Clerical

- Exam 1 on Feb 18th
- It will cover up to today's (Feb 11th) material and problem set 4 (due Feb 16th)
- You will design the exam today
- Meetings for student-led sessions have been created in a new channel in Teams called "Student-led study groups"
- Let me know if you want to lead your own group
- 5 points for leading a study group, 2 points for attending a group
- Blanca will also have reviews, 2 point for attending each Blanca review
- You can attend as many reviews/study groups as you want, but the maximum number of participation points you can get in the course is 9 (still, that's a letter grade)

Exam 1 design

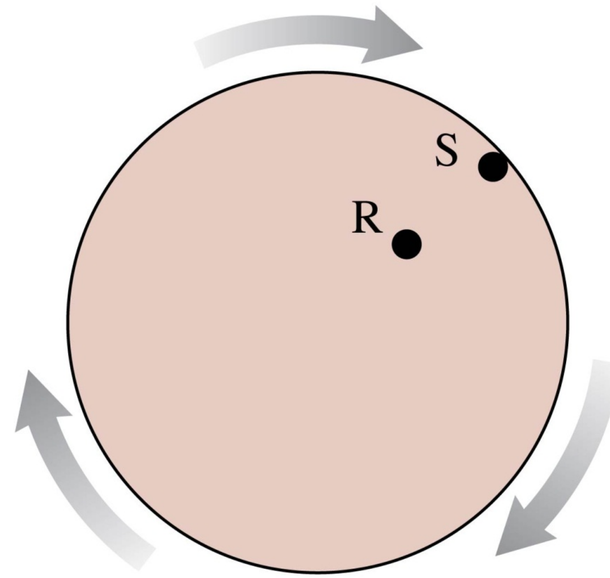
Constraints:

- 10 points total (so I suggest 3 problems and 1 plug-and-play, open to change)
- Some problems should be easy, almost plug-and-play, others should be more difficult (this way we see at which point each student is getting stuck, if at all)
- Problems/questions should be insightful (ideally) and probe the most important concepts (at the very least).
- The main purpose of the test is not to assess your learning, although it is used that way. Instead, it is to give you an opportunity to review the main concepts before we move on to the next stage.

- Let's do it.

$$\vec{a} = \left(\frac{v^2}{r}, \text{ toward center of circle} \right)$$

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's angular velocity is ______ that of Rasheed.



A. half

B. the same as

C. twice

D. four times

E. We can't say without knowing their radii.

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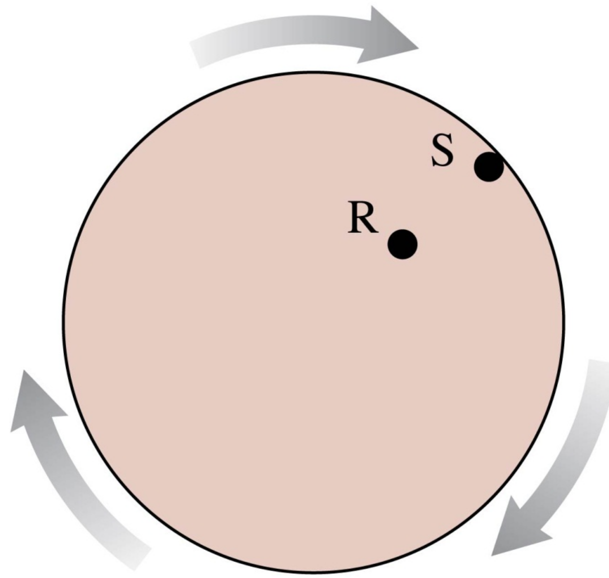

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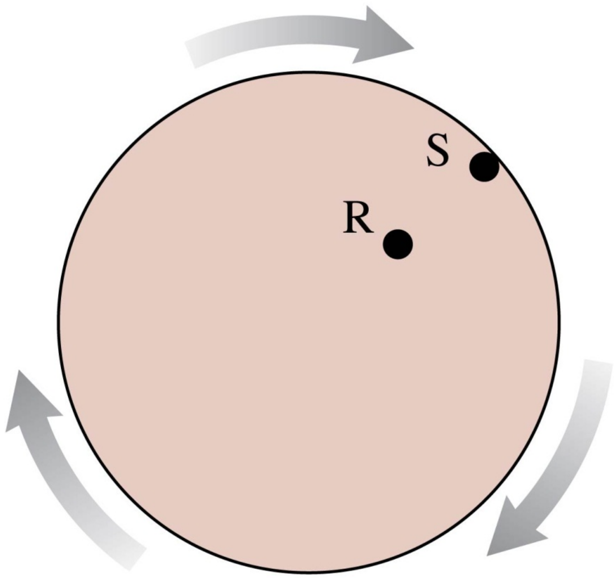
Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's speed is ______ that of Rasheed.



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$$v = \omega r$$

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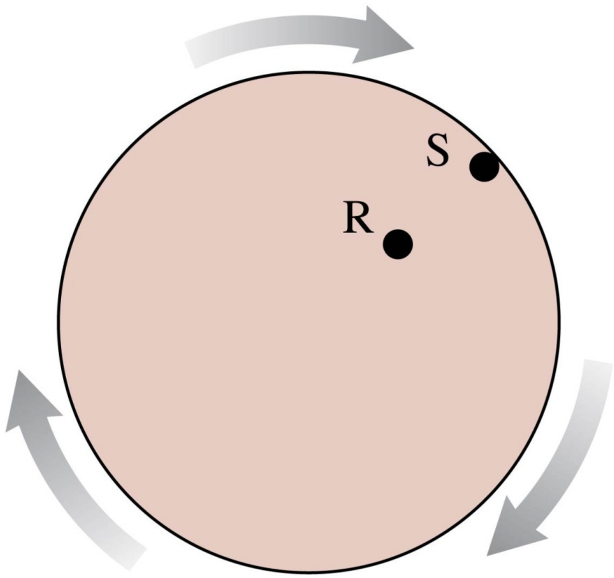

A. half

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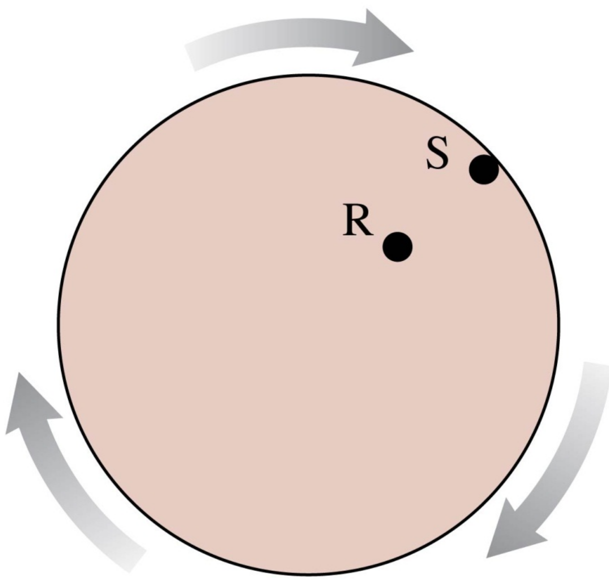
C. twice

D. four times

$v = \omega r$

E. We can't say without knowing their radii.

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's acceleration is ______ that of Rasheed.



