Digital Signal Processing
Instituto Superior Técnico
Lab assignment 2 - Spectral Analysis with the Discrete Fourier Transform
Authors: Margarida Silveira and Jorge S. Marques.
Revision: Rita Cunha.

Introduction

The Discrete Fourier Transform (DFT) is a Fourier representation used for finite duration sequences. The set of DFT coefficients is itself a finite duration sequence, which corresponds to samples, equally spaced in frequency, of the Fourier Transform of the signal. The DFT has a number of useful properties and there are efficient algorithms for its computation, which are known as Fast Fourier Transform (FFT). For these reasons, the DFT is very useful for analyzing and designing systems in the Fourier domain.

The DFT analysis and synthesis equations are the following:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}, \quad 0 \leq k \leq N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}, \quad 0 \leq n \leq N-1$$

Notes

For this lab assignment, only short answers to the questions below and the Matlab code written to answer those questions are required. The deadline for submission in fenix is one week after the day of the respective lab session at 23:59. The items that you should address are marked in bold font, e.g. **R1.a)**.

You will often find between brackets, as in {`command`}, suggestions of Matlab commands that may be useful to perform the requested tasks. You should use Matlab's help, when necessary, to obtain a description of how to use these commands.

Experimental work

1. **Spectral analysis of a synthetic signal.**

R1.a) Create a synthetic signal $x(n)$ with duration M=512, corresponding to the following sum of sinusoids:

$$x(n) = 5\cos(w_0 n + 1) + 2\cos(2w_0 n + 2) + 3\cos(5w_0 n + 3) \quad 0 \leq n \leq M-1$$

Use, in this expression, $w_0 = 5.2 \times \frac{2\pi}{M}$ rad.

R1.b) Plot the signal. Comment on what you observe. {`plot`}

R1.c) Compute the signal's DFT of length N=512. Plot the signal's magnitude and phase spectra, labelling the horizontal (frequency) axis using normalized frequency. Comment on what you observe. {`fft`}, {`abs`}, {`angle`}, {`xlabel`}, {`ylabel`}

R1.d) Identify the 3 largest peaks in the one sided magnitude spectrum and indicate their index and corresponding magnitude, frequency and phase. From this information, reconstruct the input sequence, $x_r(n)$.

R1.e) Visualize both $x(n)$ and $x_r(n)$ on the same plot. Comment on what you observe. {`plot`} {`hold`}

R1.f) Repeat the previous items using a DFT of length N=1024. Comment on the effect of the DFT length.

2. **Spectral analysis of a real voice signal.**

R2.a) Load the sound file `Howmanyroads.wav` and listen to the sound (don't forget to use the appropriate sampling frequency in the soundsc command). {`audioread`} {`soundsc`}

R2.b) Obtain a segment of the voice signal of length M=2048, starting from sample 48500.

R2.c) Compute the signal's DFT of length N=2048 and perform the necessary steps in order to reconstruct the voice signal from the three most important sinusoids.

R2.d) Compare the original signal with its reconstruction and comment.

R2.e) Compute the DFT of the complete signal and choose an adequate threshold $S_{\min}$ such that, when using only the DFT coefficients that satisfy $|X(k)| > S_{\min}$, the spoken words are still recognizable in the reconstructed sound signal {`soundsc`}. Determine the number of coefficients that were discarded.