Example sheet 2 – formative

1. Find and classify all equilibrium points for the following dynamical systems. Hence, determine the bifurcation point(s) and sketch the bifurcation diagram.
   (a) $\dot{y} = cy + 4y^3$
   (b) $\dot{x} = c + x - \dfrac{x}{1+x}$, for $c < 4$, $x \neq -1$
   (c) $\dot{x} = cx - \log(1+x)$, for $x \geqslant -1$

2. Given the nonlinear dynamical system

$$\begin{aligned} \dot{x} &= (2 - x - y)\, x, \\ \dot{y} &= (1 - 3x - 4y)\, y, \end{aligned}$$

   (a) find all the equilibrium points of the system;
   (b) determine the horizontal and vertical isoclines of the system;
   (c) establish the direction of flow along the horizontal and vertical isoclines of the system;
   (d) establish the direction of the flow in the regions defined by the vertical and horizontal isoclines;
   (e) check for any straight lines which may contain trajectories;
   (f) sketch all information obtained on a phase portrait.

3. Given the nonlinear dynamical system

$$\begin{aligned} \dot{x} &= 2x + xy - 5y, \\ \dot{y} &= x^2 - 2x. \end{aligned}$$

   (a) find all the equilibrium points of the system;
   (b) determine the horizontal and vertical isoclines of the system;
   (c) establish the direction of flow along the horizontal and vertical isoclines of the system;
   (d) establish the direction of the flow in the regions defined by the vertical and horizontal isoclines;
   (e) check for any straight lines which may contain trajectories;
   (f) sketch all information obtained on a phase portrait.