$X_{12} = tr_{N\setminus L}\left(|\vec{n}_1\rangle\langle\vec{n}_2| \right)$ $\underline{\vec{n}_1 + \vec{n}_2}$

$$\gamma(\vec{x},\vec{y}) = \gamma_1^{x_1}\gamma_{L+1}^{y_1}\gamma_2^{x_2}\gamma_{L+2}^{y_2}\cdots\gamma_L^{x_L}\gamma_{2L}^{y_L}$$

$$\gamma(\vec{x},\vec{y})\gamma(\vec{x}',\vec{y}') = e^{i\phi}\gamma(\vec{x}+\vec{x}',\vec{y}+\vec{y}')$$

$$X_{12} = {\sum_{\vec{x},\vec{y}}}' f(\vec{x},\vec{y})\,\gamma(\vec{x},\vec{y})$$

$$tr_L[X_{12}\gamma^\dagger(\vec{x},\vec{y})] = {\sum_{\vec{x}',\vec{y}'}}' f(\vec{x}',\vec{y}')\, tr_L[\gamma(\vec{x}',\vec{y}')\gamma^\dagger(\vec{x},\vec{y})]$$

$$\Rightarrow f(\vec{x},\vec{y}) = \frac{1}{2^L} tr_L[X_{12}\gamma^\dagger(\vec{x},\vec{y})]$$

$$= \frac{1}{2^L}\langle\vec{n}_2|\gamma^\dagger(\vec{x},\vec{y})|\vec{n}_1\rangle$$

$$\gamma_{i_1}\gamma_{i_2}\gamma_{i_3}\cdots\gamma_{i_K} = O_{i_1\alpha_1}O_{i_2\alpha_2}\cdots O_{i_K\alpha_K}\gamma_{\alpha_1}\gamma_{\alpha_2}\cdots\gamma_{\alpha_K}$$

$$\langle\vec{n}_2|\gamma^\dagger(\vec{x}',\vec{y}')|\vec{n}_1\rangle = e^{i\phi(\ldots)}\delta_{\vec{n}_1+\vec{x}+\vec{y},\vec{n}_2}$$

$$|\vec{n}_1\rangle = \gamma(\vec{n}_1,0)|0\rangle$$
$$|\vec{n}_1\rangle = \gamma(0,\vec{n}_1)|0\rangle \times e^{i\phi(\vec{n}_1)}$$

$$\gamma(\vec{x},\vec{y})|\vec{n}_1\rangle = |\vec{n}_1+\vec{x}+\vec{y}\rangle \times \text{fase}$$

$$\delta_{\vec{n}_1+\vec{n}_2,\vec{x}+\vec{y}}$$

$\vec{n}_1 =$ 00100110 0
$\vec{n}_2 =$ 10111001 1

1

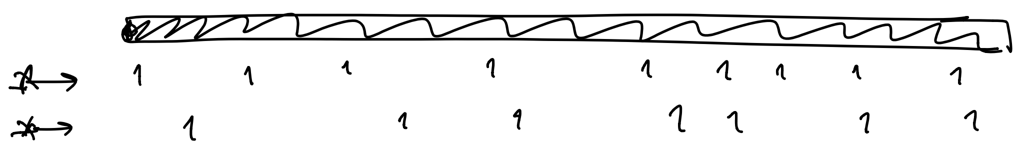
$x_i = u_{i\alpha} x_\alpha$
$y_i = u_{i\alpha} y_\alpha$

DESORDENADO

$$f(\vec{x},\vec{y}) = \frac{1}{2^L}\sum^{*}_{\vec{x}',\vec{y}'} U_{\vec{x},\vec{x}'} V_{\vec{y},\vec{y}'} \underbrace{\langle \vec{n}_2 | [\gamma^{\dagger}(\vec{x}', \vec{y}')] | \vec{n}_1\rangle}_{\propto\, \delta(\vec{n}_1+\vec{n}_2,\, \vec{x}'+\vec{y}')}$$

$$X_{12} = \cancel{\lim_{\beta\to\infty}} \frac{\rho_1\rho_2}{\sqrt{tr(\rho_1\rho_2)}}$$

