TÉCNICO LISBOA

Drives and Electrical Vehicles

LABORATORY T1

ASSOCIATION OF AN ELECTRONIC SOFT STARTER TO THREE-PHASE INDUCTION MOTORS

SOFT STARTING OR DIRECT-ON-LINE STARTING OF INDUCTION MOTORS?

Electronic soft-starter

Three-phase induction motor

Year 2019/2020

Drives and Electric Vehicles - Laboratory

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1. INTRODUCTION

When a three-phase, balanced and sinusoidal voltage source UVW is connected to a three-phase induction motor, as illustrated in **Figure 1**, a rotating magnetic field (green line) appears in the air-gap of the motor. The rotating field will be responsible for the magnetic coupling between stator coils and the rotor bars (*squirrel-cage induction machine*). Since the rotor electric conducting bars rotate in a different angular speed than the rotating field, this field will "cross" the rotor bars, which in turn will induce a set of electromotive forces through the bars. The electric currents appearing in the rotor cage will interact with the rotating field producing a net electromagnetic torque in the rotor.

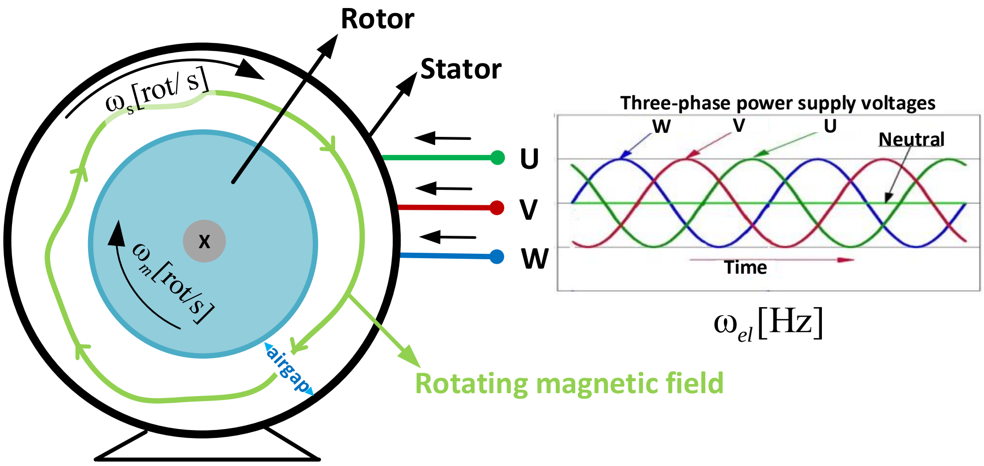


Figure 1 –

Figure 2*(a)* shows a Direct-ON-line start process (DON) of a star-connect induction motor ("direct" means that the stator coils are directly energized by the power grid). Figure 2(a) shows the stator phase voltage (dark discontinuous line) and its respective phase current (red continuous line) during the acceleration time of the motor.

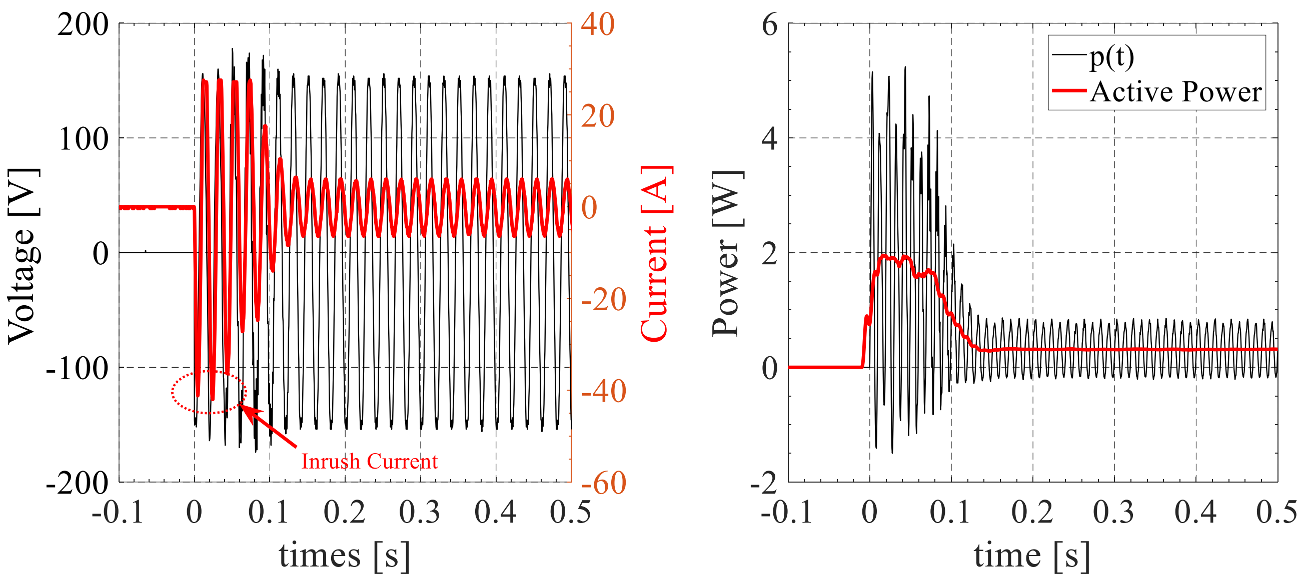
During acceleration, stator RMS currents can reach values ranging between 4 to 7 times its rated value, while the motor torque is on average 1.5 to 3.0 times its rated value. Peak current values are even higher and can reach 10 times the nominal RMS value. For example, the induction machine where the results in Figure 2*(a)* were obtained has the following nameplate data: rated current 4.8 A, rated voltage 400 V, rated power 2.2 kW, rated speed 1425 rpm, and a rated power factor equal to 0.81.

The high inrush current (indicated in Figure 1 above, or "corrente de irrupção" in Portuguese) is the starting current or peak current input. In other words, it is the maximum instantaneous input current absorbed by an electrical motor at the instant of its start.

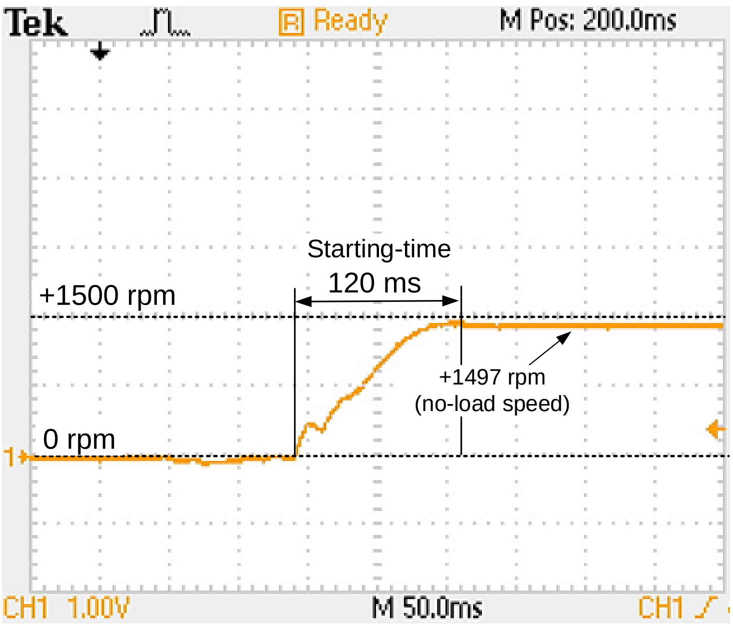
The starting time of the motor is also another important variable. The starting time depends on the inertia of the moving load driven by the motor (usually much higher than the motor), on the required rotational speed, and also on the available accelerating torque of the motor. Figure 2*(b)* shows the motor speed evolution until achieving its no-load speed of 1497 rpm. The starting time obtained took about 120 ms (0.12 s).

