Akuna Capital Options 101 - Section 3 Notes

James Evans

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Greeks Intro

Section 3 will focus on option "Greeks". Greeks is a term used in the options world to describe the characteristics and dimensions of risk associated with an option position. They are called Greeks because each is associated with a Greek symbol. Market makers use these Greeks to help them manage a portfolio of options and understand how prices (and therefore P&L) will behave with changes to the inputs of their pricing model (time, underlying price, implied volatility, interest rates, dividends, etc).

These Greeks are calculated as a first partial derivative of the option pricing model that is used. We won't go into mathematical derivations in these sections. Instead, we approach Greeks from their practical perspectives, as they apply to options market makers. The greeks we'll discuss are listed below:

Delta Δ

Gamma Γ

Theta Θ

Vega v

Rho p

Delta

Delta

The first option Greek we'll define is delta.

The **delta** of an option is the number of underlying contracts that allows one to establish a neutral hedge under current market conditions using the current theoretical value of an option.

Call options are assigned a delta between 0 and 1.00, but traders commonly drop the decimal and express the delta as between 0 and 100. Therefore, a delta of 0.57 will be denoted as 57 and verbalized as "fifty seven". This delta means that for every 100 options traded we need to trade 57 underlying contracts as a hedge.

A put option will have a delta between -1 and 0 (-100 to 0). The sign (-) of a put delta will often be understood (no need to verbalize) and dropped so that a -20 delta put might be described as "a 20 delta put" or simply "a 20 delta" if both parties understand they are discussing puts. The negative is understood, as all puts have negative deltas by definition.

In trading the 57 deltas in the example above we have now established a hedge that makes us indifferent to directional moves in the underlying (see definition 2 below).

There are 3 common definitions of delta:

1. The delta of an option is the sensitivity of an option price to changes in the price of an underlying asset. Delta will tell us how much the price of the option (or option strategy) will change as the underlying changes.
2. Hedge ratio. That is, if we buy or sell a certain number of options, how many futures must we buy/sell so that we are delta neutral? Hedging options with the correct number of underlying contracts allows us to establish a position which is indifferent to directional moves in the underlying.
3. Finally, delta can be defined as the probability that an option will expire ITM. E.g. a 0.4 (or 40) delta call is considered to have a 40% chance of finishing ITM. What would the delta of the corresponding put be?

Note: The final definition is the least mathematically accurate as interest rates and dividends distort this definition, but generally it's the most intuitive to new traders.

🔷 **Delta as Probability of Expiring In-The-Money (ITM)**

In **options trading, delta is often interpreted as an estimate of the probability that the option will expire in-the-money** (ITM).

So:

- A **call option** with **delta = 0.40 (or 40)** implies:
  - There is approximately a **40% chance** it will expire ITM
  - Meaning: the underlying stock ends up **above the strike price** at expiration

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🔷 **What about the corresponding put?**

The **put** at the same strike price and expiration will have a **delta ≈ –(1 – call delta)**.

So if the **call delta is 0.40**, then:

- The **put delta** is approximately **–0.60** (or –60)

This makes sense because:

- The **put has a 60% chance** of expiring ITM (stock ends **below** the strike)
- The **sign is negative** because put deltas are negative by convention (they move **opposite** to the underlying stock)

🔷 **Summary:**

If:

- Call delta = **0.40** → ~40% chance of expiring ITM
- Then the **put delta ≈ –0.60** → ~60% chance of expiring ITM

Because **one of them must be ITM at expiration (ignoring exact ATM)**, and their probabilities add to ~100%.

Calls

As discussed above, calls have a positive delta between 0 and 1. When you buy (sell) a call your delta will be a positive (negative) number. When the underlying increases in price, the value of the call will increase by the option's delta multiplied by the change in underlying price. Conversely, when the underlying decreases in price the value of the call will decrease by the delta value multiplied by the change in underlying price. Note, this only holds for relatively small underlying moves, as we'll discuss in the gamma section.

Example: You are trading underlying ABC (price $70.00, 1 options contract settles to 1 underlying), and a 50-delta call is worth 1.00. If ABC moves to 70.10 the call will be worth 1.05. The delta hedge for this would be 0.5 futures (if you could trade half a future, which you can't). If you sell half a future vs buying one call, your PnL would be $0 on a move in the underlying.

Puts

As stated above, puts will always have a negative delta, within the range 0 to -100. When the underlying increases, put values decrease. Holding the correct delta ratio position in the underlying will offset any option gains/losses.

Welcome. In this video, we'll define and discuss delta. We'll go through the three definitions of delta. We'll briefly discuss how delta is affected by changes in implied volatility, how the passage of time affects delta, and finally, how delta changes as we compare options through strike space. Let's first look at an options board to get our bearings on how things look. You'll notice the calls listed on the left-hand side of the middle black bar. The black bar notes our strikes. Strikes are always listed in increasing order from the top to the bottom. To the right of the strikes are put options. Now, going back to the left, you'll notice that our call deltas are prominently displayed next to our calls. Deltas are one of the most impactful and important Greeks when trading options. Traders are always conscious of the delta of each strike, as well as the delta of their position. On the right of the put theoretical values, you'll notice the put deltas are also shown. Now, you'll notice the first characteristic of deltas: call deltas are always a number between 0 and 1, while put deltas are always a number between 0 and negative 1. Traders will often refer to them as if they were a number between 0 and 100 or 0 and minus 100. For example, someone might say, "this call has a 15 delta" instead of "this call has a 0.15 delta." Now, let's look at another characteristic of delta Greeks before we dive into definitions. Note that if you take the call delta on a strike and add to that the absolute value of the put delta, you'll get one. We'll get back to this later, but it's something to think about and keep track of as we think about our option deltas. Now let's discuss the three definitions of option delta. The first of these is that the delta of an option is the sensitivity of an option to changes in the price of the underlying. Therefore, delta describes how much the price of the option changes as the underlying price changes. Let's take a look at an example where we have an underlying with a price of 100. Let's look at a call option with a price of $1 and a delta of 0.3. Remember, I may refer to this as a 30 delta from now on. What a 30 delta tells us is that if the price of the underlying goes up by, say, 20 cents from 100 to 100.20, the price of the new call option will change by 20 cents times 0.3, or six cents. So the new call value would be $1.06 in this example. Now let's look at a put option. This put has a price of $3 and a delta of –0.7. Notice that delta is negative, which means that it's negatively related to increases in the underlying price. So at the same 20 cent increase in the underlying, we would get 20 cents times 0.7, or a 14 cent decrease in the price of the option. So the new price of the option, if the underlying were to move up 20 cents, would be $2.86 for this put. Take a second and think about what the new values of these options might be if the future went from 100 to 99.50. You can pause the video and try to work these out now if you'd like. Here's the solution: if the underlying goes down by 50 cents, we would have a call value that changes by 50 times 0.3, or 15 cents, for a new value of 85 cents. And for the put, we would have a change of 50 times 0.7, or 35 cents, for a new put value of $3.35. So as we see in the definition, the delta of the option gives the change in the price of the option for changes in the price of the underlying, where the sign of the option or option strategy gives the exposure to up or down movements in the underlying asset. Now let's move to our second definition of delta: that is, the delta of an option is the hedge ratio of the option. This gives us the amount of an underlying asset we need to trade to create a balanced option strategy that is indifferent to small directional movements in the underlying. Again, let's look at the same two options with the 100 underlying. Now, we saw in example one that a call has a positive exposure to the underlying. If we buy a call, we profit if the underlying goes up, since the call increases in price. With a put and a negative delta, we profit if the underlying goes down. But we can trade some amount of underlying to neutralize this exposure. So, for example, let's pretend we've bought 10 call options. They each have a delta of 30. Now our payoff diagram would look like this: if the future goes up, the value of the calls will go up; if the future goes down, we'll lose the value we paid for these calls. But if we sell some amount of the underlying, we can offset our downside loss while giving up some upside gains. This is why the natural hedge to buying calls is to sell the underlying. The amount of underlying that we must sell against these calls can be found by multiplying the number we bought, 10, with the underlying 0.3. Therefore, we have a long three-unit underlying exposure to the future, so we must sell three futures here. We'll pretend that the underlying is the future from here on out. So, if we sell three futures now, we have a portfolio payoff diagram made up of the ten calls and the three futures we sold that looks as follows: we'll make an equal amount of profit if the future moves in either direction, and therefore we've neutralized the delta exposure of our portfolio by using the correct hedge factor, given by our delta. Let's run through our second example now with puts. Let's pretend we bought 20 puts this time. If we did that, then our payoff diagram would look as follows. Since the puts increase in value with decreases in the future price, past some point, we're positively exposed to downward moves and negatively exposed to upward moves. Therefore, to offset this, we must buy some number of underlying units or futures to offset this exposure. The correct number is found again by multiplying the 20 put units times a 0.7 delta, to see that our position in puts is short 14 futures. Therefore, we must buy 14 futures to offset this. When we do, once again, we turn our payoff diagram into something more balanced, where we don't have a preference in the direction of future moves, since our exposure—and therefore profits and losses—even out in either direction. Let me note here that all of these exposures and calculations of delta are for small moves in the underlying. As with any derivative, these characteristics will change as the underlying changes by larger amounts, and thus the delta of the options will also change. We'll get into this in future videos. I want to take a second here and make a note that's also important and often a bit confusing for those who are new to option trading. The word delta is often used in two different but very similar ways. There's the delta of an individual option, such as "this call has a 20 delta." This is how we've been using and referring to options in our previous examples. Then we can figure out the hedge by multiplying the delta times the quantity of options we traded. So if you buy

