Example sheet 4 – formative

1. Sketch the phase portraits of the following linear dynamical systems. This should include details of the equilibria, their type, eigenvalues and eigenvectors and horizontal and vertical isoclines. In each case state how all qualitatively different solutions $(x(t), y(t))$ behave as $t \to \infty$ for varying initial conditions (x_0, y_0).

   (a)

   $$\dot{x} = -x - y,$$
   $$\dot{y} = 2x - 3y.$$

   (b)

   $$\dot{x} = 2x - 5y,$$
   $$\dot{y} = x - 2y.$$

   (c)

   $$\dot{x} = 3x + 6y,$$
   $$\dot{y} = x + 2y.$$

   (d)

   $$\dot{x} = 2x - 4y,$$
   $$\dot{y} = x - 2y.$$

   (e)

   $$\dot{x} = 2x + y - 3,$$
   $$\dot{y} = 6x - 3y - 15.$$