During the initial time instants of motor acceleration, Figure 2*(b)* shows the presence of significant speed oscillations, which is an indication of strong electromagnetic torque oscillations inside the machine. If the torque load and its inertia are not significant compared with those of the motor, torque oscillations will be suppressed after some instants and none mechanical damage, either in the motor-load axis or at load level, will appear. However, direct-on-line starting of an induction motor (either being a star- or delta- stator phases connected) must take into consideration the motor and load inertia values, their axis rigidity, and also how any torque disturbance will affect the system where the motor-load are inserted: the electric or the process one.
Another important effect of the Direct-On-Line starting on industrial systems is the appearance of a voltage sag during the motor starting. As the peak current is 4 to 7 times its rated value, the voltage drop on the line feeding the motor's busbar will also increase. If the voltage-drop is sufficiently high, other equipment connected to the motor's busbar may be influenced or even harmed.

In that context, the need for a soft-starter is useful to **reduce heavy starting currents**, during the start-up of the motor, and also provide **overload and no-voltage protection.**



(a)



(b)

Figure 2 – No-load and direct on-line starting of a star-connected induction motor. (a) Above: stator current (continuous line), phase-voltage (dashed line). Below: active power (dashed line), reactive power (continuous line) evolution. (b) Motor speed evolution showing a starting-time of about 120 ms or 0.12 s until reaching its no-load speed. The speed of 1500 rpm is the synchronous speed of the motor

2. SOFT STARTERS: FUNDAMENTALS

In induction motors, the starting torque is approximately proportional to the square of the starting current drawn from the line, $T_{el_{start}} \; \alpha \; (I_{start})^2$. However, as the current is proportional to the applied voltage, <u>the starting motor torque can also be considered to be approximately proportional to the applied starting voltage</u> $T_{el_{start}} \; \alpha \; (U_{start})^2$.

When starting connected directly to the electrical supply, an induction motor will develop a very high starting torque. Therefore, at the instant of a start-up, there is some unnecessary effect on *electrical* and *mechanical* components. They are:

Unnecessary mechanical effects**:** at startup, the sudden impact on the loads driven by the motor followed by its rapid acceleration to full speed causes excessive wear on the loads.

Unnecessary electrical effects: at startup, a heavy current surge on the electrical supply can be severe enough to cause voltage dips.

Problems arising by these effects: oversized mechanical and electrical components to accommodate the power surge on start-up (see it in Figure 3).

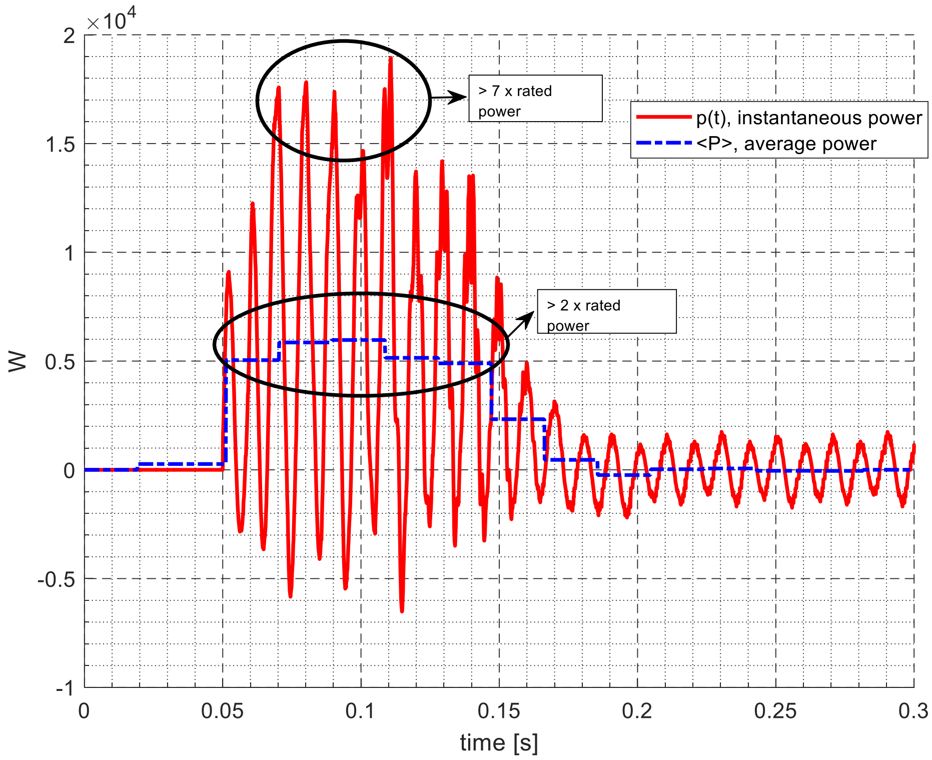


Figure 3 – Evolution of the phase power (instantaneous p(t) and average P value) during the no-load start-up of the induction motor. Rate motor power is 2.2 kW.

Unnecessary thermal effects: Due to the high startup current, a successively startup/stop of the motor can increase its temperature and therefore damaging the dielectric insulation material.

By adjusting the motor voltage during its starting, the current drawn by the motor and thus the torque produced by the motor can be reduced and increased continuously. The soft-starter equipment controls the initial voltage from a selectable initial value up to one hundred percent.

Figure 4 shows a simplified diagram of the main components in soft-starter equipment. By using six thyristors in a back-to-back configuration, the soft starter is able to regulate the voltage applied to the motor during starting from 0 Volt up to line voltage. The thyristors can also implement a soft stop by reducing motor voltage according to a set ramp time.

The motor voltage at each phase is changed with a phase cutting control, as shown in Figure 5. Through the thyristors, it is possible to apply to the motor only part of the voltage by cutting the sine semi-alternate. At the instant where the thyristor cut the sine semi-alternate, one defines the firing angle. If it is large, the RMS motor voltage becomes small. In gradually moving the firing to the left, the RMS motor voltage increases. <u>Notice, however, that frequency does not change as in a variable frequency drive, only the voltage and current change (remember Figure 1(a)).</u>

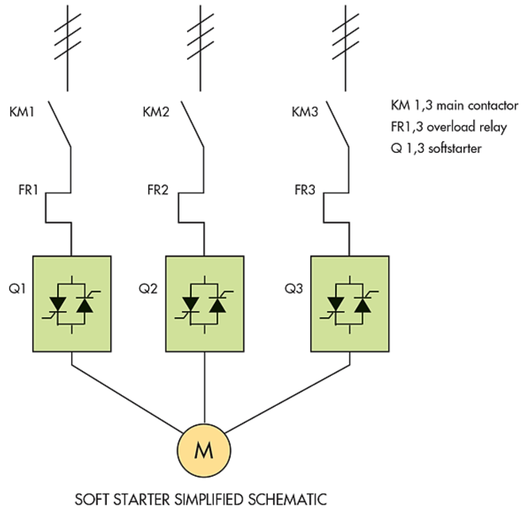


Figure 4 – Simplified diagram showing the main components in soft-starter equipment and the electric circuit where it is integrated.

