Canelo Alvarez punches Avni Yildirim during their super-middleweight title fight Saturday. (Ed Mulholland / Matchroom)

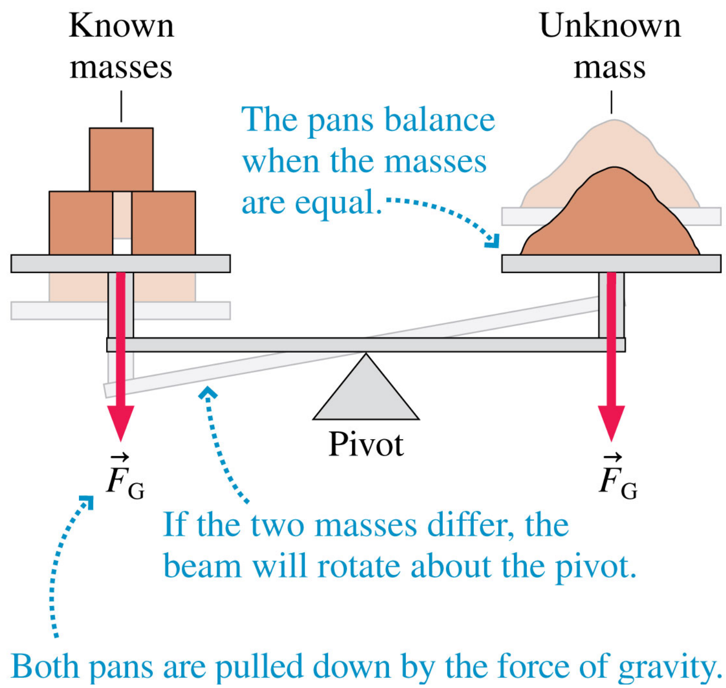
Mass: An Intrinsic Property

This has gravitas

- A *pan balance*, shown in the figure, is a device for measuring **mass**.
- The measurement does not depend on the strength of gravity.

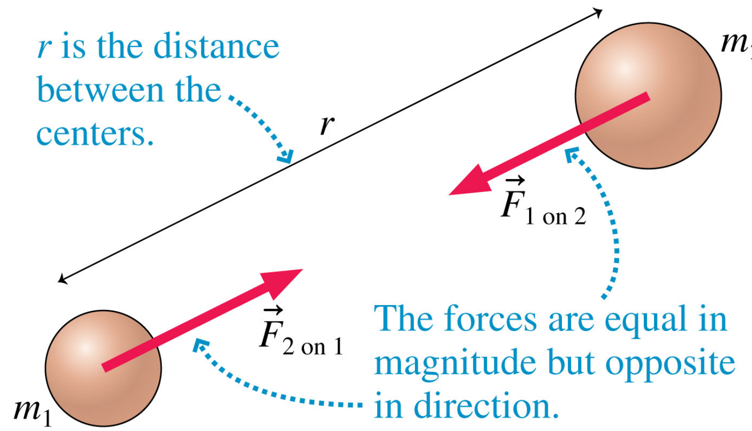
Why?

- Mass is a scalar quantity that describes an object's inertia.
- Mass describes the amount of matter in an object.
- **Mass is an intrinsic property of an object.**



Gravity: A Force Let that sink in

- Gravity is an attractive, long-range force between any two objects.
- The figure shows two objects with masses m_1 and m_2 whose centers are separated by distance r.
- Each object pulls on the other with a force:

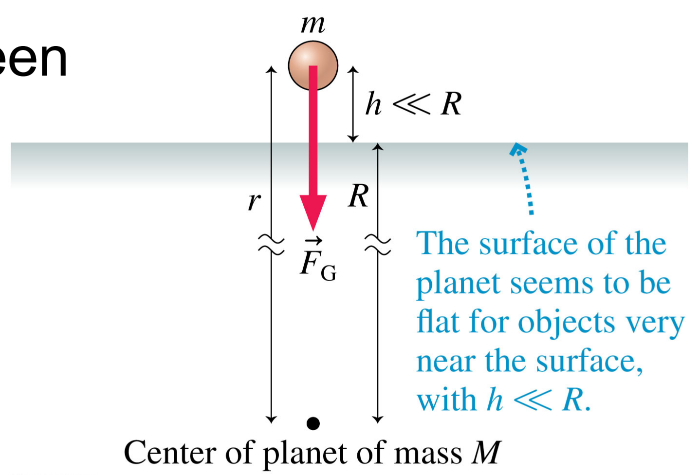


$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1 m_2}{r^2} \qquad \text{(Newton's law of gravity)}$$

where $G = 6.67 \times 10^{-11}$ N m^2/kg^2 is the gravitational constant.

Gravity: A Force

- The gravitational force between two human-sized objects is very small.
- Only when one of the objects is planet-sized or larger does gravity become an important force.



- For objects near the surface of the planet earth,

$$\vec{F}_G = \vec{F}_{\text{planet on } m} = \left(\frac{GMm}{R^2}, \text{straight down}\right) = (mg, \text{straight down})$$

where M and R are the mass and radius of the earth, and $g = 9.80\ \text{m/s}^2$.

What is g at sea level at the equator?
Sea level at the north pole? Himalayas?

Gravity: A Force

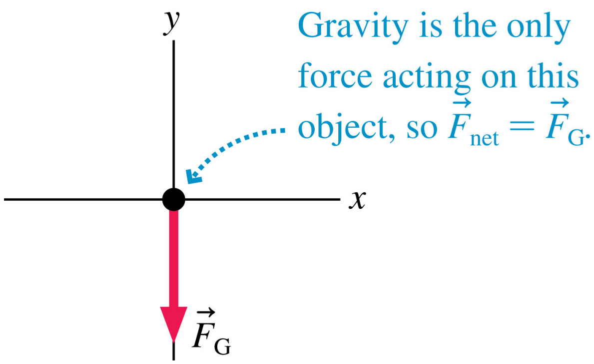
Why is R^2 the radius of the earth? What does that mean?

- The magnitude of the gravitational force is $F_G = mg$, where

$$g = \frac{GM}{R^2}$$

- The figure shows the free-body diagram of an object in free fall near the surface of a planet.
- With $\vec{F}_{\text{net}} = \vec{F}_{\text{G}}$, Newton's second law predicts the acceleration to be

$$\vec{a}_{\text{free fall}} = \frac{\vec{F}_{\text{net}}}{m} = \frac{\vec{F}_{\text{G}}}{m} = (g, \text{ straight down})$$



- All objects on the same planet, regardless of mass, have the same free-fall acceleration!

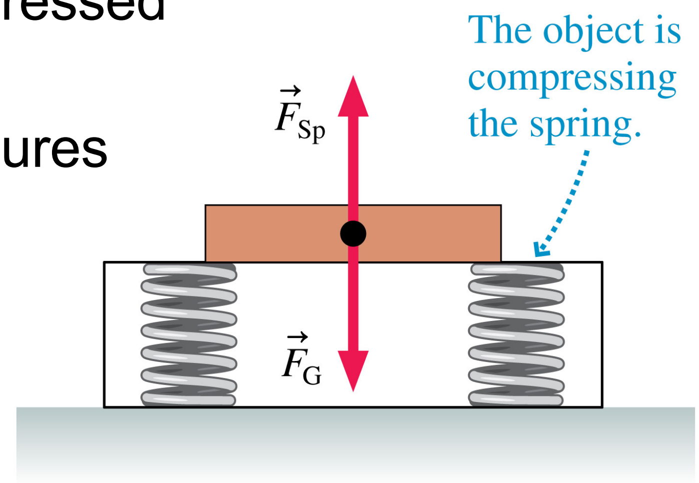
Weight: A Measurement

Wait… what?

- When you weigh yourself, you stand on a spring scale and compress a spring.
- The reading of a spring scale is F_{Sp}, the magnitude of the upward force the spring is exerting. To keep you in place
- Let's define the **weight** of an object to be the reading F_{Sp} of a calibrated spring scale when the object is at rest relative to the scale.
- That is, **weight is a measurement, the result of "weighing" an object**.
- Because F_{Sp} is a force, weight is measured in newtons.

Weight: A Measurement

- A bathroom scale uses compressed springs which push up.
- When any spring scale measures an object at rest, $\vec{F}_{\text{net}} = \vec{0}$.
- The upward spring force exactly balances the downward gravitational force of magnitude mg:



