# The Physics of Music

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Music is a very natural expression of human emotion that has been with us for thousands of years. We would be arrogant to say that it has only been with *us* for that length of time as we have all heard the 'music' that is present in nature through mating calls and birdsong etc. It is therefore quite easy and sensible to presume that music is natural—in as much as we can define natural here—as opposed to man-made. Similar is language, though there is a much larger gap between the complexity of human and animal languages.

The analysis of both language and music in a scientific sense therefore happened after they were initially used and created. In the sixth century B.C., Pythagoras noticed that when he 'stopped' a string vibrating at certain points, the notes produced sounded pleasing to the ear. The ratio between the length at which the string was 'stopped' and its total length was always a nice and easy fraction such as a half, two-thirds, three-quarters and so on. Since medieval times, the notes formed by these ratios have been called respectively the octave, fifth and forth. They are collectively known as the perfect consonances. More on these in a more modern and useful context later.

The clear link between maths, as Pythagoras saw it, and music has been known for over two thousand years. As an interesting aside, coming from the other direction by physics being explained in terms of music rather than the other way around, Johannes Kepler, who was a trained musician, used musical notation to demonstrate the harmony that existed in planetary motion.

His 'Harmonies of the World' paper is littered with musical staffs demonstrating, in his eyes (and ears), the neat and harmonic nature of physical processes. This is not dissimilar in thinking to the epicycles of Ptolemy which he created to show how simple the universe was in its use of circles and simple geometry. I will now show how easy it is, and was for Kepler, to completely blur the lines between music and physics.
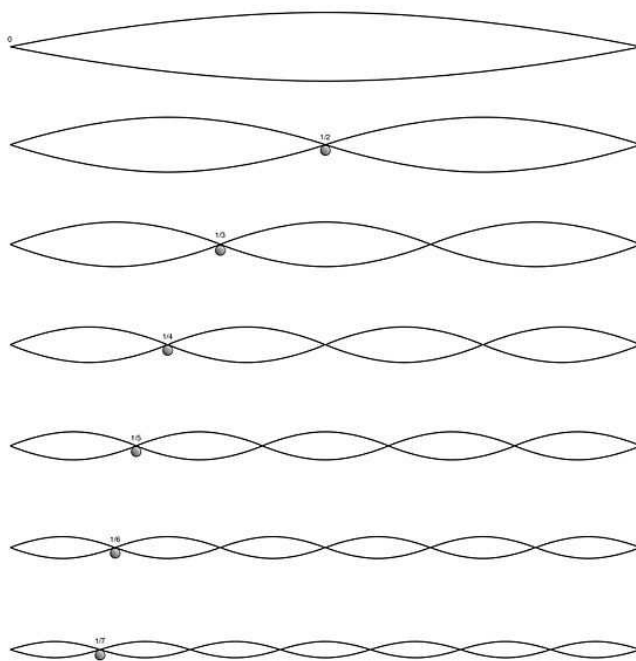


This diagram shows the first seven harmonics produced when a string is vibrated and then 'stopped' at the positions shown. This is exactly what Pythagoras did when he noted that the tones produced sounded "pleasing".

The fixed vibrating string experiment of Pythagoras and many others is well known and I will not discuss it here in great length. A fixed length of string is vibrated at a specific frequency. It is then 'stopped', as Pythagoras did, at different points. In the

second part of the diagram above, the string is 'stopped' at the half way point making the ratio mentioned earlier equal to a half. It is quite easy to see that the wavelength of the wave produced is related to the length of the string. In the case of the first diagram, the wavelength is double the length of the string. In the second, the two are equal and so on.

Assuming a constant speed of sound (which is a justified assumption and does not lead to any further problems), the frequency produced is also related to these quite simple ratios. If the wavelength is halved, the frequency of the sound produced is doubled. This frequency is very much related to the pitch of the sound that is heard by the listener and that is where the link with music comes in.

From this, it is quite simple to create a table that will then be turned into a scale that musicians will use—and, don't forget, have used long before the maths came along—to compose music.

We take a tone of a particular single frequency. Let us use 256 Hz. We shall call the note middle C. Pythagoras halved the wavelength of his vibrating string to produce a tone of double the frequency. I discussed earlier that since medieval times, this note was called the octave. This is because it sounds very similar except it is at a higher pitch. Any musician will tell you that this note will also be called C. What Pythagoras did was divide this string into different 'simple' ratios as discussed earlier.

He would have been able to produce the following table. Musicians will recognise the notes and mathematicians will find solace in the simple ratios, as Pythagoras did.

| Ratio | Note | Frequency / Hz |
|---|---|---|
| 1 | C | 256 |
| 3/2 x 3/4 | D | 288 |
| 5/4 | E | 320 |
| 4/3 | F | 341.3 |
| 3/2 | G | 384 |
| 4/3 x 5/4 | A | 426.6 |
| 3/2 x 5/4 | B | 480 |
| 2 | C | 512 |
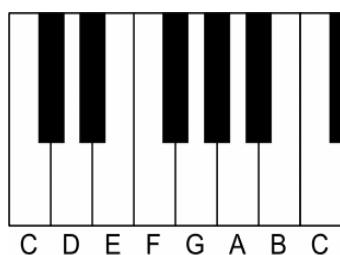


The C major scale on a keyboard. A black note separating the white notes indicates an interval of one tone between the white notes. The lack of a black note, such as between E and F, indicates an interval of a semitone.