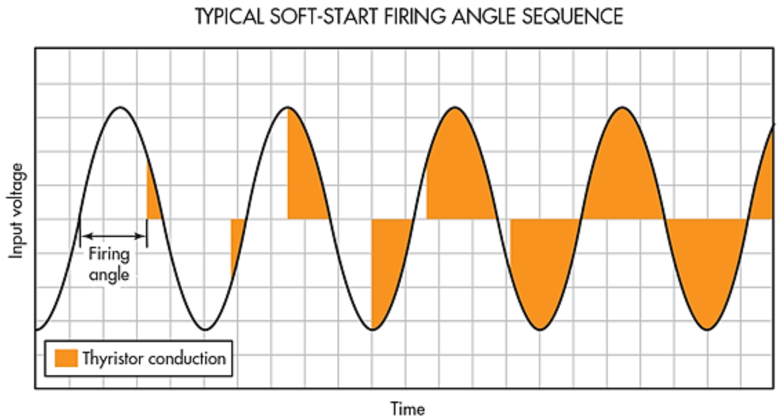


Figure 5 – Phase voltage evolution as the firing angle is decreased.

A. Motor dynamics when driven by a soft-starter

Torque-speed characteristics of the motor are plotted in Figure 6 for two cases: when it is directly connected to the electrical grid (dashed line); and when starting using a soft-starter (continuous line). This figure will allow us to analyze the progressive and slow motor starting when driven by a soft-starter.

Comparing the motor torque-speed characteristic with the torque-speed load characteristic plotted in Figure 6, these stand above the load characteristic until it intersects the latter. At this intersecting point (with and without a soft-starter), the rated load reaches the rated speed.

The difference between the load torque characteristic and motor torque is its *accelerating torque*, as indicated in Figure 6. This provides energy to the motor to begin to rotate and accelerate. If motor torque is much higher than load torque, acceleration is high and therefore the acceleration time is short. If torque is only a little higher than the load torque, it provides low acceleration and acceleration time is greater. The motor soft start is thus produced by decreasing the accelerating torque and imposing a more soft increase of it.

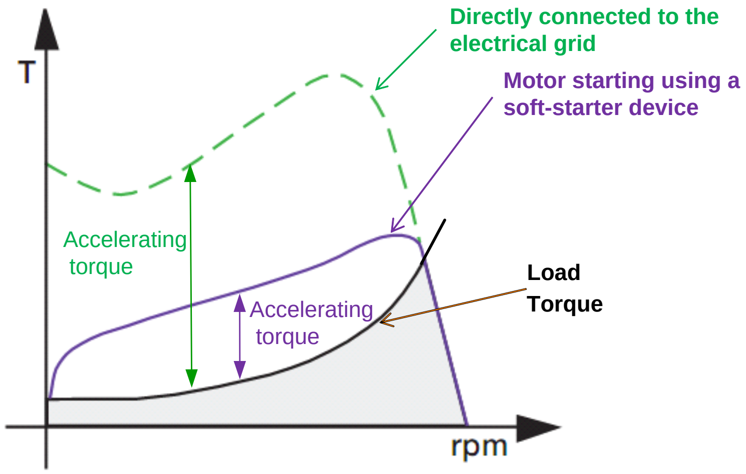


Figure 6 – Torque-speed characteristics of the motor when DOL connected and when a soft-starter. The load is represented by its torque-speed curve.

B. <u>Starting voltage</u> U_S ramp

When starting using a voltage ramp, as illustrated in Figure 7, the acceleration time t_{R_ON} and the initial starting voltage U_S must be previously set in the soft-start device. The soft-start increases the voltage at the motor terminals linearly from a predefined value U_S up to the rated voltage U_N of the electrical grid.

The low motor voltage, U_S, at the start of the acceleration process results in lower motor torque and causes a progressive acceleration, avoiding the previously stated torque oscillations. The starting value of the voltage to be applied will have to be defined by the initial torque needed to accelerate the motor plus the torque load. Usually, it is possible to choose between two profiles of soft starting with different ramp times adjusting t_{R_ON} and different starting torques when adjusting U_S.

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<u>In practice, one first defines the acceleration time and then the starting torque so that the smooth start-up is completed.</u>

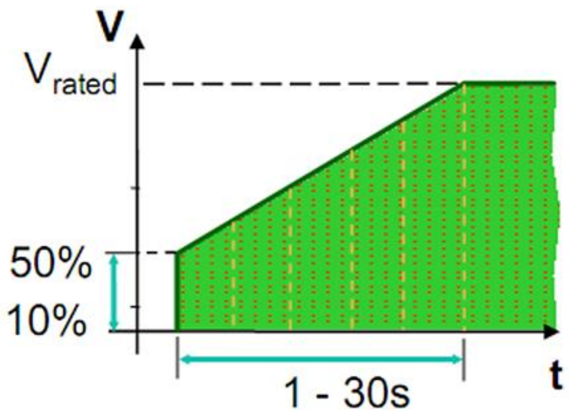
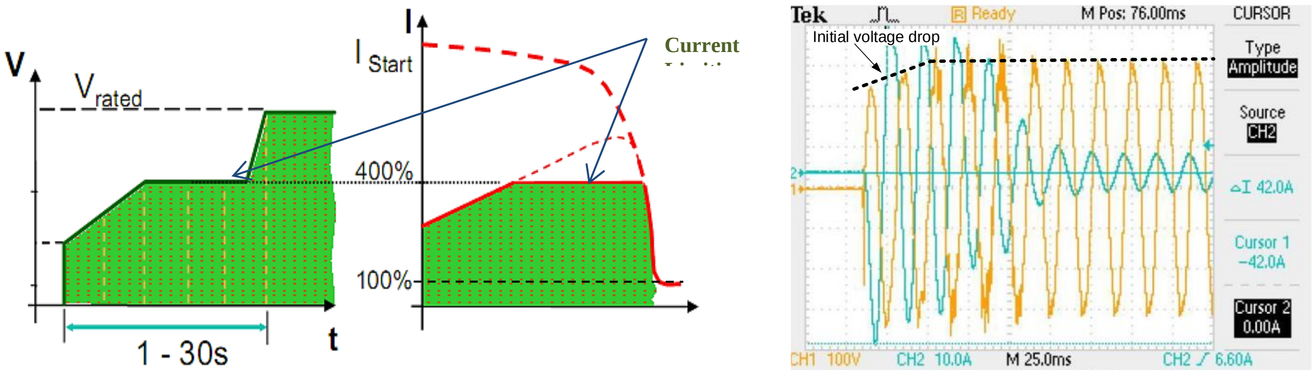


Figure 7 – Voltage/time curve previously settled in the soft-starter device (t_{R_ON} and U_S) to drive the induction motor.

C. <u>Current limiting option</u> in a soft-starter

This mode of soft starting shown in Figure 8*(a)* is generally used when it becomes necessary to limit the maximum starting current to protect the motor of high temperatures. However, notice that when a large motor power must be connected to the electrical grid, this starting method is always requested by electricity distributors to limit the current drawn from the grid and possible voltage sags appearing in vicinity equipment.

