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INTERACTION OF ROTATING WAVES IN AN ACTIVE CHEMICAL MEDIUM

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The interaction of autowave sources is experimentally studied in an active chemical medium with excitable kinetics.

Three types of vortices are considered: 1) A spiral wave (S) rotating in a simply-connected medium, 2) A spiral wave rotating around a hole (SH); and 3) A spiral wave (S_N) with topological charge N. It is found that S synchronizes SH (except very small holes), and spiral waves with lower topological charge synchronize those with higher topological charge. It is also found that the interaction of autowave sources displays some unique properties because of their ability to appear on inhomogeities, to vanish and to move in the medium.

A new phenomen of induced drift of spiral waves is demonstrated. The drift was induced by high-frequency concentrational waves propagating in the medium. A similar drift is observed upon interaction of vortices. The mechanism of the induced drift is explained in terms of wave-break translocation from one wave to another. Using this effect, one can control the location of wave sources in an active medium.

1. Introduction

The waves propagating in an active medium are governed by parabolic strongly nonlinear reaction–diffusion equations

$$u_t = f(u) + D\Delta u. \tag{1}$$

The form and amplitude of the waves (the so-called autowaves) remain constant at the expense of the energy of the active medium.

Unlike linear waves, solitons and soliton-like solutions, autowaves do not reflect when reaching the boundaries, and two waves annihilate in the collision. A very important feature of active media is that local sources of autowaves may arise [4–8]. These autowave sources may or may not be dependent on the local geometry of the medium. An example of an autowave source of the first type is provided by the source arising from micro-heterogeneity inducing autooscillations. The corresponding point system

$$u_t = f(u) \tag{2}$$

in the vicinity of a heterogeneity has a limit cycle. Such a source sends concentric waves. Another example of autowave source of the first type is the wave rotating around a hole (nonexcitable obstacle) in the medium. It excites all the points of the medium with a period $T = lv^{-1}$, where l is the perimeter of the hole and v is the wave velocity. The rotating wave has a spiral shape [3] and will be designated as SH.

Of much greater interest are sources not determined by the local structure, for example, a spiral wave similar to SH but rotating in a simply-connected medium (without a hole or an obstacle) [4–8], which will be designated as S. Recently, the rotating vortices with a multiple topological charge have been experimentally found in an active chemical medium [14, 15]. These are the vortices with two, three or four waves rotating together.

The autowave sources have a number of properties differing from those of auto-oscillation generators. Disintegration and multiplication [5] of autowave sources as well as chaos induced by their interaction [5, 6] has been described. The arising of rotating spiral waves in the heart is considered to be the principle physical mechanism of some dangerous cardiac arrhythmias [4–6, 9, 10].

Autowave sources can be controlled by changing the parameters of the medium. Another method of regulation is to use external impulses or waves, as is done for synchronization of generators in radio-electronics [1]. The aim of this paper is to study new phenomena arising in a distributed active medium.

2. Approximate analysis

During the interaction of autooscillation generators the following effects are observed: frequency pulling, beating and independent oscillations [1]. Which of these effects is realized depends on the ratios of frequencies, powers and on the degree of coupling of generators.

Synchronization of autowave sources appears simpler. Since the waves in an active medium have a constant amplitude which is dependent on the properties of the medium, the power of a source is not essential. The only requirement is that the power should exceed a minimum sufficient for emitting the waves. The degree of coupling of sources is determined by the medium, i.e. it is not an independent quantity. The interaction is determined only by the frequencies of wave sources. This follows from the important property of an active medium: the autowaves annihilate when colliding [3]. As a result, the autowave sources with unequal frequencies cannot co-exist, the most rapid sources survive.

