

1AC Algebra and Combinatorics

Problem Sheet 1: Algebra part

You should carefully write out your solutions to all the questions below.

Ensure you have read and understood the Canvas assignment for the problem sheet for instructions about submitting solutions to SUM questions.

AQ1. True or false?

- (a) $7 \mid 28$
- (b) $9 \mid 15$
- (c) $6 \mid 3$
- (d) $0 \mid 13$
- (e) $11 \mid 0$

You should justify your answers.

AQ2. 3, 5, 7 is a list of three primes of the form $p, p + 2, p + 4$.

Prove that there are no other “triplet primes”?

AQ3. (SUM)

- (a) Prove Lemma 2.3:

Lemma. Let $a, b, c, k, l \in \mathbb{Z}$.

- (a) Suppose that $a \mid b$ and $a \mid c$. Then $a \mid (kb + lc)$.
 - (b) Suppose that $a \mid b$ and $b \mid c$. Then $a \mid c$.
 - (c) Suppose that $a \mid b$ and $b \mid a$. Then $a = \pm b$.
- (b) Let $a, b, c \in \mathbb{Z}$. Suppose that $a \mid b^2$ and $a^2 \mid c$. Prove that $a^5 \mid b^3c^2$.
- (c) Let $a, b, c \in \mathbb{N}$. Give a counterexample to the following statement.
Suppose that $a^2 \mid bc$. Then $a \mid b$ or $a \mid c$.

AQ4. (SUM)

- (a) Use the Euclidean algorithm to find $\text{hcf}(671, 314)$.
- (b) Use your working to find $x, y \in \mathbb{Z}$ such that

$$\text{hcf}(671, 314) = 671x + 314y.$$