A. half

B. the same as

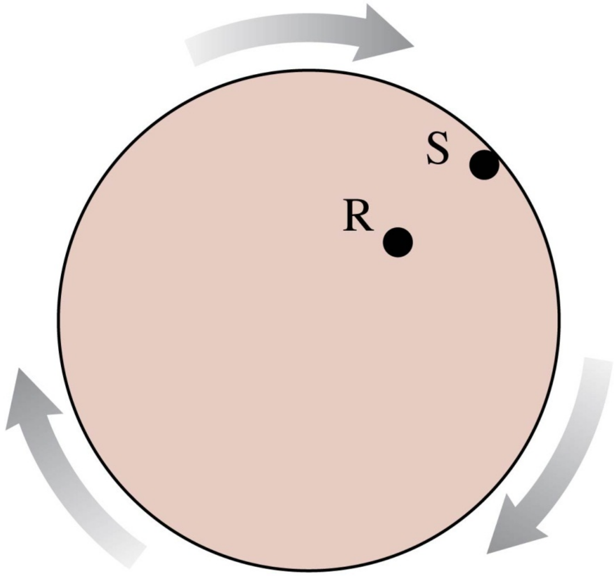
C. twice

D. four times

E. We can't say without knowing their radii.

Centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r$

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's acceleration is ______ that of Rasheed.



A. half

B. the same as

C. twice

D. four times

E. We can't say without knowing their radii.

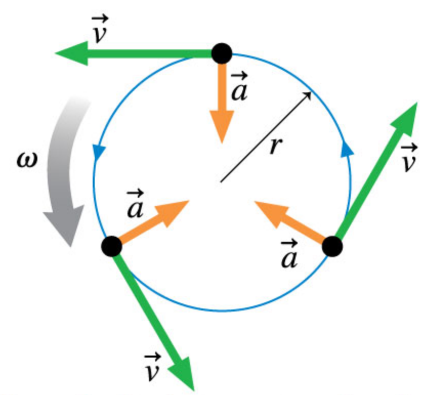
Centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r$

MODEL 4.2

Uniform circular motion

For motion with constant angular velocity ω.

- Applies to a particle moving along a circular trajectory at constant speed or to points on a solid object rotating at a steady rate.
- Mathematically:
  - The tangential velocity is $v_t = \omega r$.
  - The centripetal acceleration is v^2/r or $\omega^2 r$.
  - ω and v_t are positive for ccw rotation, negative for cw rotation.
- Limitations: Model fails if rotation isn't steady.



The velocity is tangent to the circle.
The acceleration points to the center.

Exercise 20

EXAMPLE 4.12 The acceleration of a Ferris wheel

A typical carnival Ferris wheel has a radius of 9.0 m and rotates 4.0 times per minute. What speed and acceleration do the riders experience?

MODEL Model the rider as a particle in uniform circular motion.

SOLVE The period is $T = \frac{1}{4}$ min $= 15$ s. From Equation 4.21, a rider's speed is

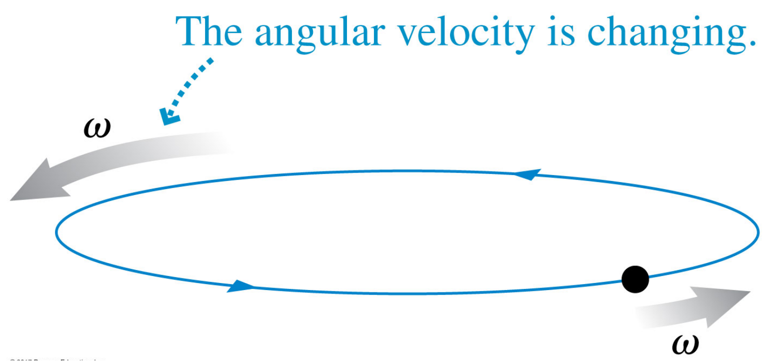
$$v = \frac{2\pi r}{T} = \frac{2\pi(9.0 \text{ m})}{15 \text{ s}} = 3.77 \text{ m/s}$$

Consequently, the centripetal acceleration has magnitude

$$a = \frac{v^2}{r} = \frac{(3.77 \text{ m/s})^2}{9.0 \text{ m}} = 1.6 \text{ m/s}^2$$

ASSESS This was not intended to be a profound problem, merely to illustrate how centripetal acceleration is computed. The acceleration is enough to be noticed and make the ride interesting, but not enough to be scary.

- The figure shows a point speeding up as it moves around a circle.
- This motion has *changing angular velocity*.

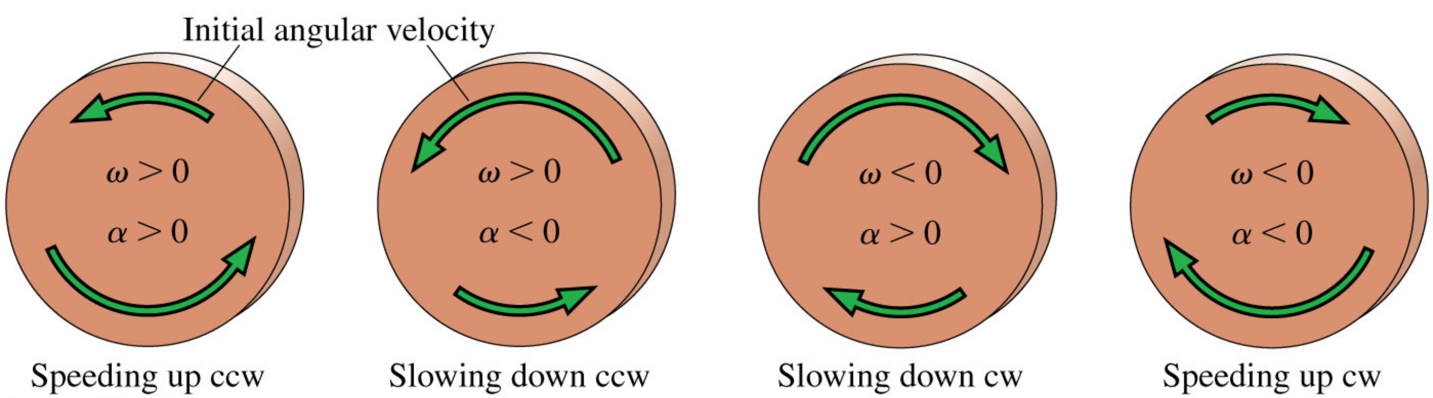


- We define the angular acceleration α (Greek alpha) of a rotating object, or a point on the object, to be

$$\alpha \equiv \frac{d\omega}{dt} \qquad \text{(angular acceleration)}$$

- The units of angular acceleration are rad/s^2.

- α is positive if $|\omega/$ is increasing and ω is counter-clockwise.
- α is positive if $|\omega/$ is decreasing and ω is clockwise.
- α is negative if $|\omega/$ is increasing and ω is clockwise.
- α is negative if $|\omega/$ is decreasing and ω is counter-clockwise.



The fan blade is slowing down.
What are the signs of ω and α?

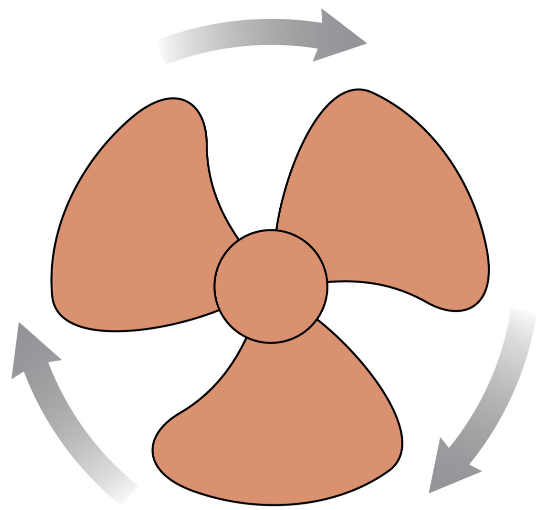
A. ω is positive and α is positive.