The time necessary for a rapid source to synchronize a slower one is easy to estimate. It equals [8]

$$T = lv^{-1}(1-\alpha)^{-1}, \tag{3}$$

where $\alpha = T_1/T_2$, $T_1 < T_2$ are the periods of the sources, l is the distance between them and v is the wave velocity. Indeed, if the two sources emit a first wave simultaneously these waves will collide and annihilate midway between the sources. The next wave will be emitted by the faster source $T_2 - T_1$ earlier than by the slower one. Therefore, the point of the wave collision will be shifted by $\delta = 0.5\, v(T_2 - T_1)$ towards the slower source. The point of collision of the subsequent waves will shift towards the slower source by steps equal to δ until a wave from the faster source covers the entire distance between the sources without colliding with a wave emitted by the slower source. After this all the points in the vicinity f the slower source will be excited with the frequency of the faster one, i.e. the slower source will be synchronized. The number of steps necessary for synchronization is $0.5\, l\delta^{-1}$, which leads to eq. (3). The time necessary for synchronization is the more, the less is the difference between the periods of wave sources.

3. Interaction of spiral wave (S) and spiral wave rotating around a hole (SH)

Synchronization of a slower source by faster one is manifested in the interaction of rotating spiral waves S and SH. The arising wave patterns are shown in fig. 1. As seen, the length of the waves sent by S is shorter than that of SH, i.e. the source S is faster than SH (the periods are directly proportional to the wavelengths in fig. 1, since the wave velocity is approximately constant). As a result, the waves emitted by S synchronize the whole medium, and all the points are excited with the period equal to the period of rotation of S (frames a, b). It is clear from topological grounds that, when synchronized, the SH-source cannot vanish because topological charge is conserved during wave interaction [14, 15] and the SH-source is characterized by a nonzero topological charge (see section 4).

This fact can be easily interpreted in terms of wave kinetics. In fig. 1b the number of anticlockwise rotating waves is always by one wave more then the number of waves rotating in the clockwise direction. After switching off the synchronizing source (S), each clockwise rotating wave will join an anticlockwise rotating wave and will move away from the obstacle. The last anti-

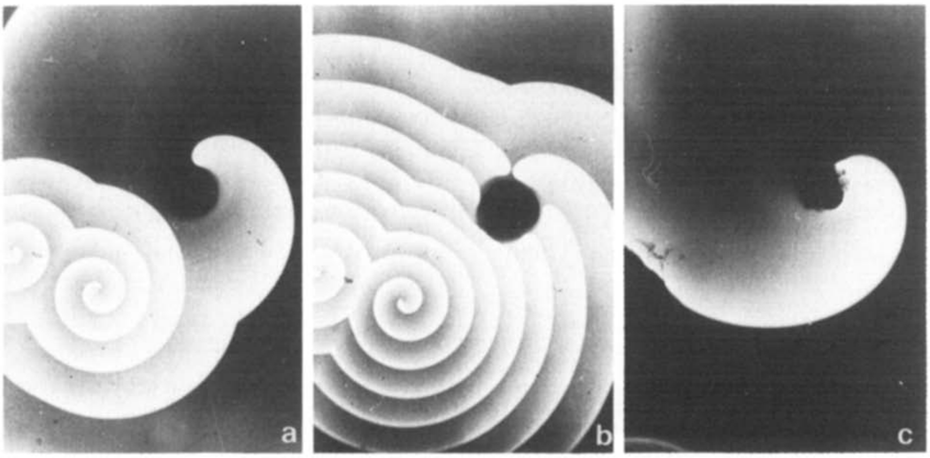

Fig. 1. Spiral waves S_1 synchronize a spiral wave SH. a) lower left, two spiral waves S_1; upper right, a spiral wave SH circulating around a hole which is seen as a dark spot; b) SH has been entrained by S_1; c) the spiral wave SH has reappeared on destroying S_1 with a large drop of KBr solution.