Figure 8*(b)* shows how current increases according to a certain stator voltage ramp-up to the set maximum voltage value. After the stator current decrease to its rated value (100%), the voltage is set to the rated motor value.



(a) (b) (c)

Figure 8 – (a) Voltage-time curve. (b) Current-speed curve when there is a current limiting option. (c) The initial voltage drop due to the starting high stator current.

D. Motor stopping using the soft-starter

For a motor soft stopping, the controller will gradually reduce the voltage to the motor, thus reducing the torque and current in a time range t_{R_OFF} set previously in the soft-starter, as illustrated in Figure 9. This process is often used with high inertia loads where a sudden stop may cause system damage.

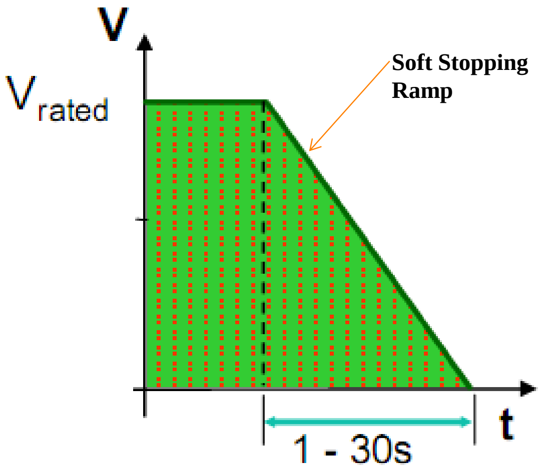


Figure 9 –Voltage-time curve used in a motor soft stopping.

E. Soft-starter: mechanical advantages

With a direct on-line starting, the motor develops a very high starting torque. Normally, the starting torque values are about 150 to 300% those of the rated one.

Depending on the start and because of the high starting torque, the mechanics of the drive may be requested excessively ("mechanical constraints"), or even the manufacturing process can be disrupted by unnecessary shocks.

- Through the implementation of a soft starter, the shock suffered by the mechanical parts of the motor is avoided.
- The characteristic of drive starting can be adapted to its application type (e.g. pump control, for example).

F. Soft-starter: electrical advantages

Direct motor starting usually demands large currents from the electrical grid (4 to 7 times the nominal current). This may result in large drop voltage that can perturb the other connected devices on this network, or even the motor voltage itself. The electricity distributors impose limits on motor currents due to that problem.

- With a soft starter, it is possible to limit the current for starting the motor, provided that a high starting torque is not necessary.

3. LABORATORY EXPERIMENT

A. Direct-On-Line (DOL) starting

i. Under no-load, the induction motor has to be started connecting its stator phases (star-connected) directly to the electrical grid (*direct-on-line*).

"No-load" means that there is no external torque applied to the induction motor axis. However, the motor has "internal" rotational losses caused by Joule losses in the ferromagnetic core due to Foucault currents and also hysteresis losses, frictional losses in roller bearings, and aerodynamic losses.

Visualize in the oscilloscope and save in a pen drive the following signals until achieving the steady-state regime: motor phase voltage and current.

ii. Under its rated load, the induction motor has to be starting up (star-connected) directly from the electrical grid (*direct-on-line*).

Visualize in the oscilloscope and save in a pen drive the following signals until achieving the steady-state regime: motor phase voltage and current.

iii. Do the start-stop cycle in Exp. 3, using the direct-on-line starting.

B. Starting-up using the soft starter.

i. Under no-load, the induction motor has to be accelerated until its no-load speed using the Siemens soft-starter 3RW30 mounted in the bench (see *Figure 9*).

a. Set the start-up time, T_{Ron}, to 5s;

Visualize in the oscilloscope and save the following signals until achieving the steady-state regime in a pen drive: motor phase voltage and current.

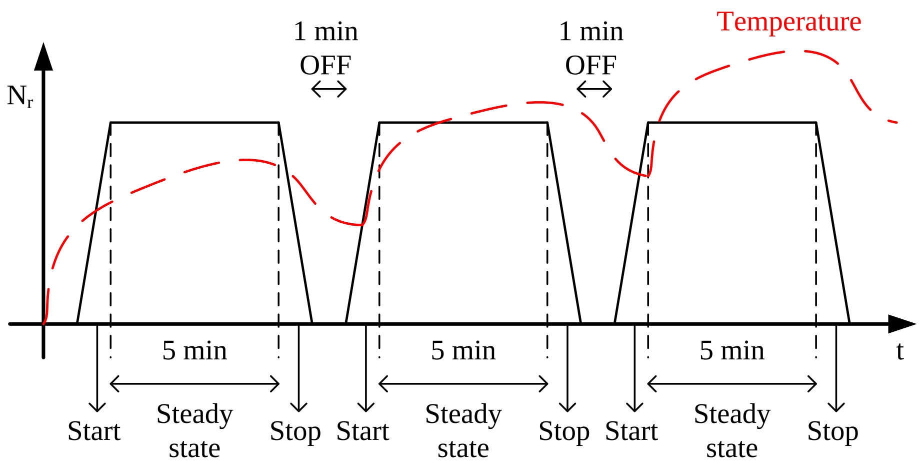
b. Repeat the same procedure, for T_{Ron} =2s and 7s.

ii. Repeat items i.a) and i.b), but under rated load.

iv. Do the start-stop cycle in Exp. 3, using the Soft Starting.

C. Intermittent operation.

The induction motor will now simulate an industrial application under a variable load and speed. The motor will perform a cycle of starting and stopping as shown in the following figure. Record motor temperature using a direct-on-line and a soft-starter (S.S.), Table 1.



Start stop start stop start stop

Time [min]	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Temp. Direct																		
Temp. S.S.																		

Table 1 – Temperature evolution for start/stop cycle, using direct-on-line and soft-starter.

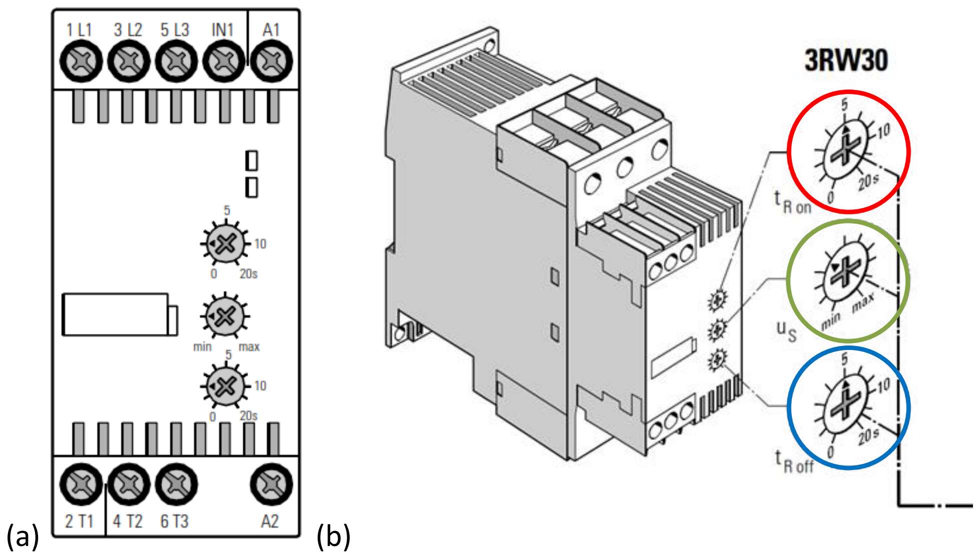


Figure 9 –. The soft-starter device that is installed in each laboratory bench.

