# ▼ Sample t test / Sample Z test

- 1. one-sample t test / one-sample Z test
- 2. two-sample t test / two-sample Z test
- · uses ztest
- used to compare mean against a claimed value

## ▼ one-sample t test / one-sample Z test

- one-sample t test is used to comapre the mean/average of a sample against a claimed value and establish whether the difference between them is statistically significant or not
- z-test
  - o population S.D. should be known
  - we work with sample S.D.
  - Sample size, n > 30
- t-test
  - Sample S.D should be known
  - ∘ Sample size, n <= 30
- · data should follow normal distribution
- ▼ import statsmodels.stats.weightstats

```
import numpy as np
import pandas as pd
# import numpy & pandas

import statsmodels
from statsmodels import stats
from statsmodels.stats import weightstats
# for ztest
```

### ▼ np.mean(ndarray)

```
1 x = [3, 7, 6, 5, 9, 8, 2, 3, 5, 4, 6, 7]
2 # create a list of Sample data
```

- Q1.
- I'm claiming that the mean of x is more than 6.5
- $H_o$ : mean(x) >= 6.5
- HA / H1 : mean(x) < 6.5

```
1 np.mean(x)
2 # actual sample Mean x̄ to be used to compare with claimed mean
3 # and then cestablish whether difference is staistically significant or not
```

5.41666666666667

▼ statsmodels.stats.weightstats.ztest(Sample, value=ClaimValue, alternative='Sign of HA')

```
1 statsmodels.stats.weightstats.ztest(x, value=6.5, alternative='smaller')
2 # statsmodels.stats.weightstats.ztest(list/ndarray, value=Claimed Value, alternative=Sign Of HA)
3 # returns (TestStatistics, P-Value)
```

(-1.744291601622091, 0.04055412767721645)

- assume C.L. = 98% means LoS = 2% = 0.02
- P-value = 0.04055 > LoS = 0.02
- since P-Value > LoS, we don't reject this H<sub>o</sub>
- P-value = 0.04055 means there is 4.055% chance that we commit Type 1 error
- means 4.055% chances of chance variation so that we might reject this hypothesis
- if p-value < 0.05 = 5%, we reject the Ho,
- it means that the difference between Sample Statistics SS & Population Parameter PP is large enough not to be ignored, so action is required

02.

- Now, I'm claiming, that mean of x is more than 6.0
- $H_o$ : mean(x) >= 6.0

• HA / H1 : mean(x) < 6.0

```
1 np.mean(x)
2 # actual sample Mean x̄ to be used to compare with claimed mean
3 # and then cestablish whether difference is stistically significant or not
5.41666666666667
```

```
1 statsmodels.stats.weightstats.ztest(x, value=6.0, alternative='smaller')
2 # statsmodels.stats.weightstats.ztest(list/ndarray, value=Claimed Value, alternative=Sign Of HA)
3 # returns (TestStatistics, P-Value)
```

(-0.9392339393349719, 0.17380532364640544)

- assume C.L. = 98% means LoS = 2% = 0.02
- P-value = 0.1738 > LoS = 0.02
- since P-Value > Los , we don't reject this Ho
- but, p-Value = 0.1735 or 17.35%, which is very high (although > 5% LoS is acceptable), so we reject this Null Hypothesis
- if p-value < 0.05 = 5%, we reject the H<sub>o</sub>,
- it means that the difference between Sample Statistics 55 & Population Parameter PP is large enough not to be ignored, so action is required
- ▼ using Sample S.D. to manually calculate Test Statistics
- ▼ np.mean(ndarray)
  - actual Population Mean (μ)

```
1 np.mean(x)
2 # actual Population Mean (μ)
3 # to be used to compare with claimed mean
4 # and then cestablish whether difference is stistically significant or not
```

5.416666666666667

- ▼ np.std(ndarray, ddof=1)
  - Sample Standard Deviation (σ)

```
1 np.std(x, ddof=1)
2 # Sample Standard Deviation (σ)
2.151461800448216

▼ len(ndaray)
• Calculate Sample Size (n)
```

```
1 len(x)
2 # Claculate Sample Size (n)
```

12

▼ Test Statistics formula

```
• TestStatistics, Z = (PopulationMean - ClaimedMean) / S.D. / sqrt(SampleSize)
```

