y}_k - \bar{Y})^2}{(K-1)}}{\sum_{k=1}^{K} \frac{(n_k - 1)S_k^2}{(n-K)}}$$

where n_k is the sample size in treatment group *k*, $\bar{Y}_k$ is the mean response in treatment group *k*, S_k^2 is the variance of responses in treatment group *k*, $\bar{Y}$ is the overall mean response, and $n = \sum n_k$is the total sample size.

The *p*-value can then be calculated based on a $F_{df_1=(K-1),df_2=(n-K)}$ distribution.

Comparing categorical variables

The classic approach to see if a categorical response variable is different between 2 or more groups is the χ^2 test for independence. A contingency table (we called it a confusion matrix) illustrates the idea:



Abortion Should be	Republican	Democrat	total
Legal	166	430	596
Illegal	366	345	711
Total	532	775	1307

If the two variables were independent, then:
$P(Y = 1 \cap X = 1) = P(Y = 1)P(X = 1)$.

How far the inner cell counts are from what they are expected to be under this condition is the basis for the test.

χ^2 test for independence

Formally, the χ^2 test for independence can be calculated as follows:

H_0: the 2 categorical variables are independent
H_A: the 2 categorical variables are not independent (response depends on the treatment).

$$\chi^2 = \sum_{all\ cells} \frac{(Obs - Exp)^2}{Exp}$$

where *Obs* is the observed cell count and *Exp* is the expected cell count:

$Exp = \frac{(row\ total)\times(column\ total)}{n}$.

The *p*-value can then be calculated based on a $\chi^2_{df=(r-1)\times(c-1)}$ distribution (*r* is the # categories for the row var., *c* is the # categories for the column var.).

Randomization test

A randomization test is the non-parametric approach to analyzing quantitative data in an experiment. It is an example of a *resampling* approach (the bootstrap is another resampling approach).

The basic assumption of the randomization test is that if the treatments are truly the same, then the measured response variable, Y_i, for subject i would not change if that subject was instead randomly assigned to a different treatment. This is sometimes called *exchangeability*.

Randomization test (cont.)

So to analyze the results, we re-randomize the individuals to treatment through simulation (keeping the sample sizes the same). We then re-calculate the statistic of interest (difference in 2 sample means or sums of squares between 3+ groups) many-many times and build a histogram of the results. This histogram is then used as the reference distribution to determine how extreme our actual observed result is.

This approach is also called a permutation test, since we are re-permuting each of the subjects into the treatment groups (and then assume this has no bearing on the response).

Fisher’s exact test

R.A. Fisher also came up with what is known as Fisher’s exact test.

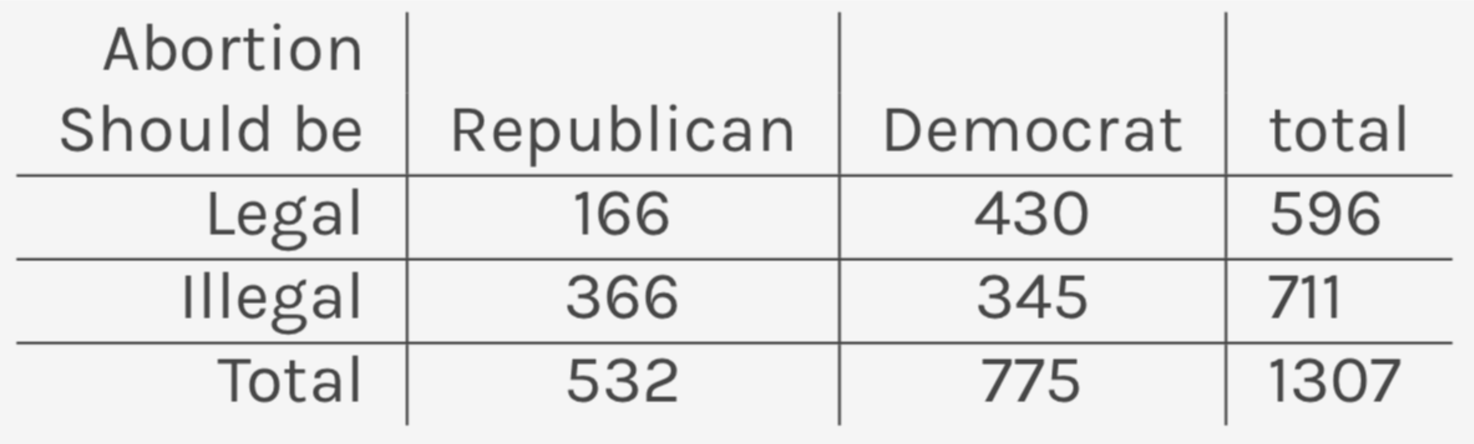
This analysis approach is useful for a contingency table, and does not need to rely on large sample size.

