Assignment 4

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Question 1: (a) Sentences in CNF (for sentence number two, I took it to mean "It is not hot, and if it's sunny, then it's pleasant" instead of "If it's hot and sunny, then it's pleasnt":

KB1. $\neg Humid \vee \neg Hot \vee Sticky$

KB2. $\neg Hot$

KB3. $\neg Sunny \vee Pleasant$

KB4. $Sunny$

KB5. $Humid$

KB6. $Sunny \vee Pleasant$

(b) i. $\neg Sticky$ (assumption)

ii. $\neg Pleasant$ (assumption)

iii. $Pleasant$ (from KB3 and KB4)

(c) i. $\neg Sticky \vee \neg Pleasant$ (assumption)

ii. $Pleasant$ (from KB3 and KB4)

iii. $\neg Sticky$ (from ii. and i.)

iv. ...Here is where I run into a dead end. I don't think this can be proven from the collection of sentences. We can conclude that it's either *Sticky* or *Pleasant*, but we can't say that it is both *Sticky* and *Pleasant*. It could be that it's both *Sticky* and *Pleasant*, but we can't tell from this collection of sentences. In other words, the statement $Sticky \wedge Pleasant$ is satisfiable, but not valid in this context.

Question 2: *Proof.* Let S be a statement in CNF such that $S = P_1 \vee P_2 \vee \cdots \vee P_n \vee Q_1 \vee Q_2 \vee \cdots \vee Q_m$ then

$$\begin{aligned} S = (P_1 \vee P_2 \vee \cdots \vee P_n) \vee (Q_1 \vee Q_2 \vee \cdots \vee Q_m) &= \neg(P_1 \vee P_2 \vee \cdots \vee P_n) \to (Q_1 \vee Q_2 \vee \cdots \vee Q_m) \\ &= (\neg P_1 \wedge \neg P_2 \wedge \cdots \wedge \neg P_n) \to (Q_1 \vee Q_2 \vee \cdots \vee Q_m) \end{aligned}$$

□

Question 3: (a) $\forall x((Big(x) \wedge House(x)) \to Expensive(x))$

(b) $\forall x((Expensive(x) \wedge House(x)) \to Big(x))$

(c) $\forall x((Big(x) \wedge House(x)) \to Expensive(x))$

(d) $\exists x \exists y(Big(x) \wedge House(x) \wedge Condo(y) \wedge (Price(y) > Price(x)))$

(e) $(\exists x \exists y(House(x) \wedge Garden(y) \wedge Has(x,y))) \wedge (\forall x \forall y \forall m \forall n(House(x) \wedge Garden(y) \wedge Has(x,y) \wedge House(m) \wedge Garden(n) \wedge Has(m,n)) \rightarrow x = m)$

Question 4: (a) Incorrect. As written, the translation says that there is at least one pink house in Minneapolis, but does not exclude the possibility that there could be others. It should be $\exists x(house(x) \wedge in(x, Minneapolis) \wedge color(x, Pink)) \wedge \forall x \forall y(house(x) \wedge in(x, Minneapolis) \wedge color(x, Pink) \wedge house(y) \wedge in(y, Minneapolis) \wedge color(y, Pink)) \rightarrow x = y$

(b) Incorrect. As written, the translation says that for every apartment there is some house that is bigger. It should be $\exists y \forall x(house(y) \wedge (appartment(x) \rightarrow bigger(y,x)))$

(c) Incorrect. As written, the translation says that for every apartment, there is some house that is cheaper. It should be $\forall x \forall y(house(x) \wedge apartment(y)) \rightarrow cheaper(x,y)$

(d) Incorrect. As written, the translation says that there is some farm and some object such that if the object is a house then the farm is cheaper than it. It should be $\exists x \exists y(farm(x) \wedge house(y) \wedge cheaper(x,y))$

(e) Incorrect. As written, the translation says that for every house, if there is a bathroom, then it is in the house. It should be $\forall x \exists y(house(x) bathroom(y) \wedge in(x,y))$

Question 5: Proof by resolution:

1. $G(B)$
2. $\neg G(x) \vee H(x)$
3. $\neg H(z) \vee I(z)$
4. $\neg H(w) \vee J(w, D)$
5. $\neg I(B) \vee J(C, B)$
6. $\neg I(q) \vee \neg J(q, y)$
7. $\neg G(B) \vee H(B)$ (from 2, B/x)
8. $H(B)$ (from 1 and 7)
9. $\neg H(B) \vee J(B, D)$ (from 4, B/w)
10. $\neg H(B) \vee I(B)$ (from 3, B/w)
11. $I(B)$ (from 10 and 8)
12. $J(B, D)$ (from 8 and 9)
13. $\neg I(B) \vee \neg J(B, y)$ (from 6, B/q)
14. $\neg I(B) \vee \neg J(B, D)$ (from 13, D/y)
15. $\neg J(B, D)$ (from 11 and 14)
16. $J(B, D) \wedge \neg J(B, D)$ (from 12 and 15)

Question 6: (a) Statements in predicate calculus:

1. $\forall x(Person(x) \wedge Rich(x) \wedge \neg Stupid(x)) \rightarrow Happy(x)$
2. $\forall x(Person(x) \wedge Reads(x)) \rightarrow \neg Stupid(x)$
3. $Person(JOHN) \wedge Reads(JOHN) \wedge Rich(JOHN)$
4. $\forall x(Happy(x) \wedge Person(x)) \rightarrow HasExcitingLife(x)$

(b) Conjunctive normal form:

KB1. $\neg Person(x) \vee \neg Rich(x) \vee Stupid(x) \vee Happy(x)$

KB2. $\neg Person(y) \vee \neg Reads(y) \vee \neg Stupid(y)$

KB3. $Person(JOHN)$

KB4. $Reads(JOHN)$

KB5. $Rich(JOHN)$

KB6. $\neg Happy(z) \vee \neg Person(z) \vee HasExcitingLife(z)$

(c) Proof of $\exists m HasExcitingLife(m)$:

1. $\neg HasExcitingLife(m)$ (assumption)
2. $\neg Person(JOHN) \vee \neg Rich(JOHN) \vee Stupid(JOHN) \vee Happy(JOHN)$ (from KB1, $JOHN/x$)
3. $\neg Person(JOHN) \vee \neg Reads(JOHN) \vee \neg Stupid(JOHN)$ (from KB2, $JOHN/y$)
4. $\neg Happy(JOHN) \vee \neg Person(JOHN) \vee HasExcitingLife(JOHN)$ (from KB6, $JOHN/z$)
5. $\neg HasExcitingLife(JOHN)$ (from 1, $JOHN/m$)
6. $\neg Rich(JOHN) \vee Stupid(JOHN) \vee Happy(JOHN)$ (from KB3 and 1)
7. $Stupid(JOHN) \vee Happy(JOHN)$ (from 6 and KB5)
8. $\neg Reads(JOHN) \vee \neg Stupid(JOHN)$ (from 3 and KB3)
9. $\neg Happy(JOHN) \vee HasExcitingLife(JOHN)$ (from 4 and KB3)
10. $\neg Stupid(JOHN)$ (from KB4 and 8)
11. $Happy(JOHN)$ (from 7 and 10)
12. $HasExcitingLife(JOHN)$ (from 9 and 11)
13. $HasExcitingLife(JOHN) \wedge \neg HasExcitingLife(JOHN)$ (from 5 and 12)