B. ω is positive and α is negative.

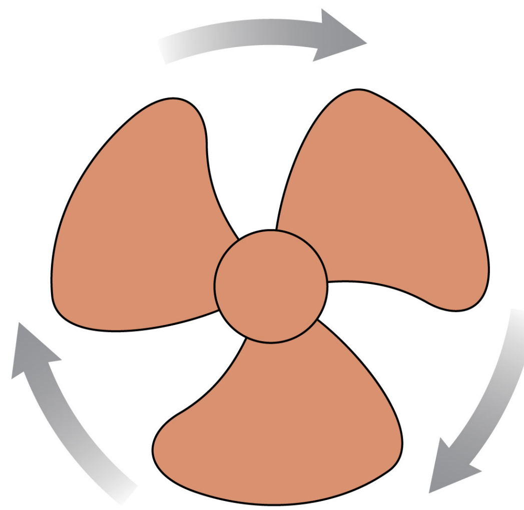
C. ω is negative and α is positive.

D. ω is negative and α is negative.

E. ω is positive and α is zero.



The fan blade is slowing down.
What are the signs of ω and α?



A. ω is positive and α is positive.

B. ω is positive and α is negative.

✔ C. ω is negative and α is positive.

D. ω is negative and α is negative.

E. ω is positive and α is zero.

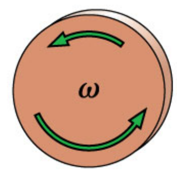
"Slowing down" means that ω and α have opposite signs, not that α is negative

MODEL 4.3

Constant angular acceleration

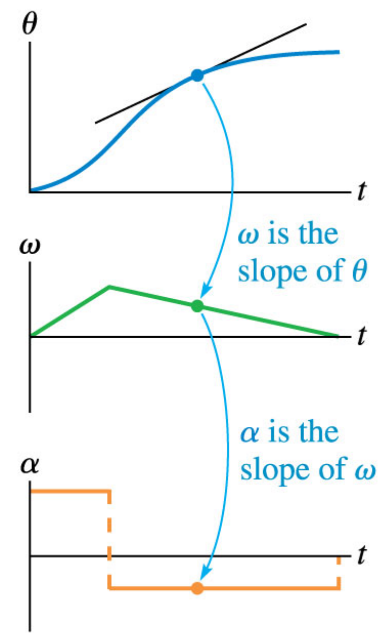
For motion with constant angular acceleration α.

- Applies to particles with circular trajectories and to rotating solid objects.



- Mathematically: The graphs and equations for this circular/rotational motion are analogous to linear motion with constant acceleration.
    - Analogs: $s \to \theta$ $v_s \to \omega$ $a_s \to \alpha$

Rotational kinematics	**Linear kinematics**
$\omega_f = \omega_i + \alpha\,\Delta t$	$v_{fs} = v_{is} + a_s\,\Delta t$
$\theta_f = \theta_i + \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$s_f = s_i + v_{is}\,\Delta t + \frac{1}{2}a_s(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$	$v_{fs}^2 = v_{is}^2 + 2a_s\,\Delta s$



Starting from rest, a wheel with constant angular acceleration turns through an angle of 25 rad in a time t. Through what angle will it have turned after time $2t$?

A. 25 rad

B. 50 rad

C. 75 rad

D. 100 rad

E. 200 rad

$$\Delta\theta \propto (\Delta t)^2$$

Starting from rest, a wheel with constant angular acceleration turns through an angle of 25 rad in a time t. Through what angle will it have turned after time $2t$?

A. 25 rad

B. 50 rad

C. 75 rad

D. 100 rad

E. 200 rad

$$\Delta\theta \propto (\Delta t)^2$$

Starting from rest, a wheel with constant angular acceleration spins up to 25 rpm in a time t. What will its angular velocity be after time $2t$?

A. 25 rpm

B. 50 rpm

C. 75 rpm

D. 100 rpm

E. 200 rpm

$\Delta\omega \propto \Delta t$

Starting from rest, a wheel with constant angular acceleration spins up to 25 rpm in a time t. What will its angular velocity be after time $2t$?

A. 25 rpm

✔ **B. 50 rpm** $\Delta\omega \propto \Delta t$

C. 75 rpm

D. 100 rpm

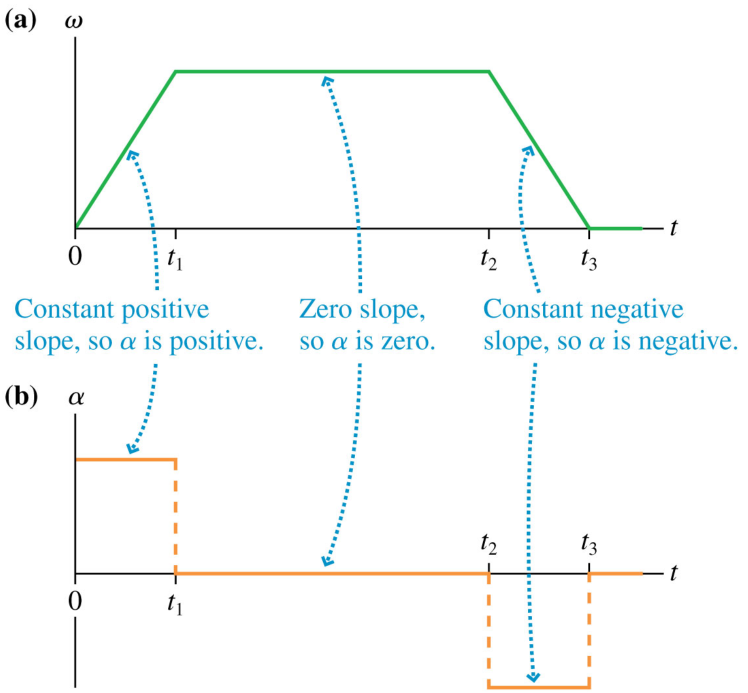
E. 200 rpm

EXAMPLE 4.13 | A rotating wheel

FIGURE 4.31a is a graph of angular velocity versus time for a rotating wheel. Describe the motion and draw a graph of angular acceleration versus time.

SOLVE This is a wheel that starts from rest, gradually speeds up *counterclockwise* until reaching top speed at t_1, maintains a constant angular velocity until t_2, then gradually slows down until stopping at t_3. The motion is always ccw because ω is always positive. The angular acceleration graph of **FIGURE 4.32b** is based on the fact that α is the slope of the ω-versus-t graph.

Conversely, the initial linear increase of ω can be seen as the increasing area under the α-versus-t graph as t increases from 0 to t_1. The angular velocity doesn't change from t_1 to t_2 when the area under the α-versus-t is zero.



EXAMPLE 4.14 | A slowing fan

A ceiling fan spinning at 60 rpm coasts to a stop 25 s after being turned off. How many revolutions does it make while stopping?

MODEL Model the fan as a rotating object with constant angular acceleration.