It fixes the row and column totals, and then determines all the ways in which the inner cells can be calculated given those row and column totals.

The probability of any of these filled out tables, given the row and column totals is fixed, is then based on a hypergeometric distribution .

Then the possible filled out tables that are less likely to occur than what was actually observed contribute to the *p*-value (by adding up their probabilities).

Fisher’s exact test



Abortion Should be	Republican	Democrat	total
Legal	166	430	596
Illegal	366	345	711
Total	532	775	1307

$$P(X_1 = 166) = \frac{\binom{596}{166}\binom{711}{366}}{\binom{1307}{532}} = 1.33\times10^{-18}$$

Then a similar calculation is done for all possible values of X_1, and these probabilities are summed up for those cases of X_1 that are not more likely to occur.

The app update roll-out problem

A company is interested in updating their app/program, so they start a ‘pilot program’ to test the waters to see how this update will affect some important measure (like revenue or usage). How should they do this?

They select a sample of users and ask them to voluntarily update the app on their phones in order to estimate the affect of this update.

Any issues with this design?

Volunteers will always be the most excited, dedicated users: a biased sample from all of their users.

We can potentially check for this bias via a χ^2 test for goodness-of-fit.

χ^2 test for goodness-of-fit

Formally, the χ^2 test for goodness-of-fit can be calculated as follows:

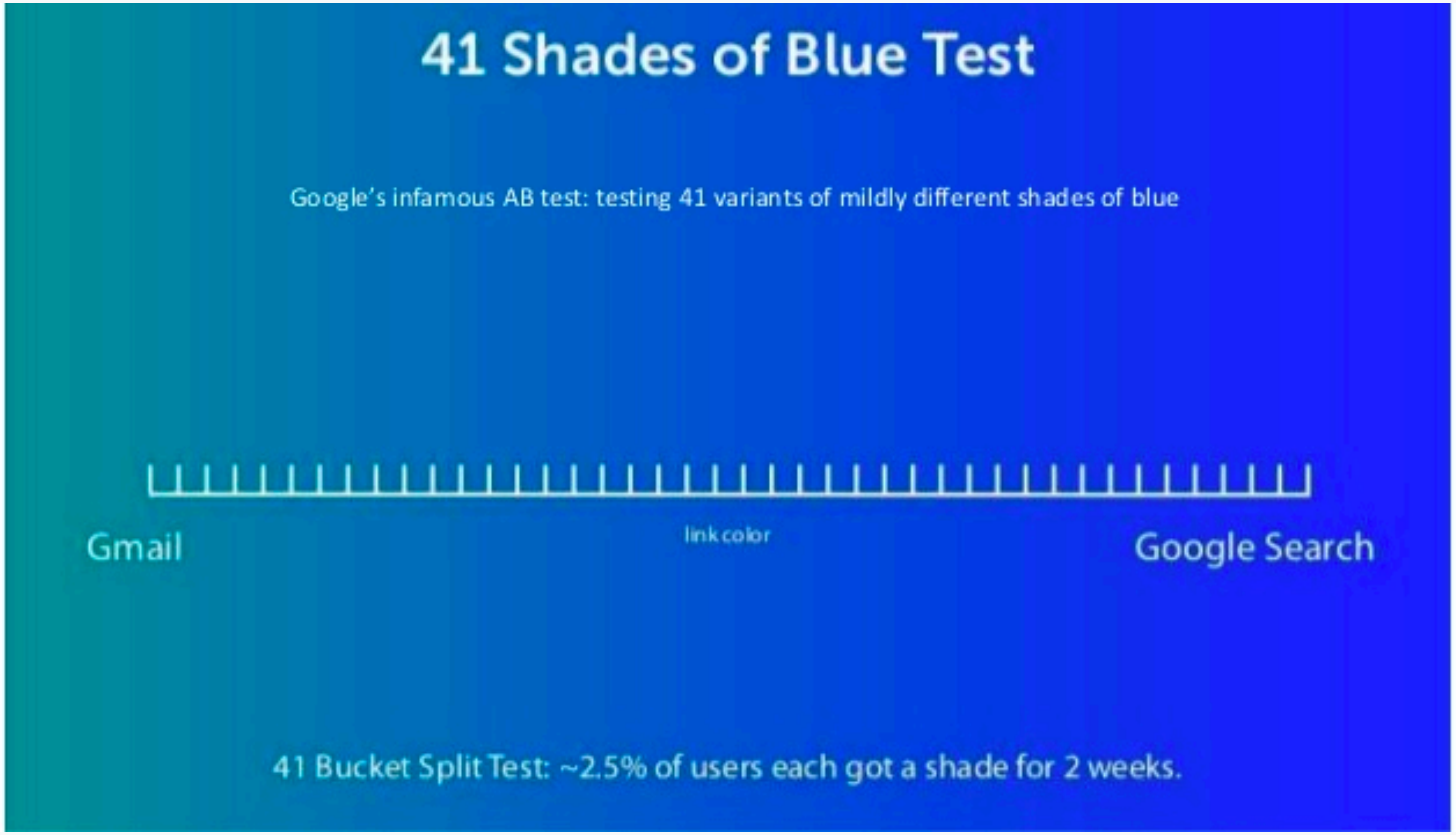
H_0: the variable follows some known distribution in the population
H_A: the variable does not follow this distribution