- TestStatistics,  $Z = (\mu X) / (\sigma/sqrt(n))$
- TestStatistics,  $Z = (\mu X) / (\sigma/(n)**0.5)$

```
1 (5.416666 - 6.5) / (2.151461800448216/12**0.5)
2 # (μ - X) / (S.D./n**0.5)
3 # test statistics if claimValue=6.5
```

-1.7442926750323076

```
1 (5.416666 - 6) / (2.151461800448216/12**0.5)
2 # (μ - X) / (S.D./n**0.5)
3 # test statistics if claimValue=6.0
```

-0.9392350127451884

▼ import scipy.stats.norm

```
1 import scipy
2 from scipy import stats
3 from scipy.stats import norm
```

▼ norm.cdf(Z)

1 norm.cdf(-1.7442926750323076)

0.17380504814940417

```
2 # to find out area under curve using value on x-axis
3 # returns P-Value when claimValue=6.5

0.040554034138080716

1 norm.cdf(-0.9392350127451884)
2 # to find out area under curve using value on x-axis
3 # returns P-Value when claimValue=6.0
```

### 95% upper bound

- 5.41 is the Population mean and because error is on left side, we would prefer this to be as high as possible
- 95% upper bound : the highest possible value at 95% C.L., is called as 95% upper bound

#### norm.ppf(area/prob)

```
1 norm.ppf(0.05)
2 # value on x-axis fo area under curve for 0.05 area
3 # value on x-axis is on left side of mean so it returns -ve value
    -1.6448536269514729
  • 95% upper bound = PopulationMean + (TestStatistics * S.D / Sqrt(SampleSize) )
 • 95% upper bound = \mu + (Z * \sigma / Sqrt(n))
 • 95% upper bound = \mu + (Z * \sigma / n**0.5)
15.41 + 1.645*(2.151461800448216 / 12**0.5)
2 # 95% upper bound = Population Mean + TestStatistics * S.D / Sqrt(SampleSize)
3 # 95% upper bound = \mu + Z * \sigma / Sqrt(n)
4 # 95% upper bound = \mu + Z * \sigma / n**0.5
6 # if considering chance variation or benefit of doubt,
7 # actual mean can go till 6.431665948328879
8 # but it cannot be exactly 6.431665948328879
9 # which is answer for highest possible sample mean or 95% upper bound
   6.431665948328879
```

```
1 statsmodels.stats.weightstats.ztest(x, value=6.0, alternative='smaller')
(-0.9392339393349719, 0.17380532364640544)

1 statsmodels.stats.weightstats.ztest(x, value=6.5, alternative='smaller')
(-1.744291601622091, 0.04055412767721645)

1 statsmodels.stats.weightstats.ztest(x, value=7.0, alternative='smaller')
(-2.5493492639092104, 0.005396207746961571)
```

Note: ClaimedValue vs. P-Value

 As we keep on increasing the claimedValue, P-Value / Probability-Value keeps on decreasing because actual Population mean is getting away from claimedValues

# ▼ two-sample t test / two-sample Z test

- Two Sample t Test is used to compare the difference of the averages of two independent samples with the claimed value and establish whether the difference is statistically significant or not
- two separate samples, a & b, calculate mean for a & b, and compare the difference of mean with claimed value
- 01.
- claiming that student of C.Blr. will get 15k more salary than student of C.Kh.
- find the difference, sample/actual difference value = 12500
- (15000 12500) check whether difference between claimed and actual value is significant or not

```
1 x = [3, 7, 6, 5, 9, 8, 2, 3, 5, 4, 6, 7]
2 y = [9, 6, 8, 10, 6, 4, 7, 8, 9, 10]
3 # creating two samples
```

## ▼ Left Tail Test

- H<sub>o</sub>: mean(y) mean(x) >= 2.7
- HA / H1 : mean(y) mean(x) < 2.7 [Left Tail test]

▼ import statsmodels.stats.weightstats

```
1 import numpy as np
2 import pandas as pd
3 # import numpy & pandas
4
5 import statsmodels
6 from statsmodels import stats
7 from statsmodels.stats import weightstats
8 # for ztest
```

▼ np.mean(ndarray1) - np.mean(ndarray2)

```
1 np.mean(y) - np.mean(x) 2 # actual difference of sample Mean \bar{x} for two samples to be used to compare 3 # with claimed mean difference 4 # and then cestablish whether difference is staistically significant or not
```