EXAMPLE 4.14 A slowing fan

SOLVE We don't know which direction the fan is rotating, but the fact that the rotation is slowing tells us that ω and α have opposite signs. We'll assume that ω is positive. We need to convert the initial angular velocity to SI units:

$$\omega_i = 60 \frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 6.28 \text{ rad/s}$$

We can use the first rotational kinematics equation in Model 4.3 to find the angular acceleration:

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 \text{ rad/s} - 6.28 \text{ rad/s}}{25 \text{ s}} = -0.25 \text{ rad/s}^2$$

Then, from the second rotational kinematic equation, the angular displacement during these 25 s is

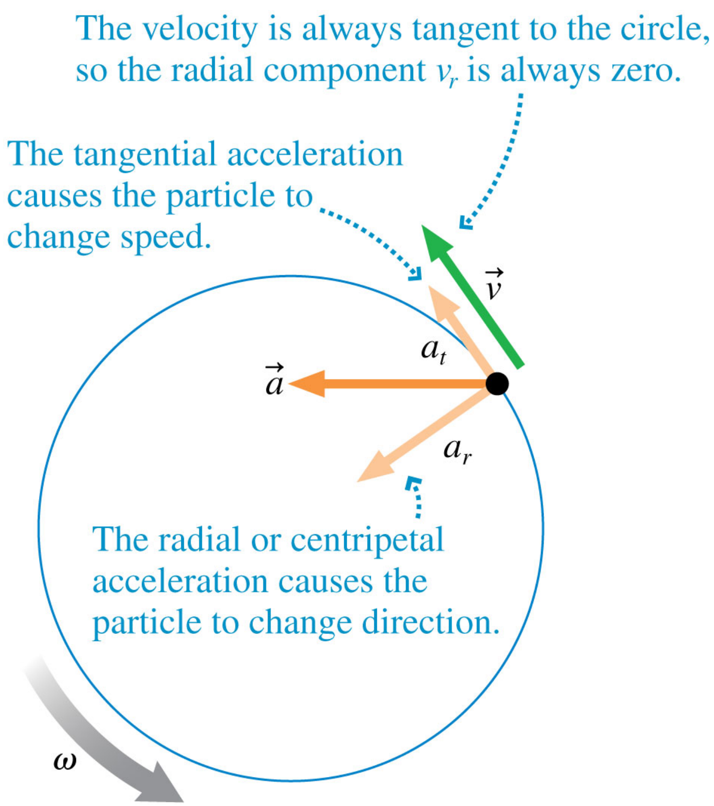
$$\begin{aligned}\Delta\theta &= \omega_i \Delta t + \tfrac{1}{2}\alpha(\Delta t)^2 \\ &= (6.28 \text{ rad/s})(25 \text{ s}) + \tfrac{1}{2}(-0.25 \text{ rad/s}^2)(25 \text{ s})^2 \\ &= 78.9 \text{ rad} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = 13 \text{ rev}\end{aligned}$$

The kinematic equation returns an angle in rad, but the question asks for revolutions, so the last step was a unit conversion.

ASSESS Turning through 13 rev in 25 s while stopping seems reasonable. Notice that the problem is solved just like the linear kinematics problems you learned to solve in Chapter 2.

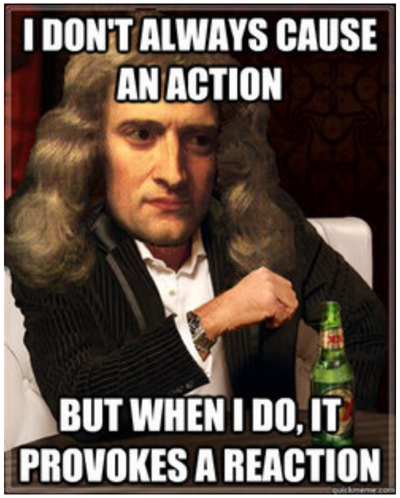
- The particle in the figure is moving along a circle and is speeding up.
- The centripetal acceleration is $a_r = v_t^2/r$, where v_t is the tangential speed.
- There is also a tangential acceleration a_t, which is always tangent to the circle.
- The magnitude of the total acceleration is

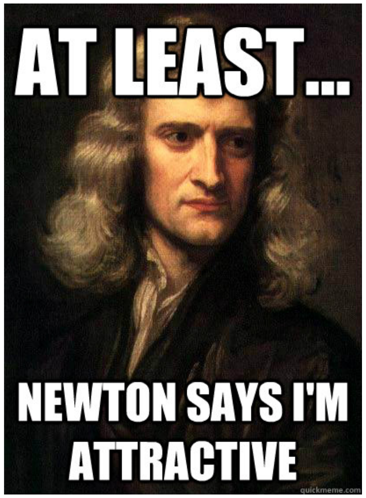
$$a = \sqrt{a_r^2 + a_t^2}$$

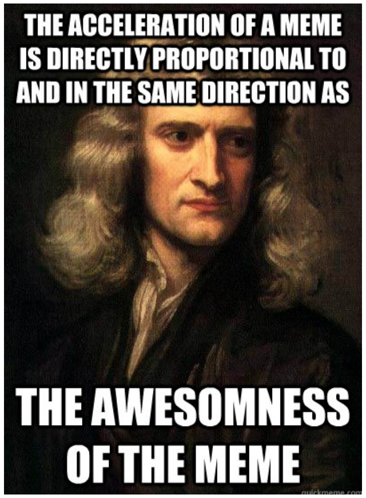


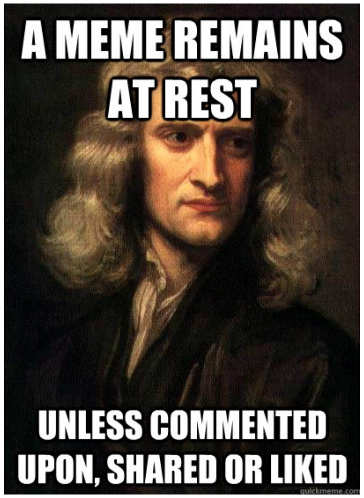
- Tangential acceleration is the rate at which the tangential velocity changes, $a_t = dv_t /dt$.
- We already know that the tangential velocity is related to the angular velocity by $v_t = \omega r$, so it follows that

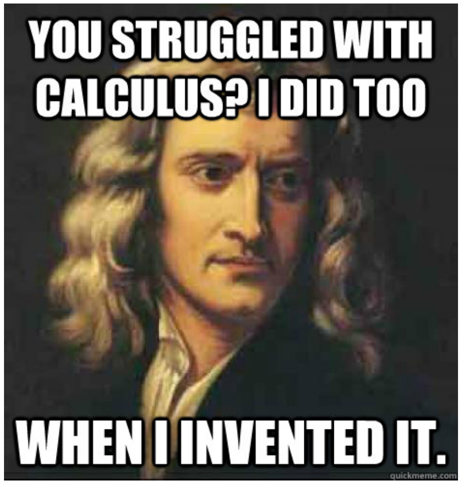
$$a_t = \frac{dv_t}{dt} = \frac{d(\omega r)}{dt} = \frac{d\omega}{dt} r = \alpha r$$

