BCNF Decomposition Example

Exercise 3.3.1, Part a

$R(A, B, C, D)$ with $AB \to C, C \to D, D \to A$

Check for BCNF violations:
$\{A, B\}^+ = \{A, B, C, D\}$ OK
$\{C\}^+ = \{C, D, A\}$ violation
$\{D\}^+ = \{D, A\}$ violation

Decompose R using $C \to D$:
$R_1(A, C, D)$: need to project dependencies
$R_2(B, C)$: 2-element relations are in BCNF

For subsets of R_1 attributes:
$\{A\}^+ = \{A\}$
$\{C\}^+ = \{C, D, A\}$ $C \to D$ and $C \to A$ (follows from other two)
$\{D\}^+ = \{D, A\}$ $D \to A$
$\{A, C\}^+ = \{A, C, D\}$
$\{A, D\}^+ = \{A, D\}$
$\{C, D\}^+ = \{C, D, A\}$
$\{A, C, D\}^+ = \{A, C, D\}$

Dependencies that project to R_1:
$\{C\}^+ = \{C, D, A\}$ OK
$\{D\}^+ = \{D, A\}$ violation

Decompose R_1 using $D \to A$:
$R_3(A, D)$: 2-element relations are in BCNF
$R_4(C, D)$: 2-element relations are in BCNF

Final answer: $R_2(B, C)$, $R_3(A, D)$, $R_4(C, D)$