March 4th

The only day of the year that is also a sentence.

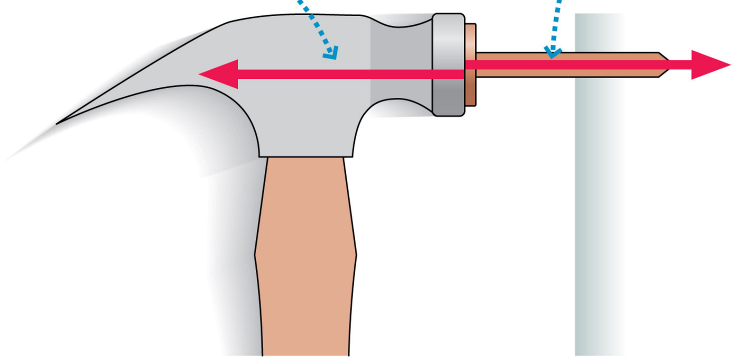
What is an *acceleration constraint*?

A. The acceleration of an object has to be positive.
B. Two objects have to accelerate in the same direction.
C. The magnitude of the accelerations of two objects have to be equal.
D. An object is prevented from accelerating.
E. Acceleration constraints preclude you from fulfilling your dreams.

What is an *acceleration constraint*?

A. The acceleration of an object has to be positive.
B. Two objects have to accelerate in the same direction.
C. **The magnitude of the accelerations of two objects have to be equal.**
D. An object is prevented from accelerating.
E. Acceleration constraints preclude you from fulfilling your dreams.

How do we use them?



- When a hammer hits a nail, it exerts a forward force on the nail.
- At the same time, the nail exerts a backward force on the hammer.

- If you don’t believe it, imagine hitting the nail with a glass hammer.
- It’s the force of the nail on the hammer that would cause the glass to shatter!
- This force also causes blisters on your hand!

Why does the ball travel farther?

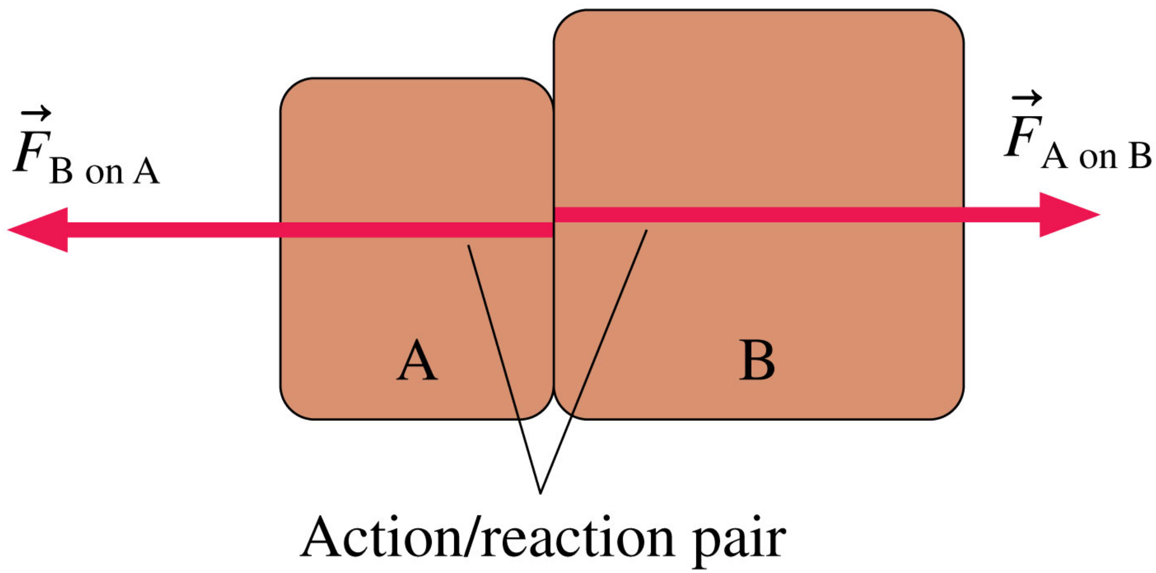
- When a bat hits a ball, the ball exerts a force on the bat.
- When you pull someone with a rope in a tug-of-war, that person pulls back on you.

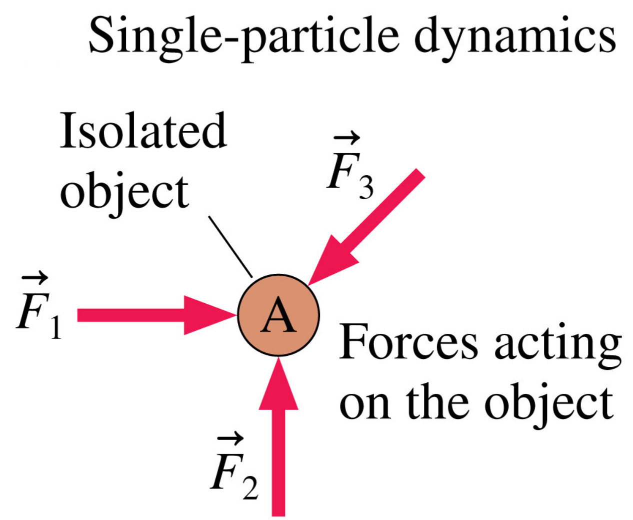
The bat and the ball are interacting with each other.

- When your chair pushes up on you (the normal force), you push down on the chair.
- All forces come in pairs, called **action/reaction pairs**.
- These forces occur simultaneously, and we cannot say which is the "action" and which is the "reaction."


I GOT
BLISTERS
ON MY
FINGERS!
CRASH

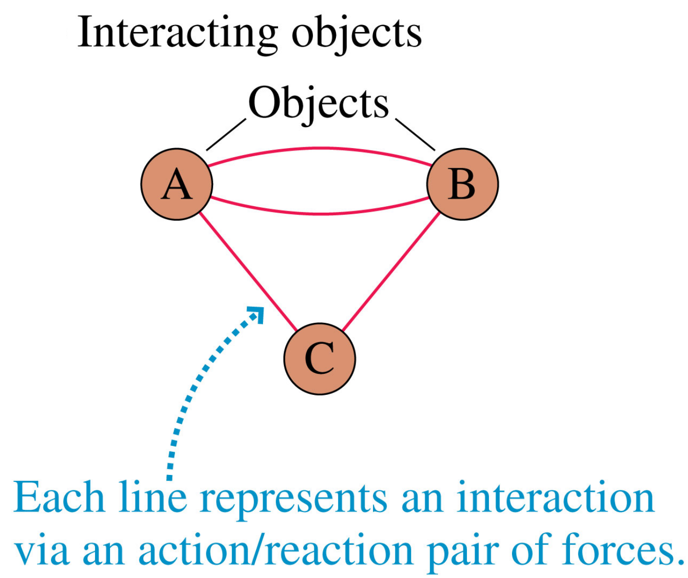
- If object A exerts a force on object B, then object B exerts a force on object A.
- The pair of forces, as shown, is called an **action/reaction pair**.





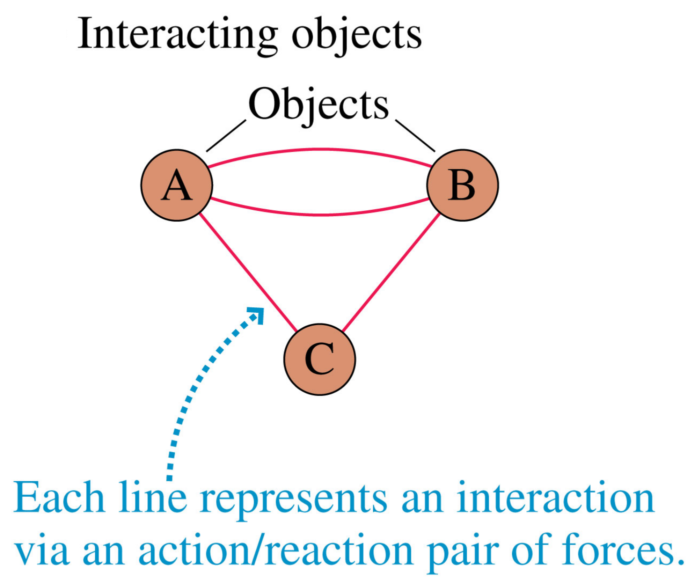
- In previous lectures we considered forces acting on a single object, modeled as a particle.
- The figure shows a diagram representing single-particle dynamics.
- We can use Newton's second law, $\vec{a} = \vec{F}_{\text{net}}/m$, to determine the particle's acceleration.

- Now we extend the particle model to include two or more objects that interact.
- The figure shows three objects interacting via action/reaction pairs of forces.



