

## Example sheet 6 – formative

1. Locate the equilibrium points of the nonlinear dynamical system,

$$\begin{aligned}\dot{x} &= \mu - xy^2, \\ \dot{y} &= xy^2 - y,\end{aligned}$$

where  $\mu > 0$  is a constant. Classify the equilibrium points for each  $\mu > 0$ .

2. Consider the nonlinear dynamical system

$$\begin{aligned}\dot{x} &= 2x - ay + y^2, \\ \dot{y} &= 4bx + y - \frac{1}{2}x^2,\end{aligned}$$

where  $(x, y) \in \mathbb{R}^2$ , and  $a, b$  are constants.

- (a) Determine the nature of the equilibrium point  $(0, 0)$  for all  $a, b \geq 0$ . Sketch the  $(a, b)$ -plane.
- (b) For the case  $a = 5, b = \frac{1}{8}$ , classify each of the equilibrium points and sketch the global phase portrait of the system. You should make sure to note the following on your sketch:
- Location of equilibrium points;
  - Straight line paths and which eigenvector they correspond to (if applicable);
  - Horizontal and vertical isoclines, and the direction of flow along them.

Make sure to give yourself enough room to sketch the phase portrait clearly, with clear labelling.