6c) Prove that

$1^2+3^2+5^2+\dots+(2n-1)^2=\dfrac{n(2n-1)(2n+1)}{3}$ for all $n \geq 1$

Base case $n=1$:

$(2(1)-1)^2 = 1^2 = \dfrac{1(2(1)-1)(2(1)+1)}{3} = \dfrac{1(1)(3)}{3} = \dfrac{3}{3}$

$1^2 = \dfrac{3}{3} \Rightarrow 1=1 \leftarrow$ Base case is true

Induction step:

- Suppose it is true for some integer K

  $1^2+3^2+5^2+\dots+(2K-1)^2=\dfrac{K(2K-1)(2K+1)}{3}$

- we want to prove:

  $1^2+3^2+5^2+\dots+(2[K+1]-1)^2=\dfrac{(K+1)(2[K+1]-1)(2[K+1]+1)}{3}$

$\Rightarrow 1^2+3^2+5^2+\dots\ (2K+2-1)^2=$

$\underbrace{1^2+3^2+5^2+\dots(2K-1)^2}_{\frac{K(2K-1)(2K+1)}{3}}+(2K+1)^2=$ (by induction hypothesis)

$\dfrac{K(2K-1)(2K+1)}{3}+(2K+1)^2=\dfrac{(K+1)(2[K+1]-1)(2[K+1]+1)}{3}$

$\dfrac{K}{3}\left[(2K-1)(2K+1)\right]+(2K+1)^2=\dfrac{(K+1)(2K+1)(2K+3)}{3}$

$\dfrac{K}{3}(4K^2-1)+(4K^2+4K+1)=\dfrac{2K^2+K+2K+1\,(2K+3)}{3}$

$\dfrac{4K^3}{3}-\dfrac{K}{3}+\dfrac{12K^2}{3}+\dfrac{12K}{3}+\dfrac{3}{3}=\dfrac{4K^3+6K^2+2K^2+3K+4K^2+6K+2K+3}{3}$

$$\underbrace{\boxed{\dfrac{4K^3-K+12K^2+12K+3}{3}=\dfrac{4K^3+12K^2+11K+3}{3}}}_{\text{it is true!}}$$