# VGLA: Sets and Notation Practice Questions

The following revision type questions relate to Appendix 1, Sets and Notation. Questions are ranked in difficulty from A (basic) to C (challenging).

**(A) Question 1.** Let $A = \{1, 2, \text{apple}\}, B = \{2, \text{apple}, 1, \text{apple}\}, C = \{2, 1\}, D = \{1, 2, \text{pear}\}$.

(a) Is $1 \in A$ ?

(b) Is $10 \in A$ ?

(c) Is $A = B$ ?

(d) Is $A = C$ ?

(e) Is $C \subseteq A$ ?

(f) Is $C \subset A$ ?

(g) Is $A \subseteq C$ ?

(h) Is $D \subseteq B$ ?

**(A) Question 2.** Write each of the following sets by listing its elements:

(a) $\{x \in \mathbb{Z} \,:\, -1 \leq x < 6\}$,

(b) $\{x \in \mathbb{Z} \,:\, x^2 < 25\}$,

(c) $\{x \in \mathbb{N} \,:\, x^2 < 25\}$,

(d) $\left\{ \dfrac{p}{q} \,:\, p \in \mathbb{N} \text{ and } q = 2p + 1 \right\}$,

(e) $\{x \,:\, x$ is the name of a month containing the letter "R"$\}$,

(f) $\{x \,:\, x$ is letter in "Mississippi"$\}$,

(g) $\{x \,:\, x$ is a month with 33 days$\}$.

**(A) Question 3.** Let $P$ be the set of prime natural numbers, $E$ the set of even natural numbers and $O$ the set of odd natural numbers. Describe the sets $E \cup O, E \cap O, E \cap P$ and $O \cap P$.

**(A) Question 4.** Determine $|A|$ when

(a) $A = \{1, 2, 3\}$, (b) $A = \{\{1, 2, 3\}\}$, (c) $A = \{1, \{2, 3\}\}$.

**(A) Question 5.** Put the correct sign $\Rightarrow, \Leftarrow$ or $\Leftrightarrow$ between the following pairs of conditions on a real number $x$. Briefly explain your answers.

(a) $x \in \mathbb{N}$; $1/x \in \mathbb{Q}$.

(b) $3x \in \mathbb{Q}$; $x \in \mathbb{Q}$.

(c) $3x \in \mathbb{Z}$; $x + 3 \in \mathbb{Z}$.

(d) $x \in \mathbb{Q}$; $1/(1 - x) \in \mathbb{Q}$.

**(A) Question 6.** Considering the answers to the previous exercise, give the correct relationship between the following pairs of sets. Use only the symbols $\subset, \subseteq$ or $=$. Explain your answers.

(a) $\mathbb{N}$ and $\{x \in \mathbb{R} : 1/x \in \mathbb{Q}\}$.

(b) $\{x \in \mathbb{R} : 3x \in \mathbb{Q}\}$ and $\mathbb{Q}$.

(c) $\{x \in \mathbb{R} : 3x \in \mathbb{Z}\}$ and $\{x \in \mathbb{R} : x + 3 \in \mathbb{Z}\}$.

(d) $\mathbb{Q}$ and $\{x \in \mathbb{R} : 1/(1 - x) \in \mathbb{Q}\}$.

**(A) Question 7.** In this question, the universal set is

$$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, \text{fish}, \text{fowl}\}.$$

Let $A = \{1, 2, 3\}$, $B = \{2, 3, \text{fish}\}$ and $C = \{2, \text{fowl}, 7, 8\}$. Work out each of the following sets using only the definition of union, intersection and complement.

(a) $A \cap (B \cup C)$.

(b) $(A \cap B) \cup (A \cap C)$.

(c) $A \cup (B \cap C)$.

(d) $(A \cup B) \cap (A \cup C)$.

(e) $C'$.

(f) $(C')'$.

(g) $(A \cap C)'$.

(h) $A' \cup C'$.

(i) $(A \cup B)'$.

(j) $A' \cap B'$.