50 of these 20 delta options, you would need to hedge by selling 10 futures. Additionally, the word delta can refer to the delta of an overall position. For example, "my position is long 20 deltas." This means that you have a book that you must sell 20 futures against to hedge that delta exposure. These 20 deltas can be a mixture of options or other futures. You're just talking about a general book that has long 20 deltas in this case. So the delta of an option is one thing. The delta of an option multiplied by the quantity of options gives you the delta of the position. Finally, we have our third definition of delta. Now, this one I'll note is more of a guideline or rule of thumb rather than a hard-and-fast definition with mathematical derivations, but it's often used—especially by non-professionals—so it's definitely worth mentioning. That is, an option delta gives us the approximate percentage chance that the option finishes in the money. So a 40 delta call would have a 40% chance of finishing in the money. Remember, in the money means that the underlying price would finish above the strike price for calls or below for puts, and that the options would be worth some amount instead of being worthless at expiration. Let's take a second to talk about the terms "in the money" and "out of the money." We'll define them here. Out of the money means that the future price is currently in a location where, if it were to finish here, the option at expiration would be worthless—hence, "out of the money." In the money means options where the future price is currently at a location where, if it finishes at this location at expiry, the option would be worth something. A good rule of thumb is that anything below a 50 delta, or negative 50 delta, corresponds to an out-of-the-money option, while anything from a 50 delta to a 100 delta is in the money. So for the same 100 underlying, let's examine our two options. The call has a 30 delta, which means it has about a 30% chance of finishing in the money by expiration. And for our put option with a negative 70 delta, it has about a 70% chance of finishing in the money. Again, I'll note that this is an approximation or guideline and not a mathematical definition. This is why you'll notice that each pair of options on a strike, when added together, adds up to one—or a 100% chance that the future finishes either above or below or at the strike—since it has to finish somewhere, so either the put or the call will be in the money. So now that we've given the three definitions of delta, we can talk about how deltas change and are affected by a few different factors. Now, this isn't an exhaustive or intensive deep dive into deltas, but instead, a quick run-through of a few of these dynamics as a refresher might be worthwhile. We'll look at how delta changes with changes in implied volatility, how delta changes with the passage of time or as we approach expiry, and finally, how delta changes as we move through strike space and compare the deltas of different strikes along the same expiry. So first, let's look at how the delta of an option would change with changes in implied volatility. Remember, implied volatility is the input into our model that dictates the option's price. Let's think about what happens when we increase volatility and also when we decrease it, to see how the delta of the option is affected. For our call example, with an increase in volatility, what we're saying is that the future is expected to be much more volatile in the future. So the delta of the option would increase and approach 0.5, or 50. You can think of this by using the third definition—with so much volatility, the chance of the out-of-the-money option finishing in the money moves closer to 50%. Now let's look at what happens to our out-of-the-money call as vol decreases. Since we're lowering the implied volatility, we expect that all options below a 50 delta will approach zero, because there's less of a chance that they finish in the money. There's less of a chance that the future moves around and finishes above the strike. Note this call is out of the money because its delta is below 50, so the strike will be above 100. Let's think about how the 70 delta put would behave with increases and decreases in volatility. Take a minute to think about how the delta would change—pause the video if you want—and we can go over these together. For an increase in implied volatility, the delta of the put moves towards -0.5. Since this put is currently in the money, this means that with increased volatility, there is less of a chance—since the underlying is projected to move a lot more—that the option will finish in the money. For decreases in implied volatility, we see the opposite effect. If the future is currently in a place where the put is in the money, and if we're decreasing the implied volatility (saying that the future price movement will be less), then there is more of a chance that the option finishes in the money. So the delta of the put approaches -1. Now let's talk about what happens as we change the time to expiry for an option. How does the delta change? Well, what we find for our out-of-the-money call is that the delta approaches 50 when we increase the time to expiry. We can think about this as follows: if there's a longer time to expiry, then there is more chance for our underlying to move around on a random walk and finish above the strike or in the money. If we decrease time to expiry, we get the opposite effect. Since there's now less time for the future to move around and finish above the strike, the chances of finishing in the money decrease and approach zero. Now take a moment and think about how our in-the-money, 70 delta put is affected. Pause the video if you want, and try to work out what

happens with the delta as we increase and decrease the time to expiry. We can see that if we increase the time to expiry, the delta of the in-the-money put approaches –0.5. This makes intuitive sense because with an increase in time to expiry, there is more time for the future to move around and possibly finish out of the money. Similarly, if we decrease the time to expiry, we're saying that the current in-the-money put will have a higher chance of finishing in the money, and therefore the delta approaches –1. If you are paying attention, you'd notice that if we increase the time to expiry, it affects the delta of options similar to an increase in implied volatility. And a decrease in time affects delta similar to a decrease in implied volatility. You can use these relationships to help you think about how deltas—and future Greeks in general—behave. Finally, let's look at how delta changes as we move through and compare deltas through strike space. Now, we haven't defined the strikes of these options, but let's pretend that we have some set of strikes for call options: the 120 strike, the 125 strike, and finally the 130. As we move up in strike space, how do the deltas of the call options behave? Well, we can see that the delta of strikes decreases as we increase call strikes. The delta of the 125 call will always be less than the delta of the 120 strike and greater than the delta of the 130 strike. I'll leave you to think about how the delta of the puts compare, but it should be a trivial exercise if you remember that the delta of the call and the absolute value of the put add up to one. Finally, I'll leave you to examine the calls and puts on this option board again to solidify the rules and relationships we just discussed in this lesson. Feel free to pause and think about the three definitions of delta and how deltas change across time, volatility, and strike space.

Call and put deltas for the same strike have a relationship. The call delta plus the absolute value of the put delta will equal 1 (excluding early exercise premium and dividends, both advanced topics which we'll table for now).

Most traders will only look at either a call or put delta column (not both) on their trading screens and use the relationship noted above if they need the corresponding option greek. For example, since the 940 calls have a 91 delta (left-most orange column labeled SplDlt), we know the corresponding puts have a -9 delta.

SplDlt	TO Qt	Bid Qt	Bid Px	Theo	Ask Px	Ask Q	Str ^	T Pos	PBid Q	PBid P	PTheo	PAsk Px	PAsk Q	PTO Q	Vega
91				16.50			940	3			0.76				0.261
69				8.37			950	-20			2.63			20	0.478
35	17			3.29			960	0			7.54			51	0.506
12	31			1.14			970	0			15.38				0.321
4	3			0.43			980	-7			24.67				0.168
2				0.18			990	1			34.41				0.083
1	5			0.08			1000	11			44.31				0.042
0				0.04			1010				54.27				0.022

Hedge Ratio Explained

In this section we'll describe why we hedge options by trading the underlying instrument on a certain hedge ratio, or delta. As we mentioned in the previous section, calls always have a positive delta and puts a negative delta. Therefore, the correct direction of the hedge for a future (or any underlying) is always:

Option Trade		Hedge
buy call	→	sell future
sell call	→	buy future
buy put	→	buy future
sell put	→	sell future

We can break down an option trade into two parts to make it even more apparent how the action on the option is a multiplicative result of two signs. We know that calls have a positive exposure to the underlying. If futures go up, their price increases. Similarly, puts have a "positive exposure" to downward moves in the underlying. Therefore, we can split up the action in options multiplied by the exposure the option has to moves in the future to get a multiplicative result which indicates our overall exposure.

Option Trade	(O Action	•	O Exposure)	=	Mult Result		Hedge	
buy call	+	•	+	=	+	→	-	sell future
sell call	-	•	+	=	-	→	+	buy future
buy put	+	•	-	=	-	→	+	buy future
sell put	-	•	-	=	+	→	-	sell future

Next, we'll add some numbers and step through an example of why delta corresponds to an accurate hedge ratio. **Remember, the key is that the hedge offsets the option position so that a change in the underlying results in no directional bias, or P&L.** We'll use an example where we buy 1 put option with a 30 delta for a price of 1.00.

The action of buying (+) a put (-) yields a negative result (+ multiplied by - = -) so we must buy futures as a hedge. Specifically, we must buy 0.30 futures for every 1 put option we buy.

Let's take a future price of 100.00 as an example. Refer to the table below; middle row. Our total portfolio is +1 put for 1.00 and +0.3 futures (the hedge) for a price of 100.00.

Remember, our goal is to maintain a portfolio that is not sensitive to directional moves in the underlying. After our hedge we should be indifferent to an upward or downward price change in the underlying.

Using our 1st definition of delta above we calculate a new option price for changes in the underlying. Let's calculate scenarios where the future moves up by 0.10 and down by 0.10 and look at the resulting option and hedge P&Ls:

	Future Price	Option Price	Future P&L	Option P&L	Portfolio P&L
Future up 0.10:	100.10	0.97	0.1*0.3 = 0.03	-0.03 * 1 = -0.03	0.00
Future at 100.00	100.00	1.00	0.00	0 * 1 = 0	0.00
Future down 0.10:	99.90	1.03	-0.1 * 0.3 = -0.03	0.03 * 1 = 0.03	0.00

Notice, that in each scenario, the Portfolio P&L remains $0. What we make in our option, we lose in our future or vice versa. The definition of delta as the change in option value for a change in underlying price makes it the right hedge ratio.

Covered Trades

There is a special type of trade that we will briefly mention here that allows market participants to trade an option and also automatically trade the hedge that offsets the option(s) delta. A **covered trade** (aka, a tied-up trade, or a tied trade) is one where the hedge is already included with the option as a package. Any market participant can request a market in a call with a specific delta and future price. If someone trades these calls, they will also trade an appropriate amount of the underlying instrument with the other party automatically. This takes the directional element out of the trade since both parties would automatically get their hedge as part of the trade.

Here is an example: The contract below is the HE (Hogs) Covered Call, V18 = Oct2018, on the 53 strike. It comes as a package with the Oct2018 future at a price of 52.25 with a 52 delta.

Instrument	SplDlt	Bid Qty	Bid Px	Theo	Ask Px	Ask Qty
HE CV-C V18 53 [0.52 V18 52.250]	1	206	2.675	2.719	2.750	189

This means that for every 100 options that trade you'll also trade 52 futures at the specified price. You can see the market on the right side. If we trade this option and future, what does our payoff diagram look like and how does it contrast with the payoff diagrams we constructed earlier for outright options?

For example, below is the outcome of a buyer lifting the 2.75 ask (offer) price shown above from a seller.

There can also be multiple parties offering different quantities of this instrument. Below, we display the resulting trades if there were 2 parties that had priority for 30 and 20 tied up packages. Each would get a percentage of the options and futures shown.

Note futures are rounded to whole numbers (30 calls X 0.52 = 15.6 futures -> 16, 20 calls X 0.52 = 10.4 futures -> 10) since participants can't transact fractional futures in the marketplace.

✅ **Summary**

This is how **multi-party package trades** are executed:

- Futures are split **proportionally by delta**
- Rounding is necessary because futures must be whole
- It ensures the **hedge ratio is preserved** across partial fills

Let me know if you want the math formalized in LaTeX.

Theo and Delta Hedging Basics Exercise

Exercise - Theo and Delta Hedging Basics

Work quickly and efficiently, your time is being recorded for this quiz and weighed against accuracy!

This exercise will provide some practice in calculating theoretical values and delta hedges for some common option strategies. For each question, calculate the theoretical price of each combo. Additionally, calculate the delta hedge (how many futures we must trade and whether we buy/sell them) if we traded 100 of each combo. Do NOT use a calculator!

