

Example sheet 5 – formative

1. Consider the nonlinear dynamical system

$$\begin{aligned}\dot{x} &= x - 2y, \\ \dot{y} &= 4x - x^3.\end{aligned}$$

- (a) Determine the equilibrium points of the system.
- (b) By constructing the Jacobian matrix, classify the equilibrium points found in part (a).
- (c) Sketch the phase portrait of the system. Note the location of the isoclines as broken red lines on the phase portrait.

2. Locate the equilibrium points of the nonlinear dynamical system,

$$\begin{aligned}\dot{x} &= x(x - y - 3), \\ \dot{y} &= y(x - 5).\end{aligned}\tag{1}$$

Classify the equilibrium points and sketch the phase portrait of (1).

3. The damped pendulum equation

$$I\ddot{\theta} + \mu\dot{\theta} + mgl \sin \theta = 0,$$

can be written in the form

$$\ddot{x} + \varepsilon\dot{x} + k^2 \sin x = 0,$$

where $\varepsilon > 0$.

- (a) Write the damped pendulum equation as a system of first order differential equations, and establish the location of the equilibrium points.
- (b) Calculate the Jacobian matrix for the nonlinear system.
- (c) Use the Jacobian matrix to classify the equilibrium points.
- (d) In the case $\varepsilon = k = 1$, sketch the phase portrait of the system for $x \in (-4, 4)$. Note the location of the isoclines as dotted lines on the phase portrait.