Problem sheet 3

Question 9. (SUM)

(a) An airline estimates that among the plane tickets it sells, 5% of the customers do not show up. Aiming to take advantage of this the airline sells 80 tickets for a plane with 78 seats. What is the probability that nobody will be denied boarding?

Find the exact value and an estimate, using the law of small numbers.

(b) Every Friday night I order a delivery takeaway. Everything usually goes perfectly, but sometimes things go wrong :(. The food is cold with probability 0.1. Some part of the order is forgotten with probability 0.2. The delivery driver gets lost and calls for directions with probability 0.3.

Let Y denote the number of Fridays needed until everything goes perfectly with my order. Assuming that all the events are independent:

(i) find the probability that everything goes perfectly on the first Friday night, i.e. that $Y = 1$.

(ii) determine the distribution that Y follows.

(iii) calculate the probability that Y is a multiple of three.

(c) Suppose X is a Poisson random variable with parameter $\lambda > 0$, that Y is a geometric random variable with parameter $1/2$, and that X and Y are independent. Show that $\mathbb{P}\big(X = Y\big) = e^{-\lambda/2} - e^{-\lambda}$.

Question 10. Let X, Y be independent discrete random variables with $X \sim \mathrm{Poi}_\lambda$ and $Y \sim \mathrm{Poi}_\mu$ where $\lambda, \mu > 0$.

(a) Show that $X + Y \sim \mathrm{Poi}_{\lambda+\mu}$.

(b) If $\lambda = \mu > 0$, show that for integers $n \geq k \geq 0$, the conditional probability

$$\mathbb{P}\big(X = k \,|\, X + Y = n\big) = \binom{n}{k} 2^{-n}.$$