Notice, the call and put theoretical values are shown (call left, put right). Similarly, the call and put deltas are shown in the yellow columns "**skew delta**".

Note: + indicates you bought the combo, - indicates you sold the combo.

Example: +85 call. Theo = **2.11***. Hedge = 0.51*100 = +51, so,* **sell 51 futures** *to hedge.* **Answer: 2.11, sell 51 futures**

Theor.		Theor.	Vega	kew de	kew delt	Act.Vol	kew gamm
13.08	H 72 T	0.09	0.01722	0.97	-0.03	29.02	0.0067
12.60	H 72.5 T	0.10	0.01932	0.97	-0.03	28.75	0.0072
12.11	H 73 T	0.12	0.02156	0.97	-0.03	28.46	0.0078
11.63	H 73.5 T	0.13	0.02394	0.96	-0.04	28.15	0.0084
11.15	H 74 T	0.15	0.02647	0.96	-0.04	27.81	0.0090
10.67	H 74.5 T	0.17	0.02914	0.95	-0.05	27.45	0.0097
10.19	H 75 T	0.20	0.03196	0.95	-0.05	27.07	0.0104
9.72	H 75.5 T	0.22	0.03494	0.94	-0.06	26.67	0.0113
9.25	H 76 T	0.25	0.03806	0.94	-0.06	26.24	0.0122
8.78	H 76.5 T	0.28	0.04134	0.93	-0.07	25.80	0.0132
8.31	H 77 T	0.31	0.04478	0.93	-0.07	25.34	0.0203
7.85	H 77.5 T	0.35	0.04844	0.91	-0.08	24.88	0.0232
7.39	H 78 T	0.39	0.05237	0.90	-0.10	24.45	0.0263
6.94	H 78.5 T	0.44	0.05654	0.89	-0.11	24.03	0.0298
6.49	H 79 T	0.49	0.06094	0.87	-0.13	23.63	0.0336
6.06	H 79.5 T	0.56	0.06551	0.86	-0.14	23.25	0.0378
5.63	H 80 T	0.63	0.07022	0.84	-0.16	22.89	0.0421
5.21	H 80.5 T	0.71	0.07500	0.81	-0.19	22.55	0.0467
4.81	H 81 T	0.81	0.07976	0.79	-0.21	22.22	0.0515
4.42	H 81.5 T	0.92	0.08442	0.76	-0.24	21.92	0.0563
4.04	H 82 T	1.04	0.08887	0.73	-0.27	21.62	0.0610
3.67	H 82.5 T	1.17	0.09299	0.70	-0.30	21.35	0.0656
3.32	H 83 T	1.33	0.09667	0.67	-0.33	21.09	0.0698
2.99	H 83.5 T	1.49	0.09978	0.63	-0.37	20.85	0.0735
2.68	H 84 T	1.68	0.10220	0.59	-0.41	20.62	0.0766
2.39	H 84.5 T	1.89	0.10384	0.56	-0.44	20.41	0.0790
2.11	H 85 T	2.11	0.10460	0.51	-0.48	20.21	0.0822
1.86	H 85.5 T	2.36	0.10444	0.47	-0.53	20.03	0.0844
1.63	H 86 T	2.63	0.10332	0.43	-0.57	19.88	0.0837
1.42	H 86.5 T	2.92	0.10126	0.39	-0.61	19.75	0.0818
1.23	H 87 T	3.23	0.09832	0.35	-0.65	19.64	0.0788
1.06	H 87.5 T	3.56	0.09456	0.31	-0.69	19.55	0.0748
0.91	H 88 T	3.91	0.09012	0.27	-0.73	19.48	0.0701
0.78	H 88.5 T	4.28	0.08512	0.24	-0.76	19.43	0.0647
0.66	H 89 T	4.66	0.07971	0.20	-0.79	19.41	0.0589
0.56	H 89.5 T	5.06	0.07406	0.18	-0.82	19.40	0.0530
0.48	H 90 T	5.47	0.06829	0.15	-0.85	19.41	0.0471
0.40	H 90.5 T	5.90	0.06254	0.13	-0.87	19.44	0.0413
0.34	H 91 T	6.34	0.05694	0.11	-0.89	19.50	0.0359
0.29	H 91.5 T	6.78	0.05156	0.09	-0.91	19.57	0.0308
0.24	H 92 T	7.24	0.04649	0.08	-0.92	19.66	0.0262
0.20	H 92.5 T	7.70	0.04176	0.06	-0.94	19.76	0.0222
0.17	H 93 T	8.17	0.03741	0.05	-0.95	19.89	0.0186
0.15	H 93.5 T	8.64	0.03345	0.04	-0.96	20.03	0.0155
0.13	H 94 T	9.12	0.02988	0.03	-0.96	20.19	0.0128
0.11	H 94.5 T	9.60	0.02668	0.03	-0.97	20.37	0.0106
0.09	H 95 T	10.09	0.02383	0.02	-0.98	20.56	0.0087
0.08	H 95.5 T	10.58	0.02131	0.02	-0.98	20.77	0.0071
0.07	H 96 T	11.07	0.01909	0.02	-0.98	21.00	0.0058
0.06	H 96.5 T	11.56	0.01715	0.01	-0.99	21.24	0.0048
0.05	H 97 T	12.05	0.01545	0.01	-0.99	21.50	0.0039
0.05	H 97.5 T	12.54	0.01396	0.01	-0.99	21.77	0.0032
0.04	H 98 T	13.04	0.01267	0.01	-0.99	22.06	0.0026

+ 76 put. What is the theoretical value and hedge for this trade?

- 9.25, sell 94 futures
- 9.25, buy 94 futures
- 0.25, sell 6 futures

correct
- **0.25, buy 6 futures** ✔
- 0.28, buy 7 futures
- 0.038, sell 6 futures

Continue ›

🔍 **Step 1: Locate the 76 strike row**

From the table:

Column	Value
Strike	H 76 T
Theo.	**0.25**
Delta	**–0.06**

This means the **theoretical value** of the 76 **put** is **0.25**, and its **delta is –0.06**.

🔍 **Step 2: Interpret the delta**

- A **put** has **negative delta**, meaning its value increases when the underlying falls.
- Since delta = –0.06, and we are **long 1 put**, our **net delta is –0.06**.
- To delta hedge, we must neutralize this.

✅ **Step 3: Hedge the delta**

To hedge –0.06 delta:

- You need to **buy 0.06 units** of the underlying (futures).
- If each futures contract is 1 unit (standard assumption), you'd **buy 6 futures** for 100 options (standardized contracts).

So the answer must be:

> **0.25, buy 6 futures**

-(82 call). What is the theoretical value and hedge for this trade?

4.04, sell 73 futures

correct

4.04, buy 73 futures ✓

1.04, sell 27 futures

1.04, buy 27 futures

1.04, sell 73 futures

1.04, buy 73 futures

4.04, sell 27 futures

4.04, buy 27 futures

‹ Back

Continue ›

Table Structure:

- The **first "Theor." column** (to the left of Vega) is indeed the **call** theoretical value.
- The **second "Theor." column** (just left of Vega, on the right side) is the **put** theoretical value.

Example: –(82 call)

- You're **short the 82 call**.
- **Look at the first "Theor." column in the 82 row**:
  - That value is **4.04** — the call's theoretical price.
- **Delta** (from the skew delta column, right after Vega): **+0.73**
  - As shorts, your position delta is –0.73 per contract.
  - For 100 contracts: –73 total delta.
  - To hedge, you **buy 73 futures**.

✅ **Answer**: **4.04, buy 73 futures**

-(72 put). What is the theoretical value and hedge for this trade?

correct

0.09, sell 3 futures ✔

0.09, buy 3 futures

0.09, sell 97 futures

0.09, buy 97 futures

13.08, sell 97 futures

13.08, buy 97 futures

‹ Back

Continue ›

🔍 Step 1: Locate the 72 strike row in the table

From the row labeled **H 72 T**:

- **Call Theo.** (left column) = **13.08** ← ignore, since we are dealing with a **put**
- **Put Theo.** (right "Theor." column, before Vega) = **0.09**
- **Skew Delta (put)** = **–0.03** (yellow column)

🧮 Step 2: Interpret the trade

We are **short** the 72 **put**

- Put delta is **–0.03** → shorting it gives you a **+0.03** delta exposure
- For 100 contracts: **+0.03 × 100 = +3 delta**

To hedge +3 delta, we must **sell 3 futures**

✅ Final Answer:

0.09, sell 3 futures

+ 89/92 call spread (call spread = 89 call minus 92 call). What is the theoretical value and hedge for this trade?

correct

- 0.42, sell 12 futures ✔
- 0.42, buy 12 futures
- 0.42, buy 28 futures
- 0.42, sell 28 futures
- 0.90, buy 12 futures
- 0.90, buy 28 futures
- 0.90, sell 28 futures

‹ Back | Continue ›

🔎 Step 1: Pull from the table

H 89 T (89 call):

- **Theo:** 0.66
- **Skew Delta: +0.20**

H 92 T (92 call):

- **Theo:** 0.24
- **Skew Delta: +0.08**

🧮 Step 2: Calculate Net Theo

$$\text{Theo (spread)} = 0.66 - 0.24 = 0.42$$

🧮 Step 3: Calculate Net Delta

$$\text{Delta (spread)} = 0.20 - 0.08 = 0.12$$

For **100-lot** position:

$$0.12 \times 100 = 12 \Rightarrow \text{Sell 12 futures to hedge}$$

✅ Final Answer:

0.42, sell 12 futures

-(85/86/87 call fly). call fly = +1 85, -2 86, +1 87. What is the theoretical value and hedge for this trade?

0.15, sell 22 futures

0.15, sell 22 futures

1.71, buy 22 futures

1.71, buy 22 futures

-0.08, sell 2 futures

-0.08, buy 2 futures

0.08, sell 1 future

correct

0.08, no hedge

‹ Back

Continue ›

✅ Step 1: Use the correct call theos:

Strike	Call Theo
85	2.11
86	1.63
87	1.23

$$\text{Theo (fly)} = 2.11 - 2(1.63) + 1.23 = 2.11 - 3.26 + 1.23 = 0.08$$

✅ Step 2: Use correct call deltas from yellow column:

Strike	Delta
85	0.51
86	0.43
87	0.35

$$\text{Delta (fly)} = 0.51 - 2(0.43) + 0.35 = 0.51 - 0.86 + 0.35 = 0.00$$

✅ Step 3: Because you are short the fly:

- Total delta of fly = **0.00**
- Shorting the fly doesn’t change that: the position still has **zero net delta**
- So **no hedge is needed**

-(85 straddle). What is the theoretical value and hedge for this trade?

