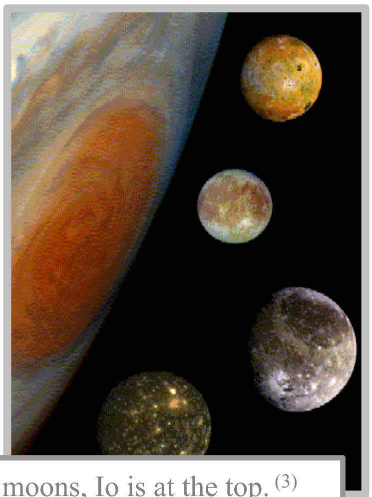
Consequences, Explanation and A History of
Einstein's 'Special Theory of Relativity'

Unfortunately, the reader will be familiar with the idea that time and space vary from what is expected as velocities approach that of light. I say unfortunately because this was not the case one hundred years ago when this was not assumed and so Einstein's paradox of the mirror was understandably a paradox. The teenager's idea was that if you and a mirror were travelling at the speed of light, light that you emitted would never reach the mirror as it would always be just ahead of the light approaching it. Einstein spent a long time not quite believing or understanding this. (1)

There is not even a similar analogy with sound as it requires a medium to travel through so you can't even imagine travelling on Concorde with a microphone (faster than the speed of sound) and speaking but the microphone not recording because the medium for sound travel (air between you and the microphone) is travelling at the same speed so there is no relative 'speed of sound' speed.

Philosophers and scientists had always wondered about light. Sound was easy. You knew that a thunder clap sounded a short while after its lightning bolt. It was clear that the speed of sound was not infinite. The Greeks even worked out that it was some sort of mechanical vibration. Light, however, has (what is now known to be) a huge velocity that led some people, Aristotle being one, to think that its speed was infinite. In Aristotle's day, most people, including Hero (the Greek, of Alexandria), believed that we could see because our eyes emitted light that was reflected back to us—the proof of this being in the bright eyes of cats at night. Hero said that if he looked at the sky on a starry night, he could close his eyes and reopen them to see the stars instantly, therefore the speed of light (emitted from his eyes and reflected back) was infinite.

But these ideas had no real scientific basis though neither did those that suggested that the speed of light was finite. Some ideas were well thought out though. I will not go into them here but René Descartes and Galileo both had good reason to believe that the speed of light was infinite. Galileo even hypothesised an experiment but the distances were so relatively small (and timekeeping so inaccurate) that the huge but finite speed of light was not revealed.



Jupiter and its moons, Io is at the top. (3)

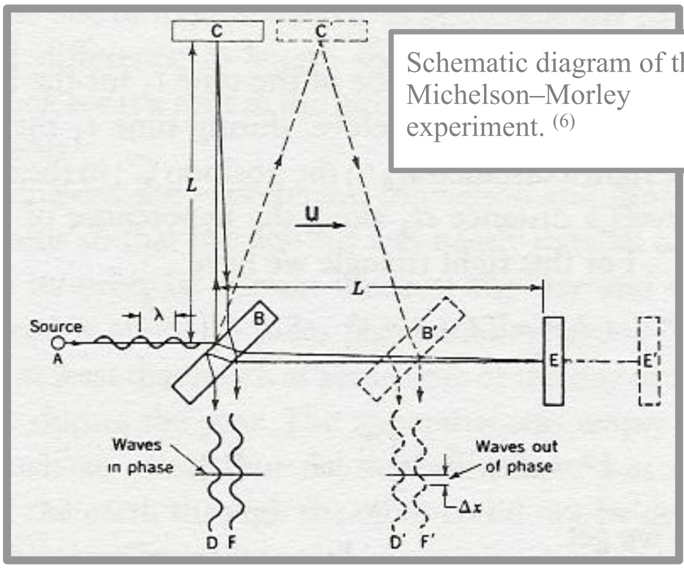
Robert Hooke did realise these problems in Galileo and Descartes' experiments and believed that light travelled "exceedingly quickly". Dane Ole Roemer proved Hooke's vague hypothesis to be correct. He observed Io, a moon of Jupiter and found that when the Earth was closest to the planet in its orbit, an eclipse of Io occurred sixteen minutes earlier than when the Earth's orbit was furthest away from Jupiter. Therefore light took sixteen minutes to traverse the diameter of the Earth's orbit. It is not thought that Roemer ever actually found an accurate distance of the orbit for the eventual speed measurement but the answer does not come out to quite today's accepted value, however, it was the first time a logical method and logical proof that the speed of light was not infinite had been used to determine it. (2)