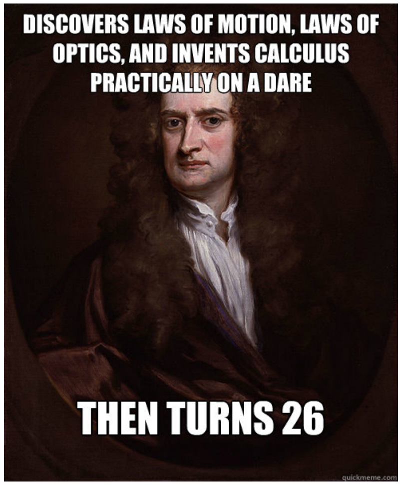


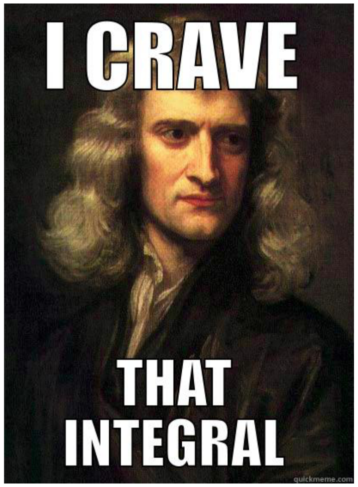


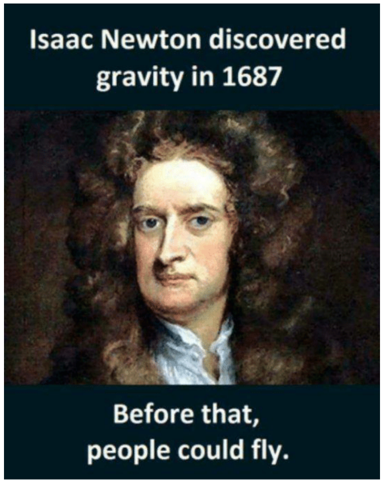




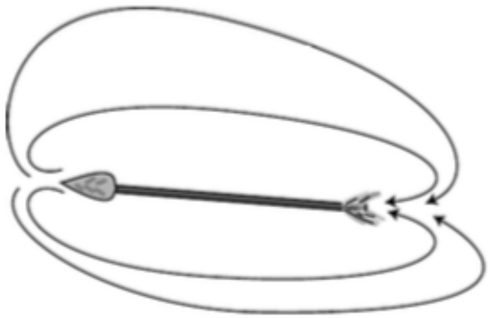




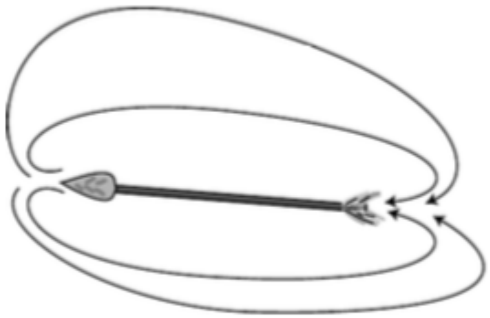




Research suggests there are more memes about his contributions to physics than to math



a / Aristotle said motion had to be caused by a force. To explain why an arrow kept flying after the bowstring was no longer pushing on it, he said the air rushed around behind the arrow and pushed it forward. We know this is wrong, because an arrow shot in a vacuum chamber does not instantly drop to the floor as it leaves the bow. Galileo and Newton realized that a force would only be needed to change the arrow's motion, not to make its motion continue.



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Newton discovered the relationship between force and motion, and revolutionized our view of the universe by showing that the same physical laws applied to all matter, whether living or nonliving, on or off of our planet's surface. His book on force and motion, the **Mathematical Principles of Natural Philosophy**, was uncontradicted by experiment for 200 years, but his other main work, **Optics**, was on the wrong track, asserting that light was composed of particles rather than waves. Newton was also an avid alchemist, a fact that modern scientists would like to forget.

Newton's first law

If the total force acting on an object is zero, its center of mass continues in the same state of motion.

Newton's second law

$$a = F_{total}/m,$$

where

m is an object's mass, a measure of its resistance to changes in its motion

F_{total} is the sum of the forces acting on it, and

a is the acceleration of the object's center of mass.

An elevator *example 1*

▷ An elevator has a weight of 5000 N. Compare the forces that the cable must exert to raise it at constant velocity, lower it at constant velocity, and just keep it hanging.

An elevator *example 1*

▷ An elevator has a weight of 5000 N. Compare the forces that the cable must exert to raise it at constant velocity, lower it at constant velocity, and just keep it hanging.

▷ In all three cases the cable must pull up with a force of exactly 5000 N. Most people think you'd need at least a little more than 5000 N to make it go up, and a little less than 5000 N to let it down, but that's incorrect. Extra force from the cable is only necessary for speeding the car up when it starts going up or slowing it down when it finishes going down. Decreased force is needed to speed the car up when it gets going down and to slow it down when it finishes going up. But when the elevator is cruising at constant velocity, Newton's first law says that you just need to cancel the force of the earth's gravity.

Terminal velocity for falling objects *example 2*

▷ An object like a feather that is not dense or streamlined does not fall with constant acceleration, because air resistance is nonnegligible. In fact, its acceleration tapers off to nearly zero within a fraction of a second, and the feather finishes dropping at constant speed (known as its terminal velocity). Why does this happen?

Terminal velocity for falling objects *example 2*

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▷ Newton's first law tells us that the total force on the feather must have been reduced to nearly zero after a short time. There are two forces acting on the feather: a downward gravitational force from the planet earth, and an upward frictional force from the air. As the feather speeds up, the air friction becomes stronger and stronger, and eventually it cancels out the earth's gravitational force, so the feather just continues with constant velocity without speeding up any more.

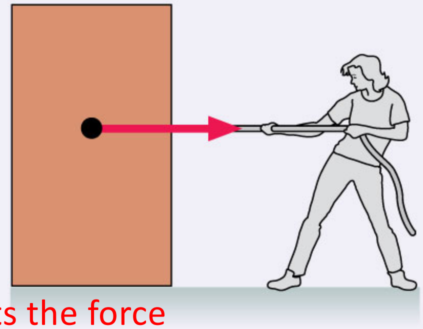
The situation for a skydiver is exactly analogous. It's just that the skydiver experiences perhaps a million times more gravitational force than the feather, and it is not until she is falling very fast that the force of air friction becomes as strong as the gravitational force. It takes her several seconds to reach terminal velocity, which is on the order of a hundred miles per hour.

What is a force?

The fundamental concept of **mechanics** is force.

Attraction or repulsion

- A force is a push or a pull.
- A force acts on an object.
- A force requires an agent.
- A force is a vector.



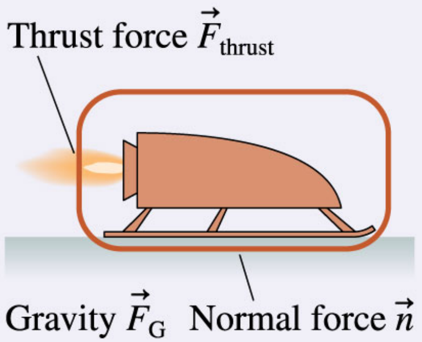
Something that exerts the force

Has magnitude and direction

How do we identify forces?

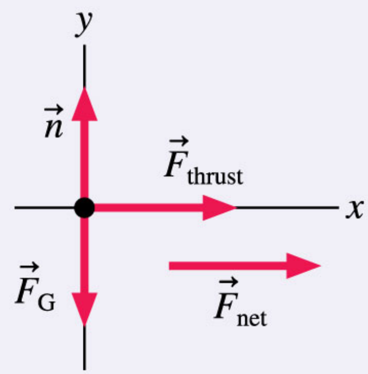
A force can be a contact force or a long-range force.

- Contact forces occur at points where the environment touches the object.
- Contact forces disappear the instant contact is lost. Forces have no memory.
- Long-range forces include gravity and magnetism.



How do we show forces?

