

Problem Sheet 2 (Combinatorics part)

Assignment available: Friday 13 October 2023 (Week 3).

Submission deadline: 1700 on Wednesday 27 October 2023 (Week 5).

Required content: All necessary content will be covered by the end of the Week 3 lectures.

About problem sheet questions: The comments made about the first problem sheet apply equally to this one. In particular, please don't be reluctant to seek help if you are unsure how to proceed towards a solution, or how to express your ideas, as (unlike exam questions) the questions are set on the basis that you have access to this support.

Question 1 (SUM). Let X be the set of positive integers which are factors of 63504. Calculate:

- (a) the size of X ;
- (b) the number of elements of X which are divisible by 4;
- (c) the number of elements of X which are divisible either by 4 or by 9.

Question 2 (SUM). How many integers n are there with $1,000 \leq n < 10,000$ which do not have 8 as a digit?

Question 3. Let $X = \{1, 2, 3\}$ and $Y = \{1, 2, \dots, 10\}$.

- (a) How many functions are there from X to Y ?
- (b) How many of these functions are injective?
- (c) How many injections from X to Y have 1 as a fixed point?
- (d) How many injections from X to Y have no fixed points?

(A fixed point of a function $f : X \rightarrow Y$ is an element $x \in X$ with $f(x) = x$, i.e. which maps to itself.)

Question 4. Prove the general form of the inclusion-exclusion formula (this is Theorem 2.7 from the Week 2 lecture notes; you can use either form of the theorem).

An outline of how to proceed was given in Lecture 4, and also appears in the Week 2 lecture notes. As noted there the main difficulty is notational: you need to handle a large number of terms in a clear and understandable way.