I will skip the other methods of determining the speed of light as they are irrelevant to the point that the speed is finite that I am making. Jumping to 1865, James Clerk Maxwell came up with some complex equations that linked electricity and magnetism, and therefore electromagnetic radiation, of which light is a form. From these equations, the speed of electromagnetic radiation would have to be given by:

$$v = \frac{1}{\sqrt{\varepsilon \times \mu}} = c$$

If you add a constant velocity to Newton's equations of motion, they come out with the same results. The implication of this being that unless you are experiencing an acceleration, the laws of physics when standing on the ground are the same as when travelling at a constant velocity in a car. This seems fairly obvious as the Earth itself can be seen as a moving body. When the same was done with Maxwell's equations, this did not appear to be the case. As the equation above shows, the velocity of electromagnetic radiation appears to have to be constant, taking no account of your own velocity. The speed of light therefore had to be relative to something, the ether. (3)

Maxwell believed that light travelled through this ether—an invisible substance that most physicists believed had to exist as a medium for electromagnetic waves to fluctuate in. The consensus at the time was that the Earth moved relative to the stationary ether. Maxwell had complained that there was no way to measure the Earth's flow through the apparently stationary ether as the only way to measure *c* was by using mirrors on the Earth's surface. Maxwell's problem was that light speed would be aided by ether flow (or Earth flow through ether looking from the ether's 'stationary' point of view) in one direction but hindered in the other so effects of the ether would cancel out. (4)



Schematic diagram of the Michelson–Morley experiment. (6)

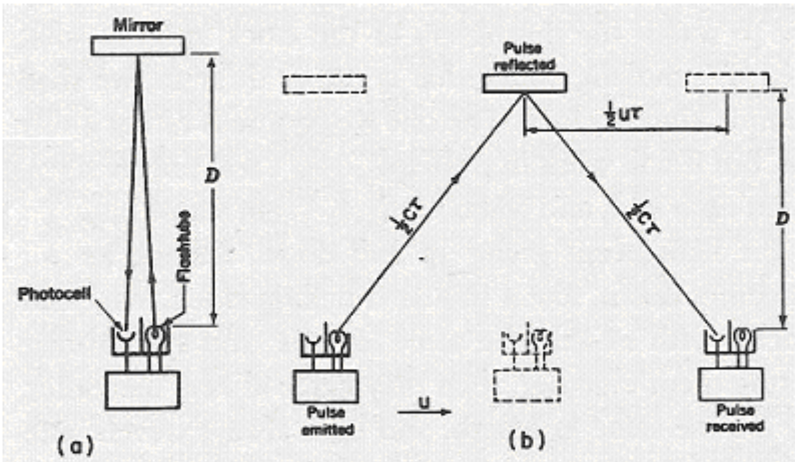
Albert Michelson, after finding the hitherto most accurate value for the speed of light, became interested in the ether. Michelson and colleague Edward Morley (a chemist somewhat surprisingly) conducted an experiment that would measure the so-called ether drift (or flow or wind as it is sometimes called) that Maxwell had complained was immeasurable. The experiment was a complex and well thought out one. The diagram to the left will help to explain it.

A source of light from the point *A* would reach a half-silvered (so half-mirrored) pane of glass *B* at 45° to it. The glass would split the beam perpendicularly (as part would be reflected and the other half would pass through). One of the beams would travel towards a mirror *C* at length *L* from *B*. The other beam would travel the same length *L* but in a perpendicular direction towards another mirror *E*. Both beams would be reflected off of their respective mirrors (*C* and *E*) to arrive back at *B* and become superimposed as beams *D* and *F*. If they were in phase, which at present they would have no reason not to be, the waves would constructively add up. However, if one distance varied, the light would have had different distances to travel (in both the 'up' direction and the 'across' direction) so would be out of phase and waves would destructively add up. If the apparatus were moving through the ether (at speed *u* to the right on the diagram), the beams would arrive at *B* out of phase. The experiment could be turned to different angles, performed at different times of day or year to prove that there was this ether that the apparatus and Earth were flowing through. (5)

