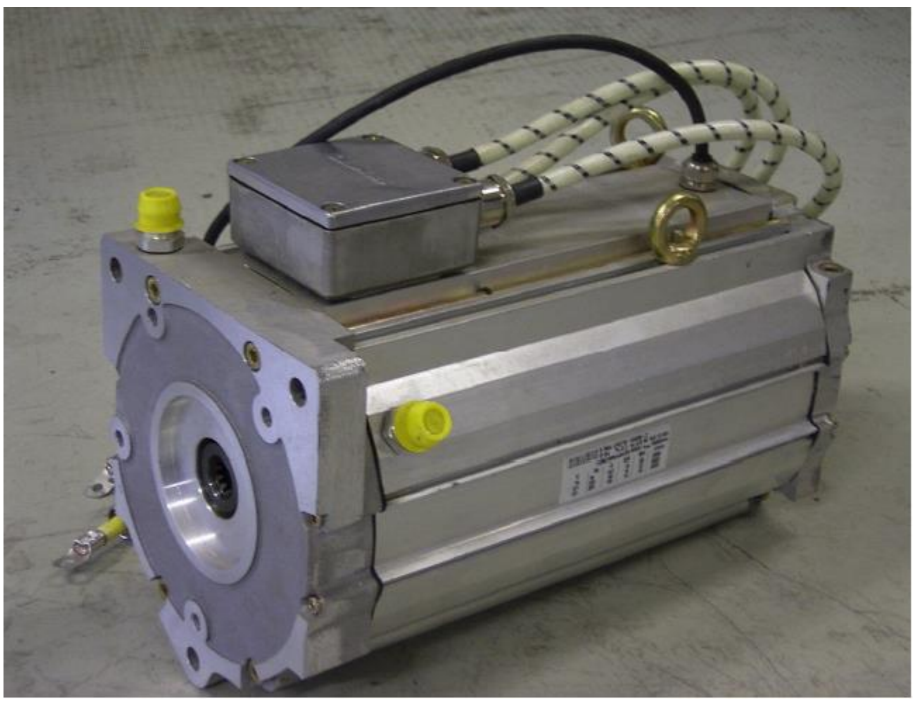
Electrical Drives and Electrical Vehicles

Project – VIENA

#4 – dq Model for the Induction Motor



I. Objectives

In this fourth and final part of the VIENA project, the objectives are:

- To develop a *d-q* dynamic model for the VIENA's induction motor, using the synchronous reference frame.
- Apply Field Oriented Control, using the speed reference from part one.

II. Background

A. *Induction machine dynamic model*

Figure 1 shows a simplified model of the induction machine, assuming that the rotor is composed of three-phases, there is a symmetry in the rotor and stator, and there is no neutral connection (homopolar currents are zero).

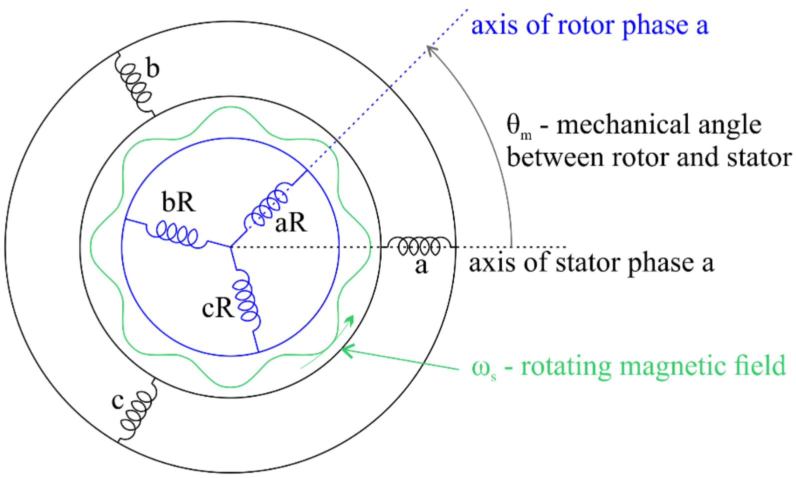


Figure 1 - Model of the induction machine.