L'Hôpital



$$\mathbb{P}(d - \langle d\rangle \geq \epsilon) \leq e^{-NS(\epsilon)}$$ ~~Chernoff~~ ?

$$\frac{d}{N} \approx \frac{\langle d\rangle}{N} \pm O(1/\sqrt{N})$$

$d < L$ $\qquad$ $L < \langle d\rangle$

$$tr(X_{12}X_{12}^{\dagger}) = \frac{1}{2^L}\sum'_{\vec{x},\vec{y}} |f(\vec{x},\vec{y})|^2$$

$$= \frac{1}{2^L}\sum'_{\substack{\vec{x},\vec{y}\\ \vec{x}'',\vec{y}''}} (U_{\vec{x}\vec{x}'} U_{\vec{x}\vec{x}''})(V_{\vec{y}\vec{y}'} V_{\vec{y}\vec{y}''})\, e^{\phi(\vec{x}',\vec{y}')\phi(\ldots}$$
$$\delta(\vec{n}_1+\vec{n}_2, \vec{x}'+\vec{y}')\, \delta(\vec{n}_1+\vec{n}_2, \vec{x}''+\vec{y}'')$$

$$\downarrow$$

$$\leq \frac{1}{2^L}$$

$$\sum' U_{\vec{x}\vec{x}'} U_{\vec{x}\vec{x}''} = \sum'_{i_1 \cdots i_K} U_{i_1 j_1} U_{i_2 j_2} \cdots U_{i_K j_K} U_{i_1 k_1} \cdots U_{i_K k_K}$$

$$\sum'_{i_1} (U^T)_{j_1 i_1} U_{i_1 k_1} = (U^T \Pi_L U)_{j_1 k_1}$$
$$\leq L/N$$

$$\mathrm{tr}(X_{12} X_{12}^\dagger) \leq \frac{1}{2^L} \left(\frac{L}{N}\right)^K 2^K$$
$$\leq \frac{1}{2^L} \left(\frac{2L}{N}\right)^K$$

$$\mathbb{P}(x > \alpha) \leq \frac{\langle x \rangle}{\alpha}$$

$$\mathbb{P}(e^{\lambda x} > e^{\lambda \alpha}) \leq \langle e^{\lambda x} \rangle e^{-\lambda \alpha} \qquad \underline{\lambda > 0}$$

$$\parallel$$

$$\mathbb{P}(x > \alpha) \leq \langle e^{\lambda x} \rangle e^{-\lambda \alpha}$$
$$\leq \min_\lambda \underbrace{\langle e^{\lambda x} \rangle} e^{-\lambda \alpha}$$
$$\leq \min_\lambda \underline{e^{\sum_{i=1}^N f(\lambda x_i) - \lambda \alpha}}$$

$$e^{N\left[\frac{1}{N}\sum f(\lambda x_i) - \lambda \frac{\alpha}{N}\right]}$$

$$\underline{P}\left(X - \langle X \rangle > \alpha \right) \leq e^{-N\alpha^2}$$

$$p \qquad \text{prob} = p^2 + (1-p)^2 = 1 - 2p + 2p^2$$

$$p'$$

$$\text{Prob}(K \text{ aciertos}) = \sum_{(A,\bar{A})} \underbrace{\underbrace{p'_{i_1} p'_{i_2} \cdots p'_{i_A}}_{\text{Aciertos}}}_{K} \underbrace{\underbrace{(1-p'_i)(1-p)}_{N-K}}_{} \cdots (1-p)$$

$$\geq \binom{N}{K} p_{min}^K (1-p_{max})^{N-K}$$

$$\leq \binom{N}{K} p_{max}^K (1-p_{min})^{N-K}$$

$$\langle K \rangle \approx N \tilde{p} \pm O(\sqrt{N})$$

$$\text{Base} = \left\{ \gamma_{i_1} \gamma_{i_2} \gamma_{i_3} \cdots \gamma_{i_K} \mid i_1 < i_2 < \cdots i_K ,\ K \leq 2L \right\}$$