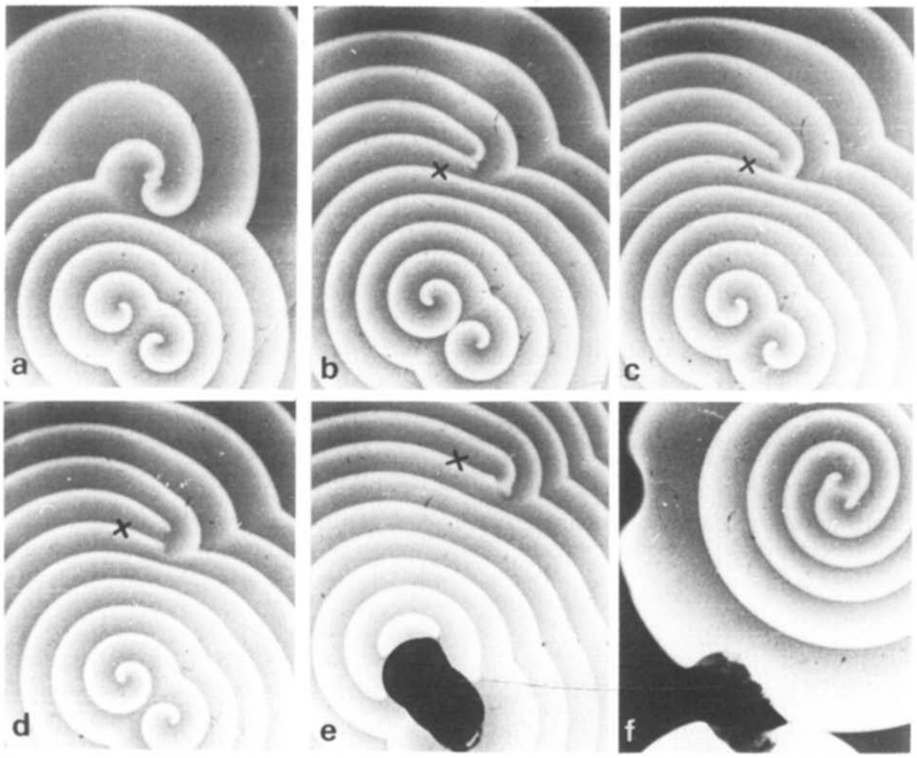

Fig. 2. Spiral waves S_1 synchronize a spiral wave with the topological charge 2 (S_2) inducing its drift. a) two spiral waves S_1 (bottom) and spiral waves S_2 (top); b–e) separated by a 15 sec interval demonstrate the transfer of a wave-break from one wave to another. One of the waves is labeled with a cross to facilitate observation; e) spiral S_1 are destroyed with a large drop of KBr solution; frame f) spiral wave S_2 is restored in a new place.

clockwise rotating wave cannot meet a pair wave and will continue, therefore, rotate around the hole (fig. 1c).

The experiment was carried out as follows. The rotating SH wave was obtained from the rotating S wave by placing a drop of KBr solution (the inhibitor of the B–Z reaction [12]) in the centre of the wave S. Then two additional rotating spiral waves S were initiated (fig. 1a), and the interaction of spiral waves S and SH began. After the SH-source was synchronized by the S sources (fig. 1b) we destroyed the sources S (fig. 1c). For this purpose, a sufficiently large drop of KBr solution was placed so that it covered the centres of both spiral waves S causing them to disappear and we observed the process of restoration of SH.

Note that a single source S cannot be destroyed in this way since the topological charge is conserved. However it is possible to destroy two oppositely rotating waves with the total topological charge equal to zero and that is the reason why two spirals S were used in our experiments on synchronization (figs. 1 and 2).

4. Interaction of spiral waves with unequal topological charges

4.1. *Topological charge of a spiral wave*

Multiarmed rotating spiral waves were found in the B–Z reaction [14, 15]. The number N of waves rotating around a common centre is a topological invariant and is termed the topological charge of a vortex [11, 14, 15]. Vortices can be represented as the solutions of the reaction–diffusion equation of the form

$$u = F(N\theta - \omega t, r), \tag{4}$$

where r and θ are the polar coordinates, ω is the angular velocity and an integer number N is the topological charge of a rotating wave. As seen from eq. (4), the topological charge N characterizes both the number of waves in a vortex and the direction of its rotation ($N = \pm 1$ corresponds to a single wave rotating clockwise or anticlockwise; $N = 2$ is a twoarmed rotating wave; $N = 0$ corresponds to a wave which is topologically equivalent to a circle). A spiral wave with the topological charge N will be designated as S_N.