$$\chi^2 = \sum_{all\ cells} \frac{(Obs - Exp)^2}{Exp}$$

where *Obs* is the observed cell count and *Exp* is the expected cell count:
$Exp_i = n\pi_i$ (π_i is the theoretical probability of being in category/bucket *i*).

The *p*-value can then be calculated based on a $\chi^2_{df=(k-1)}$ distribution (*k* is the # categories in the population).

An infamous AB Test: 41 Shades of Blue



41 Shades of Blue Test

Google's infamous AB test: testing 41 variants of mildly different shades of blue

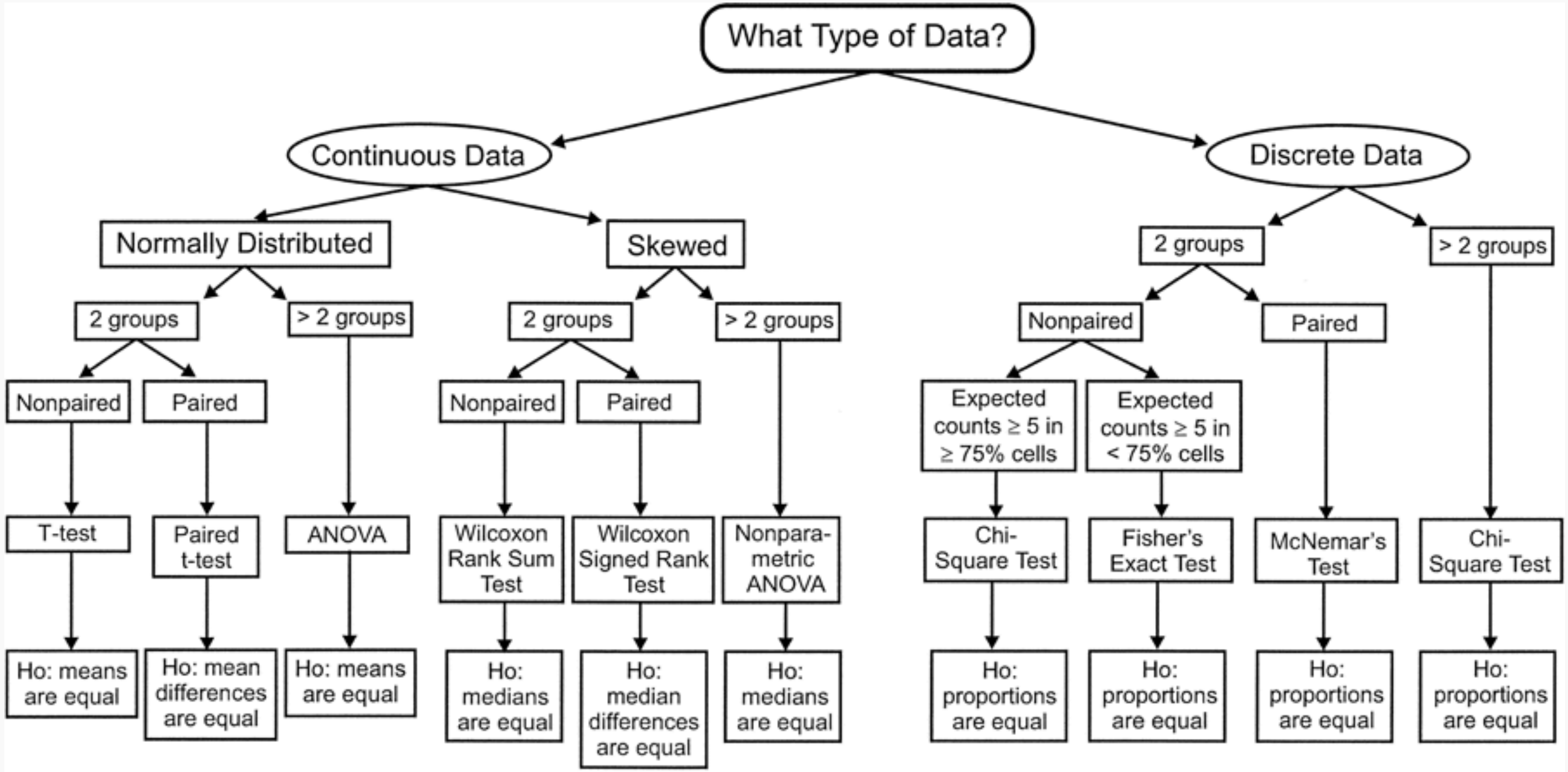
41 Bucket Split Test: ~2.5% of users each got a shade for 2 weeks.

How should the study proceed? How should the data be analyzed?

looking to see whether click-thru was uniformly distributed among colors or not

A Decision Tree for testing.

Inference



Source: Waning B, Montagne M: *Pharmacoepidemiology: Principles and Practice*: http://www.accesspharmacy.com

Obama’s 2008 Campaign

The 2008 Obama Campaign

In 2008, the Obama campaign raised much of its money via online donations through its website.

They wanted to optimize the launch page that visitors saw when they came to the campaign website. They were attempting to maximize the number of visitors that would sign up for their emailing list.

There were 2 treatments they attempted to vary:

