

Problem Sheet 3 (Combinatorics part)

Assignment available: Friday 27 October 2022 (Week 5).

Submission deadline: 1700 on Wednesday 8 November 2022 (Week 7).

Required content: All necessary content will be covered by the end of the Week 5 lectures.

About problem sheet questions: The comments made about the first problem sheet apply equally to this one. In particular, please don't be reluctant to seek help if you are unsure how to proceed towards a solution, or how to express your ideas, as (unlike exam questions) the questions are set on the basis that you have access to this support.

Question 1 (SUM).

- (a) If I choose a non-negative integer solution to the equation $x_1 + x_2 + x_3 + x_4 = 20$ uniformly at random, what is the probability that I have $x_1 = 6$?
- (b) How many integer solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 \leq 43$ (note carefully that this is an inequality, not an equality) with $x_1 \geq 3$, $x_2 \geq 1$, $x_3 \geq 2$, $x_4 \geq 5$ and $x_5 \geq 0$?
- (c) How many integer solutions are there to $x_1 + x_2 + x_3 \leq 50$ with $x_1 \geq 27$, $x_2 \geq -11$ and $x_3 \geq 35$?

Question 2 (SUM). A D8 is an eight-sided die whose sides are numbered one to eight; when rolled, each side is equally likely to be selected. If I roll three D8, what is the probability that the numbers shown form an arithmetic progression (not necessarily in the order in which they are rolled, and not including the degenerate case when the numbers are all the same)?

Question 3. A card player is dealt a hand of 13 cards from a standard 52 card deck.

- (a) What is the probability that the hand contains no hearts?
- (b) What is the probability that the hand contains 5 spades, 3 diamonds and 5 clubs?
- (c) What is the probability that the hand contains at most 3 hearts?

Question 4. Say that integers n and m are of the same *type* if you can form n by rearranging the digits of m (so for example 134366 and 663314 are of the same type, but 1255 and 1225 are not). How many different types of six-digit integers are there?