The vortices with the topological charge $N > 1$ were obtained in an active chemical medium as follows [14, 15]: a number (N) of waves were made to rotate around a hole in the medium. Then the size of the hole was gradually decreased up to zero, giving rise to a vortex with the topological charge N equal to the number of waves that rotated around the hole. The period of a spiral wave S_N was experimentally found to increase with the topological charge (for $|N| \leqslant 4$). Thus it might be expected that spiral waves with a lower topological charge can synchronize those with a greater topological charge.

4.2. *Interaction of spiral waves with topological charges* 1 *and* 2

The interaction of S_1 and S_2 sources is demonstrated in fig. 2 Waves S_1 are seen to synchronize S_2 sources, and only two closely situated wave-breaks remain of source S_2 (fig. 2b). When interacting with an S_1 source, these wave-breaks drift away from S_1 (frames b–d). This drift is slow as compared with velocity of the waves propagating from S_1.

After the sources have been destroyed with a large drop of KBr solution (fig. 2e), the two drifting wave-breaks evolved into a spiral with the topological charge two (S_2). It is seen that this S_2 source is shifted from its previous position (compare figs. 2f and 2a). We shall call this shift "induced drift" of a spiral wave. This effect makes it possible to control the position of a spiral wave in the medium.

Similarly, the drift of a spiral wave with topological charge 3 (S_3) was observed during its interaction with source S_1. It seems reasonable to suppose that the drift of all other types of spiral waves can be induced by interacting with faster

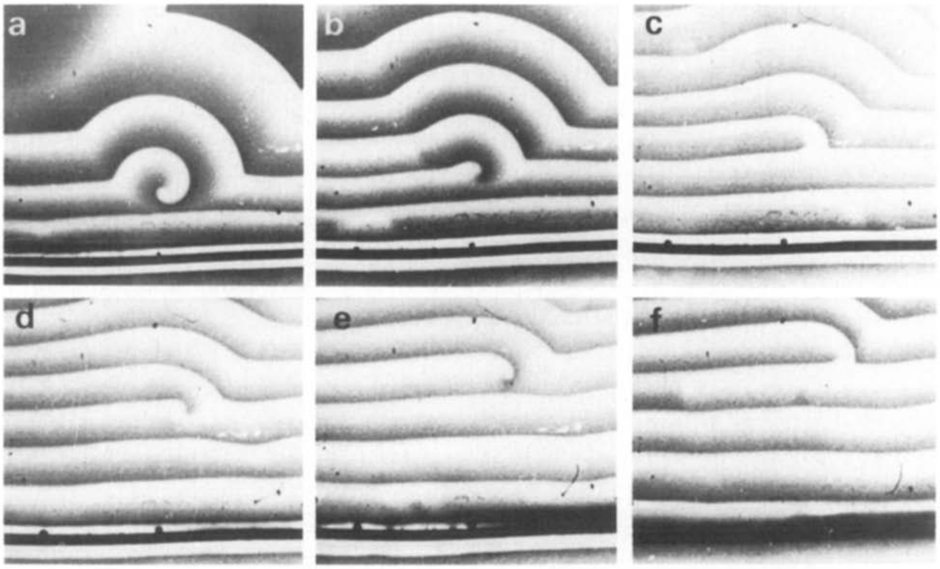

Fig. 3. The drift of spiral wave S_1 induced by high-frequency waves. Plane waves are propagating from the electrode which is seen as a black bar at the bottom of the figure. Plane waves with a higher frequency convert spiral wave a) into a wave-break b). This wave-break closes on the following wave c), which results in the break of this latter d). This process is repeated periodically (compare c and f, b and e), causing a slow drift of the break from the electrode. The period of spiral wave S_1 is 50 s and that of the plane waves is 40 s. The time interval between the frames is 4 min.

sources. We checked this supposition by attempting to induce the drift of the fastest rotating spiral wave – S_1.

5. Induced drift of spiral wave S_1

No local source of waves faster than S_1 in the active medium is known, therefore we synchronized S_1 with external waves. These were initiated by delivering electrical pulses to a wire electrode immersed into the reagent (see appendix). The interaction of spiral wave S_1 with synchronizing waves is shown in fig. 3. The spiral wave S_1 (fig. 3a) turn into a wave-break slowly moving away from the electrode (figs. 3b–f). The velocity of the wave drift was 4–8 times slower than the velocity of wave propagation in the medium, increasing with the frequency of the synchronizing waves (fig. 4). When the source of synchronizing waves was switched off, the drifting wave-break evolved into a spiral situated in a new place.