**(A) Question 8.** Express each of the following sets as a single interval.

(a) $(1,3) \cup (2,15)$. (b) $[1,8] \cap [4,16]$. (c) $[66,76] - [72,100]$
(d) $[0,\infty) \cup (-10,10)$ (e) $[27,29] - (26,28)$

**(A) Question 9.** If $X$, $Y$ are sets containing $m$, $n$ elements respectively, how many elements are there in the Cartesian product $X \times Y$?

**(B) Question 10.** (a) Find, if possible, infinite sets $A$ and $B$ such that $A \cap B = \{0\}$ and $A \cup B = \mathbb{Z}$.

(b) Find, if possible, sets $C$ and $D$ such that $C \cup D = \{b, i, g\}$ and $C \cap D = \{s, m, a, l, l\}$.

**(B) Question 11.** Recall that, if $S$ is a finite set we let

$$|S| \quad = \quad \text{the number of elements in } \ S.$$

$|S|$ is called the **cardinality** of $S$; it is always a non-negative integer.

(a) Find $|\{200, 2, \sqrt{2}\}|$, $|\{\text{fish,pear}\}|$ and $|\{200, 2, \sqrt{2}, 200\}|$.

(b) Let $A$ and $B$ be finite disjoint sets. Express $|A \cup B|$ in terms of $|A|$ and $|B|$.

(c) Let $A = \{0, 1, 2, 3, 4\}$ and $B = \{2, 5, 6, 7, 8\}$. What is $|A \cup B|$? Compare it with $|A| + |B|$.

(d) Let $A$ and $B$ be finite sets. Find a formula for $|A \cup B|$ in terms of $|A|$, $|B|$ and $|A \cap B|$. Explain why your formula works.

**(B) Question 12.** Let $l$ and $m$ be integers with $l < m$. Write down formulae for the cardinalities of the following sets in terms of $l$ and $m$.
(a) $\{x \in \mathbb{Z} \,:\, \ell \leq x \leq m\}$, (b) $\{x \in \mathbb{Z} \,:\, \ell < x \leq m\}$,
(c) $\{x \in \mathbb{Z} \,:\, \ell \leq x \leq 2\ell\}$, (d) $\{x \in \mathbb{Z} \,:\, 2^\ell \leq x < 2^{\ell+1}\}$.

**(B) Question 13.** Let $A$ and $B$ be finite sets. Suppose that $A \subseteq B$ and that $|A| = |B|$. Explain why $A = B$.

**(B) Question 14.** (a) On a Venn diagram, indicate the set $A \cup (B \cap C)$.

(b) On another Venn diagram, indicate the set $(A \cup B) \cap (A \cup C)$.

(c) Persuade yourself that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

[This is the another version of distributive law.]

**(B) Question 15.** (a) Let $A, B, C$ and $D$ be sets. Prove that

$$A \cap (B \cup C \cup D) = (A \cap B) \cup (A \cap C) \cup (A \cap D).$$

(b) Let $A, B, C$ and $D$ be sets. State and prove a formula for

$$A \cup (B \cap C \cap D).$$

**(B) Question 16.** The **symmetric difference** of two sets $A$ and $B$, written $A\Delta B$ is defined by

$$A\Delta B \;=\; (A \cup B) - (A \cap B).$$

(a) Illustrate this set using a Venn Diagram. On another Venn Diagram, illustrate the set $(A - B) \cup (B - A)$ and deduce that

$$A\Delta B \;=\; (A - B) \cup (B - A).$$

(b) Is $\Delta$ commutative?

(c) What is $A\Delta A$?

(d) What is $A\Delta\emptyset$?

(e) Suppose $A \subseteq B$, what is $A\Delta B$?

**(C) Question 17.** Let $A$, $B$ and $C$ be finite sets. Let $D = B \cup C$. By substituting for $D$ in the formula

$$|A \cup D| = |A| + |D| - |A \cap D|$$

and making repeated use of the formula for $|X \cup Y|$, prove that

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|.$$

**(C) Question 18.** Let $U$ be a universal set and $A_i \subset U$ be sets for $i \in \mathbb{Z}$. Prove that

$$\left(\bigcup_{i\in\mathbb{Z}} A_i\right)' = \bigcap_{i\in\mathbb{Z}} A_i'.$$

Hint - Prove the result directly. Avoid mathematical induction.