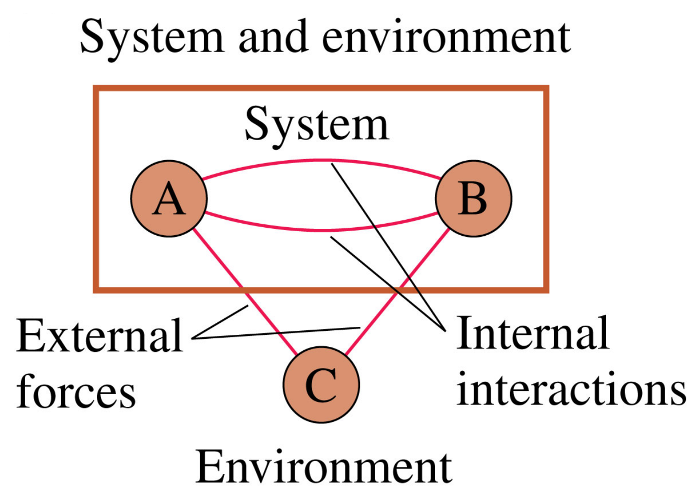
- The forces can be given labels, such as $\vec{F}_{\text{A on B}}$ and $\vec{F}_{\text{B on A}}$.

- For example, set:
  - Object A = the hammer
  - Object B = the nail
  - Object C = the earth
- The earth interacts with both the hammer and the nail via gravity.



- Practically, the earth remains at rest while the hammer and the nail move.
- Define the **system** as those objects whose motion we want to analyze.
- Define the **environment** as objects external to the system.

- The figure shows a new kind of diagram, an **interaction diagram**.
- The objects of the system are in a box.
- Interactions are represented by lines connecting the objects.

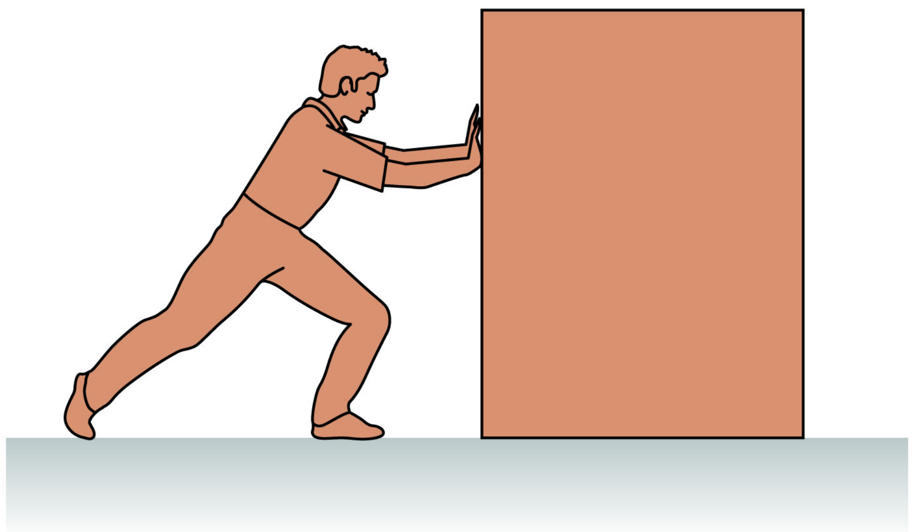


This is an *interaction diagram.*

- Interactions with objects in the environment are called **external forces**.

EXAMPLE 7.1 Pushing a crate

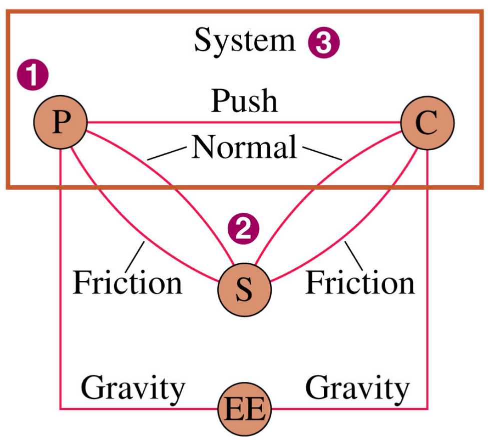
FIGURE 7.4 shows a person pushing a large crate across a rough surface. Identify all interactions, show them on an interaction diagram, then draw free-body diagrams of the person and the crate.



EXAMPLE 7.1 Pushing a crate

VISUALIZE

1. The person and crate are obvious objects, with a pushing force connecting them.
2. There are normal and friction contact forces between the person and crate and the surface. Also there is the long-range force of gravity between the person and crate and the entire earth.
3. The person and crate are the System; these are the objects whose motion we wish to analyze.

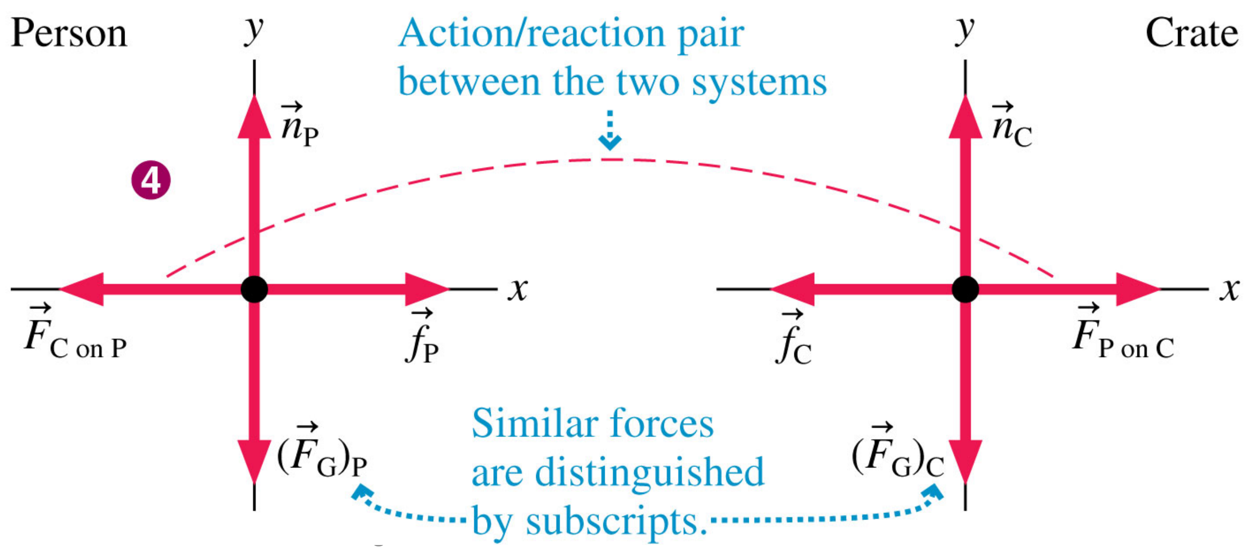


P = Person
C = Crate
S = Surface
EE = Entire earth

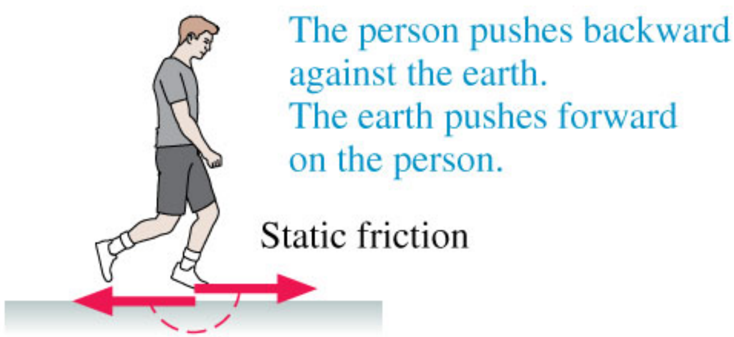
EXAMPLE 7.1 Pushing a crate

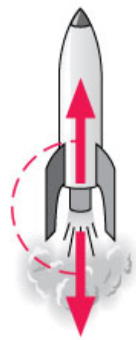
VISUALIZE

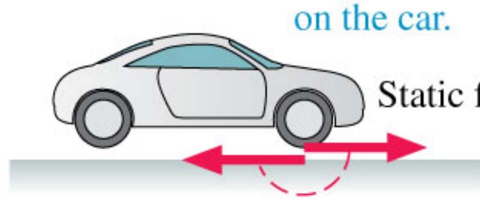
4. Below are the free-body diagrams of the person and the crate. For each, three forces are external forces. Subscripts label which object each force acts on. There is one internal interaction, labeled as an action/reaction pair.



- If you try to walk across a frictionless floor, your foot slips and slides *backward*.
- In order to walk, your foot must *stick* to the floor as you straighten your leg, moving your body forward.
- The force that prevents slipping is *static friction*.
- The static friction force points in the *forward* direction.
- It is static friction that propels you forward!