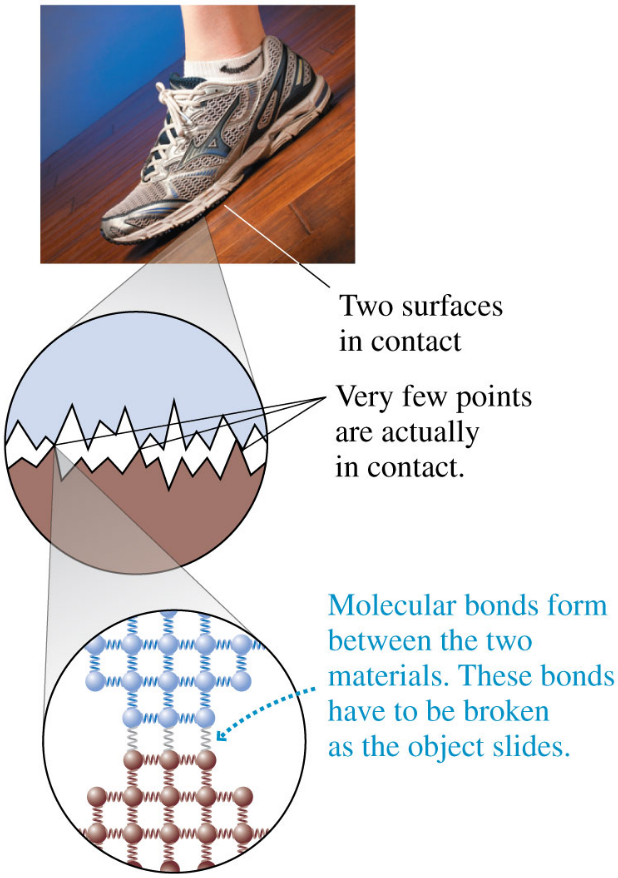
$$F_{\text{Sp}} = F_{\text{G}} = mg$$

- Weight is defined as the magnitude of F_{Sp} when the object is at rest relative to the stationary scale:

$$w = mg \qquad \text{(weight of a stationary object)}$$

Static Friction

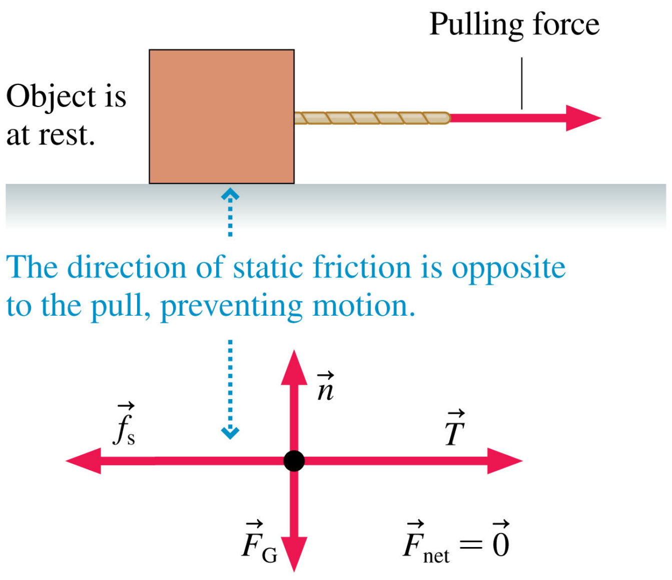
- A shoe pushes on a wooden floor but does not slip.
- On a microscopic scale, both surfaces are “rough” and high features on the two surfaces form molecular bonds.
- These bonds can produce a force *tangent* to the surface, called the **static friction** force.
- Static friction is a result of many molecular springs being compressed or stretched ever so slightly.



The friction model is phenomenological, it hides much of the the physics and replaces it with a measurement. All models are wrong, some models are useful

Static Friction

- The figure shows a rope pulling on a box that, due to static friction, isn't moving.
- Looking at the free-body diagram, the x-component of Newton's first law requires that the static friction force must exactly balance the tension force:



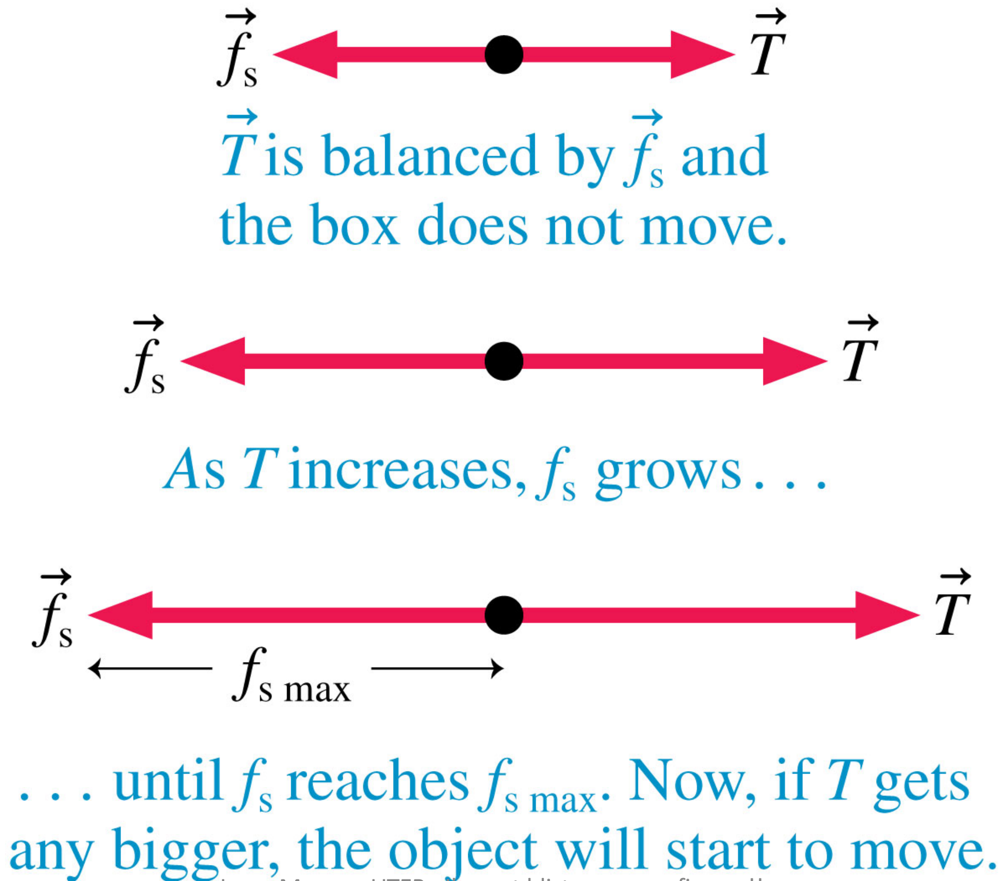
$$f_s = T$$

So the static friction is *variable*?

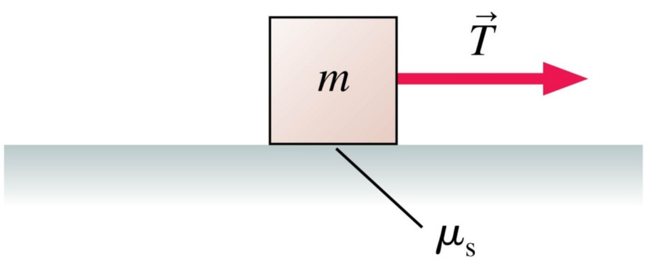
- $\vec{f}_s$ points in the direction *opposite* to the way the object would move if there were no static friction.

Static Friction

- Static friction acts in *response* to an applied force.



A box on a rough surface is pulled by a horizontal rope with tension T. The box is not moving. In this situation,



A. $f_s > T$

B. $f_s = T$

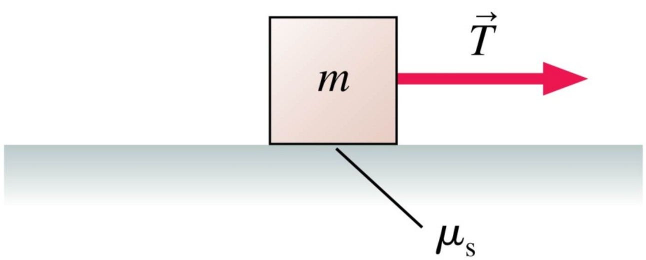
C. $f_s < T$

D. $f_s = \mu_s mg$

E. $f_s = 0$

What can we say about the static friction?

A box on a rough surface is pulled by a horizontal rope with tension T. The box is not moving. In this situation,



A. $f_s > T$

B. $\boldsymbol{f_s = T}$ Newton's first law.

C. $f_s < T$

D. $f_s = \mu_s mg$

E. $f_s = 0$

Static Friction

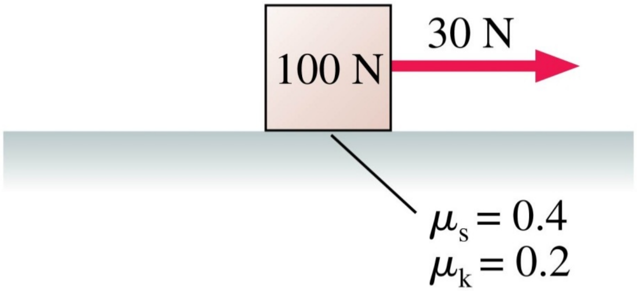
- Static friction force has a *maximum* possible size $f_{s\,max}$.
- An object remains at rest as long as $f_s < f_{s\,max}$.
- The object just begins to slip when $f_s = f_{s\,max}$.
- A static friction force $f_s > f_{s\,max}$ is not physically possible:

$$f_{s\,max} = \mu_s n$$

where the proportionality constant μ_s is called the **coefficient of static friction**.

A box with a weight of 100 N is at rest. It is then pulled by a 30 N horizontal force.

Does the box move?



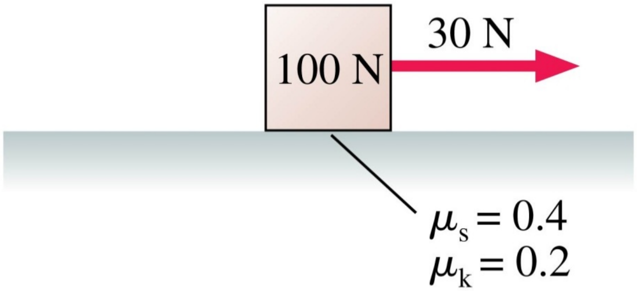
A. Yes

B. No

C. Not enough information to say.

A box with a weight of 100 N is at rest. It is then pulled by a 30 N horizontal force.

Does the box move?