We mentioned above that a single spiral wave cannot be destroyed by synchronizing waves because of the conservation of the topological charge during wave interaction. However, effect of induced drift permits one to destroy the spiral wave S_1 by causing it to drift to an edge of the active

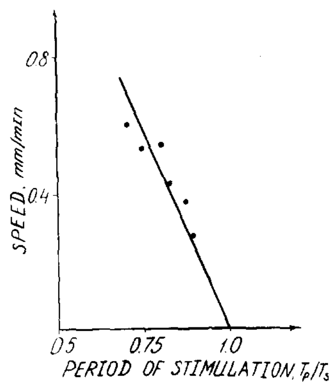

Fig. 4. The dependence of the drift velocity of S_1 on the period of plane waves. The dots designate experimental data and the straight line is drawn according to formula (6).

medium where it vanishes (in contrast to rotating spiral SH-wave).

6. Mechanism of spiral wave drift

The drift of a wave-break is schematically shown in fig. 5. The slow drift is seen to be a result of two rapid opposite motions. Firstly, the wave-break evolves into a spiral (frames a and b), thus moving downwards oppositely to the other waves. The circling part of wave 1 and wave 2 annihilate when colliding and thus the wave-break is transferred to the next wave (frame c).

Secondly, the wave-break moves upwards at the same velocity as all other waves in the medium, covering the distance $\delta_{\uparrow}$ which is determined by the time of the recovery process, whereupon the involution occurs again (frame d). Thus the whole process is reiterated. The resulting shift of the wave-break per one cycle is $\delta = \delta_{\uparrow} - \delta_{\downarrow}$ where $\delta_{\downarrow}$ is downward shift of the wave-break. Fig. 5 shows that $\delta_{\downarrow} = x_{1a} - x_{2c}$, and $\delta_{\uparrow} = x_{2d} - x_{2c}$, where x_{1a} is the coordinate of wave 1 in frame a, x_{2c} is that of wave 2 in frame c, etc. Assuming that the wave velocity in the medium does not depend on the direction and that involution of the wave-break (frame d) begins when restoration processes are finished, we obtain $\delta_{\downarrow} = \lambda_p/2$, $\delta_{\uparrow} = \lambda_s - \lambda_p/2$, and finally

$$\delta = \lambda_s - \lambda_p, \tag{5}$$

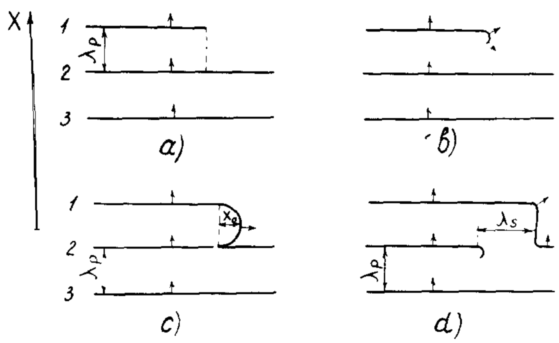

Fig. 5. The scheme of the induced drift of spiral wave S_1.

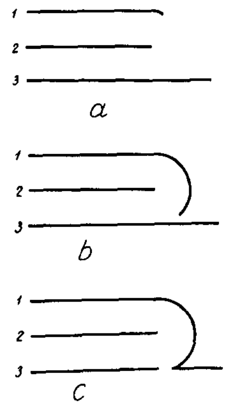

Fig. 6. The scheme of the induced drift of spiral wave S_2.

$$v_{dr} = v(1 - T_p/T_s) \tag{6}$$

where λ_s and T_s are the length and period of the waves emitted by S_1, and λ_p and T_p are the length and period of synchronizing planar waves and v is wave velocity in the medium. Expression (6) for the drift velocity follows from $v_{dr} = \delta/\Delta t$, where Δt is the time of one cycle, equal $\Delta t = t_{\uparrow} + t_{\downarrow}$ and $t_{\uparrow} = \delta_{\uparrow} v^{-1}$, $t_{\downarrow} = \delta_{\downarrow} v^{-1}$.

