Problem sheet 2

Question 5. A lie detector returns a positive result when a person is lying in 90% of all cases. Unfortunately, it also results a positive result when a person is telling the truth in 20% of all cases. Statistically, it is estimated that 10% of all test subjects lie.

(a) A randomly chosen test subject takes a test and obtains a positive result. What is the probability that the subject is lying?

(b) A randomly chosen test subject takes a test and obtains a negative result. What is the probability that the subject told the truth?

Question 6. (SUM)

(a) Three couples are invited to a dinner party and attend independently with probabilities 0.9, 0.7 and 0.5. Determine the probability

(i) that no couple attends;

(ii) that exactly one couple attends.

(b) Let $\mathbb{P}$ be a probability distribution on a sample space Ω and suppose that $A, B, C \subseteq \Omega$ are three independent events.

(i) Show that A and $B \cup C$ are independent.
Hint. It is helpful to write $A \cap (B \cup C)$ as the union of two events.

(ii) Must $A \cup B$ and $B \cup C$ also be independent?
(You should either show that this is true or give a counterexample.)

(c) Every week a family visits their favourite restaurant. The restaurant has 15 tables, and on each visit the family is seated at a table chosen uniformly at random, with all choices being independent. Show that the probability is at least $1/2$ that the family will sit at every table in the restaurant over the course of a year.

Question 7. Two balls are drawn one-by-one without replacement from an urn containing ten balls labelled from 1 to 10.

(a) Find an appropriate sample space Ω and a suitable probability distribution $\mathbb{P}$ to model this experiment. Define random variables $X, Y : \Omega \to \mathbb{R}$, where X, Y accounts for the smaller (larger, respectively) of the two drawn balls. State S_X and S_Y.

(b) State the two events $\{Y = 2\}$ and $\{2 \leq X \leq 4\} \cap \{3 \leq Y \leq 5\}$ explicitly as a collection of outcomes and find their probabilities.

(c) Find $\mathbb{P}(X \geq k)$ for $k = 1, \ldots, 10$ and deduce the mass function of X.

(d) Find the mass function of Y.

Question 8. A student takes an exam with fourteen yes/no multiple choice questions. Ten correct answers are needed to pass.

(a) The student is clueless and tosses a fair coin for each question to decide an answer. What is the probability that the student passes?

(b) Suppose instead that the student knows that exactly half the answers are yes and decides to answer yes to a random seven questions and no to the rest. What is the probability that the student passes?