In this context, the electric equations that describe the induction machine are

$$\begin{cases} [v_{abc}] = R_S[i_{abc}] + \dfrac{d[\lambda_{abc}]}{dt} \\ [v_{abcR}] = R_R[i_{abcR}] + \dfrac{d[\lambda_{abcR}]}{dt} \end{cases} \tag{1}$$

where:

- v_{abc} - stator three-phase voltages;
- i_{abc} - stator three-phase currents;
- R_S - stator coil resistance;
- λ_{abc} – stator linked fluxes;

- v_{abcR} – rotor three-phase voltages;
- i_{abcR} – rotor three-phase currents
- R_R – rotor coil resistance;
- λ_{abcR} – rotor linked fluxes;

The linked fluxes can be described as a linear relationship with the stator and rotor currents[1]

$$\begin{cases} [\lambda_{abc}] = L_S[i_{abc}] + L_M[i_{abcR}] \\ [\lambda_{abcR}] = L_R[i_{abcR}] + L_M[i_{abc}] \end{cases} \tag{2}$$

with L_S and L_R the self-inductance coefficient of the stator and the rotor, respectively, and L_M the mutual inductance coefficient between the stator and the rotor, defined as

$$\begin{cases} L_S = L_1 + L_M \\ L_R = L_2 + L_M \end{cases} \tag{3}$$

with L_1 and L_2 the leakage inductance coefficients of the stator and rotor, respectively, and $R_1 = R_S$ and $R_2 = R_R$ the stator and rotor resistances, respectively, as shown in Figure 2.

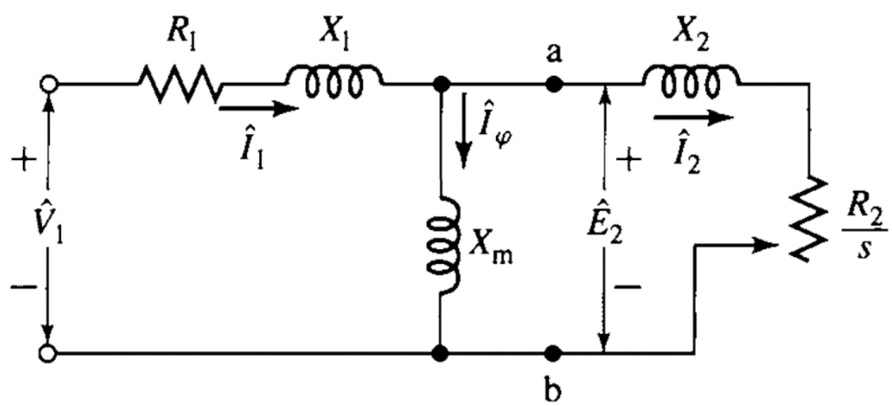


Figure 2 - Per-phase equivalent circuit of the induction machine. (Adapted from Fitzgerald, p. 316, Figure 6.9).

[1] This is only true with a homogeneous and isotropic magnetic material, without saturating it! If saturation happens, the (B-H) curve of the material needs to be taken into account.

B. dq0 Reference Frame

Figure 3 shows the electric relationship between the stator, rotor and *d-q* reference frame. Here, the relationship between the stator angle θ_S, the rotor angle θ_R and θ_{me} is

$$\theta_s = \theta_r + \theta_{me} \tag{4}$$

Mechanically, there is a relationship between the mechanical angle θ_m and the electrical angle θ_{me}, described as

