Instituto Superior Técnico - UL

Digital Signal Processing

Lab 1: Sampling

Group 63, Friday, 12:30

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Sampling and aliasing

Question 1

In this section, we're going to study a chirp described by the signal

$$x_c(t) = cos[2\pi(\frac{1}{3}k_2t^3 + \frac{1}{2}k_1t^2 + F_0t + \Phi_0)] \tag{1}$$

We have $k_2 = 1000$, $k_1 = 0$, $F_0 = 0$, $\Phi_0 = 0$ and therefore the signal simplifies to $cos[\frac{2000\pi}{3}t^3]$.
The instantaneous frequency, $\Omega(t)$, is equal to $\Omega(t) = 2000\pi t^2\ rad.s^{-1}$.

Comment on the relationship between what you heard and the signal that you created in x

The signal $x(t)$ plotted below is obtained by sampling $x_c(t)$ at a sampling frequency $f_S = 8kHz$.

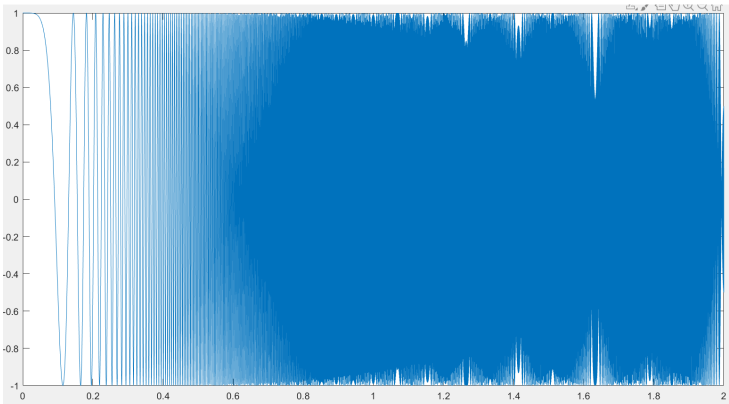

Figure 1: Plot of $x(t)$

As seen above, the sampled signal has a frequency that varies quadratically with time. This translates to a sound that gets more high-pitched over the time.

What is the relationship between the spectrogram and the sound you heard?

The spectogram of the signal is shown below.

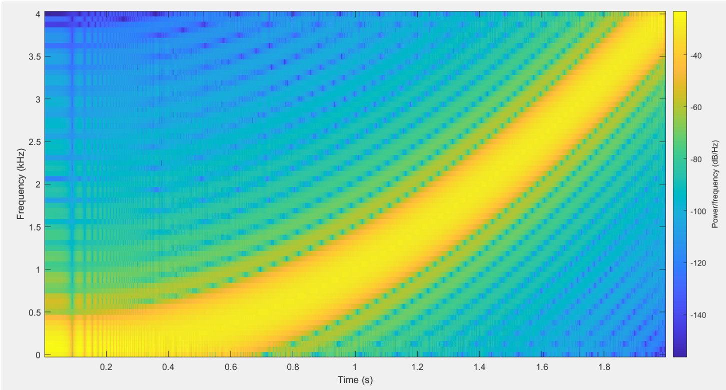

Figure 2: Spectogram of $X_c(t)$

As we can see, the shape of the $\Omega(t)$ appears to be quadratic over time, for positive frequencies, which is consistent with the mathematical expression. As far as the sound, the chirp gets sharper and faster over time.

Question 3

Indicate the sampling frequency of the signal y[n]. Justify your answer

The signal $y[n]$ is a decimated version of x$[n]$, using a factor of decimation of 2. What that means is that only every other sample of $x[n]$ is kept for $y[n]$. Therefore the sampling frequency of $y[n]$ will be half the one of $x[n]$, 4 kHz.

The same conclusion can be obtained using a more mathematical approach, through the Fourier Transform.
If a discrete-time signal $f[n]$ has a discrete time Fourier Transform (DFT) $F(e^{j\omega})$ then it is known that the DFT of the time-scaled version $f(an)$ is $\frac{1}{|a|}F(e^{j\frac{\omega}{|a|}})$

For $|a| > 1$ the frequency is reduced by $|a|$. Thus for a $= 2$, the frequency is halved.

Explain what you have heard and observed

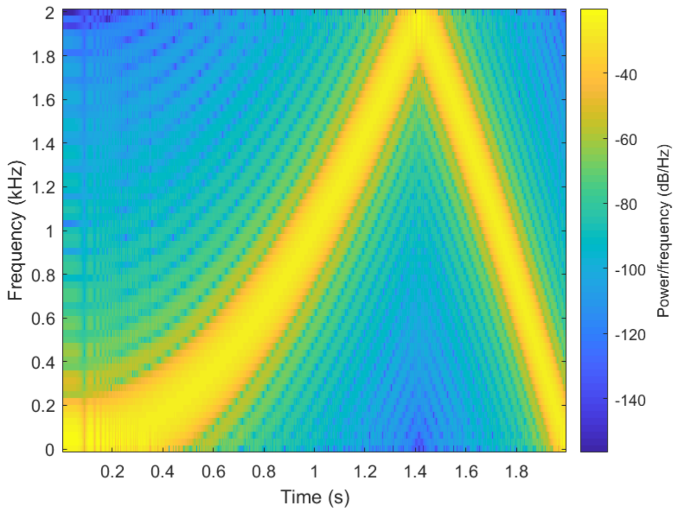

Figure 3: Spectogram of $y[n]$

The sound starts growing in pitch (agudo) but then after some time it starts to get deeper (grave).

By looking at the spectogram, the transition point heard can be seen at around 1.4s, which is when the frequency attains the value 2Khz ($F_s/2$). Beyond this frequency, aliasing occurs and the higher frequencies get reflected into lower ones. This is why frequencies are decreasing on the spectogram after 1.4s.