The rocket pushes the hot gases backward.
The gases push the rocket forward.
Thrust

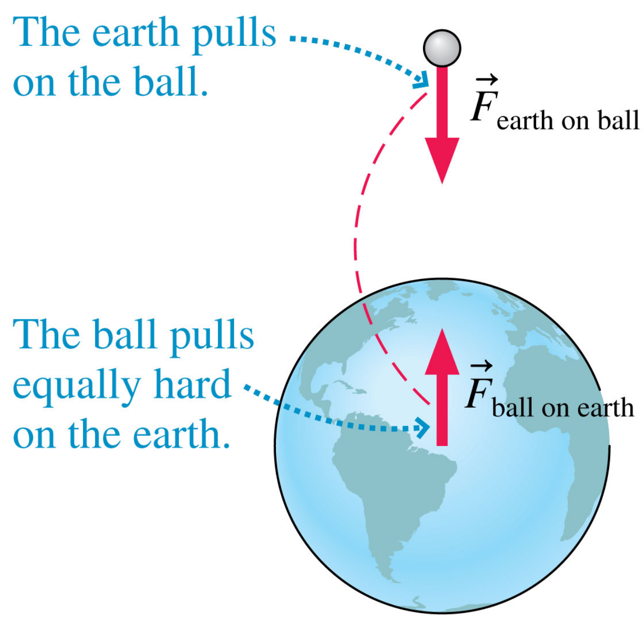

The car pushes backward against the earth.
The earth pushes forward on the car.
Static friction

Newton's third law Every force occurs as one member of an action/reaction pair of forces.

- The two members of an action/reaction pair act on two *different* objects.
- The two members of an action/reaction pair are equal in magnitude but opposite in direction: $\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$.

- When you release a ball, it falls down.
- The action/reaction *forces* of the ball and the earth are *equal* in magnitude.
- The acceleration of the ball is

$$\vec{F}_{\text{earth on ball}} = (\vec{F}_G)_B = -m_B\, g\hat{\jmath}$$



- The acceleration of the earth is

$$\vec{a}_E = \frac{\vec{F}_{\text{ball on earth}}}{m_E} = \frac{m_B\, g\hat{\jmath}}{m_E} = \left(\frac{m_B}{m_E}\right) g\hat{\jmath}$$

- If the ball has a mass of 1 kg, the earth accelerates upward at 2×10^{-24} m/s^2.

A mosquito runs head-on into a truck. Splat! Which is true during the collision?

A. The mosquito exerts more force on the truck than the truck exerts on the mosquito.
B. The truck exerts more force on the mosquito than the mosquito exerts on the truck.
C. The mosquito exerts the same force on the truck as the truck exerts on the mosquito.
D. The truck exerts a force on the mosquito but the mosquito does not exert a force on the truck.
E. The mosquito exerts a force on the truck but the truck does not exert a force on the mosquito.

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E. The mosquito exerts a force on the truck but the truck does not exert a force on the mosquito.

A mosquito runs head-on into a truck. Which is true during the collision?

A. The magnitude of the mosquito's acceleration is larger than that of the truck.
B. The magnitude of the truck's acceleration is larger than that of the mosquito.
C. The magnitude of the mosquito's acceleration is the same as that of the truck.
D. The truck accelerates but the mosquito does not.
E. The mosquito accelerates but the truck does not.

The mosquito will deaccelerate very quickly then will have the same velocity/acceleration as the truck

A mosquito runs head-on into a truck. Which is true during the collision?

- **A. The magnitude of the mosquito's acceleration is larger than that of the truck.** ✔
- B. The magnitude of the truck's acceleration is larger than that of the mosquito.
- C. The magnitude of the mosquito's acceleration is the same as that of the truck.
- D. The truck accelerates but the mosquito does not.
- E. The mosquito accelerates but the truck does not.

Same for both

Newton's second law: $a = \frac{F}{m}$

Huge difference

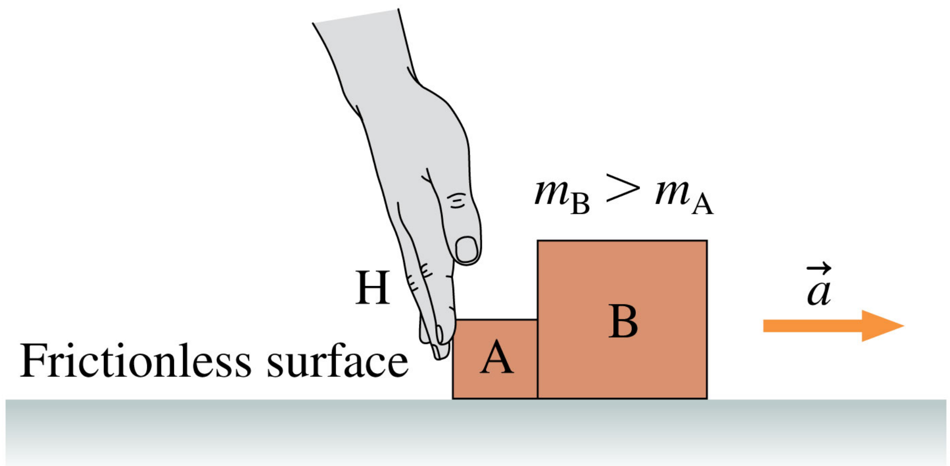
Don't confuse cause and effect! The same force can have very different effects.

EXAMPLE 7.3 | **The forces on accelerating boxes**

The hand shown in **FIGURE 7.12** pushes boxes A and B to the right across a frictionless table. The mass of B is larger than the mass of A.

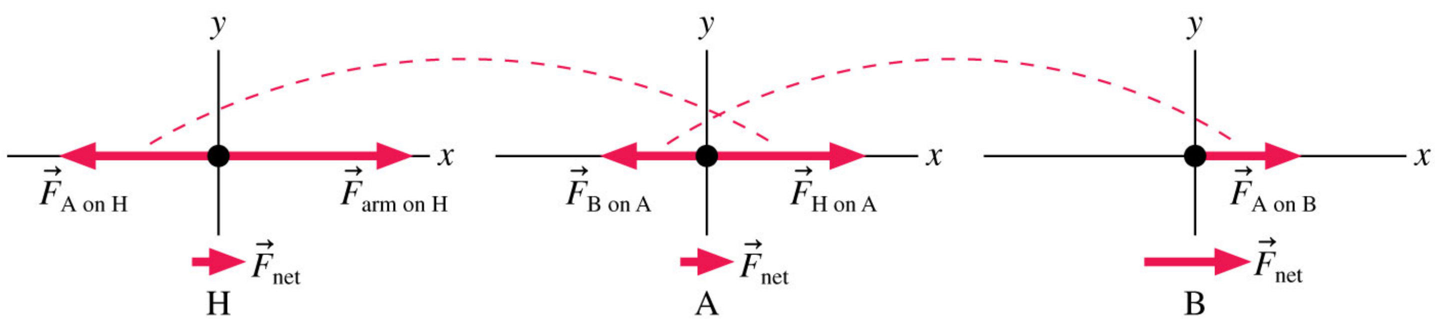
a. Draw free-body diagrams of A, B, and the hand H, showing only the *horizontal* forces. Connect action/reaction pairs with dashed lines.

b. Rank in order, from largest to smallest, the horizontal forces shown on your free-body diagrams.



EXAMPLE 7.3 | **The forces on accelerating boxes**

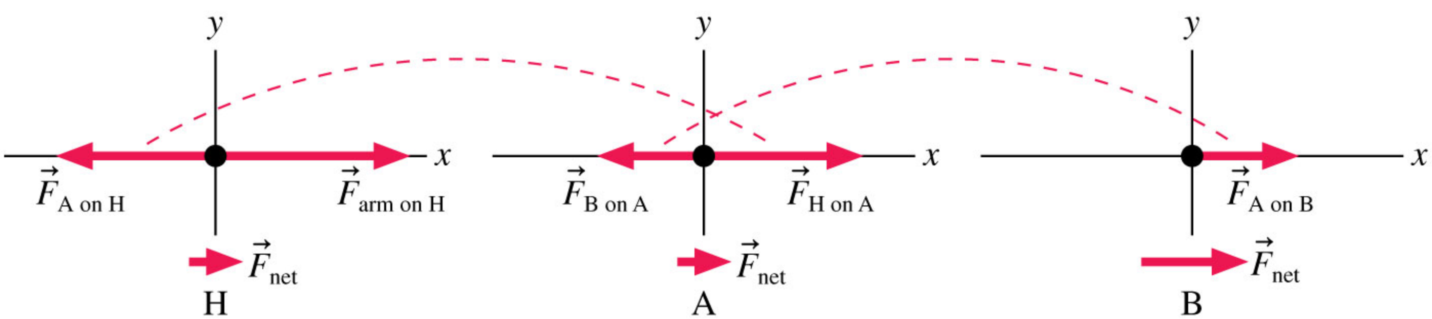
VISUALIZE a. The hand H pushes on box A, and A pushes back on H. Thus $\vec{F}_{\text{H on A}}$ and $\vec{F}_{\text{A on H}}$ are an action/reaction pair. Similarly, A pushes on B and B pushes back on A. **The hand H does not touch box B, so there is no interaction between them.** There is no friction. FIGURE 7.13 on the next page shows five horizontal forces and identifies two action/reaction pairs. Notice that each force is shown on the free-body diagram of the object that it acts *on*.