$$\theta_{me} = n_{pp}\theta_m \tag{5}$$

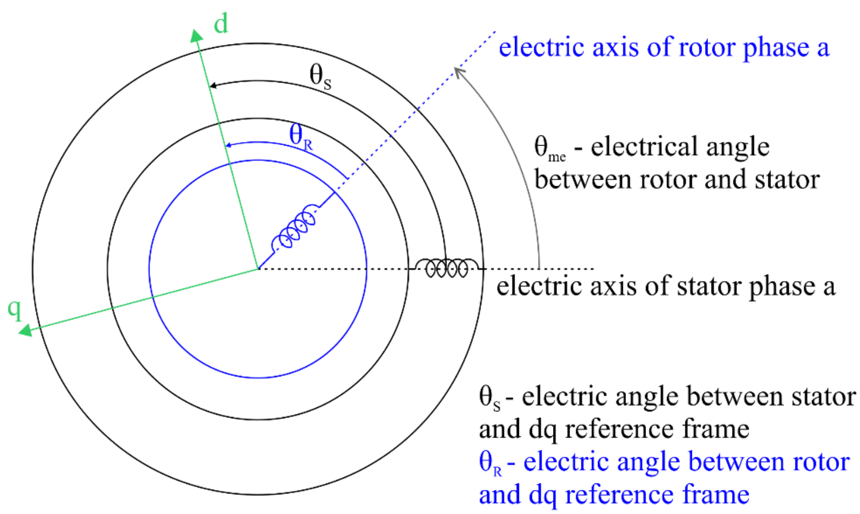


Figure 3 - Electric angle relationship between stator, rotor and *d-q* reference frame.

$$[v_{\mathrm{dq0}}] = [P(\theta)][v_{\mathrm{abc}}], \tag{6}$$

with $[P(\theta)]$ the Park transformation matrix

$$[P(\theta)] = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{7}$$

Using (6) in (1) and (2), one gets

$$\begin{cases} [P(\theta_S)]^{-1}[v_{\mathrm{dq0}}] = R_S[P(\theta_S)]^{-1}[i_{\mathrm{dq0}}] + \dfrac{d}{dt}\left([P(\theta_S)]^{-1}[\lambda_{\mathrm{dq0}}]\right) \\ [P(\theta_R)]^{-1}[v_{\mathrm{dq0}R}] = R_R[P(\theta_R)]^{-1}[i_{\mathrm{dq0}R}] + \dfrac{d}{dt}\left([P(\theta_R)]^{-1}[\lambda_{dq0R}]\right) \end{cases} \tag{8}$$

Developing the derivative term in the right hand side of (8)

$$\frac{d}{dt}\left([P(\theta)]^{-1}[\lambda_{\mathrm{dq0}}]\right) = \frac{d}{dt}([P(\theta)]^{-1})[\lambda_{\mathrm{dq0}}] + [P(\theta)]^{-1}\frac{d}{dt}\left([\lambda_{\mathrm{dq0}}]\right) \tag{9}$$

and knowing that

$$\frac{d}{dt}([P(\theta)]^{-1}) = [P(\theta)]^{-1}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\frac{d\theta}{dt} \tag{10}$$

(8) becomes

$$\begin{cases} [v_{\mathrm{dq0}}] = R_S[i_{dq0}] + \dfrac{d[\lambda_{dq0}]}{dt} + [W(\dot{\theta}_S)][\lambda_{dq0}] \\ [v_{\mathrm{dq0R}}] = R_R[i_{dq0R}] + \dfrac{d[\lambda_{dq0R}]}{dt} + [W(\dot{\theta}_R)][\lambda_{dq0R}] \end{cases} \tag{11}$$

where $[W(\dot{\Psi})] = d([P(\Psi)^{-1}])/dt$, $\dot{\theta}_S = d\theta_S/dt$ and $\dot{\theta}_R = d\theta_R/dt$:

$$[W(\dot{\theta}_S)] = \begin{bmatrix} 0 & -\dot{\theta}_S & 0 \\ \dot{\theta}_S & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } [W(\dot{\theta}_R)] = \begin{bmatrix} 0 & -\dot{\theta}_R & 0 \\ \dot{\theta}_R & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

For the synchronous reference frame, $\theta_S = \omega_e t$, where ω_e is the synchronous electric frequency (equal to the $2\pi f$), and $\theta_R = \theta_S - n_{pp}\theta_m$.

Regarding (2), the same reasoning used in (8) can be applied, resulting in

$$\begin{cases} [\lambda_{dq0}] = L^s[i_{dq0}] + L_M[i_{dq0R}] \\ [\lambda_{dq0R}] = L^r[i_{dq0R}] + L_M[i_{dq0}] \end{cases} \tag{12}$$

(11) and (12) can now be unfolded to write

$$\begin{bmatrix} v_d \\ v_q \\ v_{dR}=0 \\ v_{qR}=0 \end{bmatrix} = \begin{bmatrix} R_S & 0 & 0 & 0 \\ 0 & R_S & 0 & 0 \\ 0 & 0 & R_R & 0 \\ 0 & 0 & 0 & R_R \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_{dR} \\ i_{qR} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_{dR} \\ \lambda_{qR} \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta}_S & 0 & 0 \\ \dot{\theta}_S & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dot{\theta}_R \\ 0 & 0 & \dot{\theta}_R & 0 \end{bmatrix} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_{dR} \\ \lambda_{qR} \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_{dR} \\ \lambda_{qR} \end{bmatrix} = \begin{bmatrix} L_S & 0 & L_M & 0 \\ 0 & L_S & 0 & L_M \\ L_M & 0 & L_R & 0 \\ 0 & L_M & 0 & L_R \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_{dR} \\ i_{qR} \end{bmatrix} \tag{14}$$