Question 3C

The 4 frequencies chosen were 1,4,8 and 15 kHz. Since the sampling frequencies range from 4kHz to 20kHz, we chose 2 frequencies equal to the sampling frequencies (4 and 8 kHz), 1 under the sampling frequencies range (1 kHz) and 15 kHz which is in the range but is more than half 20 kHz. From the plots of the signal (MATLAB) the conlusion is simple: the signal is distorted everytime the frequency of z(t) is less than $F_s/2$.

When f_i is bigger than f_S and multiple, the signal shown is constant.

Otherwise the signal is shown just fine.

Question 4

Indicate which window duration you used for the computation of the spectrogram

At the end of the *audioread* operation, the value stored at the **Fs** variable is 44100 Hz.

Below is the spectogram of the acquired signal $x(t)$. It was obtained using a window of 15s of duration, with $N = 1024$. Since there are 44100 samples in a second, then the whole spectogram corresponds to 661500 samples shown.

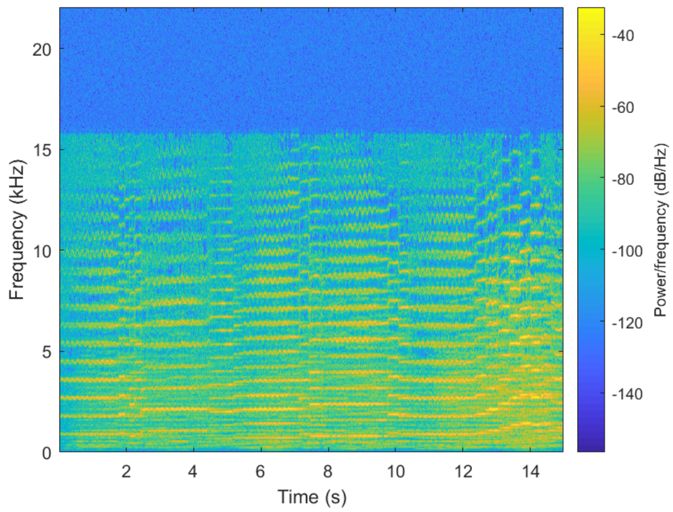

Figure 4: Spectogram of the signal $x(t)$

Question 5

Describe, and try to explain, what you heard and observed

After sampling the the signal from the *romanza* audio using $F_s/5$ as sampling rate we note that the new signal is much more noisy, and the added noise sounds like soft screams.

By analysing both spectograms, we see that the added noise is actually made up of reflected higher frequencies components, which is due to aliasing. In fact, by reducing so much the sampling frequency, the Shannon-Nyquist theorem quickly holds off.

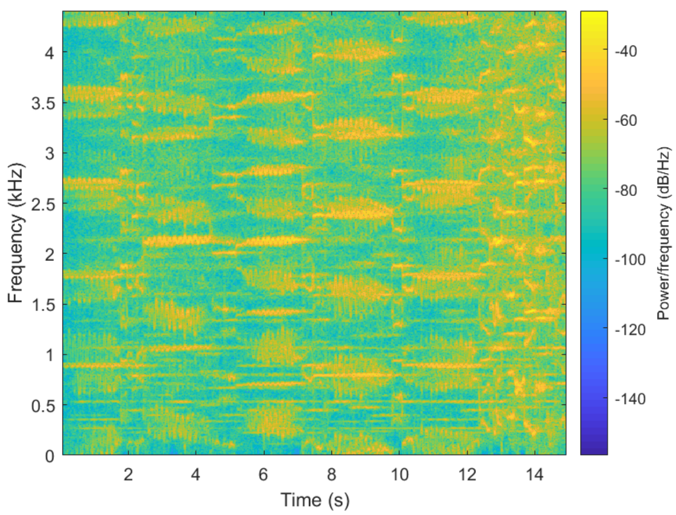

Figure 5: Spectogram of the new sampled signal

Question 6

By filtering the signal using this low-pass FIR filter, frequencies above the Nyquist frequency ($F_s/10$), are cut off. The higher frequencies that would be reflected are therefore removed. While this means some information (resolution and some tones in this case) is lost, the sound is still much clearer than its unfiltered sampled version. This is also visible on x_f's spectogram below where we can clearly see that many of the reflected components were removed. In fact, below the cutoff frequency, the signal is quite similar to the signal in Q5.

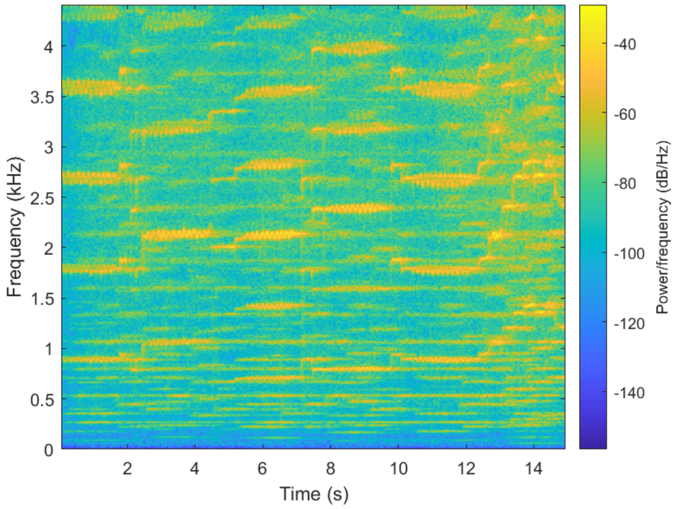

Figure 6: Spectogram of the filtered signal $x_f(t)$