QUESTIONARY FOR THE LABORATORY REPORT

Using the set of electrical signals measured during the start-up and steady-state conditions, obtain for each case, direct-on-line, soft starter, no-load and rated load, the following:

i. The evolution of the current and voltage RMS during the start-up (use the moving rms methodology over one period):

RMS Voltage for No-load: - Direct-on-line - Soft starter 2s - Soft starter 5s - Soft starter 7s	RMS Voltage for rated-load: - Direct-on-line - Soft starter 2s - Soft starter 5s - Soft starter 7s
RMS currents for No-load: - Direct-on-line - Soft starter 2s - Soft starter 5s - Soft starter 7s	RMS currents for rated-load: - Direct-on-line - Soft starter 2s - Soft starter 5s - Soft starter 7s

ii. The active power coming from the electric grid (use a moving average of the instant power over one period);

Active power for No-load: - Direct-on-line - Soft starter 2s - Soft starter 5s - Soft starter 7s	Active power for rated-load: - Direct-on-line - Soft starter 2s - Soft starter 5s - Soft starter 7s

iii. The reactive power coming from the electric grid (compute first the apparent power and, from it and with the active power, compute the reactive one);

Reactive power for No-load: - Direct-on-line - Soft starter 2s - Soft starter 5s - Soft starter 7s	Reactive power for rated-load: - Direct-on-line - Soft starter 2s - Soft starter 5s - Soft starter 7s

iv. The active and reactive energy consumed by the motor during its start, for each case.

v. Compare the temperature evolution obtained in the start/stop cycle, using the direct-on-line and soft-starter.

vi. Discuss the results. Specifically, comment on:

   a. The peak current value obtained during the start-up.

   b. The active and reactive energy consumed on the start-up.

   c. The difference between the temperature evolution in the start/stop cycles.

   d. Identify the main advantages and disadvantageous of using Soft Start.

4. ANNEX – SCRIPT FOR RESULT'S ANALYSIS

You can find the script in the course's page, in laboratory section. You will need to change the directory path. This script imports ".csv files obtained directly from the oscilloscope. Do not change anything in the .csv files.

This is just the first page of the script!

Change the directory path for all measurements.

Direct start -> U_ds + I_ds
Soft Start:
2s: U_2s + I_2s
5s: U_5s + I_5s
7s: U_7s + I_7s

```
%% main file
clc
close all
clear all

%% direct start-up

U_ds = readtable('D:\IST\IST-aulas\AVE\2019_20\LAB 1&2\Direct
Start\F0020CH1.csv');
I_ds = readtable('D:\IST\IST-aulas\AVE\2019_20\LAB 1&2\Direct
Start\F0020CH2.csv');

time_ds = table2array(U_ds(:,4));
volt_ds = table2array(U_ds(:,5));
curr_ds = table2array(I_ds(:,5));
power_ds=volt_ds.*curr_ds;



% calc time_step
step_ds=time_ds(2)-time_ds(1);
N_ds=round(0.02/step_ds);

power_ds_av=movmean(power_ds,N_ds)';

Ids_rms=zeros(1,length(curr_ds));
Uds_rms=zeros(1,length(curr_ds));
for i=N_ds+1:length(curr_ds)
    Ids_rms(i)=rms(curr_ds(i-N_ds:i));
    Uds_rms(i)=rms(volt_ds(i-N_ds:i));
end

apparent_ds_av=(Uds_rms.*Ids_rms); %Q=sqrt(S^2-P^2)
reactive_ds_av=(apparent_ds_av.^2-power_ds_av.^2).^(1/2);

figure
subplot(1,2,1)
plot(time_ds, volt_ds);
hold on
yyaxis right
plot(time_ds, curr_ds);
subplot(1,2,2)
plot(time_ds, power_ds);
hold on
plot(time_ds, power_ds_av);

%% SS start-up

% T_up=2s

U_2s = readtable('D:\IST\IST-aulas\AVE\2019_20\LAB 1&2\SS tup_2s\F0020CH1.csv');
I_2s = readtable('D:\IST\IST-aulas\AVE\2019_20\LAB 1&2\SS tup_2s\F0020CH2.csv');

time_2s = table2array(U_2s(:,4));
```

Potential of *Electric/Hybrid Propulsion Systems* to Reduce Petroleum Use and Greenhouse Gas Emissions

Objective: Comparison of energy consumption of different powertrain configurations in light-duty urban vehicles

Consider the four configurations for a light-duty urban vehicle (passenger cars and light commercial vehicles). Figures 1,2, and 3 are electric vehicles (EV) powertrains for, respectively, a **pure EV**, a **series EV**, and a **parallel-hybrid EV**.

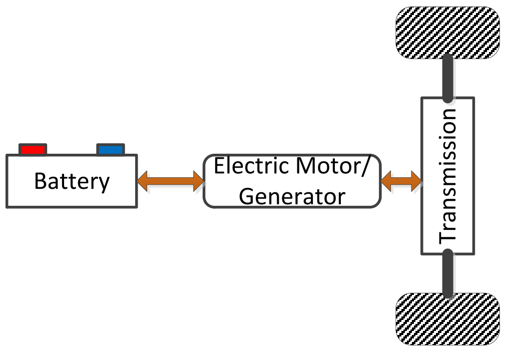


Fig. 1 – Pure EV powertrain

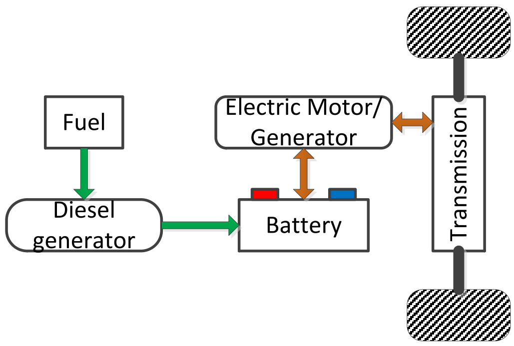


Fig. 2 – Series EV powertrain

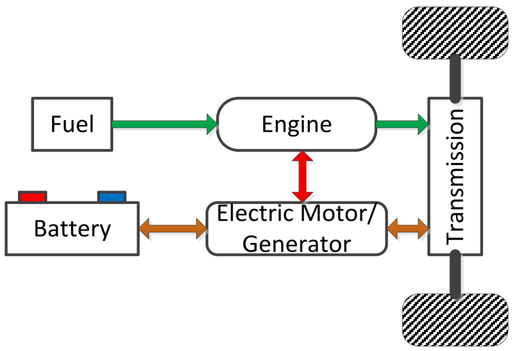


Fig. 3 – Parallel hybrid EV configuration

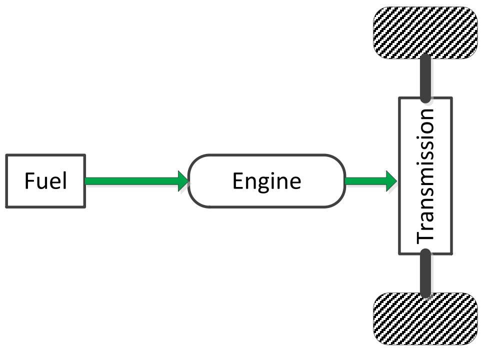


Fig. 4 – ICE vehicle

A typical set of characteristics of a light-duty urban vehicle is summarized in Table I and Table II.