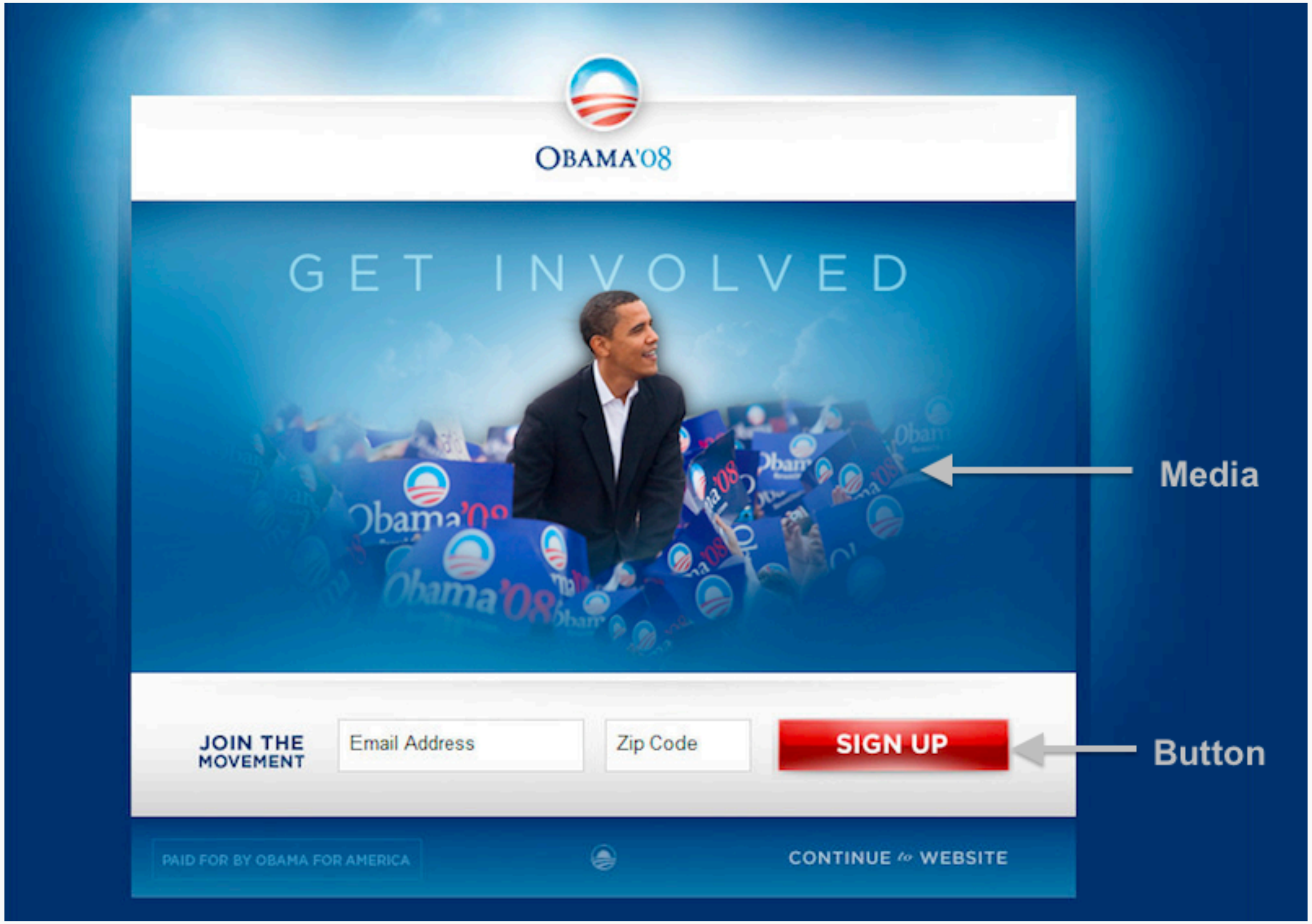
- the image or video the user saw.
- the words on the click-through button.

Media choices (6 of them):

- 3 images and 3 videos were possibly shown

Click- through button

- one of 4 choices:



How to design the experiment?

How should this experiment unfold?

1. What was the response variable?
2. What were the treatments? What were the treatment groups?
3. How many observations (sample size) needed to be selected in order to determine which treatment group is most effective?
4. What analysis should be performed?

Obama 2008: the specifics

1. What was the response variable?
   - sign-up rate
2. What were the treatments? What were the treatment groups?
   - 2 treatments (media and). 24 treatment groups
3. How many observations (sample size) needed to be selected in order to determine which treatment group is most effective?
   - The campaign decided to run the experiment on 310,382 visitors!!!
4. What analysis should be performed?
   - Classically, a χ^2 goodness-of-fit test could be performed.