EXAMPLE 7.3 | **The forces on accelerating boxes**

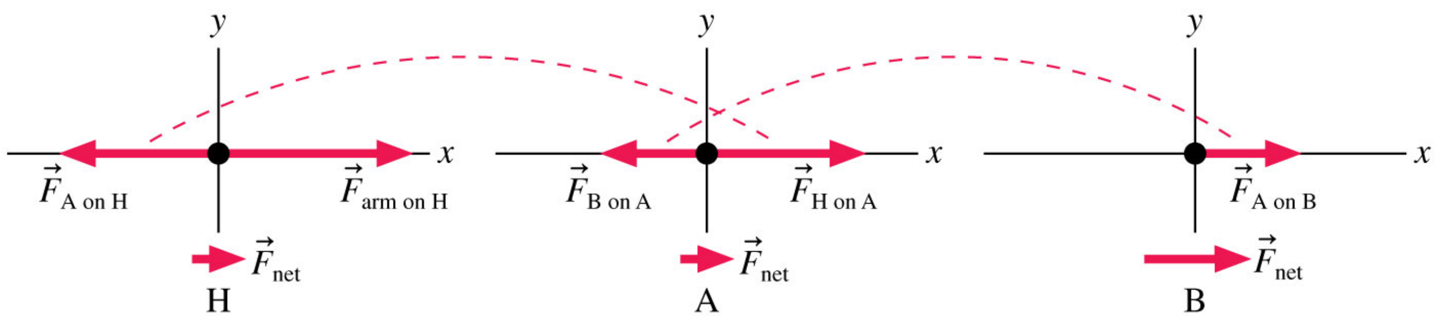
VISUALIZE b. According to Newton's third law, $F_{\text{A on H}} = F_{\text{A on B}}$ and $F_{\text{A on B}} = F_{\text{B on A}}$. But the third law is not our only tool. The boxes are *accelerating* to the right, because there's no friction, so Newton's *second* law tells us that box A must have a net force to the right. Consequently, $F_{\text{H on A}} > F_{\text{B on A}}$. Similarly, $F_{\text{arm on H}} > F_{\text{A on H}}$ is needed to accelerate the hand. Thus

$$F_{\text{arm on H}} > F_{\text{A on H}} = F_{\text{H on A}} > F_{\text{B on A}} = F_{\text{A on B}}$$

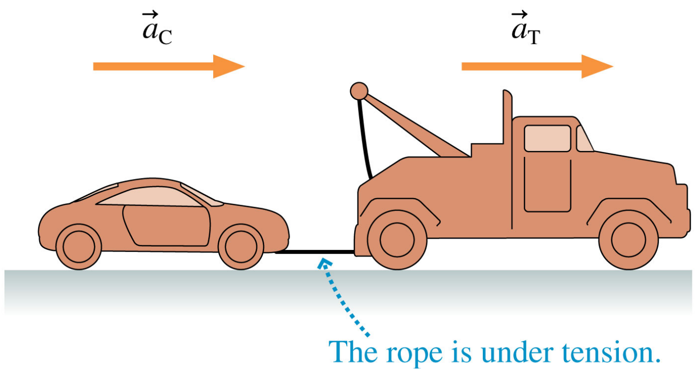


EXAMPLE 7.3 | **The forces on accelerating boxes**

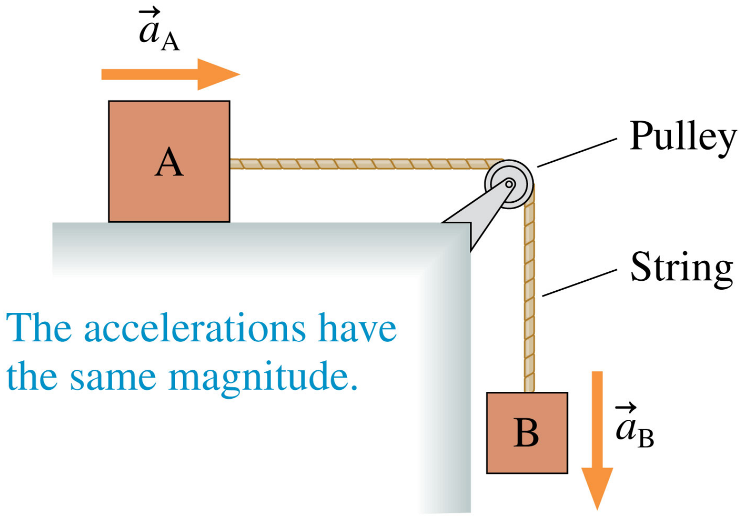
ASSESS You might have expected $F_{\text{A on B}}$ to be larger than $F_{\text{H on A}}$ because $m_B > m_A$. It's true that the *net* force on B is larger than the *net* force on A, but we have to reason more closely to judge the individual forces. Notice how we used both the second and the third laws to answer this question.



- If two objects A and B move together, their accelerations are *constrained* to be equal: $a_{\mathrm{A}} = a_{\mathrm{B}}$
- This equation is called an **acceleration constraint**.
- Consider a car being towed by a truck.
- In this case, the acceleration constraint is $a_{\mathrm{C}x} = a_{\mathrm{T}x} = a_x$.
- Because the accelerations of both objects are equal, we can drop the subscripts C and T and call both of them a_x .



- Sometimes the acceleration of A and B may have different signs.
- Consider the blocks A and B in the figure.
- The string constrains the two objects to accelerate together.



- But, as A moves to the right in the $+x$ direction, B moves down in the $-y$ direction.
- In this case, the acceleration constraint is $a_{Ax} = -a_{By}$.

Problem-Solving Strategy: Interacting-Objects Problems

PROBLEM-SOLVING STRATEGY 7.1 MP

Interacting-objects problems

MODEL Identify which objects are part of the system and which are part of the environment. Make simplifying assumptions.

VISUALIZE Draw a pictorial representation.

- Show important points in the motion with a sketch. You may want to give each object a separate coordinate system. Define symbols, list acceleration constraints, and identify what the problem is trying to find.
- Draw an interaction diagram to identify the forces on each object and all action/reaction pairs.
- Draw a *separate* free-body diagram for each object showing only the forces acting *on* that object, not forces exerted by the object. Connect the force vectors of action/reaction pairs with dashed lines.

Problem-Solving Strategy: Interacting-Objects Problems

What is the 2nd law? 3rd law? Mathematically?

PROBLEM-SOLVING STRATEGY 7.1

Interacting-objects problems

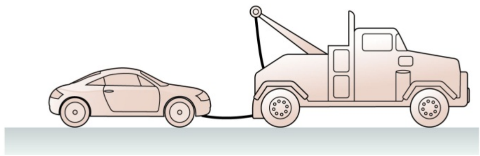
SOLVE Use Newton's second and third laws.

- Write the equations of Newton's second law for *each* object, using the force information from the free-body diagrams.
- Equate the magnitudes of action/reaction pairs.
- Include the acceleration constraints, the friction model, and other quantitative information relevant to the problem.
- Solve for the acceleration, then use kinematics to find velocities and positions.

ASSESS Check that your result has the correct units and significant figures, is reasonable, and answers the question.

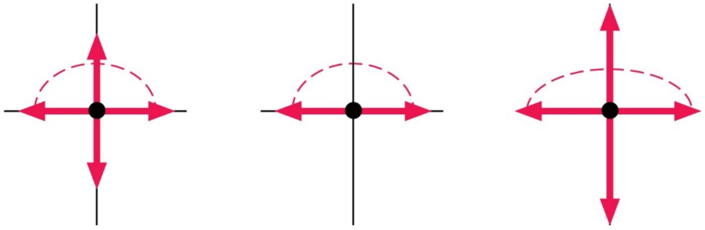
What are possible friction models and which ones will you need to know for the exam?

