

Example sheet 6 – formative

1. Locate the equilibrium points of the nonlinear dynamical system,

$$\begin{aligned}\dot{x} &= \mu - xy^2, \\ \dot{y} &= xy^2 - y,\end{aligned}$$

where $\mu > 0$ is a constant. Classify the equilibrium points for each $\mu > 0$.

2. Consider the nonlinear dynamical system

$$\begin{aligned}\dot{x} &= 2x - ay + y^2, \\ \dot{y} &= 4bx + y - \frac{1}{2}x^2,\end{aligned}$$

where $(x, y) \in \mathbb{R}^2$, and a, b are constants.

- (a) Determine the nature of the equilibrium point $(0, 0)$ for all $a, b \geq 0$. Sketch the (a, b) -plane.
- (b) For the case $a = 5$, $b = \frac{1}{8}$, classify each of the equilibrium points and sketch the global phase portrait of the system. You should make sure to note the following on your sketch:
 - Location of equilibrium points;
 - Straight line paths and which eigenvector they correspond to (if applicable);
 - Horizontal and vertical isoclines, and the direction of flow along them.

Make sure to give yourself enough room to sketch the phase portrait clearly, with clear labelling.