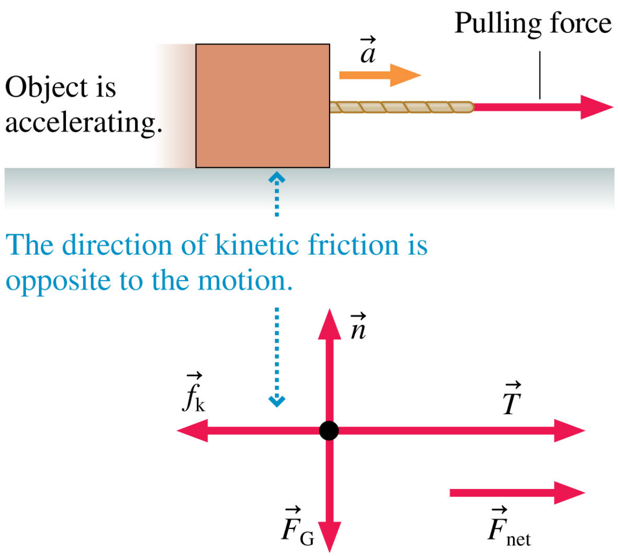


A. Yes

B. No $30\text{ N} < f_{s\,\max} = 40\text{ N}$

C. Not enough information to say.

Kinetic Friction



Is the kinetic friction *variable*?

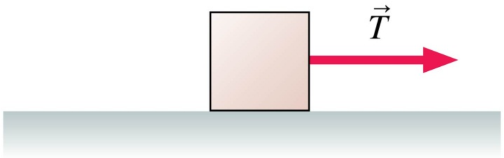
- The **kinetic friction** force is proportional to the magnitude of the normal force:

$$f_k = \mu_k n$$

where the proportionality constant μ_k is called the **coefficient of kinetic friction.**

- The kinetic friction direction is opposite to the velocity of the object relative to the surface.
- For any particular pair of surfaces, $\mu_k < \mu_s$.

A box is being pulled to the right over a rough surface. $T > f_k$, so the box is speeding up. Suddenly the rope breaks.



What happens? The box

A. Stops immediately.

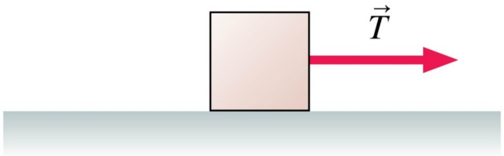
B. Continues with the speed it had when the rope broke.

C. Continues speeding up for a short while, then slows and stops.

D. Keeps its speed for a short while, then slows and stops.

E. Slows steadily until it stops.

A box is being pulled to the right over a rough surface. $T > f_k$, so the box is speeding up. Suddenly the rope breaks.



What happens? The box

1st law?

Without friction

A. Stops immediately.

B. Continues with the speed it had when the rope broke.

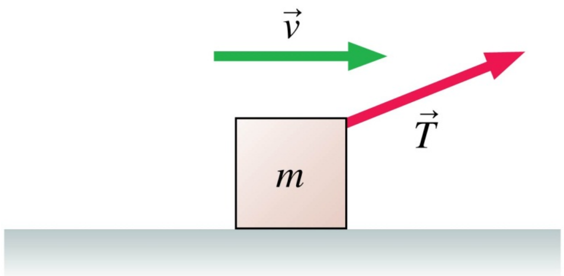
C. Continues speeding up for a short while, then slows and stops.

What?

D. Keeps its speed for a short while, then slows and stops.

✔ **E. Slows steadily until it stops.**

A box is being pulled to the right at steady speed by a rope that angles upward. In this situation:



What can we say about the normal force?

A. $n > mg$

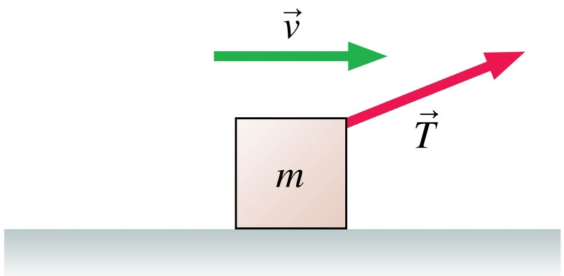
B. $n = mg$

C. $n < mg$

D. $n = 0$

E. Not enough information to judge the size of the normal force.

A box is being pulled to the right at steady speed by a rope that angles upward. In this situation:



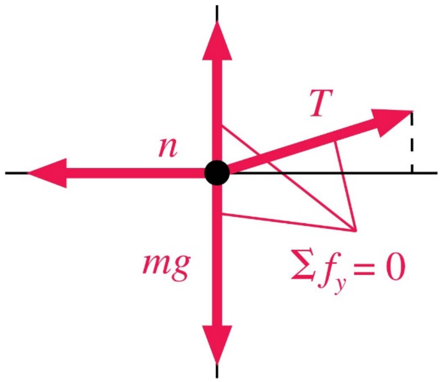
A. $n > mg$

B. $n = mg$

✔ **C.** $\boldsymbol{n < mg}$

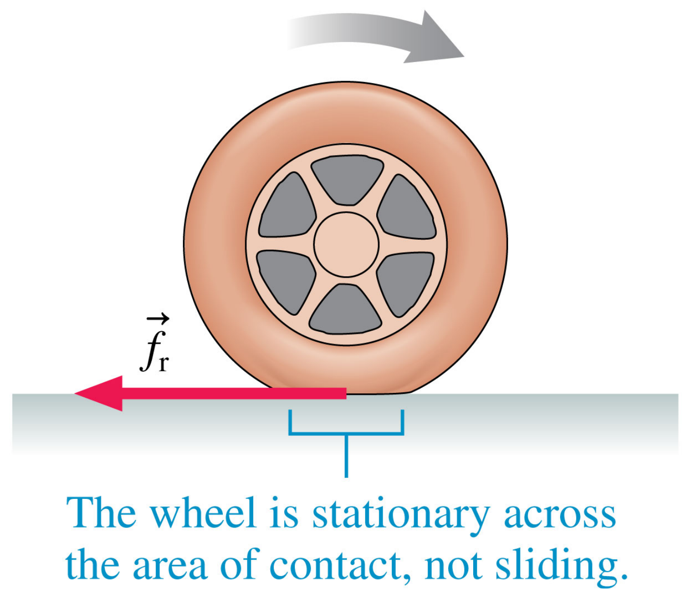
D. $n = 0$

E. Not enough information to judge the size of the normal force.



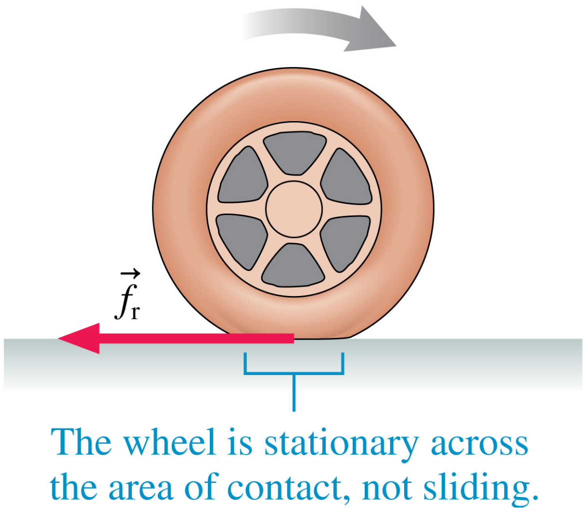
Rolling Motion

- If you slam on the brakes hard enough, your car tires slide against the road surface and leave skid marks. This is kinetic friction.
- A wheel *rolling* on a surface also experiences friction, but not kinetic friction.



- The portion of the wheel that contacts the surface is stationary with respect to the surface, not sliding.
- The interaction of this contact area with the surface causes **rolling friction**.

Rolling Friction



- A car with no engine or brakes applied does not roll forever; it gradually slows down.
- This is due to **rolling friction**.
- The force of rolling friction can be calculated as

$$f_r = \mu_r n$$

where μ_r is called the coefficient of rolling friction.
- The rolling friction direction is opposite to the velocity of the rolling object relative to the surface.

Coefficients of Friction

TABLE 6.1 Coefficients of friction

Materials	Static μ_s	Kinetic μ_k	Rolling μ_r
Rubber on dry concrete	1.00	0.80	0.02
Rubber on wet concrete	0.30	0.25	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

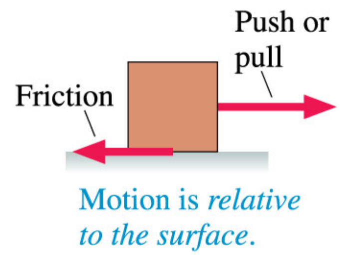
A Model of Friction

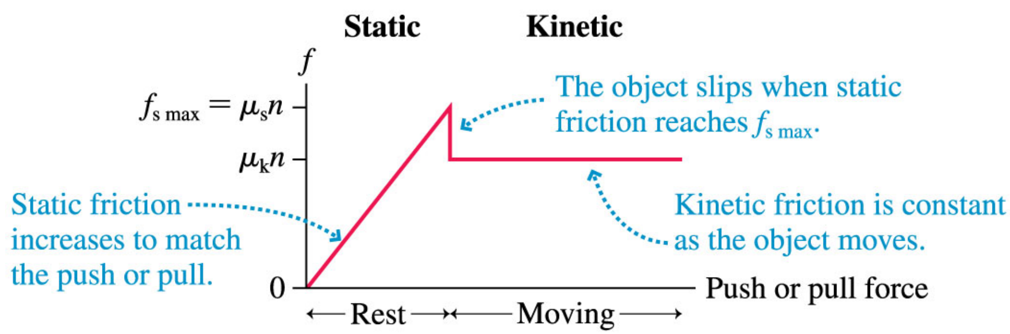
MODEL 6.3

Friction

The friction force is *parallel* to the surface.