What, if anything, is wrong with these free-body diagrams for a truck towing a car <u>at steady speed</u>? The truck is heavier than the car and the rope is massless.



$$\sum F_x = 0$$

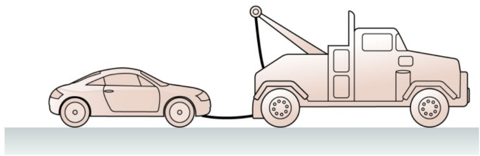
$$\sum F_y = 0$$

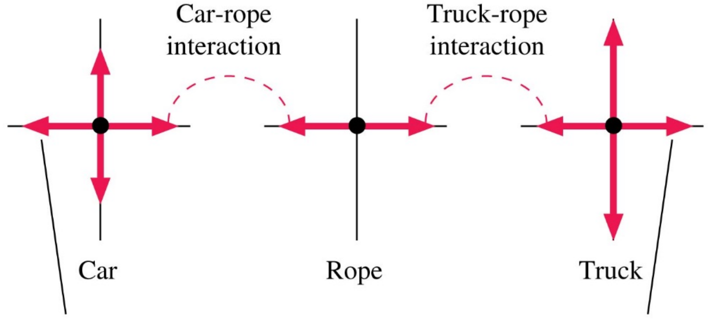


A. Nothing is wrong.
B. One or more forces have the wrong length.
C. One of more forces have the wrong direction.
D. One or more action/reaction pairs are wrong.
E. Both B and D.

What does it mean in terms of acceleration and forces that they are moving at a constant speed?

What, if anything, is wrong with these free-body diagrams for a truck towing a car at steady speed? The truck is heavier than the car and the rope is massless.





A. Nothing is wrong.
B. One or more forces have the wrong length.
C. One of more forces have the wrong direction.
D. One or more action/reaction pairs are wrong.
E. Both B and D.

Force produced by the engine of the truck does not act on the truck, it acts on the ground.

A car is parked at rest on a horizontal road. The upward force of the road on the car (the normal force) is the same size as the downward pull of gravity

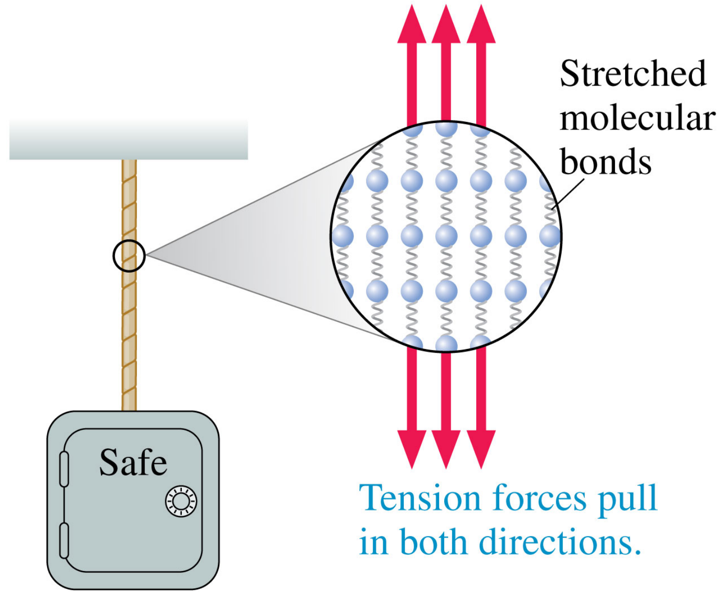
A. Because they are an action/reaction pair.
B. Because of Newton's first law.
C. Both A and B.
D. Neither A nor B. Google it.

A car is parked at rest on a horizontal road. The upward force of the road on the car (the normal force) is the same size as the downward pull of gravity

A. Because they are an action/reaction pair.
✔ **B. Because of Newton's first law.**
C. Both A and B.
D. Neither A nor B. Google it.

What is Newton's first law?
If the acceleration is zero, sum of forces is zero, forces have to be equal
What are the action/reaction pairs in the system?

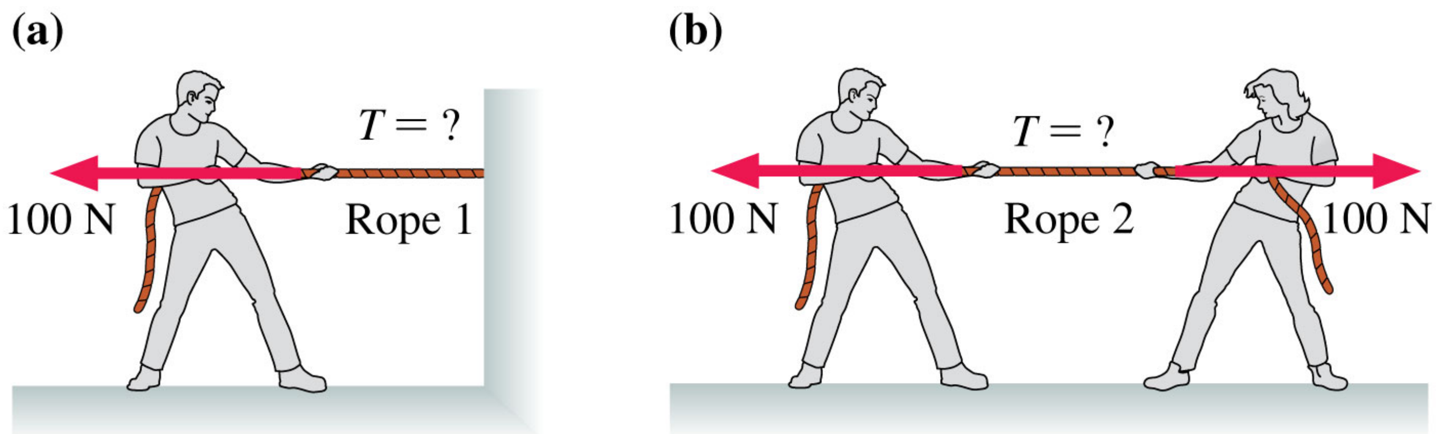
- The figure shows a heavy safe hanging from a rope, placing the rope under tension.
- We introduced an atomic-level model in which tension is due to the stretching of spring-like molecular bonds within the rope.



- Stretched springs exert pulling forces, and the combined pulling force of billions of stretched molecular springs in a string or rope is what we call **tension.**
- An important aspect of tension is that it pulls equally in both directions. Can you picture this?

EXAMPLE 7.5 Pulling a rope

FIGURE 7.18a shows a student pulling horizontally with a 100 N force on a rope that is attached to a wall. In **FIGURE 7.18b**, two students in a tug-of-war pull on opposite ends of a rope with 100 N each. Is the tension in the second rope larger than, smaller than, or the same as that in the first rope?



EXAMPLE 7.5 | **Pulling a rope**

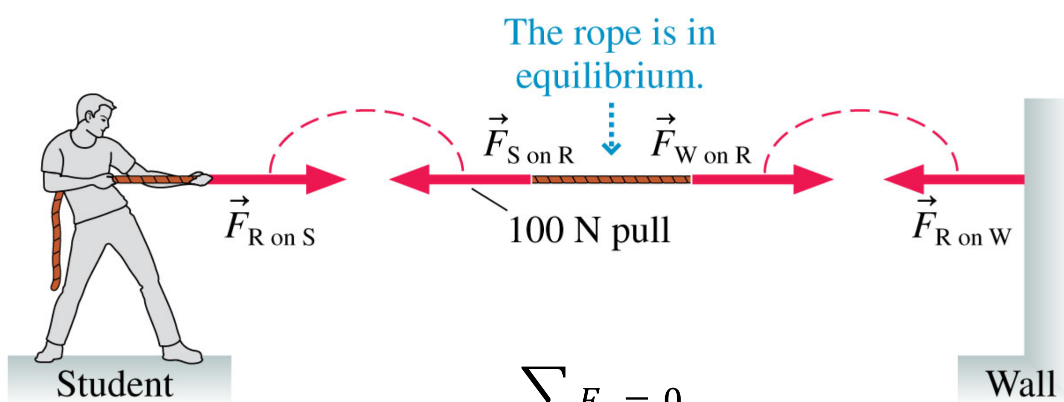
SOLVE Surely pulling on a rope from both ends causes more tension than pulling on one end. Right? Before jumping to conclusions, let's analyze

FIGURE 7.19a shows the first student, the rope, and the wall as separate, interacting objects. Force $\vec{F}_{\text{S on R}}$ is the student pulling on the rope, so it has magnitude 100 N. Forces $\vec{F}_{\text{S on R}}$ and $\vec{F}_{\text{R on S}}$ are an action/reaction pair and must have equal magnitudes. Similarly for forces $\vec{F}_{\text{W on R}}$ and $\vec{F}_{\text{R on W}}$. Finally, because the rope is in static equilibrium, *force* $\vec{F}_{\text{W on R}}$ *has to balance force* $\vec{F}_{\text{S on R}}$. Thus

$$F_{\text{R on W}} = F_{\text{W on R}} = F_{\text{S on R}} = F_{\text{R on S}} = 100\ \text{N}$$

The first and third equalities are Newton's third law; the second equality is Newton's first law for the rope.

