PHYS 5321 Mechanics Fall 2021 8/24/21

☆ $\hbar$, G, k_B

- What is mechanics? • What is classical mechanics?
- Why do we care about classical mechanics? • Is the universe quantum or classical?
- Is classical mechanics "complete"? Has everything been discovered?
- What are the main concepts of classical mechanics?
- What are the formulations of classical mechanics?

There are "levels of theory." For example, Newtonian gravitation and Coulomb's law are <u>empirical</u> laws. They describe how masses behave in the presence of other masses and charges in the prescence of other charges, but their applicability is limited to masses and charges, respectively.

☆ How would a "higher level" theory look like? Higher level of abstraction? More "fundamental"?

A higher level theory would tell you, e.g., the properties that empirical laws must have, rules that they must follow, the <u>structure of the theory</u>. The more fundamental a theory is, the more general the principles it is based on must be.

☆ What is the most fundamental principle you can think of?

Conservation laws (and some definitions)

HG Eq. 1.3

Consider Newton's Second law $\vec{F} = m\vec{a} = \frac{d}{dt}\vec{p} \equiv \dot{\vec{p}}$

with $\vec{p} = m\vec{v}$, $\vec{a} = \frac{d}{dt}\vec{v}$, $\vec{v} = \frac{d}{dt}\vec{r}$. for a particle.

If $\vec{F} = 0 \Rightarrow \frac{d}{dt}\vec{p} = 0$

$\vec{p}$ is the linear momentum

$$d\vec{p} = 0$$

$$\int d\vec{p} = 0$$

Also known as Newton's first law

$$\vec{p} + C = 0$$

$$\vec{p} = \text{constant}$$

Conservation theorem for the linear momentum of a particle: if the total force is zero, linear momentum is conserved

Let's ~~k~~ now define the angular momentum as $\vec{L} = \vec{r} \times \vec{p}$ about point O

Eq 1.9

And the torque as $\vec{N} = \vec{r} \times \vec{F} = \vec{r} \times \frac{d}{dt}(m\vec{v})$

$d(u \cdot v) = u\,dv + v\,du$ ← product rule

$$\frac{d}{dt}\underbrace{(\vec{r} \times m\vec{v})}_{\vec{r} \times \vec{p} = \vec{L}} = \vec{r} \times \frac{d}{dt}(m\vec{v}) + m\vec{v} \times \frac{d}{dt}\vec{r}$$

$$\frac{d}{dt}\vec{L} = \vec{r} \times \frac{d}{dt}(m\vec{v}) + \cancel{m\vec{v} \times \vec{v}}^{\,0}$$

vanishes since angle is zero

$$\frac{d}{dt}\vec{L} = \dot{\vec{L}} = \vec{N}$$

Eq. 1.11

if $\vec{N} = 0 \Rightarrow \frac{d}{dt}\vec{L} = 0$

$$d\vec{L} = 0 \cdot dt = 0$$

$$\int d\vec{L} = 0$$

$$\vec{L} + c = 0$$

$$\vec{L} = \text{constant}$$

Conservation theorem for the angular momentum of a particle: if the total torque is zero, angular momentum is conserved

Consider, again, Newton's Second Law $\vec{F} = m\vec{a}$ and let's define the work done by an external force $\vec{F}$ on a particle going from point ① to point ② as

$$W = \int_1^2 \vec{F} \cdot d\vec{s} \quad \cancel{\Rightarrow dW = \vec{F} \cdot d\vec{s} \Rightarrow \frac{dW}{dt}}$$

For constant mass, $W = m \int_1^2 \frac{d\vec{v}}{dt} \cdot d\vec{s}$

since $\vec{v} = \frac{d\vec{s}}{dt} \Rightarrow d\vec{s} = \vec{v}\, dt$,

product rule

$$W = m \int_1^2 \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt \qquad \frac{d}{dt}(\vec{v} \cdot \vec{v}) = \frac{d}{dt}(v^2) = \vec{v}\frac{d\vec{v}}{dt} + \vec{v}\frac{d\vec{v}}{dt} = 2\vec{v}\frac{d\vec{v}}{dt}$$

$$\Rightarrow \frac{1}{2}\frac{d}{dt}(v^2) = \frac{d\vec{v}}{dt} \cdot \vec{v}$$

so $W = \frac{m}{2} \int_1^2 \frac{d}{dt}(v^2)\, dt$

$$W = \frac{m}{2} \int_1^2 d(v^2) = \frac{m}{2} v^2 \Big|_1^2 = \frac{m}{2}\left(v_2^2 - v_1^2\right) \qquad \text{Eq. 1.13}$$