2.11, sell 3 futures

2.11, buy 3 futures

2.11, sell 51 futures

2.11, buy 49 futures

2.11, buy 49 futures

2.11, buy 49 futures

4.22, buy 6 futures

incorrect

4.22, sell 3 futures ✖

none of the above

‹ Back

Continue ›

🔍 Step 1: Get data from the table (Row: H 85 T)

Type	Value
Call Theo (left "Theor.")	**2.11**
Put Theo (right "Theor.")	**2.11**
Call Delta (left yellow column)	**+0.51**
Put Delta (right yellow column)	**–0.48**

🧮 Step 2: Calculate Theoretical Value of Straddle

$$\text{Theo} = 2.11(\text{call}) + 2.11(\text{put}) = 4.22$$

🧮 Step 3: Calculate Delta of Straddle

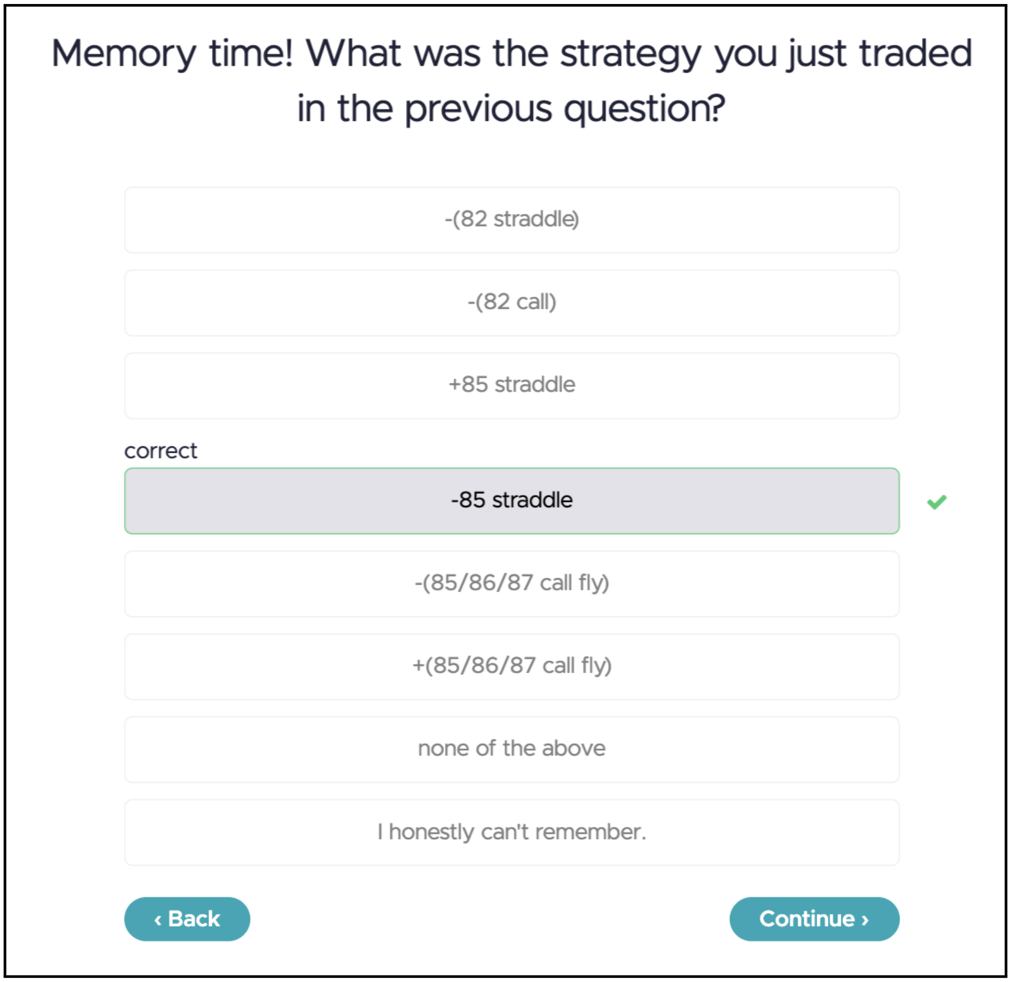
$$\Delta = +0.51(\text{call}) + (-0.48)(\text{put}) = +0.03$$

You're **short** the straddle, so your **position delta** is **–0.03 × 100 = –3**

To hedge, you must **buy 3 futures**

✅ Final Answer:

4.22, buy 3 futures


Memory time! What was the strategy you just traded in the previous question?
-(82 straddle)
-(82 call)
+85 straddle
correct
-85 straddle
-(85/86/87 call fly)
+(85/86/87 call fly)
none of the above
I honestly can't remember.
‹ Back
Continue ›

-(88/85 put spread). What is the theoretical value and hedge for this trade?

correct

1.80, sell 25 futures ✔

1.80, buy 25 futures

1.80, sell 24 futures

1.80, buy 24 futures

3.02, buy 24 futures

3.02, sell 24 futures

3.02, buy 25 futures

3.02, sell 25 futures

‹ Back

Continue ›

✅ **Step 1: Use accurate put theoretical values from the table (right-hand "Theor." column):**

Strike	Put Theo
88	**3.91**
85	**2.11**

$$\text{Theo (spread)} = 3.91 - 2.11 = 1.80$$

✅ **Step 2: Use accurate put deltas from yellow columns:**

Strike	Put Delta
88	**−0.73**
85	**−0.48**

🧮 **Step 3: Compute Delta of the Spread:**

$$\Delta_{\text{spread}} = (-0.73) - (-0.48) = -0.25$$

Since you're **short** the spread:

$$\Delta_{\text{position}} = -(-0.25) = +0.25 \Rightarrow \text{Hedge : Sell 25 futures}$$

Gamma

Gamma

We know the delta of an option for a certain underlying price. However, option deltas change as the underlying price changes. The rate at which deltas change with changes in the underlying is usually expressed per 1-point move in the underlying. This rate of change in delta is known as Gamma.

Gamma is often thought of as $\frac{dDelta}{dUnderlying}$

For example, an option with a gamma of 0.02 would have a change in delta of 0.02 for a 1-point change in the underlying. Let's say the delta of this option is 0.30 with the underlying price of $70.00. If the underlying moves from $70.00 to $71.00, the delta of that option would increase by 0.02 to 0.32. If the underlying instead moves to $69.00 the delta would decrease by 0.02 to 0.28. All options have positive gamma. Therefore, the person who owns (is "long") an option always gets delta exposure (change in delta) in the same direction the future is moving. That is to say that if the future goes, the option holder gets longer deltas; if down, shorter deltas.

Watch the following video on gamma for more insight and examples:

🔍 Breakdown of the Notation

$$\Gamma = \frac{d}{dS}\left(\frac{dV}{dS}\right) = \frac{d^2V}{dS^2}$$

Let's walk through it **piece by piece**:

✅ V = Option value

This is the price of the option — could be a call or a put.

✅ S = Underlying asset price

This is the price of the stock (or asset) that the option is based on.

✅ $\frac{dV}{dS}$ = Delta

This is the **first derivative** of the option price with respect to the underlying price.
It tells you how much the option's price changes when the underlying changes **a tiny bit**.

✅ $\frac{d}{dS}\left(\frac{dV}{dS}\right)$

This means you're taking the **derivative of Delta** with respect to S — i.e., how fast Delta is changing as the underlying moves.

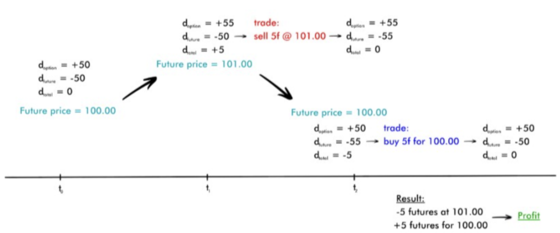
✅ $\frac{d^2V}{dS^2}$ = Gamma

This is the **second derivative** of the option value with respect to the underlying price.
It answers:

> *"If the stock moves, how much will my delta move?"*

Gamma continued:

Gamma allows us to hedge additional deltas we "pick up" to re-balance our portfolio. Since we always accumulate deltas in the direction of the underlying, we will have to re-hedge our portfolio when the underlying moves. We show this in the example below where at time 0 we have a portfolio of long options with 50 option deltas (delta * qty = 0.50*100). We have sold 50 futures against this to create a **delta neutral portfolio** ("flat", or portfolio delta = 0). If the future moves from 100.00 to 101.00 we gain +5 additional deltas from our long gamma. Therefore, we can sell 5 futures to get back to delta neutral. At time t = 2 the future goes back to 100.00 and we get shorter 5 deltas due to our gamma (-0.05*100). Notice, as the underlying does down we, once again, accumulate deltas in the direction of the underlying. To flatten, we can buy 5 futures. The end result is that we sold 5 futures at a higher price (101.00) than we bought them (100.00), and therefore profited from being long gamma. This is commonly referred to as "scalping gamma"; though it's technically scalping deltas accumulated through gamma.



Oftentimes market-makers use **1% cash gamma** (or simply **cash gamma**) as a measure of the gamma of a position of options. Cash gamma is the amount by which our cash delta will change if the underlying moves 1%. This gives us an indication of the size of our position. It also makes it much easier to compare gammas of different products. If our gamma is 30 deltas per $1 move, that position is a lot bigger in a $40 stock than a $5 stock. The cash gamma (along with the cash delta) also allow market makers to predict how many deltas they'll pick up with a 1% move in the underlying with the following relationship.

$$Deltas\ per\ 1\% = \frac{Cash\ gamma}{Cash\ delta}$$

Note that the Cash delta in the above equation is the cash delta of 1 future (since we're only looking at deltas per 1% change in the future).

Therefore, for cash gamma calculations we'd use cash delta as:

Cash Delta = future price * multiplier * 1

Examples of Cash Gamma Calculations:

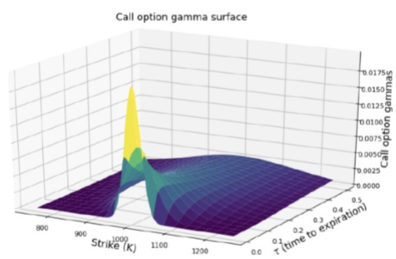
1. Crude oil future is currently $50 per barrel (one future has a cash delta of $50*1000 (1000 barrels per future) = $50,000
2. If we know we pick up 3 deltas for a 1% move, then our cash gamma per 1% is simply 3*50k = $150,000 Similarly, our trading screen can show us that we are long 150,000 cash gamma per 1%, and we can back out that we pick up the 3 deltas each 1% move.
3. In Cattle, the price of 1 future is 150 cents/pound and 1 contract equals 40,000 pounds of live cattle.
4. If for every 1% move in the underlying our delta changes by 3 deltas, so our cash gamma is simply $1.5040,0003= $180,000
5. Soybeans are currently trading at 962 cents per bushel. The unit is 5,000 bushels per future. Using the Cash Gamma number below, how many futures will we pick up if the future moves 1%?