Forces $\vec{F}_{\text{R on S}}$ and $\vec{F}_{\text{R on W}}$ are the pulling forces exerted by the rope and are what we *mean* by "the tension in the rope." Thus the tension in the first rope is 100 N.



$$\sum F_x = 0$$

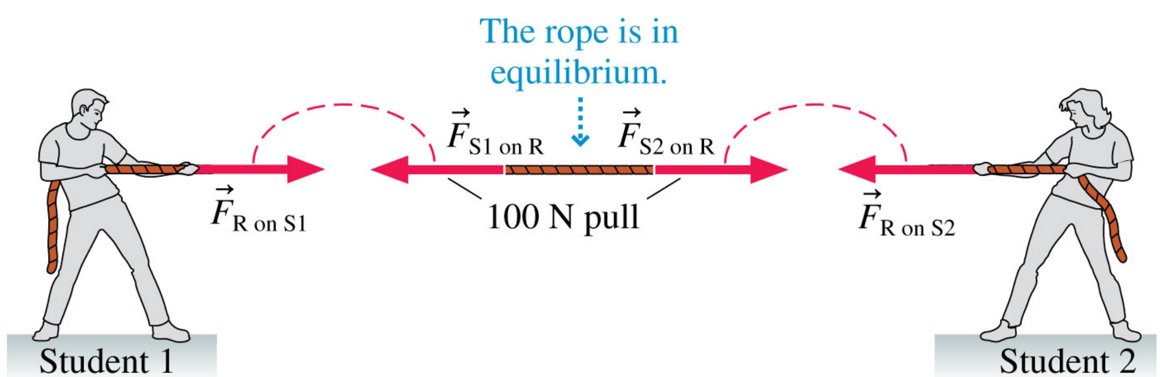
EXAMPLE 7.5 | **Pulling a rope**

SOLVE **FIGURE 7.19b** repeats the analysis for the rope pulled by two students. Each student pulls with 100 N, so $F_{\text{S1 on R}} = 100$ N and $F_{\text{S2 on R}} = 100$ N. Just as before, there are two action/reaction pairs rope is in static equilibrium. Thus

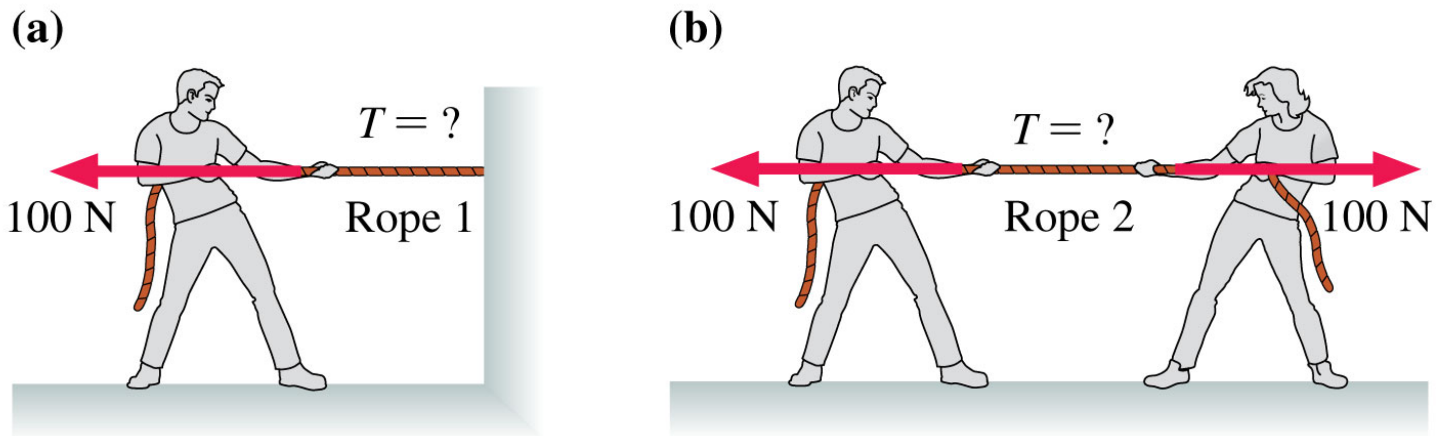
$$F_{\text{R on S2}} = F_{\text{S2 on R}} = F_{\text{S1 on R}} = F_{\text{R on S1}} = 100\text{ N}$$

The tension in the rope—the pulling forces $\vec{F}_{\text{R on S1}}$ and $\vec{F}_{\text{R on S2}}$—is still 100 N!

You may have *assumed* that the student on the right in Figure 7.18b is doing something to the rope that the wall in Figure 7.18a does not do. But our analysis finds that the wall, just like the student, pulls to the right with 100 N. The rope doesn't care whether it's pulled by a wall or a hand. It experiences the same forces in both cases, so the rope's tension is the same in both.



$$\sum F_x = 0$$

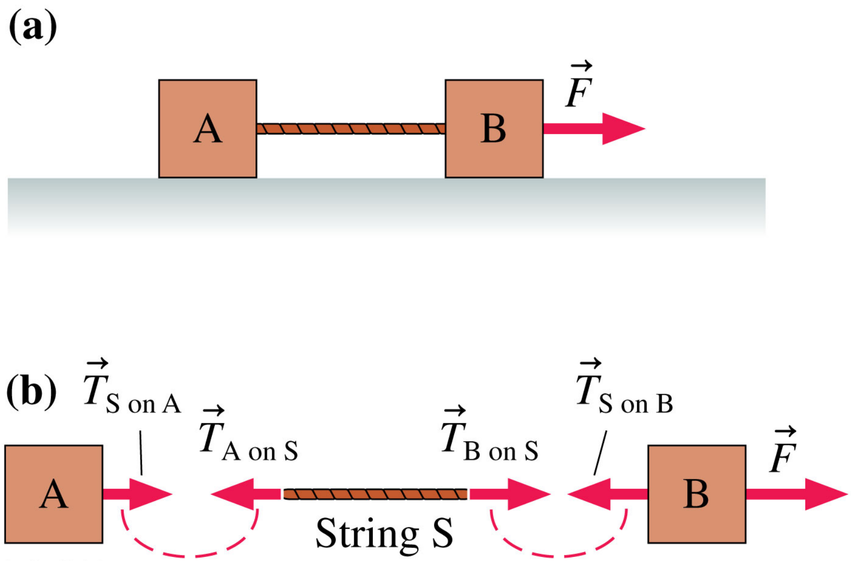


The rope’s tension is the same in both situations.

EXAMPLE 7.5 | **Pulling a rope**

ASSESS Ropes and strings exert forces at *both* ends. The force with which they pull—and thus the force pulling on them at each end—*is* the tension in the rope. Tension is not the sum of the pulling forces.

- Often in problems the mass of the string or rope is much less than the masses of the objects that it connects.
- In such cases, we can adopt the following **massless string approximation**:

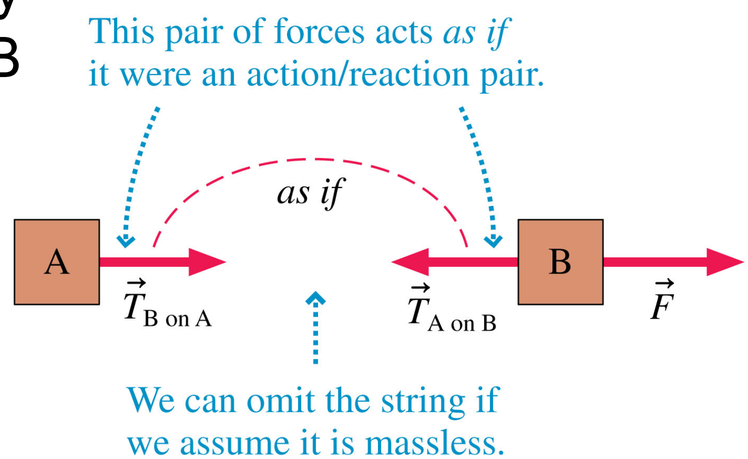


$$T_{\text{B on S}} = T_{\text{A on S}} \qquad \text{(massless string approximation)}$$

So the string is just transmitting the force

- Two blocks are connected by a massless string, as block B is pulled to the right.
- Forces $\vec{T}_{\text{S on A}}$ and $\vec{T}_{\text{S on B}}$ act *as if* they are an action/reaction pair:

$$\vec{T}_{\text{S on A}} = -\vec{T}_{\text{S on B}}$$

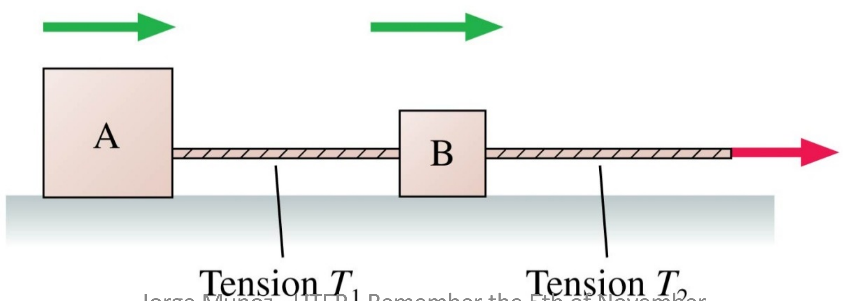


- All a massless string does is transmit a force from A to B without changing the magnitude of that force.
- **For problems in class, you can assume that any strings or ropes are massless unless it explicitly states otherwise**.