Forces can be displayed on a free-body diagram. You'll draw all forces—both pushes and pulls—as vectors with their tails on the particle. A well-drawn free-body diagram is an essential step in solving problems, as you'll see in the next chapter.



What do forces do?

A net force causes an object to *accelerate* with an acceleration directly proportional to the size of the force. This is Newton's second law, the most important statement in mechanics. For a particle of mass m,

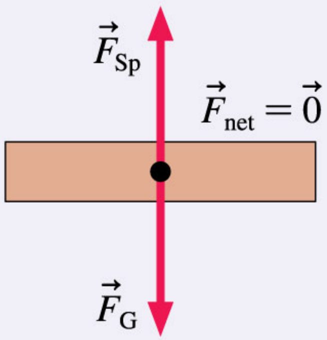
$$\vec{a} = \frac{1}{m}\vec{F}_{\text{net}}$$

« LOOKING BACK Sections 1.4, 2.4, and 3.2 Acceleration and vector addition

Before we looked at acceleration but not its cause.
Whenever there is acceleration there are forces involved.

What is Newton's first law?

Newton's first law—an object at rest stays at rest and an object in motion continues moving at constant speed in a straight line if and only if the net force on the object is zero—helps us define what a force *is*. It is also the basis for identifying the reference frames—called inertial reference frames—in which Newton's laws are valid.



What good are forces?

Kinematics describes *how* an object moves. For the more important tasks of knowing *why* an object moves and being able to predict its position and orientation at a future time, we have to know the forces acting on the object. Relating force to motion is the subject of **dynamics,** and it is one of the most important underpinnings of all science and engineering.

What is a “net force?”

A. The weight excluding the container.
B. The vector sum of all forces in a problem.
C. The vector sum of all forces acting on an object.
D. The vector force applied by a net.
E. The vector sum of all forces that add up to zero.

What is a “net force?”

A. The weight excluding the container.

Why not?

B. The vector sum of all forces in a problem.

C. **The vector sum of all forces acting on an object.**

D. The vector force applied by a net.

E. The vector sum of all forces that add up to zero.

Why not?

Which of the following are steps used to identify the forces acting on an object?

A. Draw a closed curve around the system.
B. Identify “the system” and “the environment.”
C. Draw a picture of the situation.
D. All of the above.
E. None of the above.

Which of the following are steps used to identify the forces acting on an object?

Difference between system and environment?

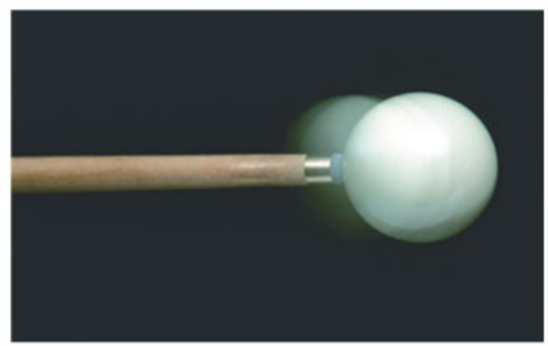
A. Draw a closed curve around the system.
B. Identify “the system” and “the environment.”
C. Draw a picture of the situation.
D. All of the above.
E. None of the above.

- A force is a *push* or a *pull*.

- A force acts on an object.
- Pushes and pulls are applied *to* something.
- From the object's perspective, it has a force *exerted* on it.

- A force requires an **agent**, something that acts or exerts power.
- If you throw a ball, your hand is the agent or cause of the force exerted on the ball.

- A force is a vector.
- To quantify a push or pull, we need to specify both magnitude and a direction.

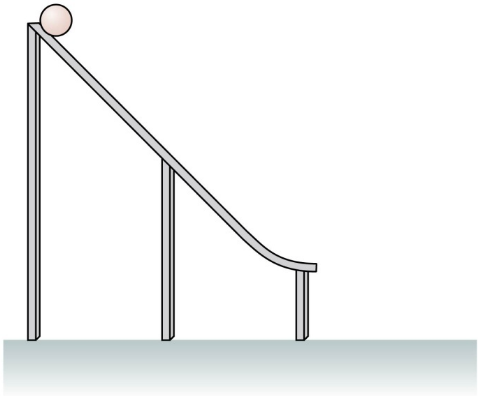


Although objects really never touch…

- **Contact forces** are forces that act on an object by touching it at a point of contact.
- The bat must touch the ball to hit it.

- **Long-range forces** are forces that act on an object without physical contact.
- A coffee cup released from your hand is pulled to the earth by the long-range force of gravity. [Force?]

A ball rolls down an incline and off a horizontal ramp. Ignoring air resistance, what force or forces act on the ball as it moves through the air just after leaving the horizontal ramp?



A. The weight of the ball acting vertically down.

B. A horizontal force that maintains the motion.

C. A force whose direction changes as the direction of motion changes.

D. The weight of the ball and a horizontal force.

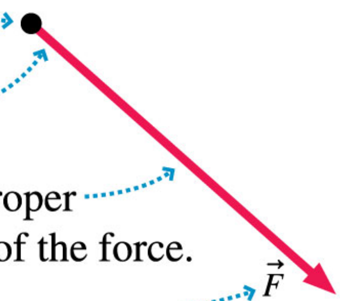
E. The weight of the ball and a force in the direction of motion.

Let’s discuss the options

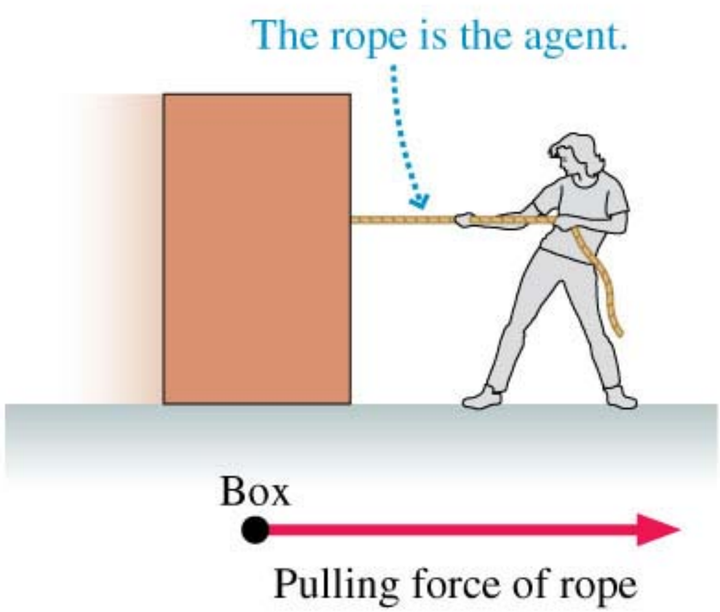
TACTICS BOX 5.1

Drawing force vectors

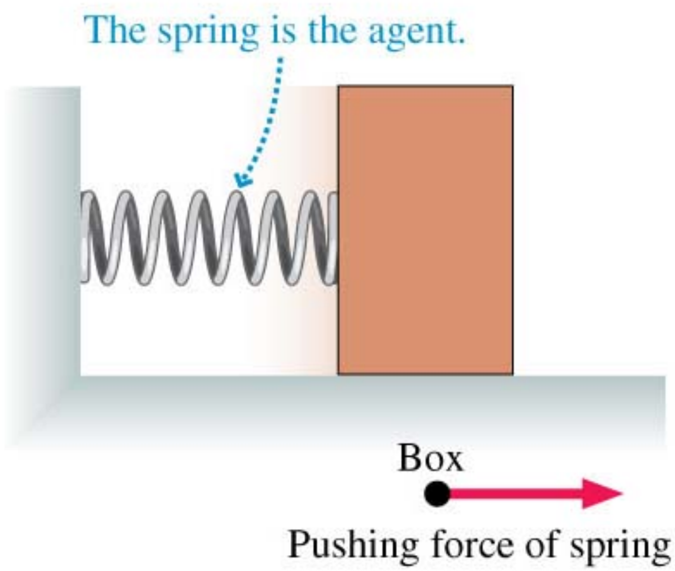
1. Model the object as a particle.
2. Place the *tail* of the force vector on the particle.
3. Draw the force vector as an arrow pointing in the proper direction and with a length proportional to the size of the force.
4. Give the vector an appropriate label.