Fig. 4 shows that this simple estimation (6) is in good agreement with experiment. A similar scheme (fig. 6) explains the mechanism of the drift of a spiral wave with the topological charge 2, S_2. The involution of wave 1 (frame a) begins earlier than of wave 2. Having circulated around wave 2, wave 1 collides with wave 3 (frame c) and the latter appears broken. Then occurs the involution of wave 2, etc. The basic mechanisms of the drift are the same as for a simple spiral wave S_1, namely 1) propagation of the wave-break together with all other waves, 2) transfer of the wave-break from one wave to another.

Appendix A

The experiments were carried out with a two-dimensional chemical active medium (the Belousov–Zhabotinsky reaction). The composition of the medium was close to that proposed by Winfree for the medium with excitable kinetics [8]. Synchronization was investigated in the medium (1): 0.3M $NaBrO_3$; 0.3M H_2SO_4; 0.03M $CH_2(COOH)_2$ 0.1M $CHBr(COOH)_2$; 0.005M ferroin. To enlarge the time of the reaction, which was necessary in the experiments with spiral waves S_2, S_3, the composition (2) was used: 0.41M H_2SO_4; 0.1M $CH_2(COOH)_2$; other components as in 1.

A Petri-dish of 90 mm in diameter was filled with 4.5 ml of the reagent giving the layer 0.6 mm thick, the temperature being 20°C. The oxidation waves were initiated by pulses of electrical current [12]. To initiate concentric waves the silver electrode was placed perpendicular to the liquid layer, while plane waves were initiated by immersing the electrode into the liquid parallel to the bottom of the dish. The voltage of displacement was 1.5 V. The amplitude and duration of the exciting pulse were much higher than the threshold values, amounting to 25 V and 150 ms for plane waves. The reference electrode was made of platinum. Unexcitable obstacles were created by a drop of KBr solution (4M). Spiral waves S_2, S_3 and S_4 were obtained as follows: 2, 3 or 4 waves were made to circulate around a drop of KCl solution (1M). Then it was removed by a sharpened strip of filter paper whereupon a multiarmed vortex arised. Details see in refs. 14 and 15.

References

[1] A.A. Andronov, A.A. Vitt and S.E. Khaikin, Theory of Oscillations (Pergamon, Oxford, 1966).
[2] B.P. Belousov, Sb. Ref. Radiats. Med. (Medgis, Moscow, 1959) p. 145.
[3] N. Winer and A. Rosenbleuth, Arch. Inst. Cardiol. Mex. 16 (1946) 205.
[4] I.S. Balakhovskii, Biofizika 10 (1965) 1063.
[5] V.I. Krinsky, Biofizika 11 (1966) 676.
[6] G.R. Ivanitskii, V.I. Krinsky and E.E. Selkov, Mathematical Biophysics of Cell (Nauka, Moskow, 1978).
[7] A.M. Zhabotinsky and A.N. Zaikin, Oscillatory Processes in Biological and Chemical Systems, vol. 2 (The Center of Biological Research, Pushchino, 1971).
[8]. A.T. Winfree, Science 175 (1972) 634.
[9] C. Brooks, B. Hoffman, E. Sucling and O. Orias, Excitability of the Heart (Grune & Stratton, New York, 1955).
[10] M.A. Allessie, F.I. Bonke and T.Y.G. Shopman, Circulation Res. 41 (1979) 7463.
[11] G.E. Volovik and V.P. Mineev, Physics and Topology (Znanie, Moskow, 1980).
[12] K. Showalter, R.M. Noyes and H. Turner, J. of Amer. Chem. Soc. 101 (1979) 7463.
[13] S.S. Jacobs and I.R. Epstein, J. of Amer. Chem. Soc. 98 (1976) 1721.
[14] V.I. Krinsky and K.I. Agladze, Dokl. Akad. Nauk USSR 263 (1982) 335.
[15] K.I. Agladze and V.I. Krinsky, Nature 296 (1982) 424.