C. *Total Power, Mechanical Power and Electromagnetic Torque*

To compute the total power of the induction machine, one just needs to multiply the voltages to their corresponding currents

$$P = \frac{3}{2}\left(v_d i_d + v_q i_q + v_{dR} i_{dR} + v_{qR} i_{qR}\right) \tag{15}$$

and substituting the voltages to their corresponding equations, which are in (13), we get

$$P = \frac{3}{2}\left(\underbrace{R_S\left(i_d^2 + {i_q}^2\right) + R_R\left({i_{dR}}^2 + {i_{qR}}^2\right)}_{joule\ losses} + \underbrace{\left(\dot{\lambda_d} i_d + \dot{\lambda_q} i_q\right) + \left(\dot{\lambda_{dR}} i_d + \dot{\lambda_{qR}} i_{qR}\right)}_{changes\ in\ M.field} \right. \\ \left. + \underbrace{\dot{\theta_S}\left(\lambda_d i_q - \lambda_q i_d\right) - \dot{\theta_R}\left(\lambda_{dR} i_{qR} - \lambda_{dQ} i_{dR}\right)}_{Mechanical\ output} \right) \tag{16}$$

The big expression in (16) can be separated in three parts: some of the power is wasted into heat because of the copper losses in the stator and rotor (R_S and R_R); another part of the power is relative to the changes in the magnetic field that may occur during the operation of the induction machine; in the end, the remaining term must be due to the mechanical power demanded by the load

$$P_m = T_e \Omega_m = \frac{3}{2}\left(\dot{\theta}_S\left(\lambda_d i_q - \lambda_q i_d\right) - \dot{\theta}_R\left(\lambda_{dR} i_{qR} - \lambda_{qR} i_{dR}\right)\right) \tag{17}$$

and with some algebraic manipulation with (14), the electromagnetic torque can be written as

$$T_e = \frac{3}{2} \frac{\left(\dot{\theta_S} - \dot{\theta_R}\right)}{\dot{\theta_m}} \frac{L_M}{L_R}\left(\lambda_{dR} i_q - \lambda_{qR} i_d\right) \tag{18}$$

Remembering that $\dot{\theta}_R = \dot{\theta}_S - (\text{poles}/2)\dot{\theta}_m$, finally the torque can be written as

$$T_e = \frac{3}{2}(n_{pp})\frac{L_M}{L_R}(\lambda_{dR} i_q - \lambda_{qR} i_d) \quad or$$
$$T_e = \frac{3}{2}(n_{pp})L_M(i_q i_{dR} - i_d i_{qR}) \tag{19}$$

III. TASK 1: TESTING OF D-Q MODEL OF THE VIENA INDUCTION MOTOR

To test the *d-q* model and compare it with the equivalent circuit model developed in part 2 and 3 of the VIENA project, an auxiliary Simulink model is provided.