The result was startling. Again, we must remember scientific thinking at the time. Though it was a strange substance and apparently contradicted itself, the ether had to exist. The result of the Michelson–Morley experiment was null, there seemed to be no detectable sign of the ether at any angle, at any time of day or year. The two beams always arrived in phase. "The result of the hypothesis of a stationary ether is thus shown to be incorrect," said Michelson. (7) Therefore, the speed of light had to be constant no matter what your speed. Albert Einstein later realised that this meant that time and length had to dilate or contract to ensure a constant speed of light but before that this was unthinkable. Hendrik Lorentz had to incorporate a parameter he called 'local time' into his electron theories when they were incorporated with Maxwell's equations. Henri Poincaré ended a lecture in 1904 with the words: "Perhaps we must construct a new mechanics of which we can only catch a glimpse…in which the velocity of light becomes an impassable limit." (8)

Lorentz, like most, did not even conceive that time was not absolute, while Poincaré envisaged something that he could not describe. It took a young patent office clerk who had spent a decade thinking about mirrors travelling at light speed to finally decide that the ether did not exist and that time and space were not as absolute as common sense seemed to dictate.

Lifted from the same book as the diagram of the Michelson–Morley experiment above, this is a diagram which will help with a derivation of the gamma equation. Lorentz noticed that this gamma equation, when substituted into Maxwell's equations, allowed them to give the same results as Newton's would in different velocity reference frames. Einstein realised that they could be used to transform Newton's equations of motion too. This was the big step and the 'new mechanics' of which up until 1905, only few had even 'caught a glimpse'. It meant that time and length would dilate and contract, something unthinkable. This will be dealt with after the maths. (9)



The diagrams above are not as schematic as the Michelson–Morley one. The first diagram, labelled (a) shows a simple light clock. Its filament emits a beam of light towards a mirror at a distance D above it. This is received after a length of time by a photocell. The time taken for this beam of light to travel to the mirror and back will be taken as one tick of our clock. The time taken can easily be shown to be:

$$t_0 = \frac{2D}{c}$$

The second diagram shows the same clock moving at a speed v to the right, therefore its mirror above and later its filament/photocell below have shifted to the right. For this diagram I will need to calculate the distance travelled by the beam of light. We will start by saying that it is the product of the speed of light and the time taken for the journey (ct).

$$ct = 2\sqrt{\frac{(vt)^2}{4} + D^2}$$

$$ct = \frac{2\sqrt{(vt)^2 + 4D^2}}{\sqrt{4} = 2}$$

$$ct = \sqrt{(vt)^2 + 4D^2}$$

$$c^2t^2 = v^2t^2 + 4D^2$$

$$t^2\left(c^2 - v^2\right) = 4D^2$$

$$t^2 = \frac{4D^2}{c^2 - v^2}$$

$$t^2 = \frac{4D^2}{c^2\left(1 - \frac{v^2}{c^2}\right)}$$

$$t = \frac{2D}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

- Express the fractions as one. Right side comes from Pythagoras' equation of a right-angled triangle applied to diagram above.

- Take out common factor of 2.

- Square everything.

- Group common factor of t^2.

- Divide by $(c^2 – v^2)$.

- Take out factor of c^2 from denominator on right.

-Then square root the lot.

Remember the t_0 we gave one tick of the clock? Substitute it into this equation and we have:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is just one derivation of the denominator part of the equation which is given the special name of the gamma equation as it comes up so frequently in special relativity. It can be shown that the mass of an object when moving increases by a factor of γ,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

while the length of an object decreases by a factor of γ^{-1},

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}}\,.$$

It can clearly be seen that when the velocity you are travelling at (v) is comparatively small compared to the speed of light (c), the gamma 'correction' factor has a value of very nearly one. Therefore, the Lorentz transformations (as they are called) reduce down to old Newtonian mechanics at velocities that Newton was, and we are, used to.

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The section above deals only with the maths of special relativity. You would have to be very narrow-minded not to see that t_0 being equal to anything but t is pretty inconceivable and contradicts common sense as well as nineteenth-century thinking in philosophy and of course physics.