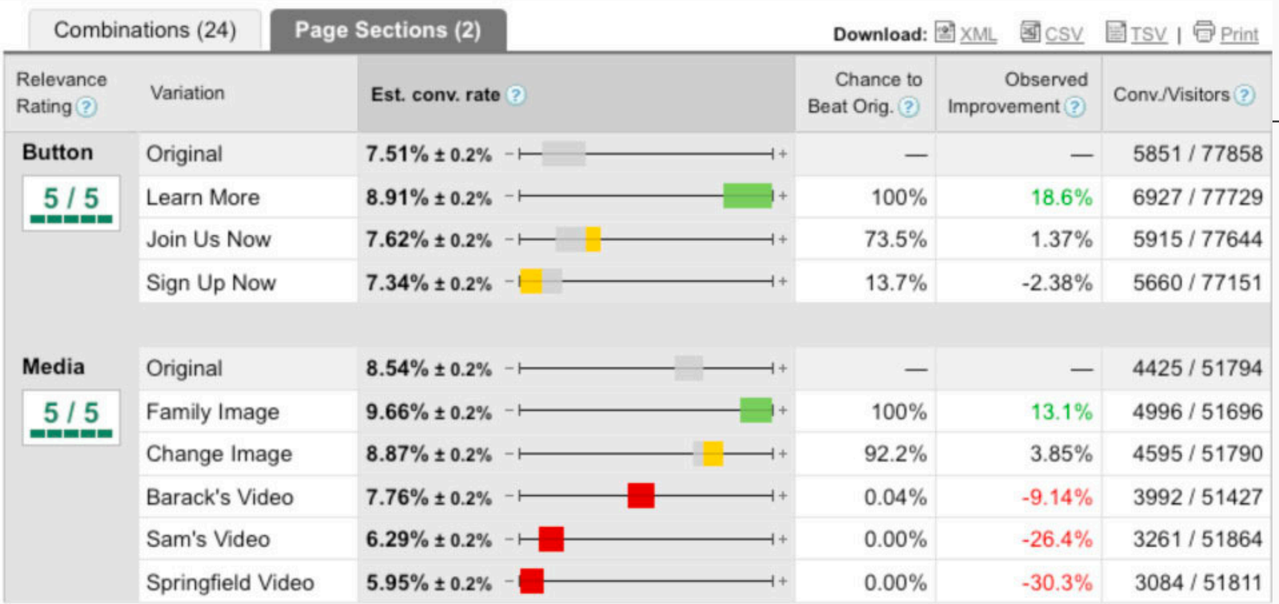
The data were overwhelming…

waiting until study is ‘over over’ wastes time and effort - you don’t want to go overboard in your testing (pick what N you want?)

The Results

The results are shown to the right (note: they are from a 3rd party site that runs AB tests for website design: Optimizely).

https://blog.optimizely.com/2010/11/29/how-obama-raised-60-million-by-running-a-simple-experiment/



Combinations (24) | Page Sections (2)