Und	Exp	Vega	Sqrt(t) Normal	SplChrm	SplDlt	1% Spline Cash Gamma
TOTAL		19,194	22,011	-9	-28	268,901

$$Deltas\ per\ 1\% = \frac{Cash\ gamma}{Cash\ delta} = \frac{Cash\ Gamma}{Price * \ Multiplier} = \frac{268{,}901}{(9.62 * 5000)} = 5.6$$

Gamma vs Underlying, Time, & Vol

The graph below shows gamma across both strike space and time to expiration. Note gamma is largest near the ATM (at-the-money) for strikes close to expiry. Notice that through time gamma ebbs and flows for most other strikes. In a given expiry, gamma will be larger near the ATM then on the wings.

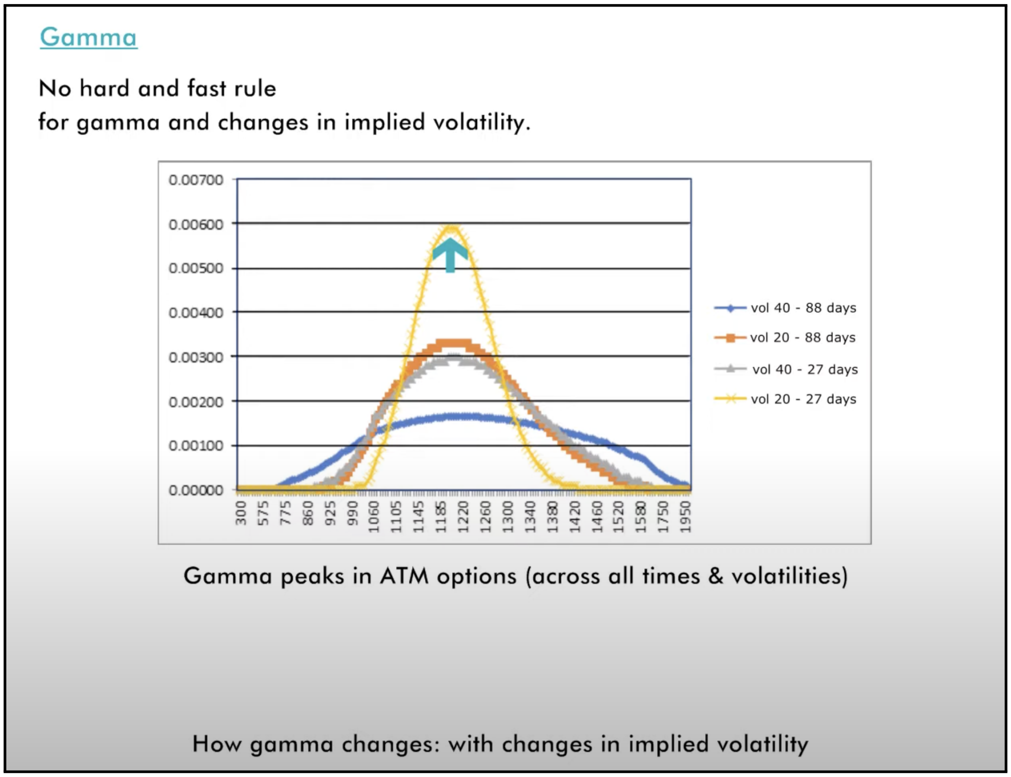


We have seen that by being long options gives the option buyer a long gamma position. That long gamma allows the option holder to make profit scalping gamma when the underlying moves. So, why would anyone ever sell options? Wouldn't we always want to be long?

To answer those questions we present the following fact: As time passes options decay in value. If you buy an option for $2.00 today (long gamma) that option will, all else equal, be worth less than $2.00 tomorrow (perhaps something like $1.90). The decrease in value of an option due to time is called theta. This is why we can't simply always be long gamma. We'll explore the Greek theta in the next section to prove the following:

A long gamma portfolio will, all other factors held constant, profit when the P&L from underlying movement exceeds the amount of theta paid. Similarly, a short gamma portfolio will, all other factors held constant, profit when the theta collected exceeds the amount lost due to movement in the underlying.

Hello and welcome to this video presented by Akununa Capital. In it, we'll be discussing the Option Greek Gamma. We'll start by defining the term "gamma." We'll then walk through a basic gamma example. Next, we'll explore the effects of time and volatility on the Greek. And finally, we'll walk through a gamma scalping example. We'll start with the definition and some of the main characteristics of Gamma. You should already have watched the video on delta and have a working knowledge of that Greek before tackling Gamma. Gamma is the rate of change of delta, with a change in the underlying asset price. The gamma of an option is expressed as the change in delta for a one-point move in the underlying, and we'll go through an example of this. All options have a positive gamma. Similar to Vega, if you buy options, you're positively exposed to gamma. Remember, for Vega, that means that if volatility went up, it'd make a profit. Here, the positive exposure is that you'll gain deltas in the direction of the underlying moves. I'll reiterate that gamma is the derivative of delta. And we can now dive into an example to show how it behaves. Let's take an example where we have a future price of $30. We have an option with the price of $0.91, a delta of 0.31, and a gamma of 0.06. Now let's see what happens when the future increases 1 point from 30 to 31. The price of the option changes to \$1.25. Let's go through the calculation to see how we get that. But first, let's remember from the Delta video that we learned that the change in price should be the change in the underlying times the delta. But we simplified things and didn't discuss that the delta wasn't held constant throughout the calculation. Now we can see that we have a starting delta of 0.31 and an ending delta of 0.37. So instead, we approximate by taking the average delta across the move and averaging 0.31 and 0.37 to get 0.34. This gets us to our price of \$1.25. But let's look at how the delta actually changed. We can see that the delta increased by 0.06 over the one-point move in the future. This is due to the gamma. You can see we had a starting gamma of 0.06 but an ending gamma of 0.065. Clearly, the gamma is dynamically changing, and these approximations will break down for larger moves in the future, as the deltas and gammas continue to change. Next, let's take a look at how gamma changes with different volatilities and times to expiry. I'll focus first on the blue and orange lines. These fix time to expiry constant at 88 days and compare vol 20 and 40 to see the effects on gamma of different volatilities. You can see that in both curves, the gamma peaks near the at-the-money options in the middle of the graph. The orange line, with a lower vol of 20, has the higher gamma in the middle points. Therefore, movement in the underlying with a lower implied volatility has a greater effect on the deltas of options near the at-the-money. Now, as we move further from the center points, we can see that this relationship doesn't hold across the curves. At some point, the blue (higher vol) is above the orange (lower vol) curve. Unlike Vega, whose options are always higher or lower with a change in implied volatilities, the gamma's across strike space don't follow a rule where the lower vol always has the higher gamma. By comparing the blue and orange curves to the gray and yellow ones, we can see what happens as we have less time to expiry. The options near the at-the-money have increased gamma as we move closer to expiry. Therefore, the delta of options is more sensitive to changes in the underlying as we move closer to expiry. Again, this only holds near the at-the-money options, not throughout the curves. Feel free to pause the video to spend some time looking at these relationships now, if needed. Now let's look at an example of how gamma affects deltas and forces us to re-hedge our portfolio of options with a change in the underlying. It's something we call gamma scalping. First, we'll remember, as we discussed in previous payoff diagram videos, market makers always want to have as little risk on as possible. Therefore, they will hedge an options portfolio to minimize this delta risk, or directional risk, whenever possible. This means that this portfolio's P&L is indifferent to a directional move in the underlying and makes the same whether the underlying moves up or down. So let's look again at the example from earlier. We have the same future price of \$30, option price of \$0.91, and a delta of plus 0.31, which means that they're calls because it's positive delta. And of course, we have the gamma of 0.06. But now let's pretend we've bought 100 of these options. This gives us a plus 31 delta, which we get by multiplying the 0.31 by 100 contracts. Now we have a positive exposure to an upward move in futures because we have a positive delta. Therefore, to hedge this portfolio, we need to go out and sell 31 futures in the marketplace. We've recorded this as a minus 31 futures delta, giving us an overall portfolio delta of zero. Now let's see what happens when the future moves up to 31. You'll notice that the price of the option changes, as we described earlier, and the gamma also means that the delta has changed from 0.31 to 0.37. Now, because we're long 100 of these options, we now have a plus 37 delta from the options, giving us an overall delta of six. This now gives us an unbalanced upside exposure to futures prices again, meaning we make more money if the futures continue to increase, but we'd lose if the futures decreased again. So we must sell more futures again to balance this risk. So we sell six additional futures at a current price of \$31. Now, what if, after some time passes, the future goes right back to \$30 again? The option price reverses, and so does the delta, due to the gamma, giving us minus six futures now because we sold this amount earlier. Therefore, we must buy six futures back again to balance the portfolio. We do this at the current price of \$30. So let's look at our end result. The options are still worth the same \$0.91 since not much time has passed in our example. We sold six futures at \$31 and bought six futures for \$30, making us a profit of \$1 times six futures, times whatever multiplier is applied. We can run through other scenarios, but the results are always the same. When you're long gamma, you gain deltas in the direction of the underlying move. These are deltas that you're going to buy or sell back out. And if the future reverts, you get to scalp the gamma. Notice if you're short the 100 options, you would have to do the inverse, buying high and selling low and locking in a loss. So the question is, why not always be long gamma, since you make profit every time the underlying moves? I'll hint at the answer by reminding you that the option price won't always stay at \$0.91, like it did in our example. With the passage of time, the option value will decrease. This is called theta, and we'll discuss it in the next video. But you can already begin to see the eternal struggle for option buyers and sellers between gamma and theta. Does the P&L made from the movement in the underlying exceed the amount of money that you'll pay from the loss of the option value over time? I'll leave you to ponder that question and tune in for our next video on theta, where we explore the topic further. That's all for this video. Thanks for watching and get in touch at akunacapital.com.



Gamma Exercise

Exercise – Gamma

Answer the questions below to understand how gamma changes option deltas with changes in underlying price.