Einstein's genius was in applying Lorentz' transformations to Newton's mechanics. This would force time and length (and of course mass) to appear to vary at speed relative to an observer.

If the new mass, increased by a factor of γ is inserted into a formula for momentum, we get:

$$\mathbf{p} = m\mathbf{v} = \frac{m_0 \mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Salvador Dali expressing the uncertainty of time in his melting clock.

At relatively low velocities, this equation obviously reduces down to the momentum of a moving body being its rest mass multiplied by its velocity. At large speeds, as your velocity approaches that of light, your momentum apparently tends towards infinity. This implies that your mass does the same. (In fact this conclusion could have been drawn from one of the equations on the previous page.)

In Newton's day a body would continue picking up speed as it was accelerated by a constant force in the same direction. However, with Einstein's special relativity this cannot happen. The body instead picks up *momentum* rather than velocity, i.e., its mass appears to increase as well. This, though, makes it even harder for the same force to accelerate the body to a greater speed as it appears to have a greater mass. (Acceleration is inversely proportional to mass, which appears to be increasing.) This is why the speed of light can never be reached by an object that has mass. Its apparent mass will increase and tend towards infinity as its velocity approaches the speed of light. It will require an acceleration that tends towards infinity which will require an energy input that tends the same way. (10)

Einstein considered molecules in a gas tank. As the tank was heated, the molecules would gain kinetic energy and would pick up speed. Due to Einstein's transformations, the tank would also pick up mass. The mass increase can be shown by dividing the original mass by γ. The equation is as follows. It has been expanded using binomial theorem. (11)

$$\frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} = m_0\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \ldots\right)$$

The series converges rapidly when v is small. Assuming later terms to be negligible,

$$m \approx m_0 + \frac{1}{2} m_0 \frac{v^2}{c^2}$$

The final term, ignoring the c^2 is the Newtonian kinetic energy of the molecule so we can say that the increase in mass of the gas tank is equal to the increase in kinetic energy divided by c^2. Einstein then went on to suggest that a body's mass can be expressed more simply than by the rest mass multiplied by γ equation above. He suggested multiplying the above equation by c^2. The result is

$$mc^2 = m_0 c^2 + \frac{1}{2} m_0 v^2 + \ldots$$

Einstein interpreted the m_0c^2 term above to be part of the total energy of the body. He knew it as an intrinsic energy, a 'rest energy'. And so he came up with the equation that defined physics in the early part of last century. When a body is not moving (relative to you measuring its mass), the total intrinsic energy contained within it is given by

$$E = m_0 c^2$$

This theory of the equivalence of mass and energy, and it seems to be based upon a large amount of assumption on Einstein's part, has been proven in atomic bombs where a gram of mass would release the predicted amount of energy and in electron–positron collisions which produced gamma rays with the predicted energy. This equation unites two completely different apparently fundamental ideas. All energy is capable of carrying a small mass while more clearly, all mass is capable of carrying energy. We see some energy released when we burn peanuts in biology, but not the intrinsic energy Einstein is talking about that we only see in large particle colliders and atomic bombs where the nucleus is broken up and mass is 'destroyed' (converted to energy governed by the above equation).

The most famous hypothetical–'real life' implication of the idea of time dilation is known somewhat strangely as the 'twin paradox'. The idea is simple, as is the effect once the equations are understood. Two twins, Speedy and Dopey, live on Earth for a number of years. One day Speedy decides to orbit the Earth in a spaceship. His spaceship is capable of travelling at $0.8c$—80% of the speed of light, so the quotient of Speedy's velocity squared against the speed of light squared is equal to 0.8-squared. He leaves Dopey on Earth for thirty years. A clock on Speedy's spaceship shows that time is running smoothly and everything takes just as long as it always has. The same is true of Dopey's clock but that would be expected as things have not changed for him.