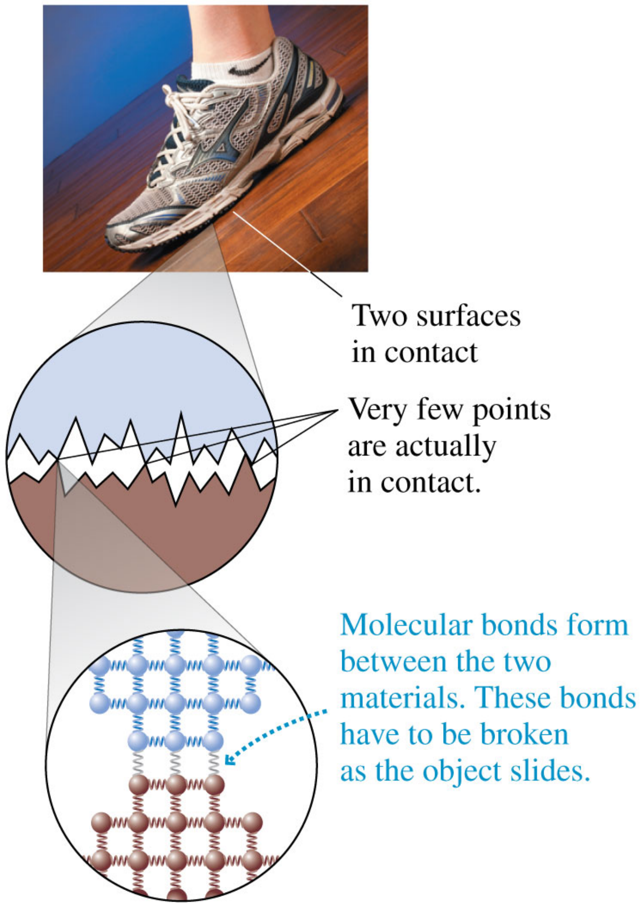
- Static friction: Acts as needed to prevent motion. Can have *any* magnitude up to $f_{s\,max} = \mu_s n$.
- Kinetic friction: Opposes motion with $f_k = \mu_k n$.
- Rolling friction: Opposes motion with $f_r = \mu_r n$.
- Graphically:





Causes of Friction

- All surfaces are very rough on a microscopic scale.
- When two surfaces are pressed together, the high points on each side come into contact and form molecular bonds.
- The amount of contact depends on the normal force n.
- When the two surfaces are sliding against each other, the bonds don't form fully, but they do tend to slow the motion.



Drag

- The air exerts a drag force on objects as they move through the air.
- Faster objects experience a greater drag force than slower objects.
- The drag force on a high-speed motorcyclist is significant.
- The drag force direction is opposite the object's velocity.

Birds

Drag

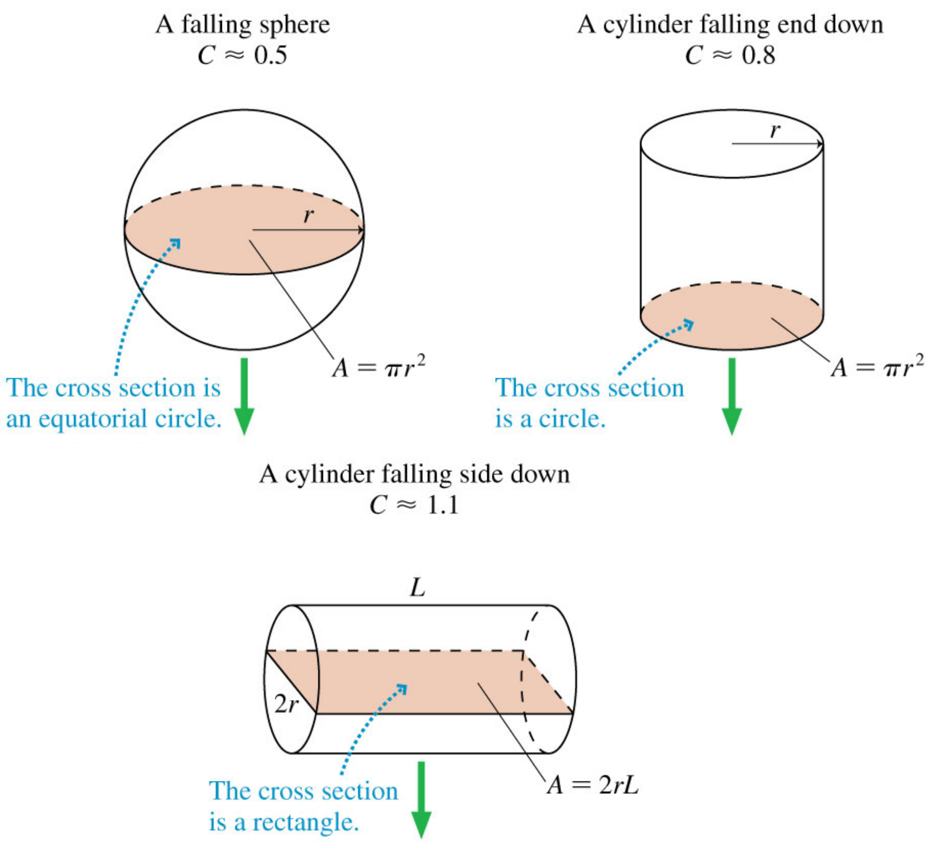
- For normal-sized objects on earth traveling at a speed v which is less than a few hundred meters per second, air resistance can be modeled as

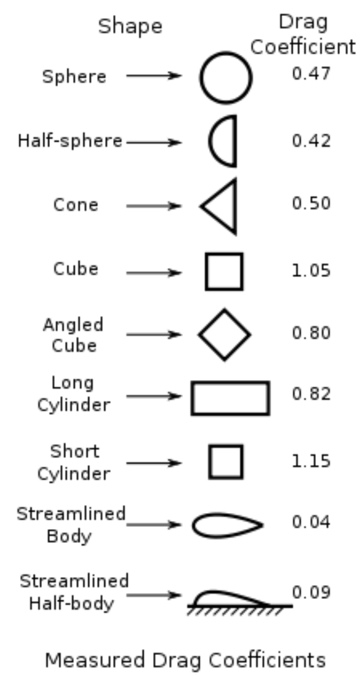
$$\vec{F}_{\text{drag}} = \left(\tfrac{1}{2}C\rho A v^2, \text{direction opposite the motion}\right)$$

- A is the *cross-section area* of the object.
- ρ is the density of the air, which is $1.3\ \mathrm{kg/m^3}$, at atmospheric pressure and $0^{\circ}\mathrm{C}$, a common reference point of pressure and temperature.
- C is the **drag coefficient**, which is a dimensionless number that depends on the shape of the object.

Drag

- Cross-section areas for objects of different shape.



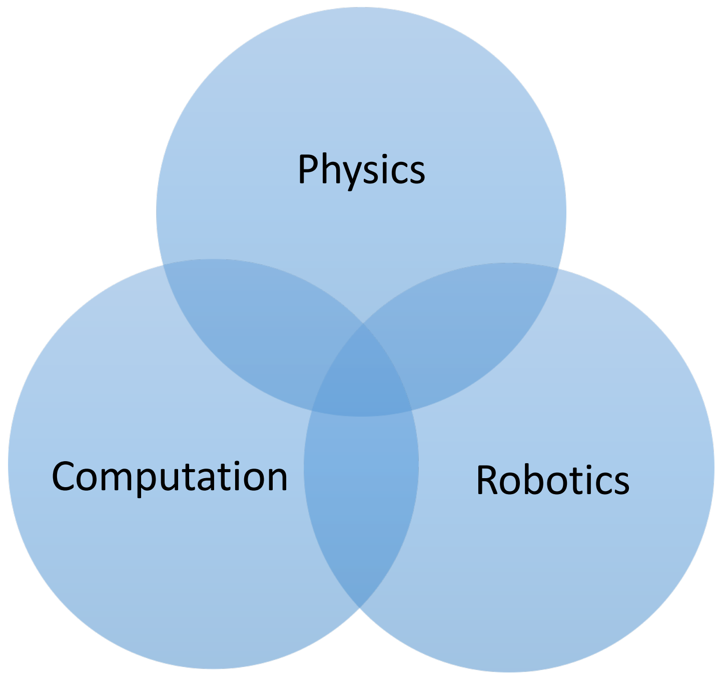


[Ferrari]

Let's assume we have mastered statics and dynamics and we can describe forces arbitrarily accurately. Can we:

- Predict parabolic motion even with drag?
- Build bridges and skyscrapers even in earthquake zones?
- Design engines and vehicles?
- Put humans on the moon and satellites around gas giants?
- Design robots that can walk?

What is a model?

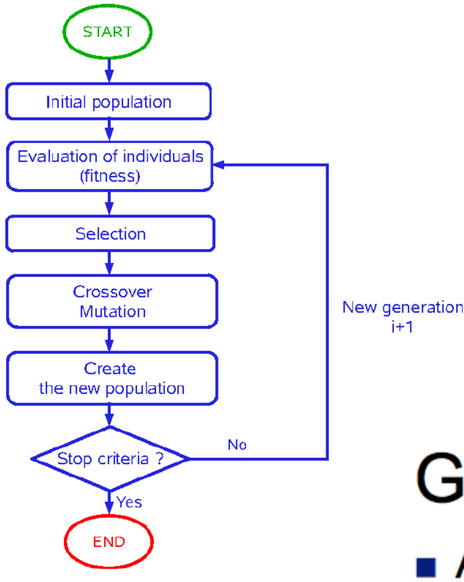


How would you go to build a model for a robot to walk? Or open a door?