Table I

Parameters	Vehicle
Frontal surface area	2.7 m^2
Aerodynamic drag coefficient	0.25
Rolling resistance coefficient	0.018
Mass	1400 Kg

Table II

Pure EV		ICE Vehicle	
Motor	AC Permanent Magnet Synchronous motor	**Motor**	Diesel
Power	80 kW	**Power**	110 CV/81 kW (1461 cm^3)
Torque	280 Nm	**Torque**	240 Nm
Maximum speed	145 km/h	**Urban consumption**	5.8 l/100 km
Batteries	48 Li-ion package, total voltage 403.2 V	**Extra-Urban consumption**	4.2 l/100 km
Battery power	90 kW	**Mix consumption**	4.8 l/100 km
Battery capacity	24 kWh	**Maximum speed/** **Acceleration (0-100 km/h)**	175 km/h 11.2 s

The typical driving cycle in the city can be characterized by a speed curve, as shown in Fig. 5. Anyway, to compare energy consumptions it is also used as a driving cycle shown in Fig. 6 and composed by an urban cycle (period 3) repeated 4 times (period 1), and an extra-urban cycle (period 2).

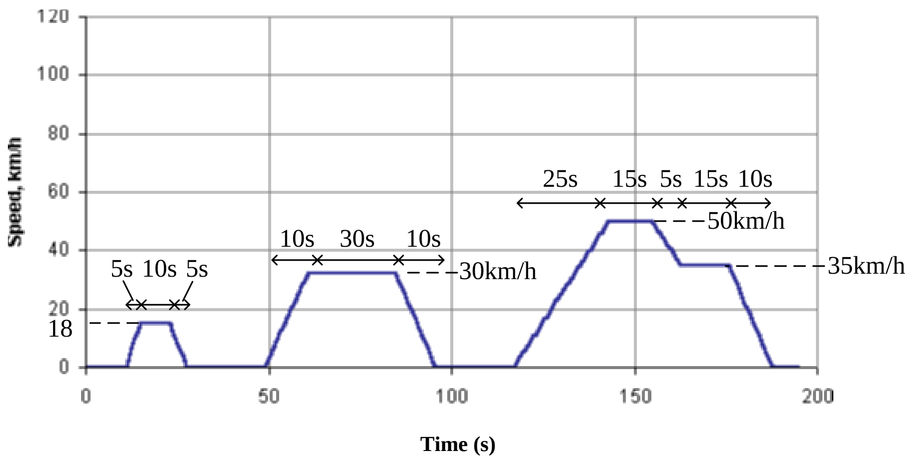
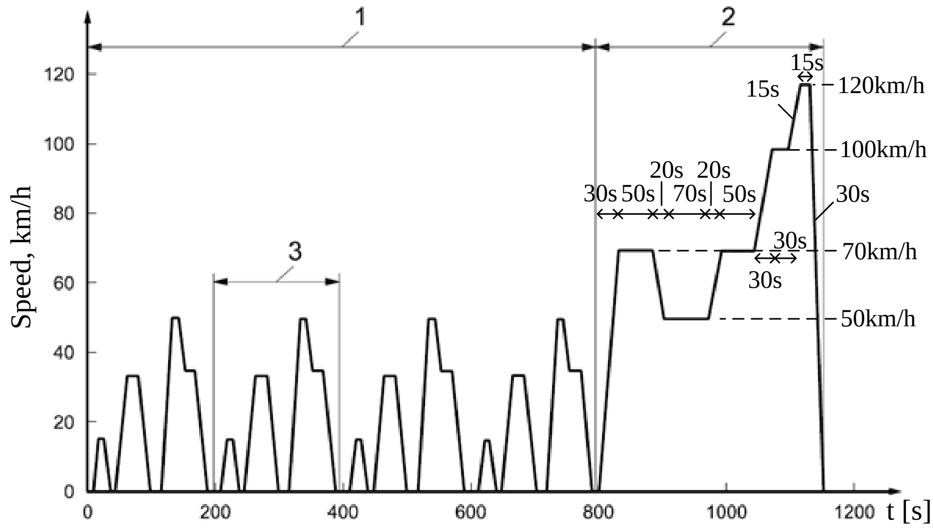


Fig. 5 - Basic urban driving cycle



Legend: X time [s]**,** Y speed [km/h]**;** 1 Cycle part 1 (urban)**;** 2 Cycle part 2 (extra-urban)**; Cycle part 3 (**elementary urban cycle).

Fig. 6 - Mix driving cycle

Under typical operation for 10 mix driving cycles (10 repetitions of Fig. 6 – lead to approximately 110km), it can be assumed that the:

- **Pure EV** (Fig. 1) gets the power from a low voltage AC grid (<u>Energy cost: 0.15 €/kWh</u>);
- **Series EV** (Fig. 2) the batteries are charged from the DIESEL Generator (<u>Fuel price: 1.4 €/l and Fuel energy density 10kWh/l</u>);
- **Parallel-hybrid EV** (Fig. 3) uses only the electric chain for the speed range [0, 20] km/h, both chains (electric + ICE) for the speed range [20, 40] km/h. and uses only the ICE engine for speeds over 40 km/h (<u>Energy cost: 0.15 €/kWh; Fuel price: 1.4 €/l</u>);
- **ICE vehicle**, as usual, is filled in with an ordinary fuel (<u>Fuel price: 1.4 €/l</u>).

For the vehicle topologies represented in Fig. 1 to Fig. 4:

1) Compare the power consumptions (electric and fuel, this once available) for each configuration when performing the referred driving cycles (Fig. 5 and Fig. 6).
2) Discuss the feasibility and value for the energy that can be recovered for each configuration and driving cycle.
3) Discuss the buying option from the economical point of view, since a car runs on average 15000 km/year. Consider that the reference price of a car is: 35 k€ for a pure EV, 30 k€ for a series EV, 25 k€ for a parallel EV and 20 k€ for an ICE vehicle.
4) Discuss the global efficiency of each solution, including the energy cost per cycle and gas emissions.