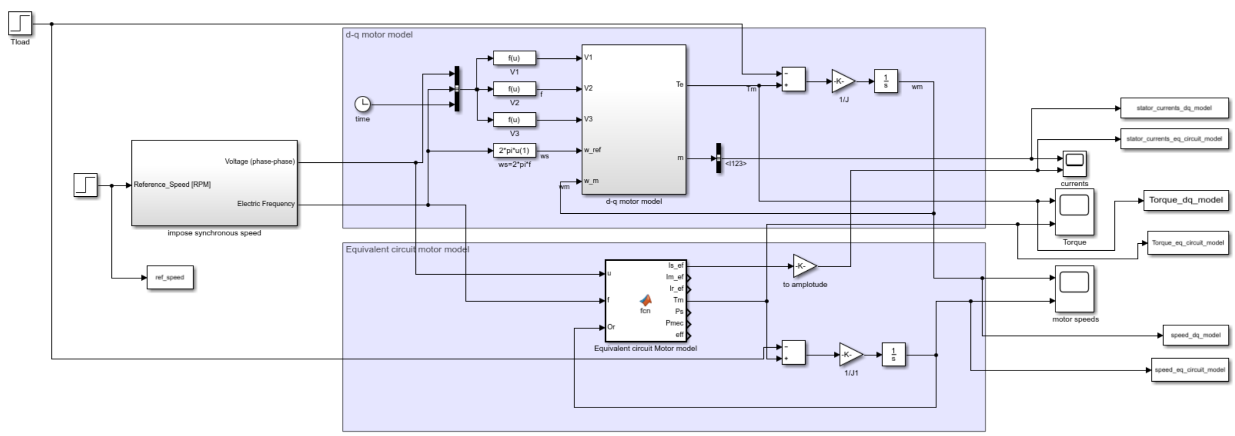


Figure 4 – Auxiliary Simulink model to compare the *d-q* and equivalent circuit models.

In this, a synchronous speed and load torque steps are applied to both models. The behaviors of each model are then compared, in terms of developed torque, mechanical speed and stator currents.

$$\Omega_{s_{ref}} = \begin{cases} 0\ rpm, & t < 1s \\ 3750\ rpm, & t > 1s \end{cases}$$

$$T_{load} = \begin{cases} 0\ Nm, & t < 5s \\ 40\ Nm, & t > 5s \end{cases}$$

The Simulink model must be started by running the Matlab script “run_to_complete.m”. **<u>This Matlab script is not completed!</u> You must fill the Ls and Lr self-inductances and the matrices [L] and [R] for the *d-q* model.**

After choosing the right values of inductances, compare the results from both models. Comment on the differences found in the Motor speeds, Electromagnetic torque and Stator currents and Active power and energy.

IV. TASK 2: IMPLEMENTATION OF FIELD ORIENTED CONTROL

Using the model developed in VIENA-part3, replace the equivalent circuit model of the induction machine by the new *d-q* model and replace the V/f command by the Field Oriented Control command (FOC).

Field Oriented Control (FOC), as the name suggests, consists in aligning the direct axis of the dq reference frame with the rotor flux, which means that

$$\lambda_{qR} = 0. \tag{20}$$

With this, (13), (14) and (19) become

$$\begin{cases} T_e = \frac{3}{2}\left(n_{pp}\right)\frac{L_M}{L_R}\lambda_{dR}i_q \\ i_{dR} = 0 \\ \lambda_{dR} = L_m i_d \\ \lambda_d = L_S i_d \end{cases} \tag{21}$$

In (21), it can be easily seen that the direct flux λ_{dR} are commanded by the direct stator current i_d and the torque T_e is commanded by the i_q current. These are the main reasons to implement a field-oriented controller, we can have a separate control of the direct flux and torque.

One question arises: how to orient the rotor flux such that $\lambda_{qR} = 0$? For this, it is necessary to calculate $\theta_S = \omega_e t + \theta_0$, using a flux observer. Figure 5 shows the FOC implementation for the induction machine, that you must implement in the VIENA project part 4.