Future price = 370.00

Gamma	Vol	QBidO	cVolum	Delta	Bid Qty	BidPx	Theo	Ask Px	Ask Qty	Strike	T Pos	Bid Qty	BidPx	Theo	Ask Px	Ask Qt	Vega
0.0001	0.245	0.155		1.00			110.02			260	0	0	0.000	0.02	0.125	50	0.009
0.0003	0.245	0.155		1.00			100.06			270	0	0	0.000	0.06	0.125	48	0.022
0.0006	0.245	0.154		0.99			90.15			280	0	25	0.125	0.15	0.250	46	0.047
0.0011	0.245	0.154		0.98			80.34			290	0	44	0.250	0.34	0.375	44	0.088
0.0017	0.238	0.154		0.97			70.61			300	0	42	0.500	0.61	0.625	42	0.138
0.0026	0.231	0.152		0.95			61.04			310	0	40	1.000	1.04	1.125	40	0.205
0.0038	0.224	0.148		0.91			51.73			320	0	38	1.625	1.73	1.750	38	0.291
0.0054	0.219	0.145		0.87			42.90			330	0	36	2.875	2.90	3.000	36	0.400
0.0071	0.216	0.140		0.80			34.67			340	0	34	4.625	4.67	4.750	34	0.516
0.0086	0.213	0.135		0.72			27.25			350	0	32	7.125	7.25	7.250	32	0.622
0.0097	0.212	0.124		0.62	26	20.750	20.87	20.875	26	360	0	30	10.75	10.87	10.875	30	0.700
0.0103	0.211	0.112		0.52	26	15.375	15.50	15.500	26	370	0	28	15.375	15.50	15.500	28	0.734
0.0100	0.213	0.108		0.42	28	11.250	11.32	11.375	28	380	0	26	21.25	21.32	21.375	26	0.720
0.0092	0.214	0.098		0.33	30	8.000	8.10	8.125	30	390	0			28.10			0.667
0.0080	0.217	0.089		0.25	32	5.625	5.72	5.750	32	400	0			35.72			0.588
0.0067	0.221	0.080		0.19	34	4.000	4.03	4.125	34	410	0			44.03			0.499
0.0054	0.225	0.073		0.14	36	2.750	2.82	2.875	36	420	0			52.82			0.411
0.0043	0.229	0.067		0.10	38	1.875	1.98	2.000	38	430	0			61.98			0.331

What is the delta of the 370 strike call?

- 0.0103
- 0.211
- 0.112

correct

- **0.52** ✔
- 15.50
- 0.48
- -0.48

Continue ›

If you buy 100 calls, how many futures must you trade to flatten your delta from the previous question?

- 0
- sell 0.48
- sell 48
- buy 480
- sell 0.52
- correct: sell 52 ✔
- buy 52
- none of the above

‹ Back

Continue ›

If the future price increases 2.00 to 372.00, what will be the delta of the 370 call? (assume constant gamma across move)

correct

- 0.5406
- 0.5206
- 0.5103
- 0.5097
- 0.4994
- none of the above

‹ Back

Continue ›

We're given:

- **Current future price = 370.00**
- **Current delta of the 370 call = 0.52**
- **Gamma of the 370 call = 0.0103**
- **Price increases by 2.00 (from 370.00 to 372.00)**

Formula:

$$\Delta_{\text{new}} = \Delta_{\text{old}} + (\Gamma \times \Delta S)$$

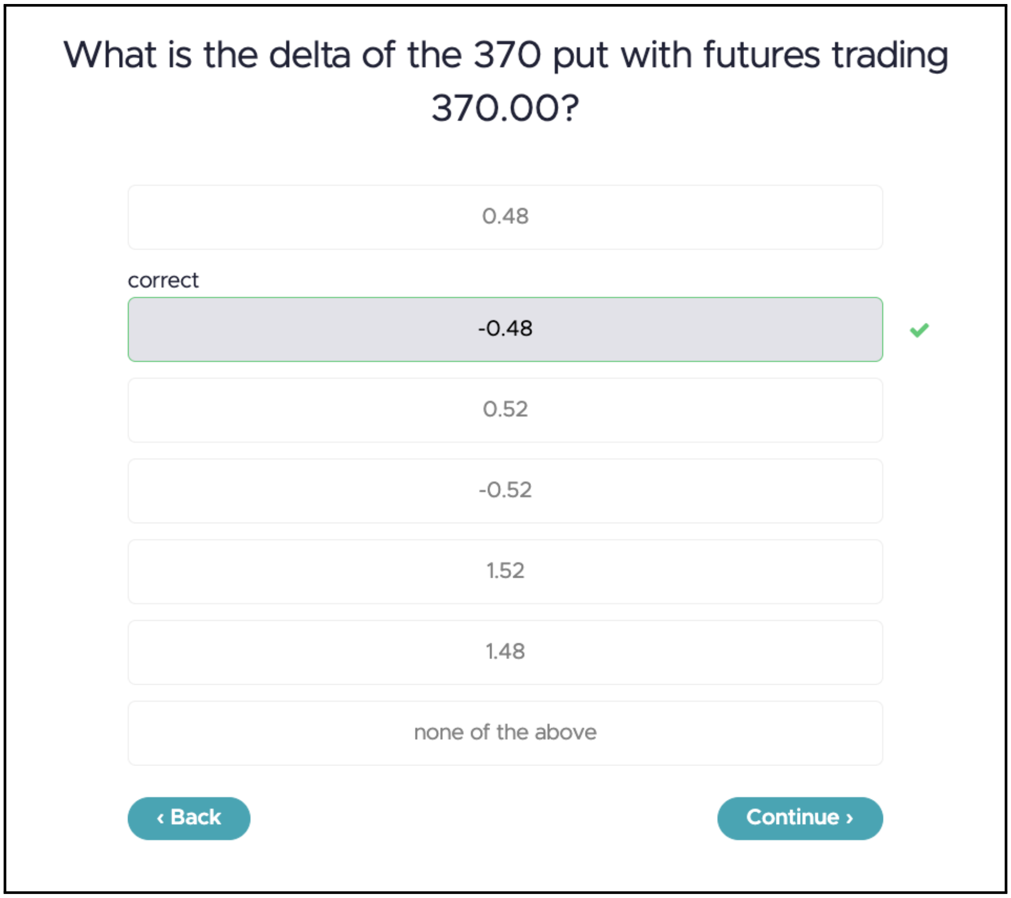
Where:

- $\Delta_{\text{old}} = 0.52$
- $\Gamma = 0.0103$
- $\Delta S = 2.00$

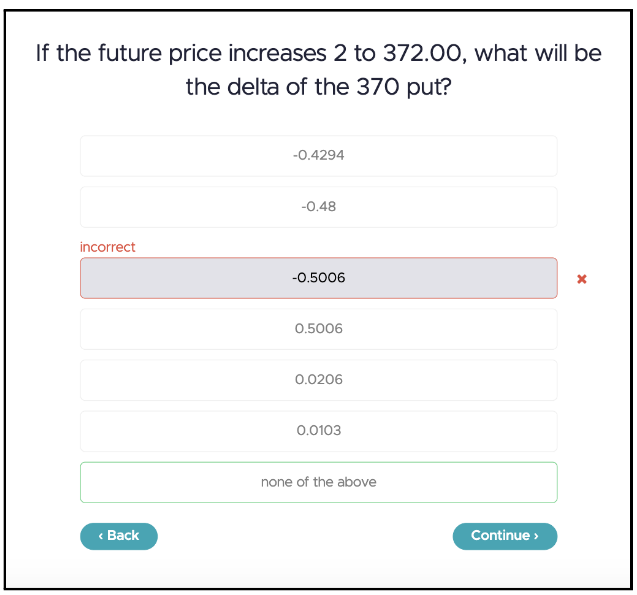
Plug in values:

$$\Delta_{\text{new}} = 0.52 + (0.0103 \times 2.00) = 0.52 + 0.0206 = 0.5406$$

✅ Correct answer: 0.5406 (Top option)



Reference page 2 of this document to understand "the why"



Given:

- Original delta (370 put) = **−0.48**
- Gamma = **0.0103**
- Futures price increases by **+2.00**
- Gamma affects call and put deltas symmetrically, but for puts it **reduces** delta as the price **rises**

$$\Delta_{\text{new}} = \Delta_{\text{old}} + (\Gamma \times \Delta S)$$

$$\Delta_{\text{new}} = -0.48 + (0.0103 \times 2.00) = -0.48 + 0.0206 = -0.4594$$

So the correct answer should be:

✅ **−0.4594**

We buy 100 of the 330 calls. We hedge our delta by selling 87 futures at 370.00. If the future increases 2 points to 372.00, what must we do to rehedge our portfolio back to a flat delta?

- nothing
- buy 1 future
- correct: sell 1 future ✔
- buy 2 futures
- sell 2 futures
- buy 20 futures
- sell 20 futures

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Setup Recap

- You **buy 100 of the 330 calls**
- At a **futures price of 370**, the 330 call has:
  - **Delta = 0.87**
  - **Gamma = 0.0054**
- You hedge by **selling 87 futures**
- The futures price **rises 2 points to 372.00**

Step 1: Compute New Delta Per Option

Gamma tells us how much **delta changes per $1 move** in the underlying.

$$\Delta_{\text{change}} = \Gamma \times \Delta S = 0.0054 \times 2 = 0.0108$$

So new delta per option:
$0.87 + 0.0108 = 0.8808$

Step 2: Compute New Portfolio Delta

You own 100 options, so:
$100 \times 0.8808 = 88.08 \text{ deltas}$

You are short 87 futures, so your **net delta**:
$88.08 - 87 = +1.08$

To flatten this, you must **sell 1.08 futures**, so:

✅ **Answer: Sell 1 future**

Memory Time: What strike call option did we ask you about in the first question on this quiz?