As the first approach evaluating the issues above, consider:

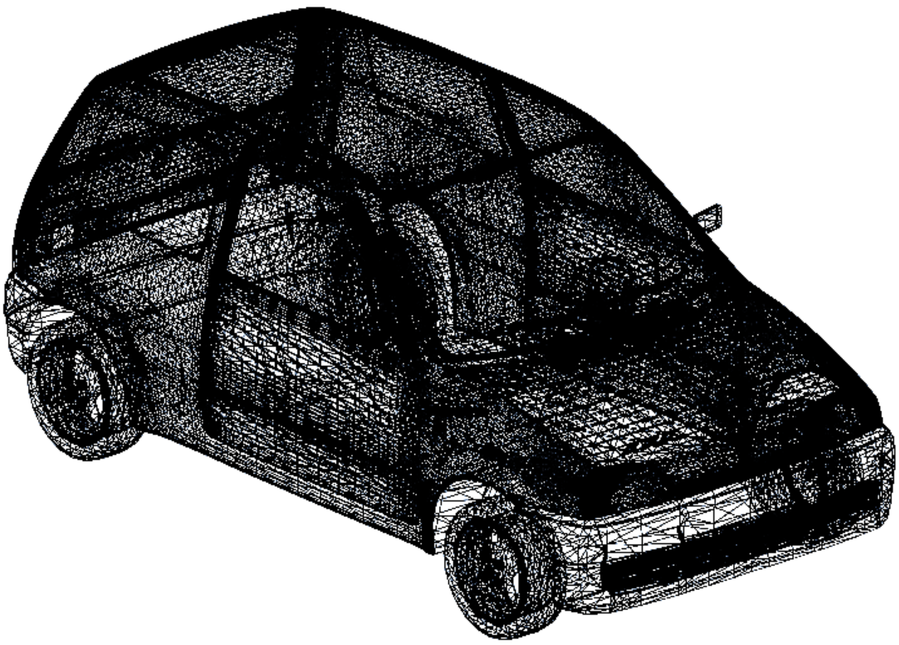
- The kinetic energy of the vehicle (rigid body) can be used as the main input.
- The battery bank of the parallel hybrid EV can be charged as a plug-in solution or from the fuel engine (point out your option).

Drives

and Electrical Vehicles

Project – VIENA

#1 – Mechanical Characteristics



Drives and Electric Vehicles – 2019/2020

João Fernandes, Francisco Silva, Bruno Tibério

1. OBJECTIVES

The main objectives of this first FIAT-competition project report are:

- To obtain the length and slope of each track;
- To obtain the linear mechanical equations that will represent the translational motion of the vehicle;

2. OVERALL SYSTEM

The overall system is divided into three subsystems, as represented in *Fig. 1*:

1. The Vehicle Dynamics;
2. The Gearbox, and;
3. The 3-phase Induction Motor.

The first part of the project will focus on the characterization of the subsystem titled "Vehicle Dynamics". The inputs of this subsystem should be the slope of the track, θ, and the traction torque acting upon the two rear wheels, T_w. The outputs should be the vehicle acceleration and linear speed, a and v_y respectively, and the wheels angular speed, ω_w.

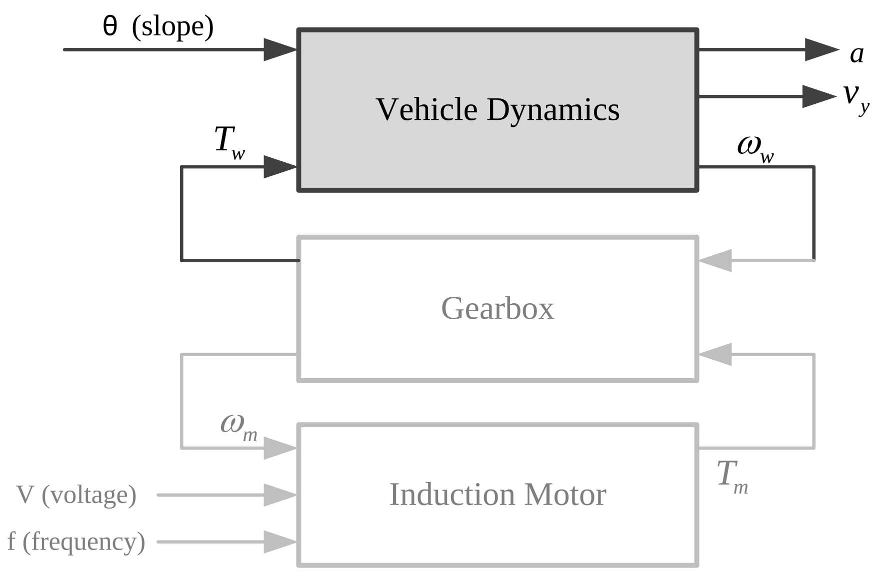


Fig. 1. Block diagram of the overall system

3. TRACK CHARACTERIZATION

The FIAT-competition will be divided into two tracks, as indicated by red bold lines in Fig. 2:

- **Track 1:** From point A to B with a high slope;

- **Track 2:** Endurance competition, with multiple turns (from C-D-E to F and repeating this cycle).

In Table 1 are listed the coordinates of each track point.

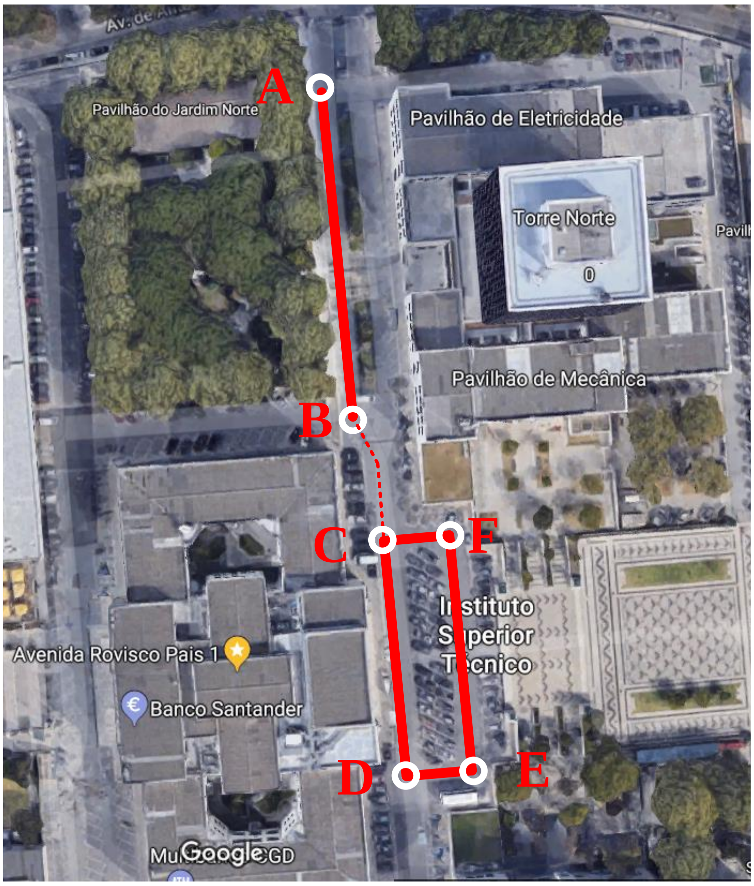


Fig. 2. Satellite view of each track.

Table 1 – Coordinates of each track point.

Point	X [m]	Y [m]
A	0.0	0.0
B	4.8	-70.0
C	13.1	-97.5
D	19.2	-159
E	33.5	-158
F	28.5	-97.9

TASK 1: OBTAIN THE LENGTH AND SLOPE OF EACH PART OF THE TRACKS. Divide the tracks as you believe fit, i.e., for example, you can divide part C to D into two parts or more.

4. VEHICLE DYNAMICS SUBSYSTEM

To obtain the vehicle dynamics, you must consider the mechanical forces on the vehicle, Fig. 3, where F_g is the gravitational force, F_d is the total friction forces (aerodynamic and roll friction) and F_T is the traction force.

