Mark Scheme Problem Sheet 3

Please note this mark scheme is only a guide of approximately how marks were allocated for the solutions to problem sheet 3. Individual markers may also use their own discretion and judgement in the allocation of marks.

4) 1/2 mark for stating that S is not open. 1/2 mark for correct justification.
   1/2 mark for stating that S is not closed. 1/2 mark for correct justification.
   1/2 mark for stating that S is not connected with an explanation of why (do not require a formal proof/justification for this one)
   1/2 mark for stating that S is not simply connected because it is not connected.
   1/2 mark for stating that S is not a domain because it is not open (or not connected).
   1/2 mark for stating that S is not bounded. 1/2 mark for a correct justification.
   1/2 mark for correct description of the set of interior points. 1/2 mark for justification of this.
   1/2 mark for correct description of the set of boundary points. 1 mark for checking that the points in these sets are indeed boundary points. It is sufficient to check carefully for points belonging to one of the "straight line parts" of the boundary and one of the "circle parts" of the boundary and then say that the other parts are "similar".
   1 mark for checking that there are no further boundary points.
   Total 8 Marks

7) (a) 2 marks for a correct $\varepsilon - \delta$ proof of the limit.
   (b) 2 marks for a correct $\varepsilon - \delta$ proof of the limit.
   Total 4 Marks

10) (a) 1 mark for correctly identifying the appropriate functions u and v and computing their partial derivatives. 1 mark for correctly identifying where the Cauchy Riemann equations are satisfied and using this to determine the points where f is non-differentiable. 1 mark for determining whether f is differentiable at the remaining points.
   (b) 1 mark for correctly identifying the appropriate functions u and v and computing their partial derivatives. 1 mark for correctly identifying where the Cauchy Riemann equations are satisfied and using this to determine the points where f is non-differentiable. 1 mark for determining whether f is differentiable at the remaining points.
   (c) 1 mark for correctly identifying the appropriate functions u and v and computing their partial derivatives. 2 mark for determining that the Cauchy Riemann equations are satisfied everywhere and that the partial derivatives of u and v are continuous, so by a theorem from the lectures f is differentiable everywhere.
   Total 9 Marks

12) 1 mark for correct computation of all partial derivatives of u up to order 2.
   1 mark for noting that all these partial derivatives are continuous (this is one of the properties needed for u to be harmonic).
   1 mark for substituting the derived formulae for $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2}$ into the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0.$

1 mark for the correct final description of the set of 4-tuples (a, b, c, d) for which u is harmonic.
Total 4 Marks