A pianist will tell you that the three notes that make up the C chord are C, E and G. A mathematician will tell you that these notes are in the ratio 4:5:6. We can again see how neatly the two subjects are intertwined.

However, there came a problem when keyboard instruments began to be manufactured. Rather than tuning by ear as orchestras did, pianists had to be able to adapt to different scales and play equally well in all. Johann Sebastian Bach was a great fan of the equal-tempered scale in which the frequencies used are slightly adjusted so that the interval in frequency between one range of tones is exactly equal to the interval in frequency between the same range of tones elsewhere on the keyboard.

If you have paid particularly enthusiastic attention to the table above, you will notice that the ratio intervals between adjacent (in alphabetical or frequency order) notes are always either 1.111, 1.125 or 1.067. These numbers are very similar and a slight deviation will not be audible. Therefore we standardise the interval to be 1.12246. The reason for this is that the octave is defined to contain twelve semitones; the twelfth root of two is 1.05946. We call this the semitone ratio. The tone ratio would be the square of this, 1.12246.

J.S. Bach went as far as to write a set of piano pieces entitled 'Das Wohltemperirte Clavier', 'The Well Tempered Clavier'. The two books in his series are considered one of the most influential works in Western music because of their use of the equal-tempered scale. 'The Well Tempered Clavier' showed that the new scale would work.

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So far, I have discussed the physics of music in terms of its impact on the notes and scales used by musicians. Of course, musical instruments are a masterwork of engineering that has physics as its core in terms of calculations of the length and thickness of a piano, violin or guitar string or the distance and size of holes in a flute.

You may think that the guitar is the instrument most suited for discussion after talking of the vibrating string experiment. It is not. You do not simply 'stop' the string, like Pythagoras, to hear a different harmonic. You are in fact altering the length of the string and so producing a different standing wave that is shorter in wavelength and therefore higher in pitch. The frets on the guitar guide your fingers to the correct position for each semitone interval. Violins and other stringed instruments are very similar. Pianos obviously have a string of certain length for each note.

One thing that you can try with your guitar is to place your finger lightly (in a style similar to muting) over each of the strings on the twelfth (and certain other) frets. Then strum. You will not hear muted notes. You will hear the different harmonics produced by the different strings. This is completely analogous to the vibrating string experiment of Pythagoras and you are 'stopping' the string as he did. This is not how you play the guitar, however!

When studying standing waves on strings, it is common to tie that in with standing waves in pipes. This adds to our range of understanding a huge number of instruments such as organs, flutes and clarinets. Again, I will not go into too much detail on the physics of standing waves in pipes except to say that they are very similar to those on strings.



The flute is an instrument ideal for further discussion. It is considered to be, physically speaking, a pipe which is open at both ends (ignoring the important holes). Blowing through it will produce a note that has a wavelength that is double the length of the pipe. Producing just one tone is pretty useless to a musician, however, so there is a mechanism for changing the pitch of the note produced. To alter the pitch, the musician needs to either be able to alter the wavelength of the first harmonic produced (i.e., change the length of the pipe) or move onto a later harmonic as a guitarist may

have tried.  You will not be too surprised to know that it is the length of the pipe that is effectively changed when one of the flute's holes is opened.

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This piece of work covers scales and then talks about how different instruments work to use those scales to produce music that is pleasing to the ear.  It is a drop in the ocean, however.  Music is huge subject matter based in a very major way on physics.  From the maths of the scales to the basic engineering of an instrument to the degree in itself of sound or audio acoustics, physics plays a huge role in music, much more so than I could hope to touch on in a vacation essay.

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**References**

'The Physics of Music', a Scientific American publication that consolidates some of its articles on the subject. The only part of the book used was the introduction which was written by Carleen Maley Hutchins. ISBN 0716700956

'On The Shoulders of Giants', a collection of 'the great works of physics and astronomy' published by Penguin. It includes selected works of Johannes Kepler which came in useful. ISBN 0141015713

The website of Professor Bryan Suits of Michigan Tech University
http://www.phy.mtu.edu/~suits/Physicsofmusic.html

Lecture notes from Professor Roger Barlow at the University of Manchester
http://www.hep.man.ac.uk/u/roger/PC1302/lecture12.pdf

An article by Peter Pesic on Johannes Kepler in Volume 11, No. 1 of the Journal of Seventeenth-Century Music which can be found online at http://sscm-jscm.press.uiuc.edu/v11/no1/pesic.html

Wikipedia's articles on the Physics of Music and 'The Well Tempered Clavier'

Wikipedia's image of the seven first harmonics on a vibrating string and an adapted version of Wikipedia's image of a keyboard.

The image of the flute is from the 1911 edition of Webster's Dictionary.

'How To Love Your Flute' by Mark Shepard. ISBN 0938497103