Ch. 4 The Kinematics of Rigid Body Motion 10/5/21

☆ What is kinematics? Study of motion without regard to forces (what ~~produces~~ does not care the motion)

☆ What is a rigid body? Represent a physical object, it contains N particles of mass m_n. Without constrains, it would have 3N degrees of freedom. Nevertheless, holonomic constraints dictate, e.g., the distance between particles and this greatly reduces the # degrees of freedom

Are human bodies rigid bodies? why yes, why not? provide examples

☆ How can you describe the motion of rigid bodies? What is # of degrees of freedom? The generalized coordinates?

I would say 6 degrees of freedom: 3 to hold the object in place ~~and three~~ or move, and 3 more to rotate it. Maybe?

The book mentions that to fix the points in a rigid body, you don't ~~~~ need to specify all the points, you just need to specify a plane ~~~~ by using 3 points. What are the degree of freedom for each ~~~~ point?

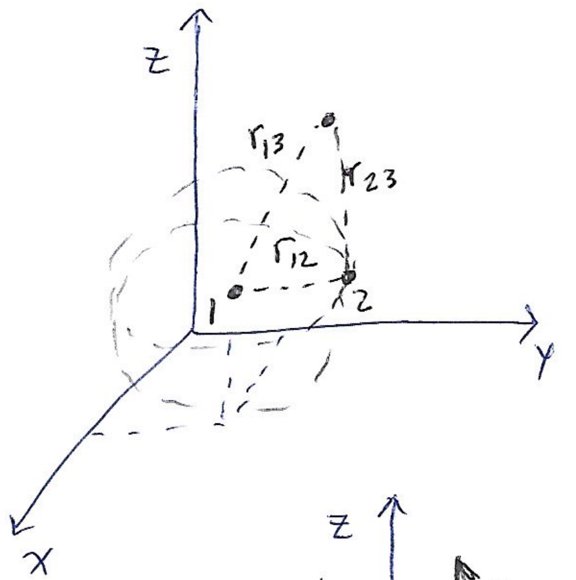
3 for the first one, nothing to do about it

2 for the second point, need to specify position of first point and relavite distance

1 for the third point. It has to maintain its distance to the other, but can still rotate

(i was wrong, correct number of 6 dof) (Actually, not wrong)

Let's draw them!



- think (x, y, z)

3 coordinates for point 1

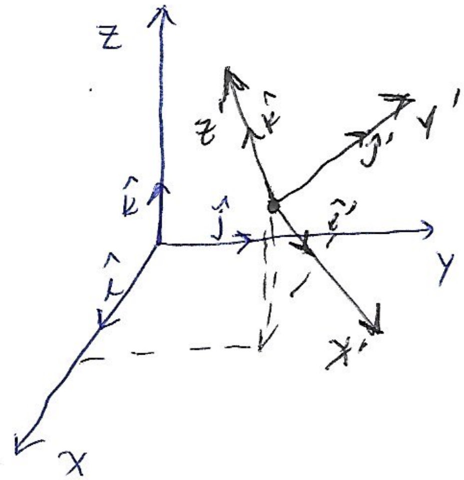
2 coordinates for point 2

- r_{12} is a constrain
- think θ and ϕ in spherical coordinates, makes a sphere about point 1

1 coordinate for point 1

- r_{13} and r_{23} constrained
- think r in spherical coordinates

we can still rotate, so gives the $\hat{r}$ direction.



Spherical coordinates still orthogonal, so it is like having another system relative to first one

From the definition of the dot product $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta_{AB}$ and magnitude of unit vectors is 1, so $\hat{i}' \cdot \hat{j} = \cos\theta_{ij}$

$$\cos\theta_{1'1} = \hat{i}' \cdot \hat{i} \qquad \cos\theta_{2'1} = \hat{j}' \cdot \hat{i} \qquad \cos\theta_{3'1} = \hat{k}' \cdot \hat{i}$$

$$\cos\theta_{1'2} = \hat{i}' \cdot \hat{j} \qquad \cos\theta_{2'2} = \hat{j}' \cdot \hat{j} \qquad \cos\theta_{3'2} = \hat{k}' \cdot \hat{j}$$

$$\cos\theta_{1'3} = \hat{i}' \cdot \hat{k} \qquad \cos\theta_{3'2} = \hat{j}' \cdot \hat{k} \qquad \cos\theta_{3'3} = \hat{k}' \cdot \hat{k}$$

$$\hat{i}' = \cos\theta_{1'1}\,\hat{i} + \cos\theta_{1'2}\,\hat{j} + \cos\theta_{1'3}\,\hat{k}$$

$$\hat{j}' = \cos\theta_{2'1}\,\hat{i} + \cos\theta_{2'2}\,\hat{j} + \cos\theta_{2'3}\,\hat{k}$$

$$\hat{k}' = \cos\theta_{3'1}\,\hat{i} + \cos\theta_{3'2}\,\hat{j} + \cos\theta_{3'3}\,\hat{k}$$

direction cosines

Consider a vector given by $\vec{r} = x'\hat{i}' + y'\hat{j}' + z'\hat{k}'$,

then
$$\vec{r} = x'\left(\cos\theta_{1'1}\,\hat{i} + \cos\theta_{1'2}\,\hat{j} + \cos\theta_{1'3}\,\hat{k}\right)$$
$$+\, y'\left(\cos\theta_{2'1}\,\hat{i} + \cos\theta_{2'2}\,\hat{j} + \cos\theta_{2'3}\,\hat{k}\right)$$
$$+\, z'\left(\cos\theta_{3'1}\,\hat{i} + \cos\theta_{3'2}\,\hat{j} + \cos\theta_{3'3}\,\hat{k}\right)$$

$$\Rightarrow \vec{r} = \left(x'\cos\theta_{1'1} + y'\cos\theta_{2'1} + z'\cos\theta_{3'1}\right)\hat{i}$$
$$+ \left(x'\cos\theta_{1'2} + y'\cos\theta_{2'2} + z'\cos\theta_{3'2}\right)\hat{j}$$
$$+ \left(x'\cos\theta_{1'3} + y'\cos\theta_{2'3} + z'\cos\theta_{3'3}\right)\hat{k}$$