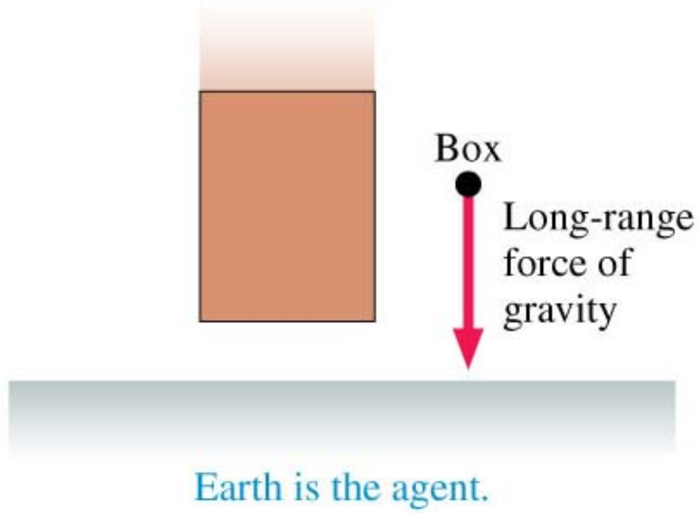
A box is pulled to the right by a rope.



A box is pushed to the right by a spring.



A box is pulled down by gravity.

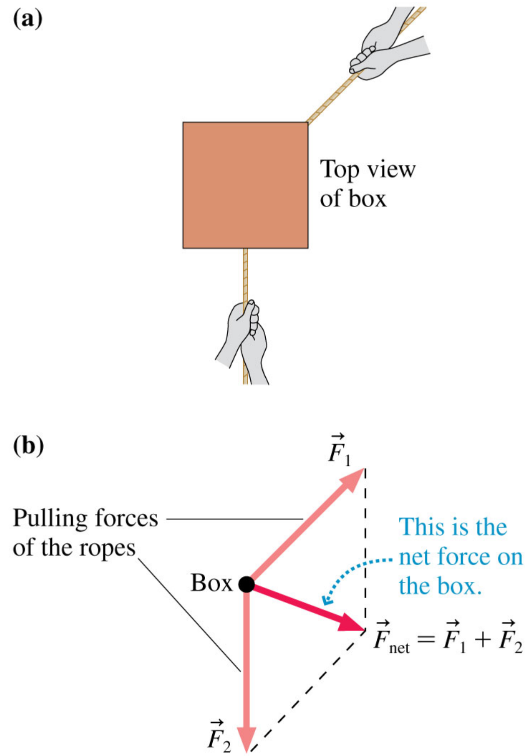


Inertial vs gravitational mass…

- A box is pulled by two ropes, as shown.
- When several forces are exerted on an object, they combine to form a **net force** given by the *vector* sum of *all* the forces:

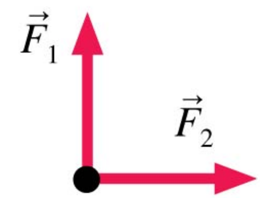
$$\vec{F}_{\text{net}} \equiv \sum_{i=1}^{N} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N$$

- This is called a **superposition of forces**.

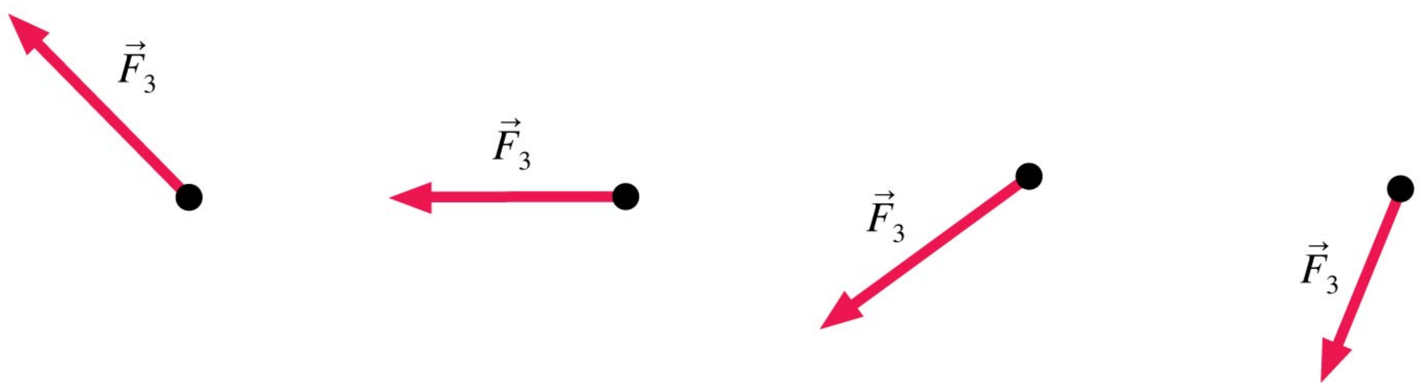


Methods that we used for velocity, etc. are the same

The net force on an object points to the left. Two of three forces are shown. Which is the missing third force?

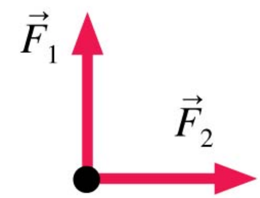


Two of the three forces exerted on an object

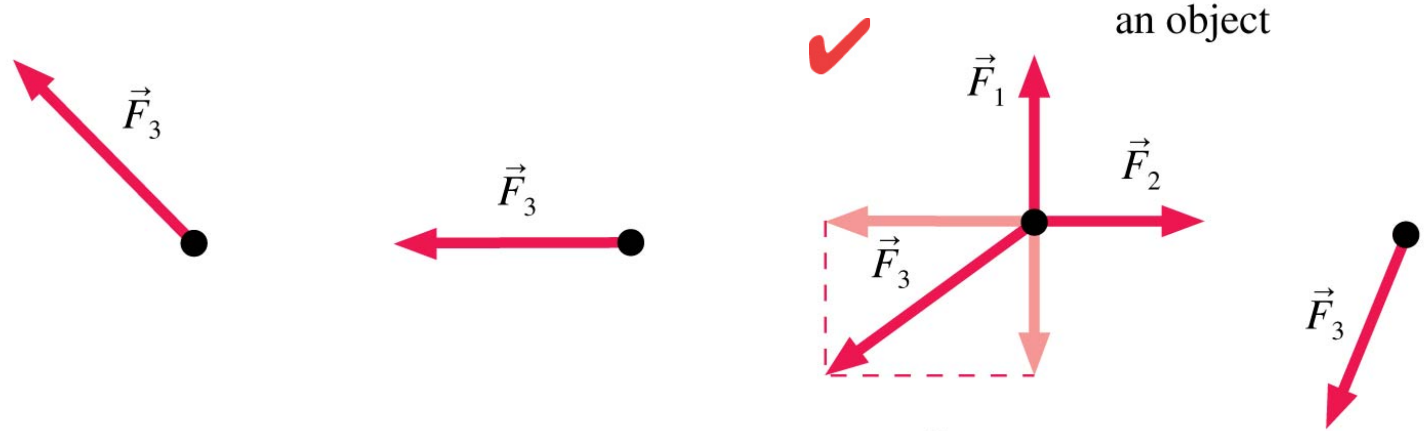


A. B. C. D.

The net force on an object points to the left. Two of three forces are shown. Which is the missing third force?



