Assignment 3

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Question 1: The last page of this document contains the game tree with evaluations. The starting move is highlighted in green and the nodes pruned assuming optimal ordering are highlighted in red.

(a) There are 9! possible ways to fill-in a tic-tac-toe board—this is an overestimate, but a good-enough approximation of the size of the search space.

Question 2: (a) This will happen when $e1$ is b times faster than $e2$.

(b) With α-β pruning, assuming optimal ordering, this will happen when $e1$ is $\sqrt{b}$ times faster than $e2$, and with random ordering, when $e1$ is $d^{\frac{3}{4}}$ times faster than $e2$.

(c) With α-β pruning, less dramatic increases in speed get the same or better performance increases than you would get in mini-max, but ordering is more important than raw speed. With mini-max, you have to visit and evaluate each node so raw speed is more important than it is with α-β pruning.

Question 3: Constraints:

$$I \neq D \neq O \neq T$$

$$I + D = O + C_{10} \cdot 10$$

$$I + C_{10} = O + C_{100} \cdot 10$$

$$D + C_{100} = T$$

$$\forall v \in \{I, D, O, T\}; 0 \leq v \leq 9$$

Using the MRC heuristic, we are always assigning values to variables with the fewest available legal assignments.

Depth 1: assign $C_{100} = 1$ as this is the only legal assignment for C_{100} (assigning $C_{100} = 0$ violates the constraint that $D \neq T$

Depth 2: assign $C_{10} = 1$ as this is the only legal assignment ($C_{10} = 0$ violates the constraint that $I \leq 9$)

Depth 3: assign $I = 9$ as this is the only legal assignment for I since $I + 1 \geq 10$

Depth 4: assign $O = 0$ as this is the only legal assignment for O since $I + 1 = O + 10$

Depth 5: assign $D = 1$ as this is the only legal assignment for D since $9 + D = 0 + 10$

Depth 6: assign $T = 2$ as this is the only legal assignment for T since $1 + 1 = T$

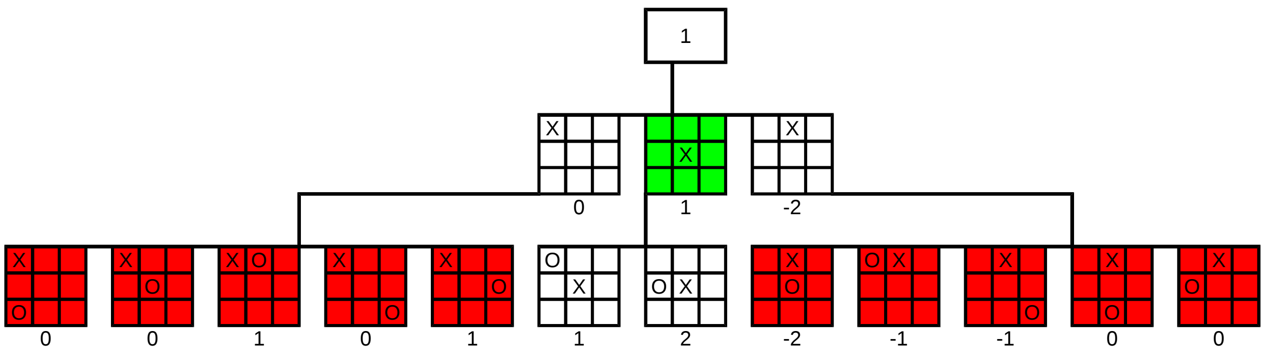
Question 4: To answer each part of this question, I will use the fact that the size of the search space s in n variables with a set of domains $D = \{D_1, D_2, \ldots D_n\}$ is equal to $\prod_{i=1}^{n} |D_i|$. From this it follows that the size of s is $\Omega(d_{\text{min}}^n)$ and $O(d_{\text{max}}^n)$ where $d_{\text{min}} = \min\{|D_i|; D_i \in D\}$ and $d_{\text{max}} = \max\{|D_i|; D_i \in D\}$. From this if follows that:

(a) The size of s is exponential in the number of variables

(b) The size of s is polynomial in the size of the domains.

Question 5: It is often a good idea to choose the most constrained variable and the least constraining value when we are only interested in finding the first solution rather than all solutions to a CSP. This is because by choosing the most constrained variable, we eliminate all the nodes in the search tree where we would run out of possible values for the most constrained variable, and by choosing the least constraining value for that variable, we eliminate fewer possibilities for the remaining variables making it more likely we will find a solution.

Question 6: There are 18 possible solutions to the map-coloring problem for the Australia map with three colors. Tazmania can take any of the three colors regardless of the colors chosen for any of the 6 other states. The colors of the rest of the states are driven by the color choice of Southern Australia which can take any of the three colors. After we have made a choice for Southern Australia, there are really only 2 possible remaining solutions—one where Western Australia is one of the remaining 2 colors or the other since the only restricition is that it can't be the color chosen for Southern Australia. After that choice has been made there is only one possible color for each of the remaining states since every state other than Tazmania borders Western Australia and one other state. This comes out to $3 \cdot 3 \cdot 2 = 18$ possible solutions.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