The traction force, F_T, is given by the rear wheels torque and radius, T_w and r_w, respectively. Usually, one only considers the mass of the vehicle to obtain its resultant force ($\sum F=Ma$). However, in a vehicle, an additional term must be added: "the equivalent mass of the rotational parts", M_{eq_rot}, resulting in ($\sum F = (M+M_{eq\_rot})a$).

This equivalent mass of the rotational parts M_{eq_rot} can be computed as in Eq. (1), where I_w and I_m are the wheels inertia and motor's rotor inertia, respectively, and g_r is the gearbox ratio.

NOTE: DEVELOP THE SUB-SYSTEM IN SIMULINK.

$$M_{eq\_rot} = \frac{1}{r_w^{\,2}}\left[I_w + I_m (g_r)^2 \right] \tag{1}$$

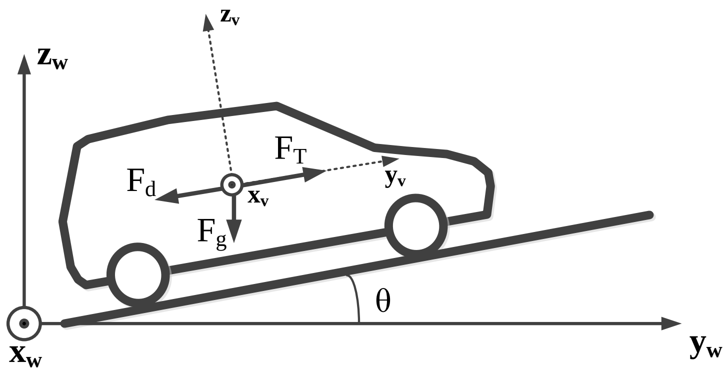


Fig. 3. Mechanical forces acting on the vehicle.

TASK 2: OBTAIN THE EQUATIONS OF THE SUBSYSTEM "VEHICLE DYNAMICS", CONSIDERING THE REQUIRED INPUTS AND OUTPUTS.

5. RESULTS

Consider the vehicle parameters listed in Table 2.

TASK 3: FOR A WHEEL TORQUE OF 150NM FOR TRACK A->B AND A WHEEL TORQUE OF 40NM FOR TRACK C->D->E->F (ONE LAP, BUT DO NOT REPEAT POINT C), COMPUTE:

- The time to finish;
- The linear velocity of the vehicle (Km/h);
- The power traction developed by the vehicle during its motion;
- The total traction energy consumed to complete each track.

Consider that the vehicle starts with zero velocity.

Table 2 – Vehicle parameters.

Characteristic	Value
Frontal area	$2.14\ m^2$
Vehicle weight + passengers	900 kg
Wheels radius, r_w	0.165 m
Roll friction coefficient, f_r (wheels on concrete)	$\begin{cases} 0.01, & \lvert v_y \rvert > 0.1\ \mathrm{m/s} \\ 0, & \lvert v_y \rvert < 0.1\ \mathrm{m/s} \end{cases}$

Aerodynamic coefficient	0.33
Gearbox ratio, g_r	8
Wheel inertia, I_w	0.25 kg.m^2
Motor's rotor inertia, I_m	0.0025 kg.m^2

6. NOTES ABOUT MECHANICAL ASPECTS

1. GEARBOX:

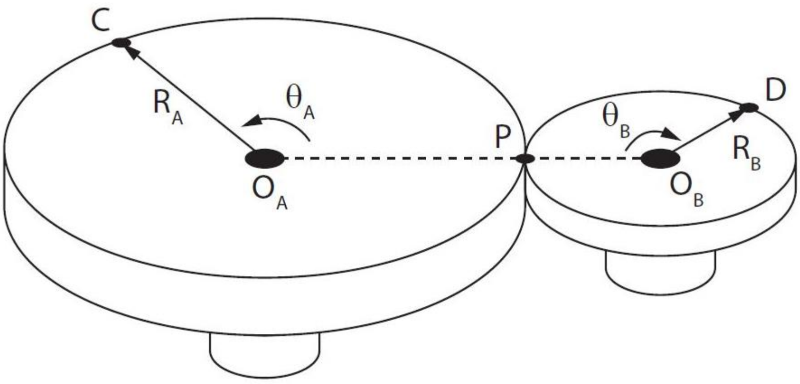


Fig. 4. Two wheels, rotating and contacting at point P.

Consider two wheels, A and B, in Fig. 4. They have an angular speed of ω_A and ω_B, respectively. The relationship between the angular velocities of the wheels is determined by

$$\omega_A R_A = \omega_B R_B \leftrightarrow \frac{\omega_A}{\omega_B} = \frac{R_B}{R_A} = g_r \quad (2)$$

Now, consider the relationship between the torques of both wheels, T_A and T_B. Assuming there are no mechanical losses in the contact between both wheels, the power from each wheel must be the same ($P_A = P_B$). The power is determined by $P = T\omega$, and so

$$T_A \omega_A = T_B \omega_B \leftrightarrow \frac{T_A}{T_B} = \frac{\omega_B}{\omega_A} = \frac{1}{g_r} \quad (3)$$

What about if we are considering a start of rotational movement from the wheels? The Euler law of motion must apply for the wheels

$$T_A = I_A \alpha_A \tag{4}$$

$$T_B = I_B \alpha_B \tag{5}$$

Since $\alpha = d\omega/dt$, applying the derivative in both sides of (2), the relationship between accelerations is the same as (2) ($\alpha_A/\alpha_B = g_r$). Dividing (4) with (5), one gets

$$\frac{T_A}{T_B} = \frac{I_A}{I_B} \frac{\alpha_A}{\alpha_B} \leftrightarrow \frac{I_A}{I_B} = \frac{1}{g_r^2} \tag{6}$$

2. EQUIVALENT MASS:

Starting with the car model in Fig. 3 (not including the weight and the friction forces), one has

$$F_T = Ma \tag{7}$$

Since the traction is made in the wheels, the traction force F_T must be applied in the rear wheels.

Fig. 5 shows the forces applied in the wheels that are connected to the motor. Since these wheels are responsible for the traction power the car has, the traction force F_T must be applied in the point of contact between the wheels and the road.

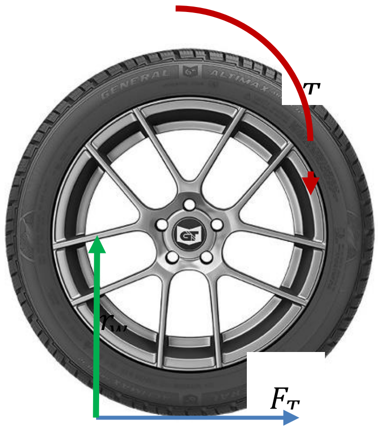


Fig. 5. Force diagram in the wheel responsible for the traction power (wheel connected to the motor).

Applying the Newton equations for the rotating parts (which is called the Euler's Laws of Motion), one has:

$$T_T - F_T r_w = I_{Total}\alpha \tag{8}$$

where I_{total} is the total inertia seen from the wheels, r_w is the wheel radius and α is the angular acceleration.

Solving (8) with respect to F_T and using the relation between the angular acceleration and the linear acceleration, (7) becomes

$$\frac{T_T}{r_w} = \left(M + \frac{I_{total}}{r_w^2} \right) a \tag{9}$$