- 330
- 340
- 350
- 360

correct

- 370 ✔
- 380
- none of the above

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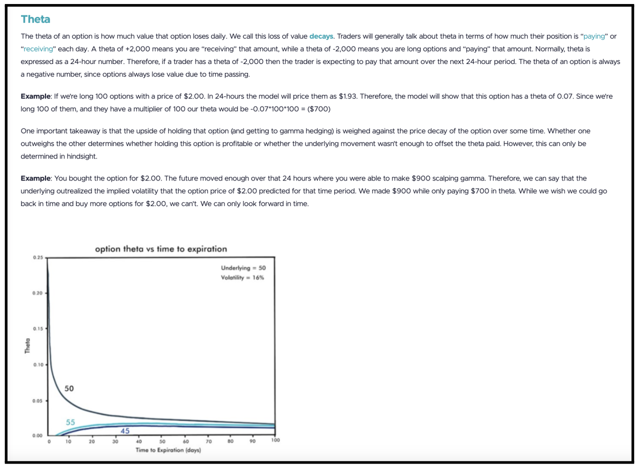
Theta



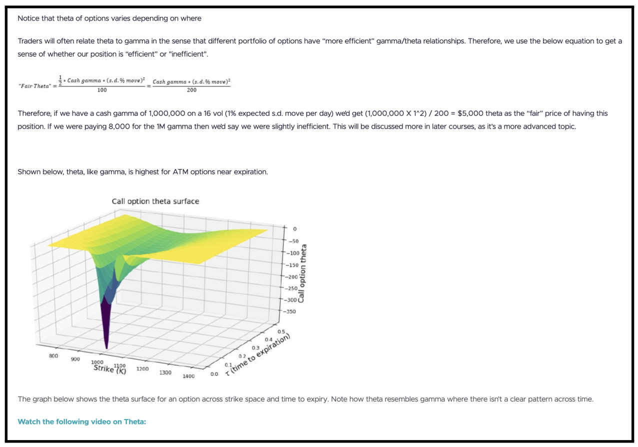
Theta

The theta of an option is how much value that option loses daily. We call this loss of value **decays**. Traders will generally talk about theta in terms of how much their position is "paying" or "receiving" each day. A theta of +2,000 means you are "receiving" that amount, while a theta of -2,000 means you are long options and "paying" that amount. Normally, theta is expressed as a 24-hour number. Therefore, if a trader has a theta of -2,000 then the trader is expecting to pay that amount over the next 24-hour period. The theta of an option is always a negative number, since options always lose value due to time passing.

Example: If we're long 100 options with a price of $2.00. In 24-hours the model will price them as $1.93. Therefore, the model will show that this option has a theta of 0.07. Since we're long 100 of them, and they have a multiplier of 100 our theta would be -0.07*100*100 = ($700)

One important takeaway is that the upside of holding that option (and getting to gamma hedging) is weighed against the price decay of the option over some time. Whether one outweighs the other determines whether holding this option is profitable or whether the underlying movement wasn't enough to offset the theta paid. However, this can only be determined in hindsight.

Example: You bought the option for $2.00. The future moved enough over that 24 hours where you were able to make $900 scalping gamma. Therefore, we can say that the underlying outrealized the implied volatility that the option price of $2.00 predicted for that time period. We made $900 while only paying $700 in theta. While we wish we could go back in time and buy more options for $2.00, we can't. We can only look forward in time.



Notice that theta of options varies depending on where

Traders will often relate theta to gamma in the sense that different portfolio of options have "more efficient" gamma/theta relationships. Therefore, we use the below equation to get a sense of whether our position is "efficient" or "inefficient".

$$\text{"Fair Theta"} = \frac{\frac{1}{2} * \text{Cash gamma} * (s.d.\%\ move)^2}{100} = \frac{\text{Cash gamma} * (s.d.\%\ move)^2}{200}$$

Therefore, if we have a cash gamma of 1,000,000 on a 16 vol (1% expected s.d. move per day) we'd get (1,000,000 X 1^2) / 200 = $5,000 theta as the "fair" price of having this position. If we were paying 8,000 for the 1M gamma then we'd say we were slightly inefficient. This will be discussed more in later courses, as it's a more advanced topic.

Shown below, theta, like gamma, is highest for ATM options near expiration.

The graph below shows the theta surface for an option across strike space and time to expiry. Note how theta resembles gamma where there isn't a clear pattern across time.

Watch the following video on Theta:

Hello and welcome to this video presented by Ak Kununa Capital. In it, we'll be discussing the Option Greek Theta. In this video, we'll start by defining theta, we'll then work through a simple example. We'll briefly talk about the theta-versus-gamma relationship. We'll plot theta across time and strike space, and then finally, I'll leave you with a summary of the main takeaways. Let's start by examining our options board to find our Greek theta. If we look to the far left, we'll find the theta on this board. So now that we've found it, let's start by defining what it is. The theta of an option is how much value the price of the option will lose over the next trading day. Traders will often use the term "decay," since the price of an option will decrease or decay slowly over time. Let's look at an example by focusing on the 380 strike call on this board. The theoretical value of the call is 11.04, and the theta of this line is negative 12 cents. This means that the price of the option in 24 hours will be 12 cents less than the current price of $11.04, giving us $10.92. Notice that the theta is expressed as a negative number, and it's also usually expressed as cents per 24-hour period. Now, let me specify here that this is holding all other factors constant, including the price of the underlying, as well as the implied volatility and other inputs into the equation we use to price the option. So for this discussion of theta, we assume that all other inputs are held unchanged. Next, let's bring your attention to the fact that theta is the same for a call and a put option on the same strike. This means we can look at the 380 strike put as well, and calculate that the theoretical value will change by the same 12 cents. This should make sense, as the call and put thetas should line up, otherwise we'd violate put-call parity equations. While most other Greeks are described by using the terms "long" for being the owner or buyer of an option, and "short" for being the seller, the term "paying theta" is used for someone who is long the option and must pay as the price of the option decreases, and traders will use the term "collecting theta" if you sold the option and are taking in money as the option price decreases, since you will collect that premium. So now that we know that the theta of an option is how much value the option loses in a day, we can start to ask some questions that we posed in the preceding video on gamma. Does the theta paid offset the gain of owning the option? And more specifically, does the theta paid offset the gains we expect due to the underlying movement of the future or stock? Let's go through an example to illustrate. Since we know that our option will be 12 cents lower over the next 24 hours, that's a known and can be thought of as a cost of holding the options over that period of time. But the question is, what do we get in return? To answer, we'll review the gamma scalping example from the previous gamma video. I'll note here that these are two different options, and we simply use the known example to work through the methodology, since we've already gone through the steps. We're long options, and the future goes from a price of 30 to 31. Our long gamma position means that we have picked up extra deltas. We sell the six futures to offset the delta change. Then the future goes back down. This results in us picking up short deltas on the way down because of our gamma. Therefore, we must buy futures again. In the end, being long options enabled us to sell futures at a higher price, 31 in this case, and buy them back at a lower price, 30, and made some amount of profit as a result. So the theta of 12 cents times 100 options, times a multiplier of 100 gives us a cost or expected decay of $1,200 for owning this position. The gamma we were able to scalp from our previous example was six futures times $1 per future times the multiplier, to give us a profit of $600. If we had performed this analysis on the same two options—and for simplicity here, we'll assume that we did—then we'd compare these two values to see that we had a total loss of $600. Therefore, the theta decay was larger than the amount we were able to make gamma scalping over this time period. We can think of it another way. The movement in the future didn't allow us to make enough to overcome the loss from the theta we paid for that time period. Now we'll leave this train of thought here, as the analysis of gamma versus theta gets us into more advanced topics that we'll talk about later. We included it briefly because this example often helps new traders understand the balance between these two Greeks. Now let's change gears and look back at our options board to examine how theta changes through strike space and with different times to expiration. Let's focus on the theta column and plot it across strikes with the black line. Note that our graph shows these as negative numbers to drive home the sign, though the convention is to graph them or think of them as positive numbers. I've also increased the time to expiry and plotted that with a green line. You can see that the theta is greatest in the at-the-money options on both graphs, and that they cross over so that the relationship between time and expiry doesn't increase with every strike as we add time to expiry. But we can see that it holds for strikes near out-of-the-money. We can examine this on another chart, where we plot the theta of options for three strikes through different times to expiry. The out-of-the-money 45 and 55 strikes will have decreasing thetas close to expiry, while the strike that is exactly at the underlying price will continue to have increasing theta as we approach expiry. I won't get into why the 55 strike is above the 45 strike in this lecture. Instead, let's focus on the takeaway that theta increases as we get close to expiry for at-the-money strikes. Theta is one of the more straightforward Greeks. So I'll summarize our main points below: the theta of an option is the amount by which the option will decrease in price. Usually, this is expressed per 24-hour period. Theta is greatest near the at-the-money strikes, and it increases as we get closer to expiry for these strikes. Finally, theta increases at an increasing rate as we approach expiry if the future value is right on strike. Hopefully, the example we presented where theta is related to gamma when trying to account for PnL is useful. Finally, I'll remind you that traders use the terms "paying theta" and "receiving theta," rather than saying "long" or "short" theta. Hopefully, this video has provided some insight about how market makers think about and work with theta.

Greeks and General Quiz Exercise

The greek that models the change in option price over time is known as _____________.

- delta
- gamma
- correct: theta ✔
- vega
- none of the above

Continue ›

We sold calls at $3.30, 40x. The theta for this call is 0.20. This option has a multiplier of 50. Which of the following best describes the theta from this position? (assume nothing else changes)

- We'll pay $3.30 in theta over the next 24 hours.
- We'll collect $3.30 in theta over the next 24 hours.
- We'll pay $200 in theta over the next 24 hours.
- We'll collect $200 in theta over the next 24 hours.
- We'll pay $0.20 in theta over the next 24 hours.
- We'll collect $0.20 in theta over the next 24 hours.
- We'll pay $8 in theta over the next 24 hours.
- We'll collect $8 in theta over the next 24 hours.
- We'll pay $400 in theta over the next 24 hours.
- correct: We'll collect $400 in theta over the next 24 hours. ✔

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We are **short** 40 calls (sold), each with:

- **Theta = 0.20** (daily decay **benefiting** us as sellers)
- **Multiplier = 50**
- So, daily theta per contract = $0.20 \times 50 = 10$ dollars collected
- Total across 40 contracts:

$40 \times 10 = 400$

✅ Because theta decay benefits sellers, **we collect** the theta.

Correct answer:

"We'll collect $400 in theta over the next 24 hours."

You are long 30 deltas in a portfolio of options. You are not hedged. You are long gamma. If the underlying moves lower in price, which of the following best describes your new delta position?

greater than 30

equal to 30

correct

less than 30 ✔

can't tell

none of the above

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You're **long 30 deltas** and **long gamma**. If the **underlying moves lower**, your **delta will decrease**.

Why?

- Gamma is the **rate of change of delta**.
- Being **long gamma** means you gain deltas when the underlying rises, and **lose deltas when it falls**.
- So if price drops, **delta goes down**.

✅ **Correct answer:**

"less than 30"

If someone offered you a call spread for price \$0, how would you respond?

I'd have to know the strikes to know whether \$0 was a good price to buy.

incorrect

I'd have to know the volatility, strikes, time to expiry and all other inputs to assess whether \$0 was a good price to buy. ✖

I'd buy it only if both the calls are in-the-money.