2.283333333333333

- statsmodels.stats.weightstats.ztest(Sample1, Sample2, value=ClaimValue, alternative='Sign of Ha')
  - alternative='smaller' for left tail Test
  - alternative='larger' for right tail Test
  - alternative='two-sided' for two tail Test

```
1 statsmodels.stats.weightstats.ztest(y, x, value=2.7, alternative='smaller')
2 # alternative='smaller' - for left tail Test
3 # alternative='larger' - for right tail Test
4 # alternative='two-sided' - for two tail Test

(-0.47198804896079605, 0.3184676595008745)

1 statsmodels.stats.weightstats.ztest(y, x, value=3.7, alternative='smaller')

(-1.6047593664667057, 0.054273385712541596)

1 statsmodels.stats.weightstats.ztest(y, x, value=4.2, alternative='smaller')
```

(-2.1711450252196607, 0.014960105716979144)

Note:

· as claimed difference keeps on increasing, P-Value / Probability Value keeps on decreasing

## ▼ Right Tail Test

- H<sub>o</sub>: mean(y) mean(x) <= 1.5
- HA / H1 : mean(y) mean(x) > 1.5 [right tail test]

```
1 statsmodels.stats.weightstats.ztest(y, x, value=1.5, alternative='larger')
(0.8873375320462958, 0.1874486022476365)
```

#### ▼ Two Tail Test

- if it was two tail test, Ho will be equality with some value, and HA will be not equal to that value
- $H_o$ : mean(y) mean(x) = 1.5
- HA / H1 : mean(y) mean(x) != 1.5 [two tail test]

```
1 statsmodels.stats.weightstats.ztest(y, x, value=1.5, alternative='two-sided')
```

## ▼ Paired t test

- two samples are dependent and paired to each other
- sample size has to be same
- whenever performance/some numbers/parameters need to be compared before and after an event
- in two-sample t test: difference of averages/means of two samples is compared with claimed value
- in paired t test: mean of differences of two samples is compared with claimed value
- Q. before & after covid started,
  - before people worked from home and after people worked from office

```
before = c(7, 9, 8, 6, 7, 8)

after = c(6, 8, 9, 5, 6, 8)

since people started working from home, their time of delivery has increased by an avg of 0.2min
```

```
- H<sub>o</sub> : avg(after-before) >= 0.2
- H<sub>A</sub> : avg(after-before) < 0.2 ---> left tail test
```

```
1 # t.test(after, before, mu=0.2, alternative='less', paired=T)
2 # run in R lang
3
4 # t.test(s1, s2, mu=claimedValue, alternative='less', paired=T)
5 # difference of means is statistically significant as P-Value 0.04786 < 0.05,
6 # so H<sub>o</sub> is rejected
```

#### ▼ one-way ANOVA

- ANOVA: ANalysis Of VAriance
- used when we have multiple samples
- ANova uses F-Distribution(Fischer Distribution)
- Ho : all the pair of means are equal
- HA / H1 : there is atleast one pair of means where the difference is not equal to zero
- response should follow normal distribution
- · variances are equal
- Predictor is attribute/categorical & Response is continuous

Sample Value

```
5
             Α
     2
                    6
             Α
     3
             Α
                    7
             Α
                    5
     7
             В
     8
                    7
     10
             С
                   10
1 import statsmodels.api as sm
2 from statsmodels.formula.api import ols
3 import statsmodels.stats.multicomp
             С
                   12
1 # 'sample' is called predictor
2 # 'value' is called as response
3 # '~' means depends on
4 mod1 = ols('Value ~ Sample', data=df).fit()
5 # ols('colWithResponse ~ ColumnWithPredictor', data=dataFrame).fit()
1 tbl = sm.stats.anova_lm(mod1)
2 tbl
3 # df : degree of freedom
4 # degree of freedom for residual = sample size 14 - sample 2 - 1 = 11
5 # sum_sq : sum of squares
6 # mean_sq : mean of squares
7 # F : F-Distribution
8 # PR(>F) : P-Value from F-distribution
```

```
        df
        sum_sq
        mean_sq
        F
        PR(>F)

        Sample
        2.0
        130.278571
        65.139286
        37.416822
        0.000012

        Residual
        11.0
        19.150000
        1.740909
        NaN
        NaN
```

X