The quantity $\frac{mv^2}{2}$ is called the kinetic energy, we will use T.

$$W = T_2 - T_1$$

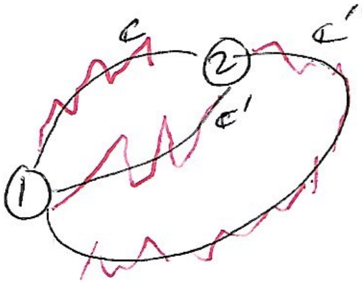
The work done is the change in kinetic energy

The value of a line integral

$$\int_{C} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_{C} (F_x\, dx + F_y\, dy + F_z\, dz) \quad *$$

depends implicitly on the time since $\vec{r}(t)$

over a path C from point ① to point ② in general depends not only on ① and ②, but also on the path C along which we integrate.



★ What are the conditions for path independence?

Define a line integral as <u>independent of path</u> in a domain D in space if for every pair of points ① and ② in D, integral $*$ has the same value for all paths in D that start at ① and end at ②.

A "well-known" theorem of vector analysis says that a line integral is independent of the path in D <u>if and only if</u>

$$\vec{F} = \nabla f, \quad \text{so} \quad F_x = \frac{\partial f}{\partial x}\ ; \ F_y = \frac{\partial f}{\partial y}\ ; \ F_z = \frac{\partial f}{\partial z}$$

I will prove the <u>if</u>, but not the <u>only if</u>.

Let C be any path in D from point ① to point ②

given by $\vec{r}(t) = \vec{x}(t)\,\hat{i} + \vec{y}(t)\,\hat{j} + \vec{z}(t)\ ; \quad a \le t \le b$

$$d\vec{r}(t) = d\vec{x}(t)\,\hat{i} + d\vec{y}(t)\,\hat{j} + d\vec{z}(t)\,\hat{k}$$

$$\int_{\mathbb{C}} \vec{F}(\vec{r})\cdot d\vec{r} = \int_{\mathbb{C}} F_x\,dx + F_y\,dy + F_z\,dz$$

we make time-explicit

$$= \int_a^b \left(F_x \frac{dx}{dt} + F_y \frac{dy}{dt} + F_z \frac{dz}{dt}\right) dt$$

Let $\vec{F}(\vec{r}) = \nabla f$, then

$$\int_{\mathbb{C}} \vec{F}(\vec{r})\cdot d\vec{r} = \int_{①}^{②} \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

$$= \int_a^b \left(\frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}\right) dt$$

$$= \int_a^b \left(\frac{\partial f_x}{dt} + \frac{\partial f_y}{dt} + \frac{\partial f_z}{dt}\right) dt = \int_a^b \frac{df}{dt}\,dt \quad V_1 - V_2$$

$$-V_2 - (-V_1)$$

$$\Rightarrow \int_a^b df = f[x(t), y(t), z(t)]\Big|_{t=a}^{t=b} = \cancel{f(2) - f(1)} \quad F_2 - F_1$$

Eq. 1.16

For convenience, let $f = -V(\vec{r})$, then $\vec{F} = -\nabla V(\vec{r})$.
~~F(r)~~ V is called the potential energy. A force field in which the work done by an external force on a particle is independent of the path taken is called a conservative force field.
Notice that this would not happen in general and friction is a particularly important case. But when it holds, then $T_2 - T_1 = V_1 - V_2$

$\Rightarrow T_1 + V_1 = T_2 + V_2$

Eq. 1.18

Conservation theorem for the energy of a particle: if the forces acting on a particle are conservative, then the total energy of the particle, $T+V$, is conserved.