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* The direction cosines could have also been expressed as

$$\hat{i} = \cos\theta_{1'1}\,\hat{i}' + \cos\theta_{2'1}\,\hat{j}' + \cos\theta_{3'1}\,\hat{k}'$$
$$\hat{j} = \cos\theta_{1'2}\,\hat{i}' + \cos\theta_{2'2}\,\hat{j}' + \cos\theta_{3'2}\,\hat{k}'$$
$$\hat{k} = \cos\theta_{1'3}\,\hat{i}' + \cos\theta_{2'3}\,\hat{j}' + \cos\theta_{3'3}\,\hat{k}'$$

$$x' = r\cdot\hat{i}' = x\cos\theta_{1'1} + y\cos\theta_{1'2} + z\cos\theta_{1'3}$$
$$y' = r\cdot\hat{j}' = x\cos\theta_{2'1} + y\cos\theta_{2'2} + z\cos\theta_{2'3}$$
$$z' = r\cdot\hat{k}' = x\cos\theta_{3'1} + y\cos\theta_{3'2} + z\cos\theta_{3'3}$$

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$$x = \vec{r}\cdot\hat{i} = x'\cos\theta_{1'1} + y'\cos\theta_{2'1} + z'\cos\theta_{3'1}$$

$$y = \vec{r}\cdot\hat{j} = x'\cos\theta_{1'2} + y'\cos\theta_{2'2} + z'\cos\theta_{3'2}$$

$$z = \vec{r}\cdot\hat{k} = x'\cos\theta_{1'3} + y'\cos\theta_{2'3} + z'\cos\theta_{3'3}$$

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~~So $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$~~

(#) Consider x.

$$x = (x\cos\theta_{1'1} + y\cos\theta_{1'2} + z\cos\theta_{1'3})\cos\theta_{1'1}$$
$$+ (x\cos\theta_{2'1} + y\cos\theta_{2'2} + z\cos\theta_{2'3})\cos\theta_{2'1}$$
$$+ (x\cos\theta_{3'1} + y\cos\theta_{3'2} + z\cos\theta_{3'3})\cos\theta_{3'1}$$

$$x = x(\cos^2\theta_{1'1} + \cos^2\theta_{2'1} + \cos^2\theta_{3'1})$$
$$+ y(\cos\theta_{1'2}\cos\theta_{1'1} + \cos\theta_{2'2}\cos\theta_{2'1} + \cos\theta_{3'2}\cos\theta_{3'1})$$
$$+ z(\cos\theta_{1'3}\cos\theta_{1'1} + \cos\theta_{2'3}\cos\theta_{2'1} + \cos\theta_{3'3}\cos\theta_{3'1})$$

Similarly for y and z.

(□) The Kronecker δ symbol δ_{lm} is defined by

$$\delta_{lm} = 1 \qquad l = m$$
$$= 0 \qquad l \neq m \text{ , so}$$

$$\sum_{l=1}^{3} \cos\theta_{lm'}\cos\theta_{lm} = \delta_{m'm}$$

$$y = \left(x \cos\theta_{1'1} + y \cos\theta_{1'2} + z\cos\theta_{1'3}\right)\cos\theta_{1'2}$$
$$+ \left(x\cos\theta_{2'1} + y\cos\theta_{2'2} + z\cos\theta_{2'3}\right)\cos\theta_{2'2}$$
$$+ \left(x\cos\theta_{3'1} + y\cos\theta_{3'2} + z\cos\theta_{3'3}\right)\cos\theta_{3'2}$$

$$y = x\left(\cos\theta_{1'1}\cos\theta_{12} + \cos\theta_{2'1} + \cos\theta_{2'2} + \cos\theta_{3'1}\cos\theta_{3'2}\right)$$
$$+ y\left(\cos^2\theta_{1'2} + \cos^2\theta_{2'2} + \cos^2\theta_{3'2}\right)$$
$$+ z\left(\cos\theta_{1'3}\cos\theta_{1'2} + \cos\theta_{2'3}\cos\theta_{2'2} + \cos\theta_{3'3}\cos\theta_{3'2}\right)$$

$z \ldots$

So we need

for x

$$\sum_{l=1}^{3} \cos^2\theta_{l1} = 1$$

for y

$$\sum_{l=1}^{3} \cos^2\theta_{l2} = 1$$

for z

$$\sum_{l=1}^{3} \cos^2\theta_{l3} = 1$$

$$\sum_{l=1}^{3} \cos\theta_{l2} \cos\theta_{l}$$

$$\Rightarrow \sum_{l=1}^{3} \cos\theta_{lm'}\cos\theta_{lm} = 0 \quad m \neq m'$$

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