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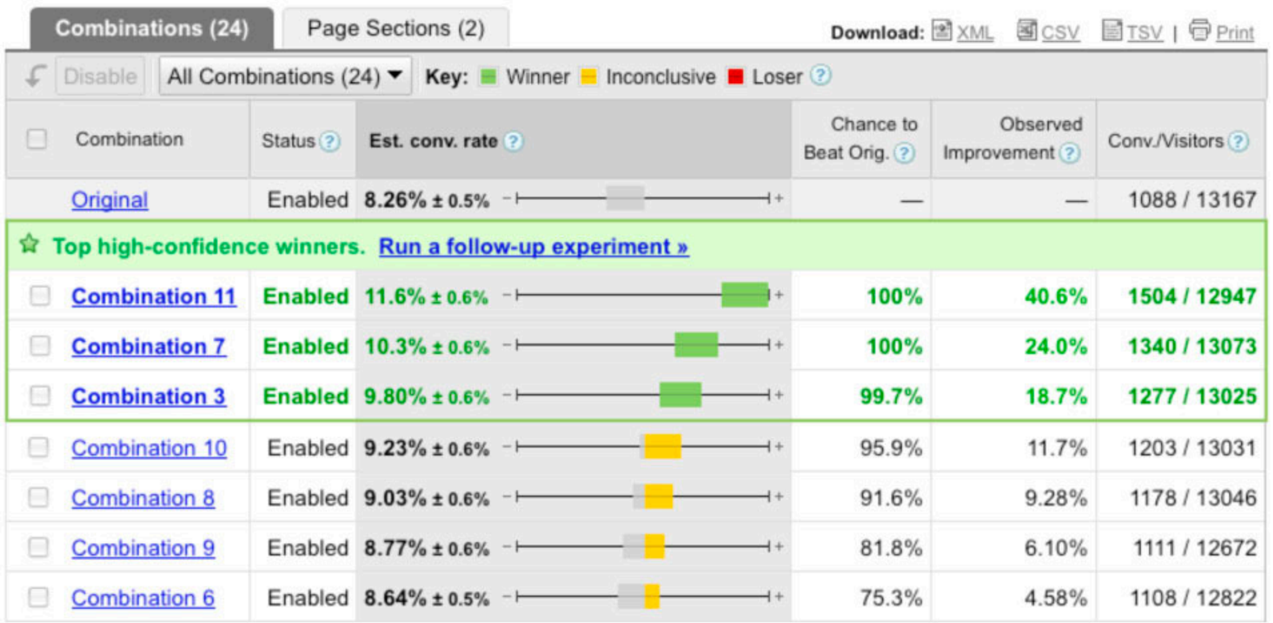
Relevance Rating ?	Variation	Est. conv. rate ?	Chance to Beat Orig. ?	Observed Improvement ?	Conv./Visitors ?
Button 5 / 5	Original	7.51% ± 0.2%	—	—	5851 / 77858
	Learn More	8.91% ± 0.2%	100%	18.6%	6927 / 77729
	Join Us Now	7.62% ± 0.2%	73.5%	1.37%	5915 / 77644
	Sign Up Now	7.34% ± 0.2%	13.7%	-2.38%	5660 / 77151
Media 5 / 5	Original	8.54% ± 0.2%	—	—	4425 / 51794
	Family Image	9.66% ± 0.2%	100%	13.1%	4996 / 51696
	Change Image	8.87% ± 0.2%	92.2%	3.85%	4595 / 51790
	Barack's Video	7.76% ± 0.2%	0.04%	-9.14%	3992 / 51427
	Sam's Video	6.29% ± 0.2%	0.00%	-26.4%	3261 / 51864
	Springfield Video	5.95% ± 0.2%	0.00%	-30.3%	3084 / 51811



Combinations (24) | Page Sections (2)

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Disable | All Combinations (24) | Key: 🟩 Winner 🟨 Inconclusive 🟥 Loser ?

□	Combination	Status ?	Est. conv. rate ?	Chance to Beat Orig. ?	Observed Improvement ?	Conv./Visitors ?
	Original	Enabled	8.26% ± 0.5%	—	—	1088 / 13167
☆	Top high-confidence winners. Run a follow-up experiment »					
□	Combination 11	Enabled	11.6% ± 0.6%	100%	40.6%	1504 / 12947
□	Combination 7	Enabled	10.3% ± 0.6%	100%	24.0%	1340 / 13073
□	Combination 3	Enabled	9.80% ± 0.6%	99.7%	18.7%	1277 / 13025
□	Combination 10	Enabled	9.23% ± 0.6%	95.9%	11.7%	1203 / 13031
□	Combination 8	Enabled	9.03% ± 0.6%	91.6%	9.28%	1178 / 13046
□	Combination 9	Enabled	8.77% ± 0.6%	81.8%	6.10%	1111 / 12672
□	Combination 6	Enabled	8.64% ± 0.5%	75.3%	4.58%	1108 / 12822