[Biped Learning to Walk Using Evolved Neural Nets]

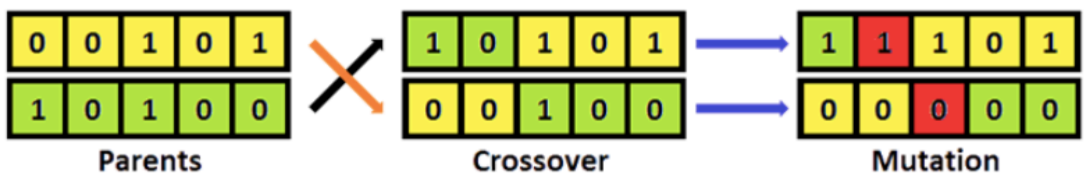
[Evolving Gaits for Legged Robots]

[NaturalMotion's Euphoria]



Genetic Algorithms

- Abstraction of evolution
  - Genes = bits, integers, reals
  - Natural selection = fitness function
  - Mutation = bit flip, integer swap, random perturbation, …
  - Crossover = parents swap substrings

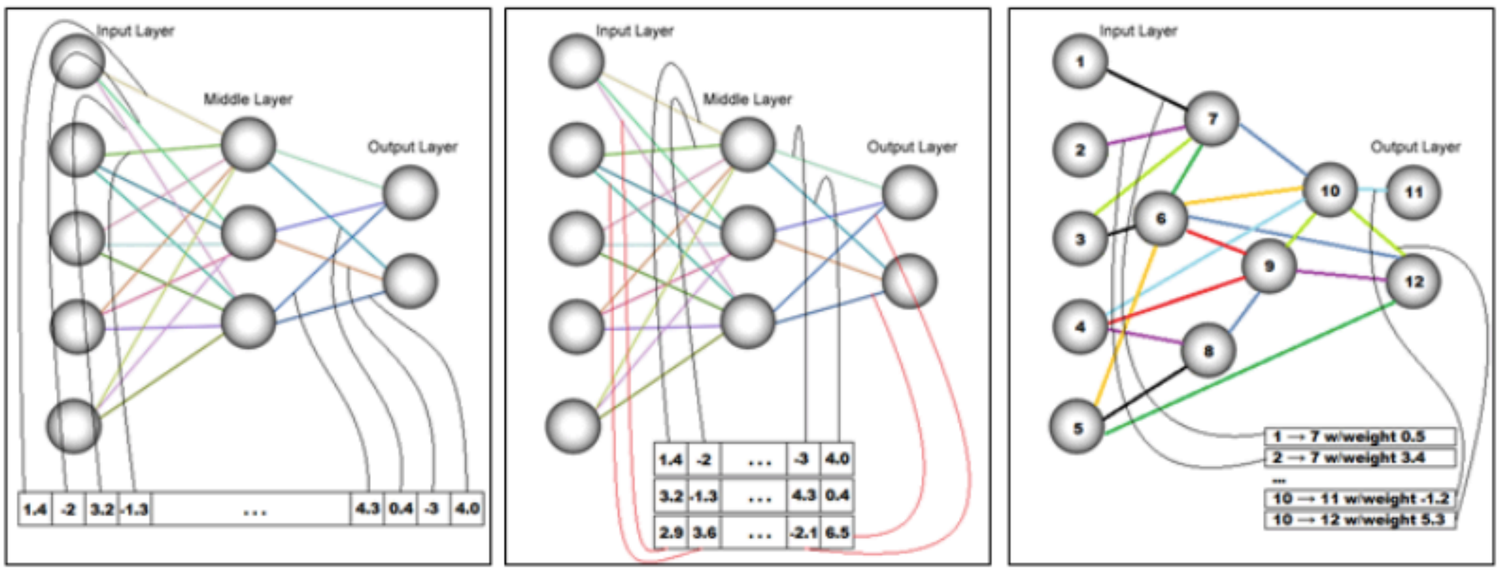


  - Other representations, mutation ops, crossover ops, …

All the physics is encoded in the neural network!

Neuro-Evolution

- Genetic Algorithms + Neural Networks
- Many different network representations



Fixed length string | Subpopulations for each hidden layer neuron [1] | Evolve topology and weights [2]

[1] F. Gomez and R. Miikkulainen, "Incremental Evolution Of Complex General Behavior" Adaptive Behavior 5, 1997.
[2] K. O. Stanley and R. Miikkulainen, "Evolving Neural Networks Through Augmenting Topologies" EC 10:2, 2002.

In this lecture we will go over the nature of several forces that we encounter in everyday situations and might go into your free-body diagram.

You DO NOT need to remember the details. What you DO need to remember is the directions in which they act (upward, downward, against the direction of motion, with the direction of motion) and that mathematically they are treated the same.

Newton's second law: $\vec{F} = m\vec{a}$

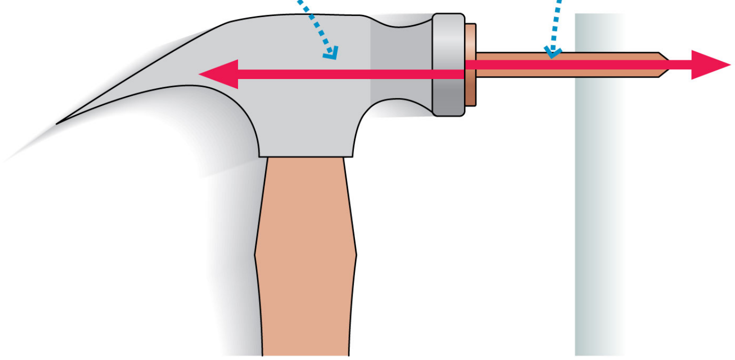
What is an *acceleration constraint*?

A. The acceleration of an object has to be positive.
B. Two objects have to accelerate in the same direction.
C. The magnitude of the accelerations of two objects have to be equal.
D. An object is prevented from accelerating.
E. Acceleration constraints preclude you from fulfilling your dreams.

What is an *acceleration constraint*?

A. The acceleration of an object has to be positive.
B. Two objects have to accelerate in the same direction.
✔ **C. The magnitude of the accelerations of two objects have to be equal.**
D. An object is prevented from accelerating.
E. Acceleration constraints preclude you from fulfilling your dreams.

How do we use them?



- When a hammer hits a nail, it exerts a forward force on the nail.
- At the same time, the nail exerts a backward force on the hammer.
- If you don’t believe it, imagine hitting the nail with a glass hammer.
- It’s the force of the nail on the hammer that would cause the glass to shatter!
- This force also causes blisters on your hand!

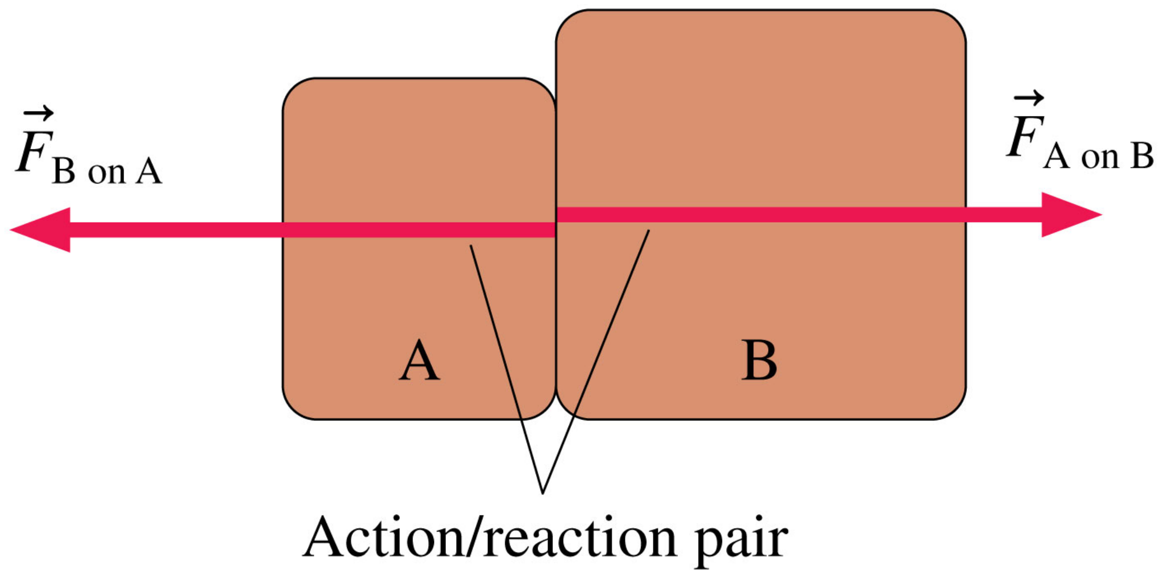
Why does the ball travel farther?

- When a bat hits a ball, the ball exerts a force on the bat.
- When you pull someone with a rope in a tug-of-war, that person pulls back on you.

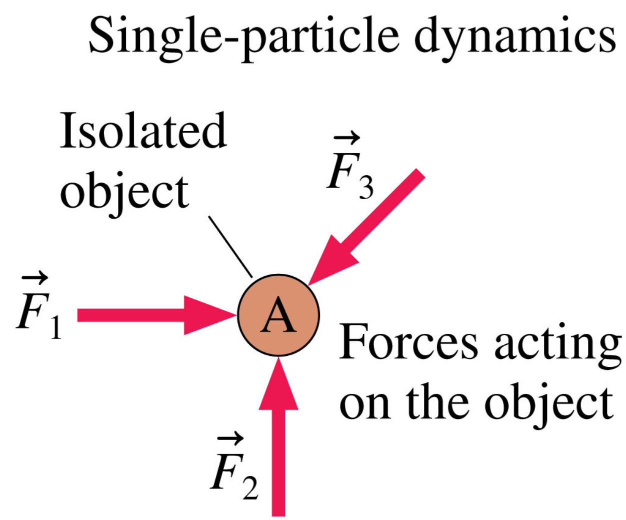
The bat and the ball are interacting with each other.

