University of Birmingham
School of Mathematics

Vectors, Geometry and Linear Algebra
VGLA

Problem Sheet 1

Weeks 1&2

$\boxed{\text{SUM}}$ **Q1**. Suppose that $\mathbf{a} = (2, 1, 5)$, $\mathbf{b} = (1, 2, 3)$ and $\mathbf{c} = (1, 1, 1)$ are vectors. Throughout your answers be careful to distinguish points, vectors and scalars.

(i) Calculate $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$.
(ii) Calculate $(\mathbf{a} \cdot \mathbf{c})\mathbf{a} - (\mathbf{b} \cdot \mathbf{c})\mathbf{b}$.
(iii) Determine $\text{proj}_{\mathbf{a}}(\mathbf{c})$.
(iv) Find $\lambda \in \mathbb{R}$ such that $\mathbf{b} + \lambda\mathbf{c}$ is perpendicular to $\mathbf{a}$.
(v) Write down the set of points on the line which has direction vector parallel to $\mathbf{a}$ and passes through the point $\mathbf{d} = (2, 3, 4)$.
(vi) Describe the points of the plane Π perpendicular to $\mathbf{a}$ containing the point $P = (1, 3, 3)$.

Q2. Suppose that $\triangle ABC$ is a triangle in $\mathbb{R}^2$ with vertices A, B and C. Show that the perpendicular bisectors of the sides go through a common point. You may like to use the following plan of a proof:

- Choose an origin to be the intersection of the perpendicular bisector of two of the sides of $\triangle ABC$ say AB and BC.
- With respect to this origin write down position vectors for all the vertices of $\triangle ABC$ and midpoints of sides of the triangle.
- Use the fact that the position vector through of the midpoint of line segment AB is perpendicular to $\vec{AB}$ and the position vector of the midpoint of the line segment BC is perpendicular to $\vec{BC}$ to deduce that the origin is the same distance from each vertex of $\triangle ABC$.
- Solve the problem.

$\boxed{\text{SUM}}$ **Q3**. Find the line of intersection of the planes

$$\{(x, y, z) \in \mathbb{R}^3 : x + 2y + z = 3\}$$

and

$$\{(x, y, z) \in \mathbb{R}^3 : x + y + 2z = 4\}.$$