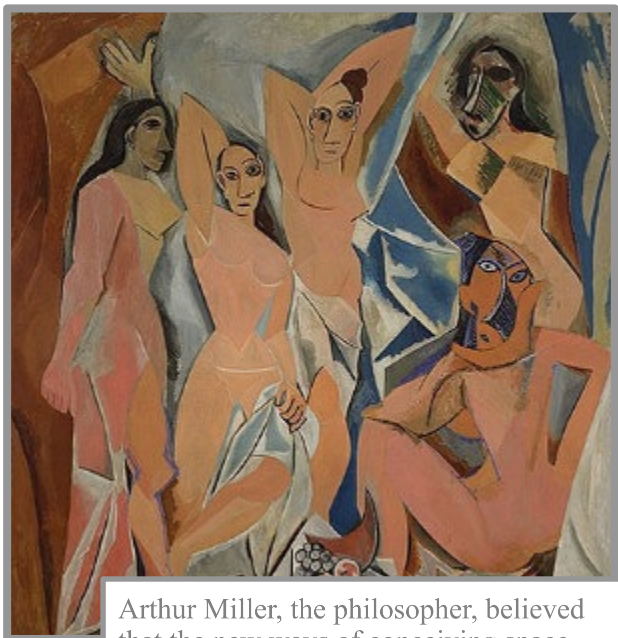
Yet, for Speedy, time has dilated by a factor of γ. When Speedy returns to Earth to meet his brother, his brother is…

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{30}{\sqrt{1 - 0.8^2}} = 50$$

…years older rather than just thirty years older. Speedy has gained twenty years on Dopey. This is not a paradox at all. It is strange and inconceivable but not a paradox. We have not invented fuels that allow spaceships to travel at eighty per cent of the speed of light. We have, however, invented aeroplanes that travel at a very small, but known, fraction of the speed of light. (12)

During October 1971, two atomic clocks were flown around the world, one eastward, the other westward. They were obviously synchronised and on return were compared with clocks at the U.S. Naval Observatory. Physicists worked out the value of the time dilation that the clocks should have shown. Both showed (to quite a high degree of accuracy) their expected values! Passengers on the planes had gained a couple of hundred nanoseconds on friends 'stationary' (obviously not stationary in the proper sense of the word but, as the theory has taught us, as stationary as anything can ever be described as being) on Earth. (13)

Muons, μ-mesons, disintegrate after a lifetime of about two millionths of a second. They shower the Earth within cosmic rays. They are created about ten kilometres above the Earth's surface and even if they travelled at the speed of light, they should not arrive at the Earth's surface as they should have disintegrated a few hundred metres *below* the point where they were created. However, they do reach the surface of the Earth. They seem to live a lot longer than two millionths of a second. The reason is that from their point of view, they do only live for two millionths of a second as time has 'dilated' for them. Using the time dilation formula it is easy to predict how long a muon should last when travelling at a certain speed. And it does last that long, just as the atomic clocks varied by a predicted factor. (14)



Arthur Miller, the philosopher, believed that the new ways of conceiving space and time, stemming from Einstein's theories, mirrored the cubist revolution in art. He cites Pablo Picasso's 'Les Demoiselles d'Avignon' as an example.

These facts both show that time dilation is real. Many people seem to think that there is no proof for Einstein's theories, that they are just theories and that time and space don't really dilate and contract. They do. It has been proven so.

These ideas led to huge changes in thinking in the early part of the century. Art apparently mirrored the ideas in cubism while what Richard Feynman calls 'cocktail party philosophers' would ponce about saying "everything is relative" while not really understanding Einstein's theories. (15)

In his 'Physics and Philosophy', Werner Heisenberg says, "The decisive change [after the theory's emergence] was in the structure of space and time. It is very difficult to describe this change in the words of common language, without the use of mathematics, since the common words 'space' and 'time' refer to a structure … that is actually an idealisation and oversimplification of the real structure." (16) Maybe this is the reason that most people do not understand the ideas fully but also are unable to believe them when told that they are true.

The trouble with this sort of fundamental physics is that we can never know how right our ideas are. After any great discovery or idea we tend to think that that's it. We know everything there is to know. But this has never been true and I don't think ever can be. We seem to forget, even after all of this on the constancy of the speed of light, that we have no idea what light is. It's not a particle and not a wave. Both the particle and wave ideas are just models that work in certain cases. How do we know that relativity is not just a (much more complex) model that works only in certain cases?