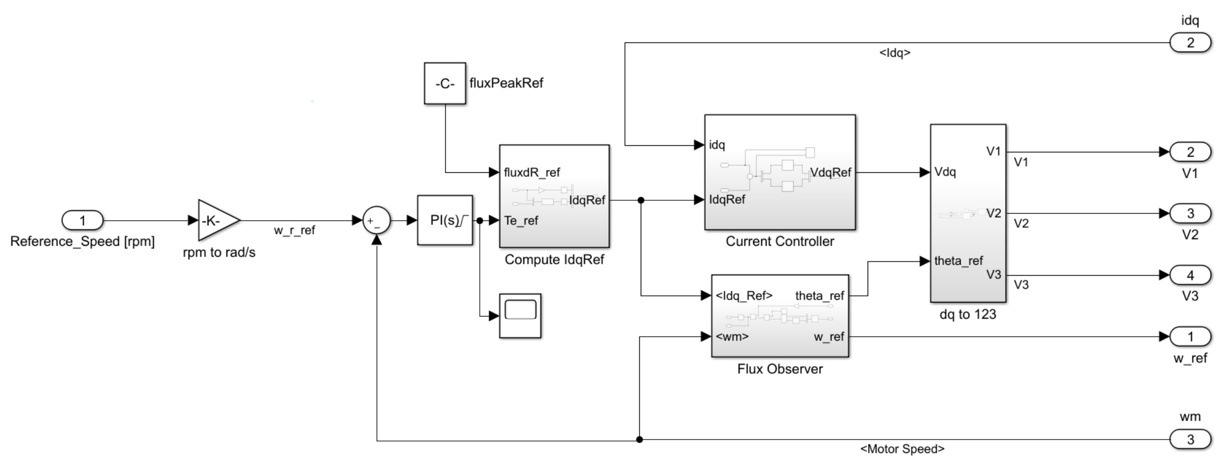


Figure 5 - FOC implementation using Matlab Simulink.

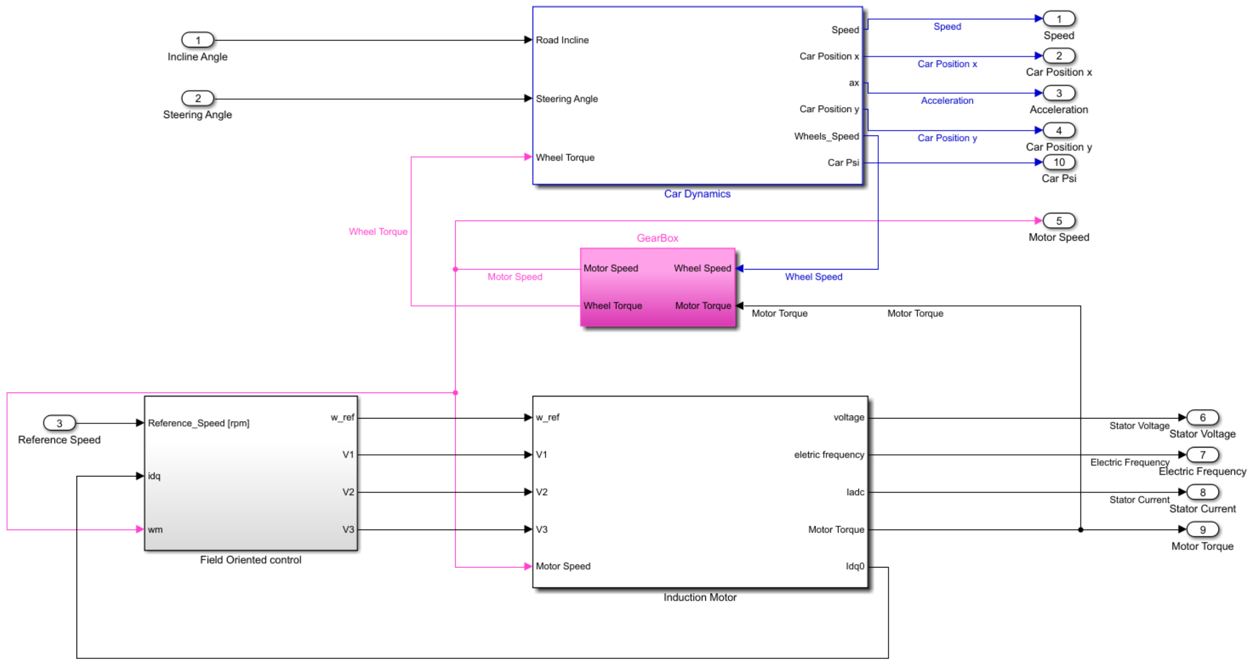


Figure 6 – Display of the overall diagram.

For more information, see the torque control of induction motors in Chapter 11 of Fitzgerald's Electric Machinery.

After implementing the *d-q* induction machine model and the FOC control:

- Obtain the evolution of the motor speed, torque and stator voltages and currents when applied a reference speed equal to the ones obtained from part 1 (the same reference speed you used for VIENA-part2 and VIENA-part3 – trackAB and trackCDEF).

- Repeat the previous question, now for different values of proportional and integral gains of the PI(s) used in the FOC (Figure 5). Choose the best ones and justify your answer.

- Compare the results, of the FOC control with the best PI(s) gains, with the ones obtained in VIENA-part2 (V/f without closed loop control) and in VIENA-part3 (V/f with closed loop control).