- When your chair pushes up on you (the normal force), you push down on the chair.
- All forces come in pairs, called **action/reaction pairs**.
- These forces occur simultaneously, and we cannot say which is the “action” and which is the “reaction.”

- If object A exerts a force on object B, then object B exerts a force on object A.
- The pair of forces, as shown, is called an **action/reaction pair**.

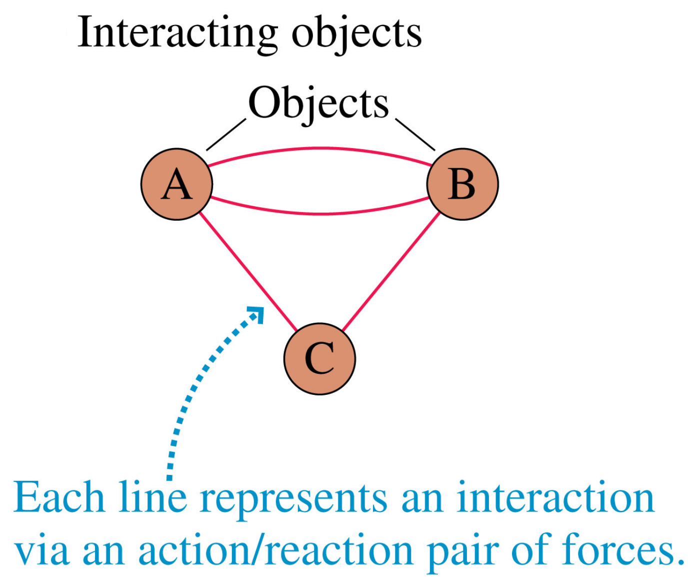


- In previous lectures we considered forces acting on a single object, modeled as a particle.
- The figure shows a diagram representing single-particle dynamics.

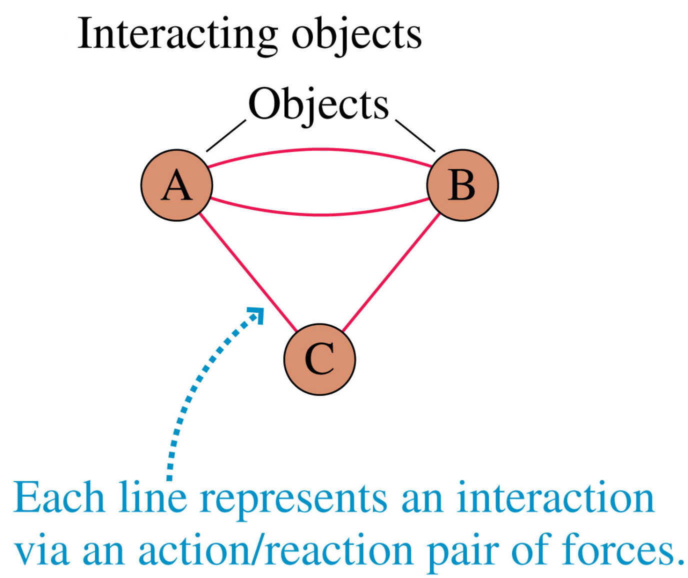


- We can use Newton's second law, $\vec{a} = \vec{F}_{\text{net}}/m$, to determine the particle's acceleration.

- Now we extend the particle model to include two or more objects that interact.
- The figure shows three objects interacting via action/reaction pairs of forces.

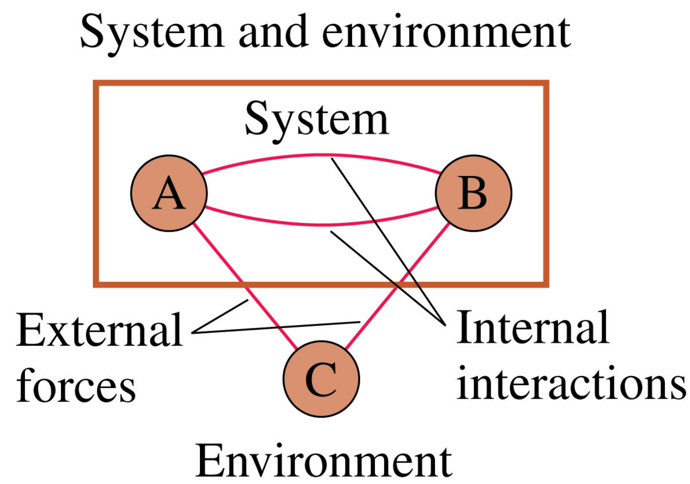


- The forces can be given labels, such as $\vec{F}_{\text{A on B}}$ and $\vec{F}_{\text{B on A}}$.



- For example, set:
  - Object A = the hammer
  - Object B = the nail
  - Object C = the earth
- The earth interacts with both the hammer and the nail via gravity.
- Practically, the earth remains at rest while the hammer and the nail move.
- Define the **system** as those objects whose motion we want to analyze.
- Define the **environment** as objects external to the system.

- The figure shows a new kind of diagram, an **interaction diagram**.
- The objects of the system are in a box.
- Interactions are represented by lines connecting the objects.

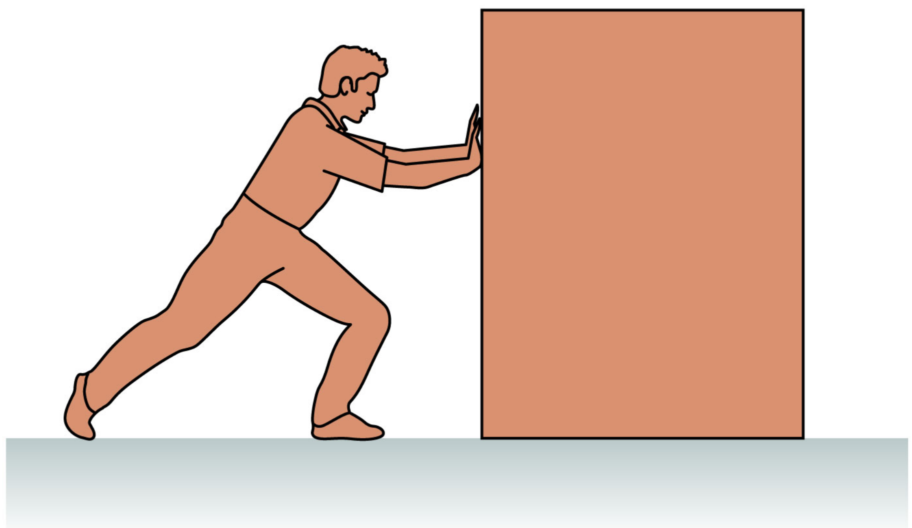


This is an *interaction diagram.*

- Interactions with objects in the environment are called **external forces**.

EXAMPLE 7.1 Pushing a crate

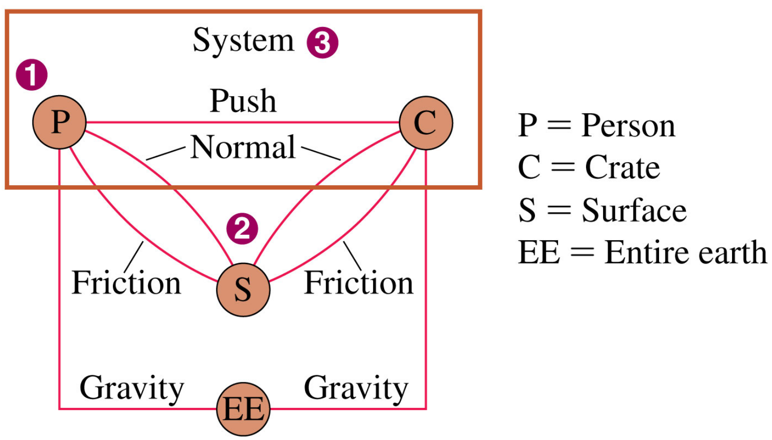
FIGURE 7.4 shows a person pushing a large crate across a rough surface. Identify all interactions, show them on an interaction diagram, then draw free-body diagrams of the person and the crate.



EXAMPLE 7.1 **Pushing a crate**

VISUALIZE

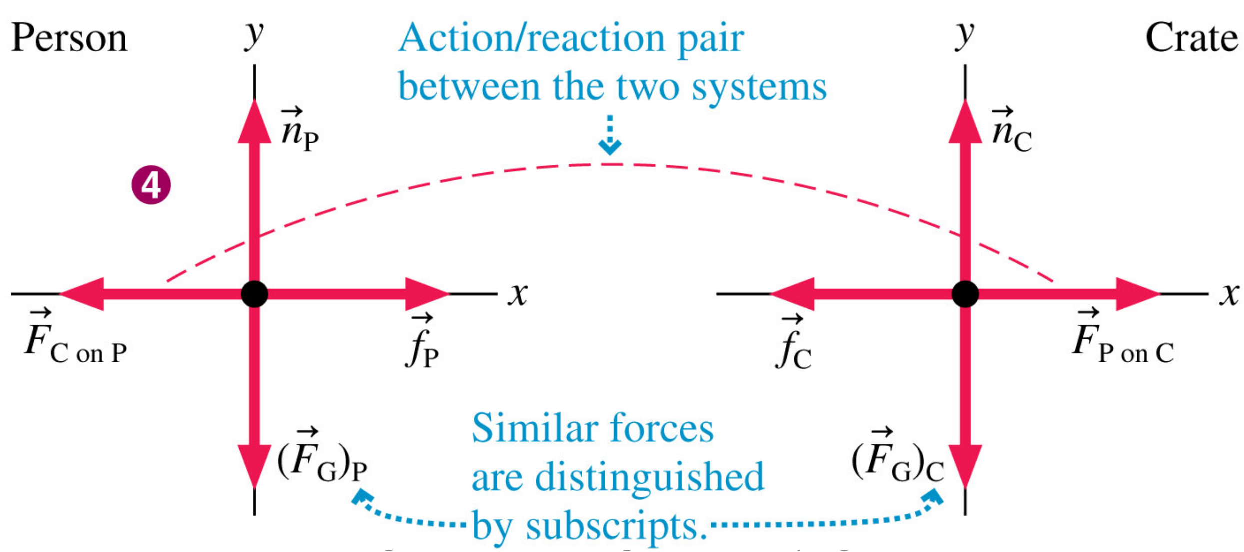
1. The person and crate are obvious objects, with a pushing force connecting them.
2. There are normal and friction contact forces between the person and crate and the surface. Also there is the long-range force of gravity between the person and crate and the entire earth.
3. The person and crate are the System; these are the objects whose motion we wish to analyze.



