

## Example sheet 1 - formative

1. Turn the following ODEs into autonomous dynamical systems:

(a)  $\ddot{x} - x^2 + x \dot{x} = 0$

(b)  $\ddot{x} - \dot{x}(1-x)(2-x)\ddot{x} = 0$

(c)  $\ddot{x} = \cos x - t x$

2. For the following autonomous systems find the equilibrium points and sketch the phase line. Then use a graphical argument to classify the stability of the equilibrium points.

(a)  $\dot{x} = x^2 - 1$

(b)  $\dot{x} = x(1-x)(2-x)$

(c)  $\dot{x} = \cos x$

3. Use linear stability analysis to classify the fixed points of the following systems. If linear stability analysis fails (because  $f'(x^*) = 0$ ), then use a graphical argument.

(a)  $\dot{x} = x(2-x)$

(b)  $\dot{x} = 1 - e^{-x}$

(c)  $\dot{x} = \log x$

(d)  $\dot{x} = x^4$

4. For the dynamical systems in question two, sketch the direction fields on the following domains,

(a)  $(x, t) \in (-5, 5) \times (0, 10)$

(b)  $(x, t) \in (-1, 3) \times (0, 10)$

(c)  $(x, t) \in (-2\pi, 2\pi) \times (0, 10)$

5. The Logistic equation,

$$\frac{dn}{dt} = n(a - bn), \quad n(t) \geq 0, b \neq 0$$

for some constants  $a$  and  $b$ , was first introduced as a way of describing population growth by Verhulst in 1838, where  $n(t)$  is the size of the population at time  $t$ .

(a) Find the equilibrium points of the logistic equation. Then, using linear stability analysis, establish the stability of the equilibrium points of the logistic equation for all non-zero values of  $a$  and  $b$ .

- (b) For the case  $a = b = 1$ , by considering  $\dot{n}(t)$  at various points, sketch the direction field for the Logistic equation for  $0 \leq n \leq 2$ ,  $t > 0$ . Then use this sketch to draw the solution curves for the system.
- (c) Finally, using part (b), describe what happens to the size of a population which at time  $t = 0$  is given by  $n(0) = 0.1$ .