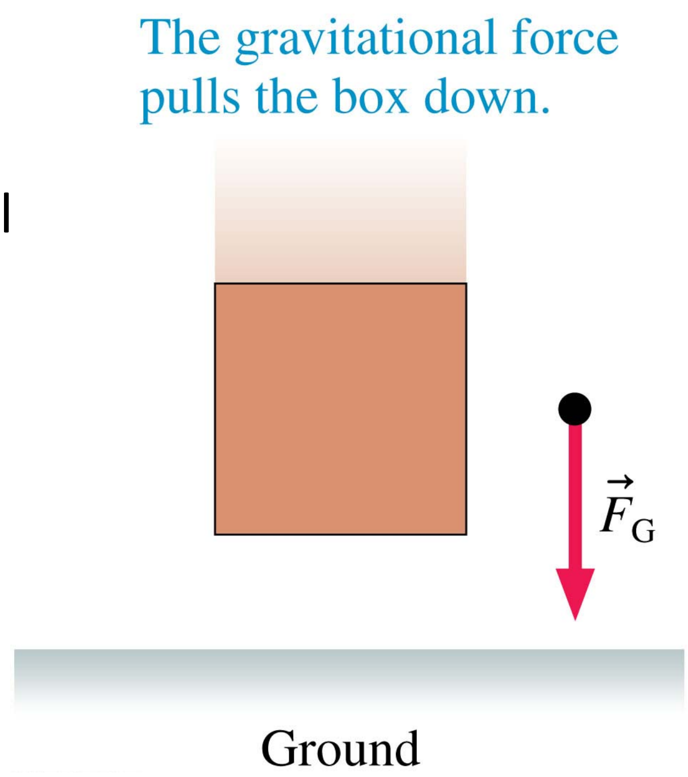
Two of the three forces exerted on an object



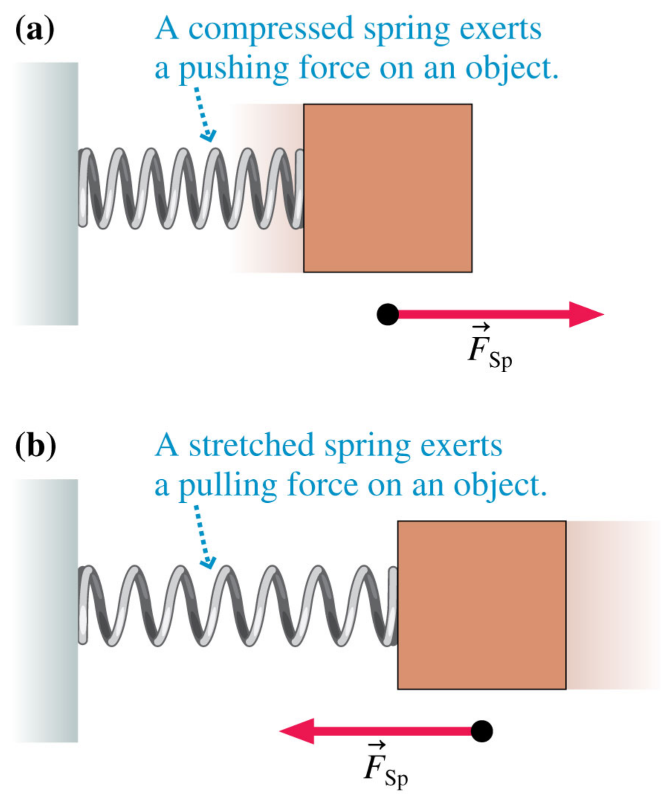
A. B. C. D.

Vertical components cancel

- The pull of a planet on an object near the surface is called the **gravitational force**.
- The agent for the gravitational force is the *entire planet*.
- Gravity acts on *all* objects, whether moving or at rest.
- The gravitational force vector always points vertically downward.

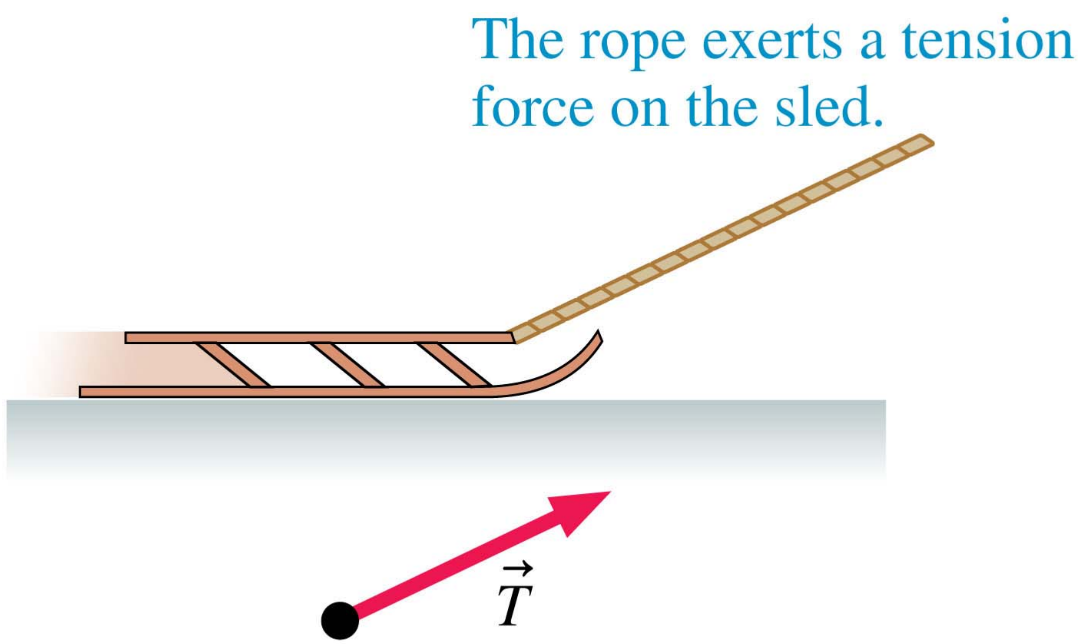


- A spring can either push (when compressed) or pull (when stretched).
- Not all springs are metal coils.
- Whenever an elastic object is flexed or deformed in some way, and then "springs" back to its original shape when you let it go, this is a **spring force**.



What are examples of this?

- When a string or rope or wire pulls on an object, it exerts a contact force called the **tension force**.
- The tension force is in the direction of the string or rope.



MODEL 5.1

Ball-and-spring model of solids

Solids consist of atoms held together by molecular bonds.

- Represent the solid as **an array of balls connected by springs.**
- Pulling on or pushing on a solid causes the bonds to be stretched or compressed. **Stretched or compressed bonds exert spring forces.**
- There are an immense number of bonds. The force of one bond is very tiny, but the combined force of all bonds can be very large.
- Limitations: Model fails for liquids and gases.