We say that mass appears to increase but we don't know what mass is. What causes an object to resist acceleration? What causes an object to attract other massive bodies? These gravity waves and gravitons are just ideas that are as 'real' as corpuscles of light. What is time? It's not an absolute, constant flow because Einstein has proved that not to be true. God knows. But then "God does not play dice," according to Einstein, yet quantum physics showed that He does play dice. Even Einstein got this and other things wrong.

"The more important fundamental laws and facts of physical reality have all been discovered and they are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote …. Our future discoveries must be looked for in the sixth place of decimals." (17) What rubbish from Michelson in 1899. In 1899 Max Planck and others were just beginning to think about quantum mechanics. Einstein was thinking about the speed of light. And then, just as now, we really do not understand the "fundamental laws and facts of physical reality" *at all*.

The 1905 theory takes away the idea that time moves forward at a constant rate. For physicists it created a link between mass and energy. Unless you delved very deeply into physics at the beginning of the century, or thought seriously and mathematically about Henri Poincaré's words at the end of that lecture in 1904, you would have had no reason to believe in such an absurd idea. One patent office clerk did, however, and that is why he is remembered for being the maverick genius that changed thinking in a steadily ticking world.

Girish Gupta
19th–23rd March 2005

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Discussion of Sources

My main sources were a small number of websites and a few books which included chapters on the derivation of the relevant formulae, the implications and philosophy of the theory and the history of ideas about light. *A Case History of the Speed of Light* helped with the introduction to the report but only really led up to Roemer's observations of Io. For Maxwell's part I looked to *Einstein's Mirror*. It also explained Albert Michelson's part. For the explanation of the Michelson–Morley experiment, and the diagram of it, I went to *Six Not So Easy Pieces*. I worked out the time dilation formula using the Feynman book and some websites. The diagram came from the Feynman book, as did most of the maths that follows.

I did not include too much background on the scientists involved as this would have taken the report off the point. I included, generally, only their names.

These were my main sources and their uses. However, I did not only use one source for each section. There were many from which I took background and related information and many others unmentioned from which I gained background knowledge. Therefore, below is a list of most of the books and websites I used in order of their help.

- *Six Not So Easy Pieces* by **Richard Feynman** [A]
Penguin, 014027667X
- *Einstein's Mirror* by **Patrick Walters** and **Tony Hey** [B]
Cambridge University Press, 0521435323
- *A Case History of the Speed of Light* by **Moti Nissani** [C]
http://www.is.wayne.edu/mnissani/a&s/light.htm
- *The Special Theory of Relativity* by **David Harrison** [D]
http://www.upscale.utoronto.ca/GeneralInterest/Harrison/SpecRel/SpecRel.html
- *A Brief History of Time* by **Stephen Hawking** [E]
Bantam, 0553175211
- *Physics and Philosophy* by **Werner Heisenberg** [F]
Penguin, 0141182156

Below are page numbers, specific websites etc. related to paragraphs, images and diagrams in the text. Letters in square brackets relate to the book or website above.

(1) This introduction is the idea behind the title of book [B].
(2) A lot of this history is mentioned on website [C].
(3) This idea is described very well on page 22 of book [E]. Also, Newton's equations working in different reference frames is illustrated and calculated in the 'Symmetry in Physical Laws' chapter of book [A].
(4) This complaint of Maxwell's is seen on pages 38 and 39 of book [B].
(5) The experiment itself is described on pages 55 to 57 of book [A].
(6) Diagram lifted from page 55 of book [A].
(7) Quote from page 40 of book [B].
(8) Good quote from page 48 of book [B].
(9) The diagram is lifted from book [A]. The book does contain an explanation of the derivation but not in very much detail. A couple of websites and pen and paper helped with the derivation.
(10) Page 67 of book [A] describes this reasonably well.
(11) This and the maths involved up until $E=mc^2$ comes from pages 67 and 68 of book [A]. It is followed in the book by going back, with $E=mc^2$, to derive the Lorentz transformation for mass using $E=Fv$.
(12) There is no one source for this twin paradox. The idea is in every book or website related to the subject.
(13) Experiment mentioned on page 18 of book [B], conducted by Hafele and Keating.
(14) Explained on page 62 of book [A].
(15) Feynman begins talking about this in the chapter entitled 'Relativistic Energy and Momentum' in book [A]
(16) Book [F], obviously, page 70.
(17) Quote from website [D].