Boxes A and B are being pulled to the right on a frictionless surface. Box A has a larger mass than B. How do the two tension forces compare?

A. $T_1 > T_2$
B. $T_1 = T_2$
C. $T_1 < T_2$
D. Not enough information to tell.

What are the constraints?
How do the free-body diagrams look like?



Boxes A and B are being pulled to the right on a frictionless surface. Box A has a larger mass than B. How do the two tension forces compare?

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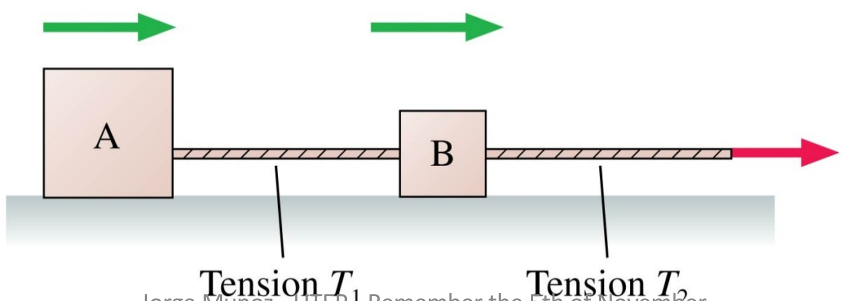
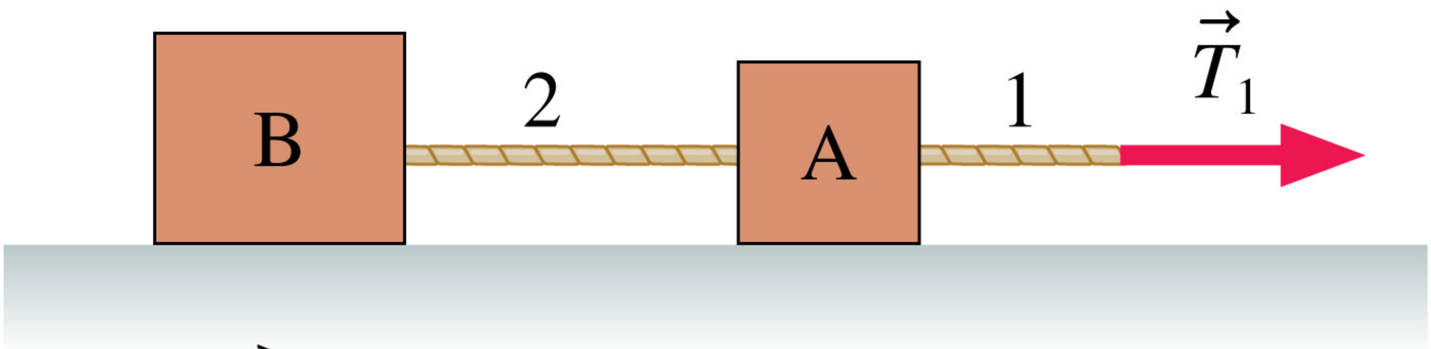
B. $T_1 = T_2$
C. $\boldsymbol{T_1 < T_2}$
D. Not enough information to tell.



EXAMPLE 7.6 Comparing two tensions

Blocks A and B in FIGURE 7.22 are connected by massless string 2 and pulled across a frictionless table by massless string 1. B has a larger mass than A. Is the tension in string 2 larger than, smaller than, or equal to the tension in string 1?

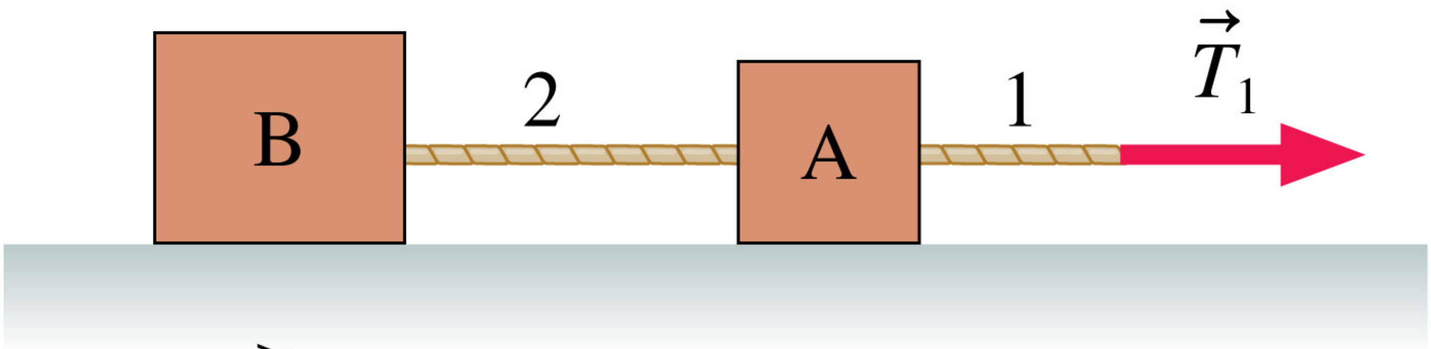


$m_B > m_A$

EXAMPLE 7.6 | **Comparing two tensions**

MODEL The massless string approximation allows us to treat A and B *as if* they interact directly with each other. The blocks are accelerating because there's a force to the right and no friction.

SOLVE B has a larger mass, so it may be tempting to conclude that string 2, which pulls B, has a greater tension than string 1, which pulls A. The flaw in this reasoning is that Newton's second law tells us only about the *net* force. The net force on B *is* larger than the net force on A, but the net force on A is *not* just the tension $\vec{T}_1$ in the forward direction. The tension in string 2 also pulls *backward* on A!



$$m_B > m_A$$

$$\sum F_x = m_A a = -T_2 + T_1$$

EXAMPLE 7.6 | **Comparing two tensions**

FIGURE 7.23 shows the horizontal forces in this frictionless situation. Because the string is massless, forces $\vec{T}_{\text{A on B}}$ and $\vec{T}_{\text{B on A}}$ act *as if* they are an action/reaction pair.

From Newton's third law,

$$T_{\text{A on B}} = T_{\text{B on A}} = T_2$$

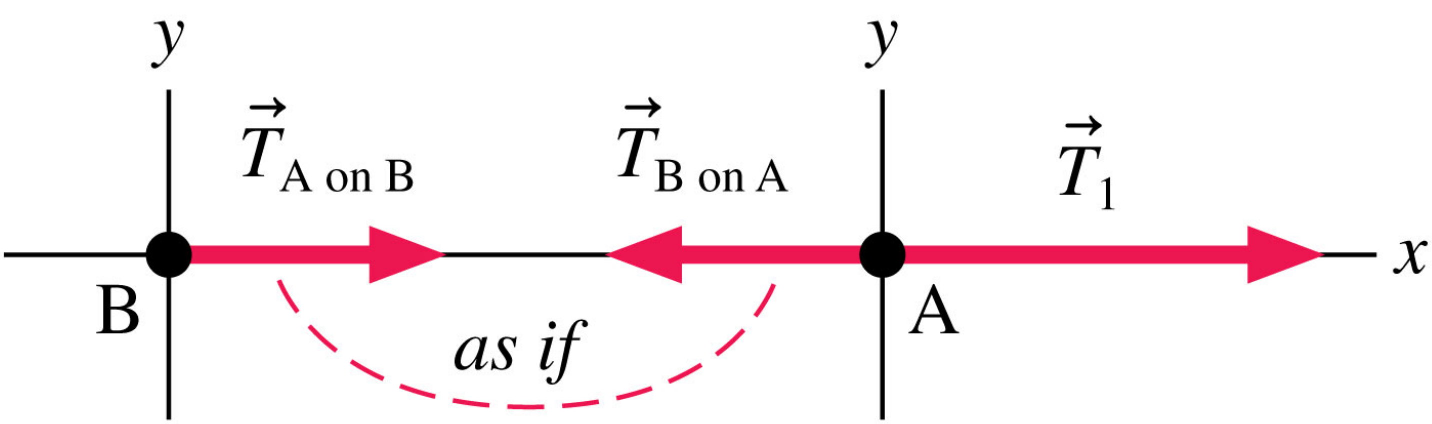
where is the tension in string 2. From Newton's second law, the net force on A is

