University of Birmingham
School of Mathematics
Vectors, Geometry and Linear Algebra
VGLA

Problem Sheet 2

Remember that there are practise questions under the materials section for each week.

SUM **Q1**. Suppose that $z_1 = 1 + i$ and $z_2 = 3 - 2i$.

(i) Calculate $z_1 + z_2$ and $z_1 z_2$ giving your answer in the form $a + bi$ for appropriate real numbers a and b.
(ii) Calculate z_1^8 in both modulus-argument form and exponential form giving the principal value of the argument.
(iii) Find all the fifth roots of z_1. Present your answer in exponential form giving the principal value of the argument.
(iv) Calculate β such that $z_1^2 + \beta z_1 + \beta = 0$.
(v) Let $\beta \in \mathbb{C}$, find all $z \in \mathbb{C}$ such that $z^2 + \beta z + \frac{\beta^2}{4} = 0$.
(vi) The circle of radius r in the xy-plane centred at $(a, b) \in \mathbb{R}^2$ is described by the equation $(x - a)^2 + (y - b)^2 = r^2$.
Describe the solution set to the equation

$$|z - 1| = 4|z - 3|$$

as a geometrically defined subset of the Argand diagram.
(vii) Suppose that $n \geq 2$ and set $R_n = \{r_1, \ldots, r_n\}$ to be the set of the n distinct roots of the equation $z^n + 1 = 0$. Let r be a root of $z^n - 1 = 0$. Show that $R_n = \{rr_1, \ldots, rr_n\}$. Deduce that $\sum_{i=1}^{n} r_i = 0$.

SUM **Q2**. (i) Using the augmented matrix and row operations, find the solution set of the following system of real simultaneous linear equations.

$$\begin{array}{ccccccc} 2x & + & y & + & z & = & 5 \\ x & + & 3y & + & 2z & = & 1 \\ 3x & + & 4y & + & 3z & = & 6. \end{array}$$

(ii) Suppose that $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 2 & 5 \end{pmatrix}$.
(a) Determine $\mathbf{A}^{-1}$.
(b) Solve the system of simultaneous linear equation

$$\mathbf{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 2 \end{pmatrix}.$$