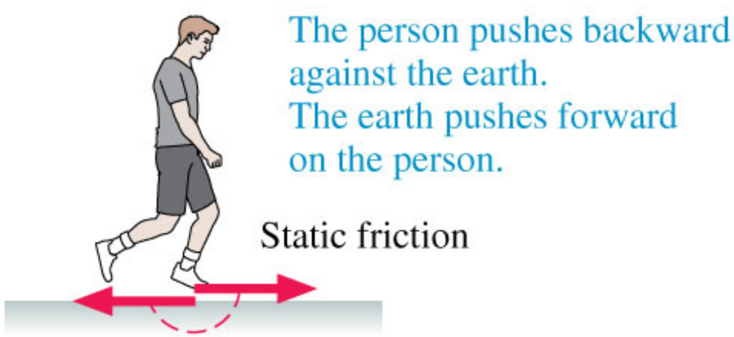
EXAMPLE 7.1 Pushing a crate

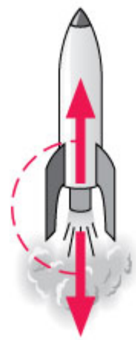
VISUALIZE

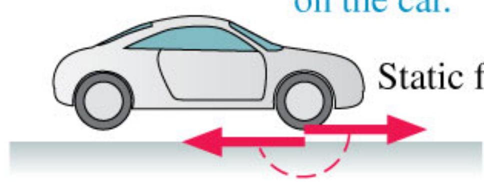
4. Below are the free-body diagrams of the person and the crate. For each, three forces are external forces. Subscripts label which object each force acts on. There is one internal interaction, labeled as an action/reaction pair.



- If you try to walk across a frictionless floor, your foot slips and slides *backward*.
- In order to walk, your foot must *stick* to the floor as you straighten your leg, moving your body forward.
- The force that prevents slipping is *static friction*.
- The static friction force points in the *forward* direction.
- It is static friction that propels you forward!




The rocket pushes the hot gases backward.
The gases push the rocket forward.
Thrust

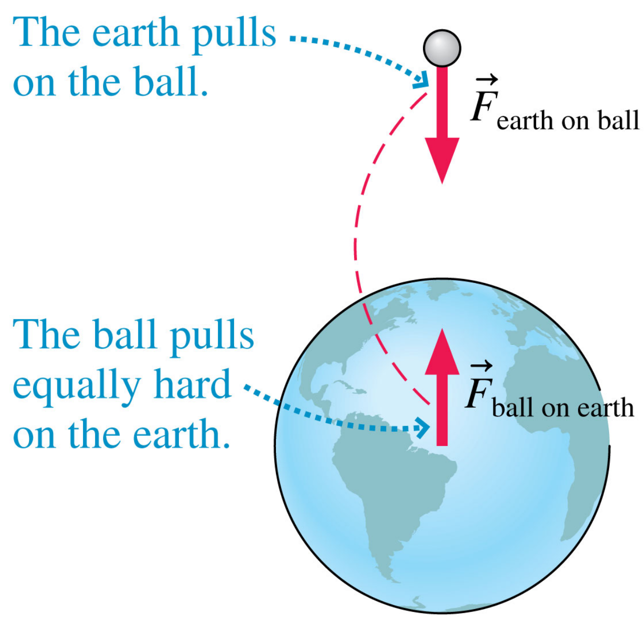

The car pushes backward against the earth.
The earth pushes forward on the car.
Static friction

Newton's third law Every force occurs as one member of an action/reaction pair of forces.

- The two members of an action/reaction pair act on two *different* objects.
- The two members of an action/reaction pair are equal in magnitude but opposite in direction: $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$.

- When you release a ball, it falls down.
- The action/reaction *forces* of the ball and the earth are *equal* in magnitude.
- The acceleration of the ball is

$$\vec{F}_{\text{earth on ball}} = (\vec{F}_G)_B = -m_B\, g\hat{j}$$



- The acceleration of the earth is

$$\vec{a}_E = \frac{\vec{F}_{\text{ball on earth}}}{m_E} = \frac{m_B\, g\hat{j}}{m_E} = \left(\frac{m_B}{m_E}\right) g\hat{j}$$

- If the ball has a mass of 1 kg, the earth accelerates upward at 2×10^{-24} m/s^2.

A mosquito runs head-on into a truck. Splat! Which is true during the collision?

A. The mosquito exerts more force on the truck than the truck exerts on the mosquito.
B. The truck exerts more force on the mosquito than the mosquito exerts on the truck.
C. The mosquito exerts the same force on the truck as the truck exerts on the mosquito.
D. The truck exerts a force on the mosquito but the mosquito does not exert a force on the truck.
E. The mosquito exerts a force on the truck but the truck does not exert a force on the mosquito.

A mosquito runs head-on into a truck. Splat! Which is true during the collision?

A. The mosquito exerts more force on the truck than the truck exerts on the mosquito.
B. The truck exerts more force on the mosquito than the mosquito exerts on the truck.
✔ **C. The mosquito exerts the same force on the truck as the truck exerts on the mosquito.**
D. The truck exerts a force on the mosquito but the mosquito does not exert a force on the truck.
E. The mosquito exerts a force on the truck but the truck does not exert a force on the mosquito.

A mosquito runs head-on into a truck. Which is true during the collision?

A. The magnitude of the mosquito's acceleration is larger than that of the truck.
B. The magnitude of the truck's acceleration is larger than that of the mosquito.
C. The magnitude of the mosquito's acceleration is the same as that of the truck.
D. The truck accelerates but the mosquito does not.
E. The mosquito accelerates but the truck does not.

The mosquito will deaccelerate very quickly then will have the same velocity/acceleration as the truck

A mosquito runs head-on into a truck. Which is true during the collision?

✔ **A. The magnitude of the mosquito's acceleration is larger than that of the truck.**

B. The magnitude of the truck's acceleration is larger than that of the mosquito.

C. The magnitude of the mosquito's acceleration is the same as that of the truck.

D. The truck accelerates but the mosquito does not.

E. The mosquito accelerates but the truck does not.

Newton's second law: $a = \frac{F}{m}$

Same for both

Huge difference

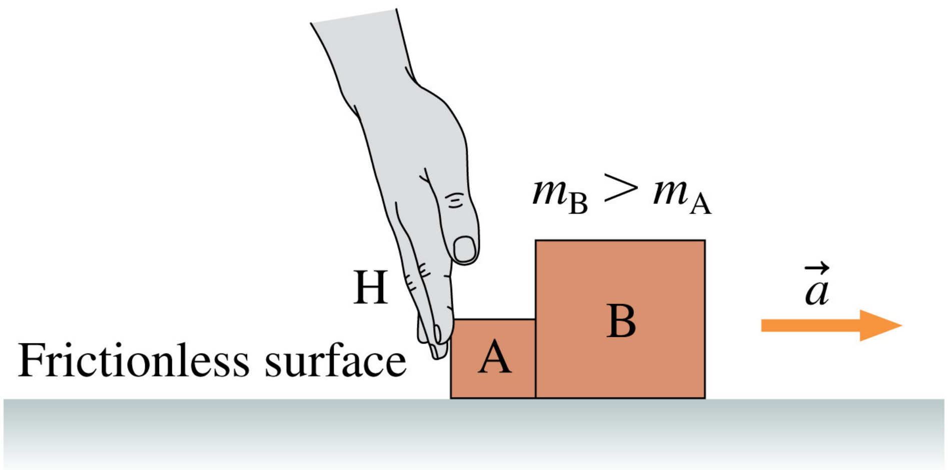
Don't confuse cause and effect! The same force can have very different effects.

EXAMPLE 7.3 | **The forces on accelerating boxes**

The hand shown in **FIGURE 7.12** pushes boxes A and B to the right across a frictionless table. The mass of B is larger than the mass of A.

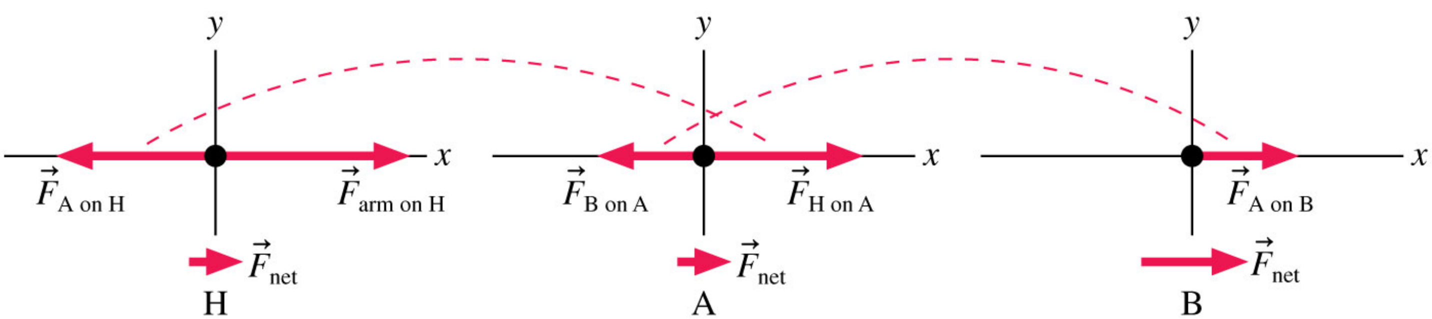
a. Draw free-body diagrams of A, B, and the hand H, showing only the *horizontal* forces. Connect action/reaction pairs with dashed lines.

b. Rank in order, from largest to smallest, the horizontal forces shown on your free-body diagrams.