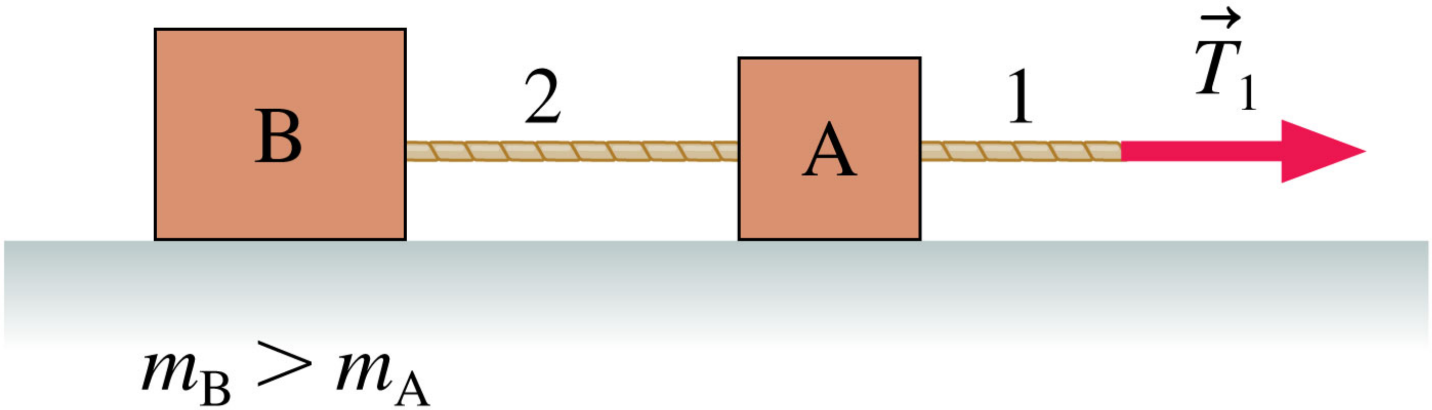
$$(F_{\text{A net}})_x = T_1 - T_{\text{B on A}} = T_1 - T_2 = m_{\text{A}} a_{\text{A}x}$$

The net force on A is the *difference* in tensions. The blocks are accelerating to the right, making $a_{\text{A}x} > 0$, so

$$T_1 > T_2$$

The tension in string 2 is *smaller* than the tension in string 1.





$$T_1 > T_2$$

EXAMPLE 7.6 **Comparing two tensions**

ASSESS This is not an intuitively obvious result. A careful study of the reasoning in this example is worthwhile. An alternative analysis would note that $\vec{T}_1$ accelerates *both* blocks, of combined mass $(m_A + m_B)$, whereas $\vec{T}_2$ accelerates only block B. Thus string 1 must have the larger tension.

Boxes A and B are sliding to the right on a frictionless surface. Hand H is slowing them. Box A has a larger mass than B. Considering only the *horizontal* forces,

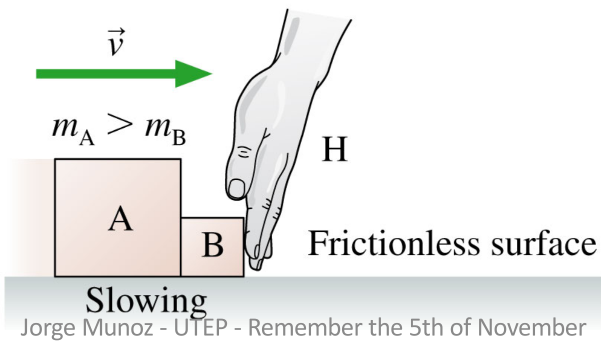
Which pairs of forces have to be the same?

A. $F_{\text{B on H}} = F_{\text{H on B}} = F_{\text{A on B}} = F_{\text{B on A}}$

B. $F_{\text{B on H}} = F_{\text{H on B}} > F_{\text{A on B}} = F_{\text{B on A}}$

C. $F_{\text{B on H}} = F_{\text{H on B}} < F_{\text{A on B}} = F_{\text{B on A}}$

D. $F_{\text{H on B}} = F_{\text{H on A}} > F_{\text{A on B}}$



Boxes A and B are sliding to the right on a frictionless surface. Hand H is slowing them. Box A has a larger mass than B. Considering only the *horizontal* forces,

Which pairs of forces have to be the same?

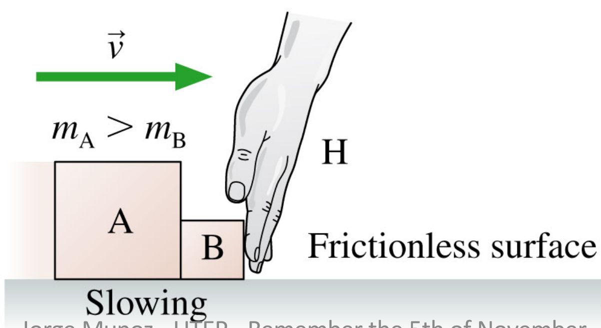
A. $F_{\text{B on H}} = F_{\text{H on B}} = F_{\text{A on B}} = F_{\text{B on A}}$

B. $F_{\text{B on H}} = F_{\text{H on B}} > F_{\text{A on B}} = F_{\text{B on A}}$

C. $F_{\text{B on H}} = F_{\text{H on B}} < F_{\text{A on B}} = F_{\text{B on A}}$

D. $\cancel{F_{\text{H on B}} = F_{\text{H on A}} > F_{\text{A on B}}}$

$F_{\text{H on B}} = F_{\text{B on H}}$
$F_{\text{B on A}} = F_{\text{A on B}}$



Boxes A and B are sliding to the right on a frictionless surface. Hand H is slowing them. Box A has a larger mass than B. Considering only the *horizontal* forces,

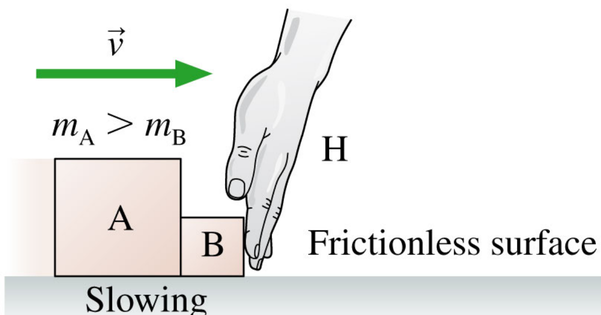
How do the free-body diagrams look like?

A. $F_{\text{B on H}} = F_{\text{H on B}} = F_{\text{A on B}} = F_{\text{B on A}}$

B. $F_{\text{B on H}} = F_{\text{H on B}} > F_{\text{A on B}} = F_{\text{B on A}}$

C. $F_{\text{B on H}} = F_{\text{H on B}} < F_{\text{A on B}} = F_{\text{B on A}}$

D. $\cancel{F_{\text{H on B}} = F_{\text{H on A}} > F_{\text{A on B}}}$



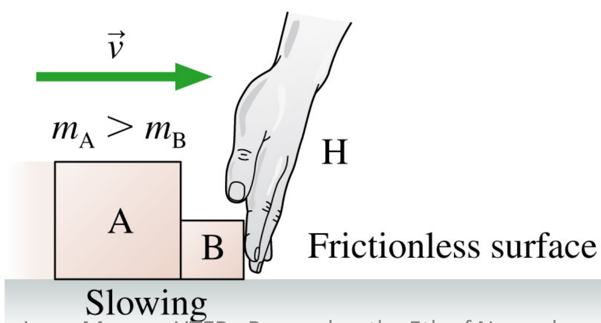
Boxes A and B are sliding to the right on a frictionless surface. Hand H is slowing them. Box A has a larger mass than B. Considering only the *horizontal* forces,

A. $\cancel{F_{\text{B on H}} = F_{\text{H on B}} = F_{\text{A on B}} = F_{\text{B on A}}}$

B. $F_{\text{B on H}} = F_{\text{H on B}} > F_{\text{A on B}} = F_{\text{B on A}}$

C. $\cancel{F_{\text{B on H}} = F_{\text{H on B}} < F_{\text{A on B}} = F_{\text{B on A}}}$

D. $\cancel{F_{\text{H on B}} = F_{\text{H on A}} > F_{\text{A on B}}}$



How do the free-body diagrams look like?

For B to accelerate in the negative direction,

$F_{\text{H on B}} > F_{\text{A on B}}$

Boxes A and B are sliding to the right on a frictionless surface. Hand H is slowing them. Box A has a larger mass than B. Considering only the *horizontal* forces,

A. $F_{\text{B on H}} = F_{\text{H on B}} = F_{\text{A on B}} = F_{\text{B on A}}$

B. $\boldsymbol{F_{\text{B on H}} = F_{\text{H on B}} > F_{\text{A on B}} = F_{\text{B on A}}}$

C. $F_{\text{B on H}} = F_{\text{H on B}} < F_{\text{A on B}} = F_{\text{B on A}}$

D. $F_{\text{H on B}} = F_{\text{H on A}} > F_{\text{A on B}}$