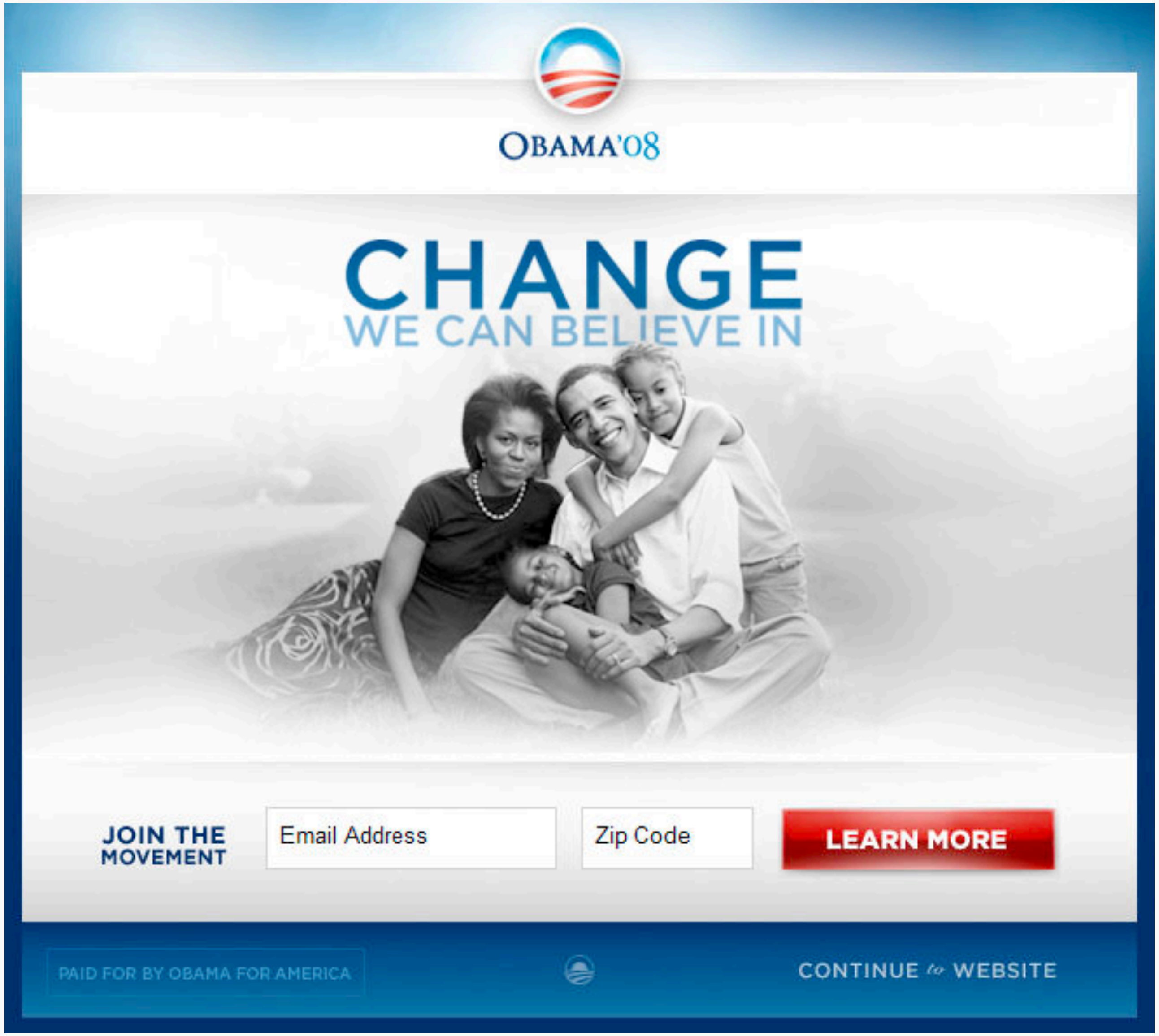
The winning variation had a sign-up rate of 11.6%. The original page had a sign-up rate of 8.26%. That's an improvement of 40.6% in sign-up rate…[leading to an] additional 2,880,000 email addresses translated into 288,000 more volunteers…and an additional $60 million in donations.

study had way too many people - could have optimized website earlier
could have done catered website offers?? personalized marketing??
projections are a little too optimistic -

See any issue in this conclusion?

But more importantly, they did learn one lesson: those intimately involved in designing websites (or medical treatments) are too biased to properly make conclusions as to what works best. They like the videos, and they performed the worst.

And the winner was:


OBAMA'08
CHANGE
WE CAN BELIEVE IN
JOIN THE MOVEMENT
Email Address
Zip Code
LEARN MORE
PAID FOR BY OBAMA FOR AMERICA
CONTINUE to WEBSITE