EXAMPLE 7.3 | **The forces on accelerating boxes**

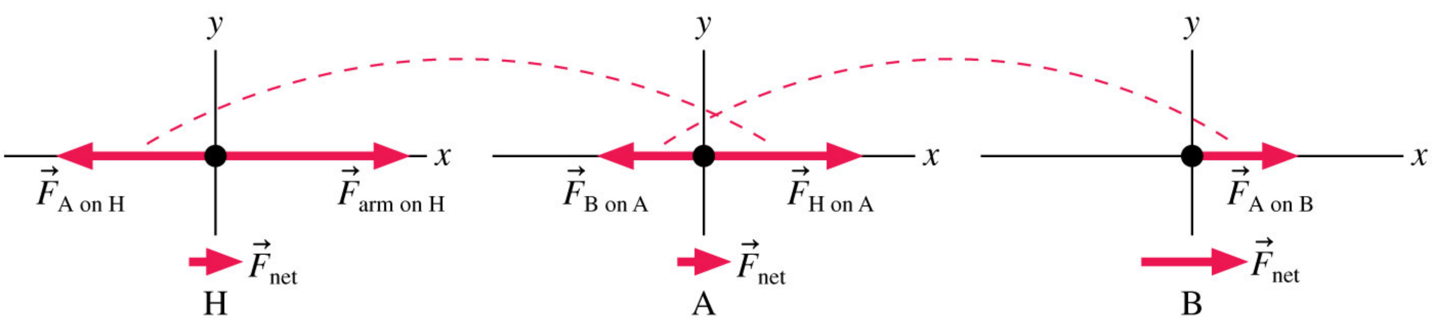
VISUALIZE a. The hand H pushes on box A, and A pushes back on H. Thus $\vec{F}_{\text{H on A}}$ and $\vec{F}_{\text{A on H}}$ are an action/reaction pair. Similarly, A pushes on B and B pushes back on A. **The hand H does not touch box B, so there is no interaction between them.** There is no friction. FIGURE 7.13 on the next page shows five horizontal forces and identifies two action/reaction pairs. Notice that each force is shown on the free-body diagram of the object that it acts *on.*



EXAMPLE 7.3 **The forces on accelerating boxes**

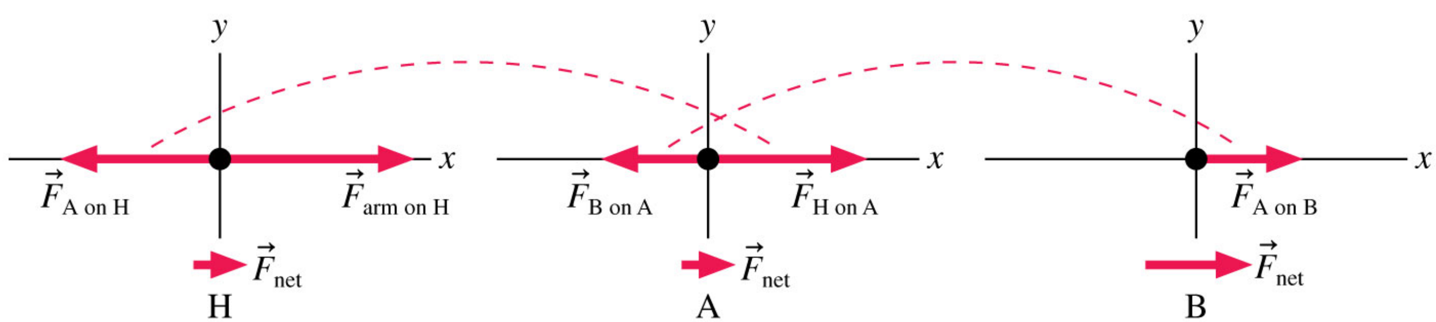
VISUALIZE b. According to Newton's third law, $F_{\text{A on H}} = F_{\text{A on B}}$ and $F_{\text{A on B}} = F_{\text{B on A}}$. But the third law is not our only tool. The boxes are *accelerating* to the right, because there's no friction, so Newton's *second* law tells us that box A must have a net force to the right. Consequently, $F_{\text{H on A}} > F_{\text{B on A}}$. Similarly, $F_{\text{arm on H}} > F_{\text{A on H}}$ is needed to accelerate the hand. Thus

$$F_{\text{arm on H}} > F_{\text{A on H}} = F_{\text{H on A}} > F_{\text{B on A}} = F_{\text{A on B}}$$

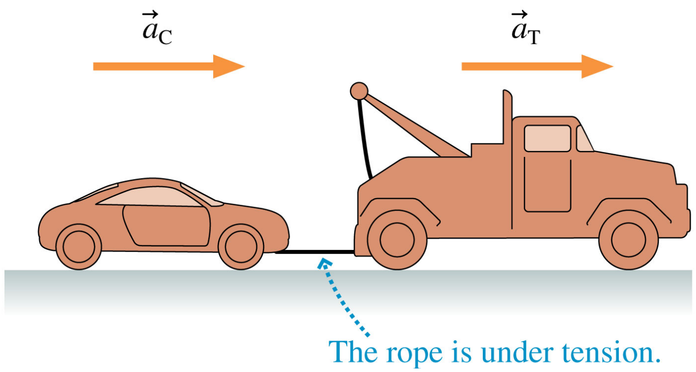


EXAMPLE 7.3 | **The forces on accelerating boxes**

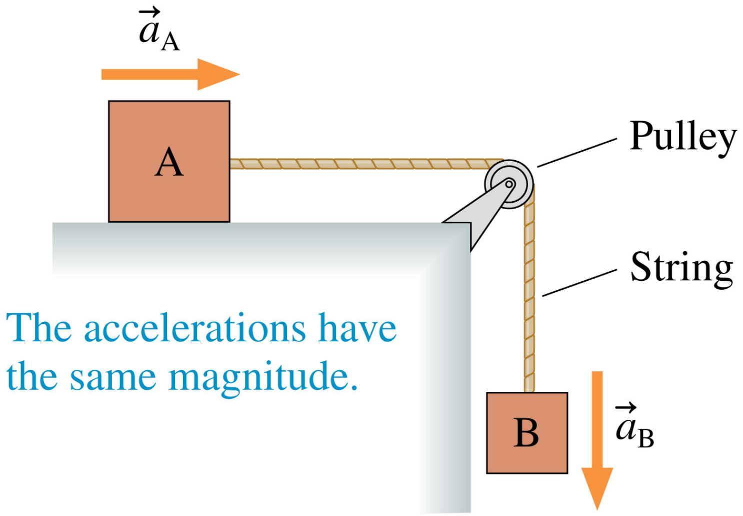
ASSESS You might have expected $F_{\text{A on B}}$ to be larger than $F_{\text{H on A}}$ because $m_B > m_A$. It's true that the *net* force on B is larger than the *net* force on A, but we have to reason more closely to judge the individual forces. Notice how we used both the second and the third laws to answer this question.



- If two objects A and B move together, their accelerations are *constrained* to be equal: $a_{\mathrm{A}} = a_{\mathrm{B}}$
- This equation is called an **acceleration constraint**.
- Consider a car being towed by a truck.
- In this case, the acceleration constraint is $a_{\mathrm{C}x} = a_{\mathrm{T}x} = a_x$.
- Because the accelerations of both objects are equal, we can drop the subscripts C and T and call both of them a_x .



- Sometimes the acceleration of A and B may have different signs.
- Consider the blocks A and B in the figure.
- The string constrains the two objects to accelerate together.



- But, as A moves to the right in the $+x$ direction, B moves down in the $-y$ direction.
- In this case, the acceleration constraint is $a_{Ax} = -a_{By}$.

Problem-Solving Strategy: Interacting-Objects Problems

PROBLEM-SOLVING STRATEGY 7.1

Interacting-objects problems

MODEL Identify which objects are part of the system and which are part of the environment. Make simplifying assumptions.

VISUALIZE Draw a pictorial representation.

- Show important points in the motion with a sketch. You may want to give each object a separate coordinate system. Define symbols, list acceleration constraints, and identify what the problem is trying to find.
- Draw an interaction diagram to identify the forces on each object and all action/reaction pairs.
- Draw a *separate* free-body diagram for each object showing only the forces acting *on* that object, not forces exerted by the object. Connect the force vectors of action/reaction pairs with dashed lines.

Problem-Solving Strategy: Interacting-Objects Problems

What is the 2nd law? 3rd law? Mathematically?

PROBLEM-SOLVING STRATEGY 7.1

Interacting-objects problems

SOLVE Use Newton's second and third laws.

- Write the equations of Newton's second law for *each* object, using the force information from the free-body diagrams.
- Equate the magnitudes of action/reaction pairs.
- Include the acceleration constraints, the friction model, and other quantitative information relevant to the problem.
- Solve for the acceleration, then use kinematics to find velocities and positions.

ASSESS Check that your result has the correct units and significant figures, is reasonable, and answers the question.

What are possible friction models and which ones will you need to know for the exam?