I'd buy it for \$0, since no call spread can be less than that price.

I'd sell it at \$0 since no call spread can be worth more than that price.

I'd need to know how far apart the two strike prices were so I could look at a payoff diagram and assess the risk vs benefit of buying the spread..

I'm not sure.

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Why is that marked right?

From a **practical market-making perspective**, a **call spread worth \$0** is **free optionality** — there's no downside, only potential upside. Since:

- A call spread can't be worth *less than zero*.
- You're not risking capital.
- Worst case: it's worthless.
- Best case: it finishes in the money and you profit.

So **buying a \$0 spread is always a favorable trade**, regardless of greeks or inputs.

Why the detailed input-based answer was marked wrong:

Even though it's **technically more complete**, the question was really testing for **trading intuition** — and in practice, **a free spread is a buy**.

> ✅ Final takeaway:
> If someone offers you **free convexity**, you take it.

A call had a 30 delta before the future moved 1.5 points. The new call delta is now 37. What is the gamma of this call? (assume gamma remains constant over the move).

- 0.02
- 0.03
- 0.045
- correct: **0.046** ✔
- 0.10
- 0.125
- 0.15
- 0.20
- 0.225
- 0.30

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We're given:

- Initial delta: **30**
- New delta: **37**
- The future moved: **1.5 points**
- We're told **gamma is constant**.

Gamma measures the **rate of change of delta** per 1-point move in the underlying.
So the formula is:

$$\gamma = \frac{\Delta_{\text{new}} - \Delta_{\text{old}}}{\Delta S}$$

Substitute:

$$\gamma = \frac{37 - 30}{1.5} = \frac{7}{1.5} = 4.\overline{6}$$

But gamma is typically expressed **per 1 unit of underlying** in **decimal form**, not as deltas out of 100. So we **convert from 4.666 deltas to 0.04666...**

✅ **Final Answer:**

$$0.046$$

That's the gamma in **decimal form per 1-point move** in the underlying.

Reference page 22 of this document to understand "the why"

Which of the following is the most likely tick increment of this product, given the following bids in the market: 0.50, 0.75, 0.875, 1.00, 1.25, 1.50, 2.00

- 1
- 0.5
- 1/4th
- 0.1

correct

- 1/8th ✓
- 1/12th

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Let's break it down.

The bids are:

```
0.50, 0.75, 0.875, 1.00, 1.25, 1.50, 2.00
```

Now look at the **differences** between them:

- 0.50 → 0.75 = 0.25
- 0.75 → 0.875 = 0.125
- 0.875 → 1.00 = 0.125
- 1.00 → 1.25 = 0.25
- 1.25 → 1.50 = 0.25
- 1.50 → 2.00 = 0.50

The **smallest common increment** is **0.125**, which is:

$$\frac{1}{8}$$

✅ **Final Answer:**

$1/8\text{th}$

A call and put on the same line have:

opposite intrinsic values

correct

equal amounts of extrinsic value

the same amount of intrinsic value

different values. The OTM option always has more extrinsic value

different values. The ITM options always has more extrinsic value

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Volatility

Before we move ahead to the Greek vega, we present the topic of volatility. To proceed with vega without first defining volatility would be like describing the ocean without first defining water.

Volatility Basics

Volatility is a measure of the deviation of an underlying's annual price movement. Formally, it is the degree of variation of a trading price series over time as measured by the standard deviation of logarithmic returns.

🔷 **"the degree of variation of a trading price series over time"**

This refers to **how much a stock's (or any asset's) price moves around** as time passes.

- If the price changes a lot, it has **high variation** (i.e., high volatility).
- If it stays steady, it has **low variation**.

🔷 **"as measured by the standard deviation"**

This means we are **quantifying** that variation using a specific mathematical tool: **standard deviation**.

- Standard deviation tells you *how spread out the numbers are* from their average.
- So in this context: **How spread out the returns are from their average return**.

🔷 **"of logarithmic returns"**

This is the key part. Instead of using simple percentage changes (called arithmetic returns), finance often uses **logarithmic returns**, which are defined as:

$$\text{log return at time } t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Where:

- P_t is the price today,
- P_{t-1} is the price yesterday,
- $\ln$ is the natural logarithm.

Why use log returns?

- They are time-additive: the log returns over multiple days can be added up to get the total log return.
- They better approximate continuously compounded returns.

🔁 **Apply to Log Returns:**

Let's say:

- Day 1: $r_1 = \ln\left(\frac{P_1}{P_0}\right)$
- Day 2: $r_2 = \ln\left(\frac{P_2}{P_1}\right)$

Then:

$$r_1 + r_2 = \ln\left(\frac{P_1}{P_0}\right) + \ln\left(\frac{P_2}{P_1}\right)$$

👉 **Use the log addition rule:**

$$\ln\left(\frac{P_1}{P_0}\right) + \ln\left(\frac{P_2}{P_1}\right) = \ln\left(\frac{P_1}{P_0} \cdot \frac{P_2}{P_1}\right)$$

🧹 **Cancel the P_1 terms:**

$$\frac{P_1}{P_0} \cdot \frac{P_2}{P_1} = \frac{P_2}{P_0}$$

Now we have:

$$r_1 + r_2 = \ln\left(\frac{P_2}{P_0}\right)$$

✔ Which means: **The total log return over 2 days is the sum of the 2 individual log returns.**

Rule 1: $\log_b(M \cdot N) = \log_b M + \log_b N$

Rule 2: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Rule 3: $\log_b\left(M^k\right) = k \cdot \log_b M$

Rule 4: $\log_b(1) = 0$

Rule 5: $\log_b(b) = 1$

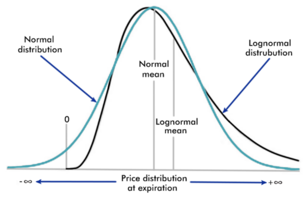
Rule 6: $\log_b\left(b^k\right) = k$

Rule 7: $b^{\log_b(k)} = k$

Where:

$b > 0$ but $b \neq 1$, and M, N, and k are real numbers but M and N must be positive!

Because of the continuous compounding of volatility moves over time we find that a lognormal curve best represents the distributions of an underlying instrument. Just as interest compounds, the volatility distribution will show a larger tail to the upside and a more compact distribution to the downside. The fact that all values are positive in a log distribution provides a well modeled floor on stocks and futures since their value can never be below 0.



There are several types of volatilities used by market makers.

Historical volatility (aka realized volatility) is a measure of how much an underlying has moved in the past. Since this volatility is found from observing a time series data of past markets, finding the historical volatility of an underlying over a given time-frame is a simple task. Usually, when measuring and calculating historical volatility the close-to-close prices are used for each day. This can miss some measure of intraday volatility since the underlying can have large moves only to finish unchanged (close to zero change on day). This measure isn't perfect and there are alternative ways to capture movement, but this is the most widely used time interval for large historical datasets.

Implied volatility is the expected/predicted future volatility of the underlying asset. This is an **input** into our pricing model and it's what is used to drive our option theoretical values. Different market participants may have different implied volatilities, but the overall market will be made up of the combination of the best bids and offers provided by the implied volatilities of everyone's market. It's important to point out that each individual can **control their option theos by changing the implied volatility in their model**.

Forward volatility is the expected average volatility between the expiration dates of two options with successive maturities. We'll leave in-depth discussion of this topic for later courses.

✅ **Implied Volatility (IV)**

- **Definition**: The market's expectation of **future volatility** for an asset over the **life of a specific option**.
- **Derived from**: The current price of the option using a pricing model like Black-Scholes.
- **Timeframe**: It's tied to a **specific option maturity** (e.g., 1 month, 3 months, 6 months).
- **What it reflects**: The **average expected volatility** from now until the option's expiration.
- **Key point**: It's **backed out from option prices**, not predicted by a model.

Example:

If a 1-month option has a 20% IV, the market expects 20% annualized volatility over the next month.

✅ **Forward Volatility**

- **Definition**: The **expected volatility between two future dates**, often derived from the term structure of implied volatility.
- **Derived from**: Two implied volatilities of different maturities using the formula for variance subtraction:

$$\sigma^2_{fwd} = \frac{\sigma_2^2 T_2 - \sigma_1^2 T_1}{T_2 - T_1}$$

where:

  - σ_1, σ_2 = implied volatilities for maturities T_1, T_2
  - σ_{fwd} = **forward volatility** between T_1 and T_2
- **Timeframe**: Refers to **volatility in the future**, not from now.
- **What it reflects**: Market's expectation of **volatility in a future interval**, not starting today.

Example:

If the 1M IV is 15% and 3M IV is 20%, then the **forward volatility between month 1 and month 3** could be computed using the formula above. This helps traders price options expiring **after** month 1 but **before** month 3.

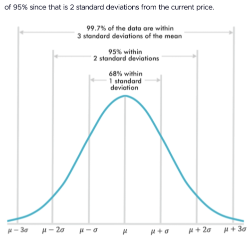
🔁 Summary

Feature	Implied Volatility	Forward Volatility
Refers to	Now until a given option's expiration	Between two future dates
Derived from	Option prices	Differences in implied vols (term structure)
Use case	Pricing vanilla options	Pricing calendar spreads, forward-start options
Time dependency	Anchored at **now**	Anchored at **future interval**

Volatility as a Measure of Movement

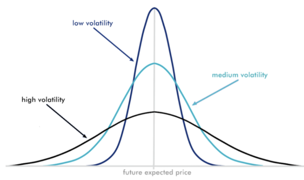
The volatility numbers used in options pricing are a measure of the standard deviations of a product's future price move.

For example, a 20% volatility would tell us that the underlying will be within 20% of the current underlying price with a 68% probability (1 standard deviation) a year from now. So, an underlying that is currently priced at $100 would be between $80 and $120 one year from today with a 68.3% probability. Similarly, it would be between $60 and $140 with a probability of 95% since that is 2 standard deviations from the current price.



(These numbers are slightly off since we assume a lognormal, not normal distribution.)

In this manner, we can translate volatilities into expected price distributions of the underlying instrument, as shown in the graph below. For example, a low volatility would have a tighter distribution with a higher peak near the current underlying forward price.



The equation for the volatility over t days is shown below. Traders oftentimes use 256 days to approximate the number of trading days in a year, because the sqrt of 256 is 16, though 252 is the more precise number used when modeling. We can calculate the expected standard deviation percent move for a period of days by:

$$\text{expected 1 s.d. move over t days} = \sigma * \sqrt{\frac{t}{252}}$$

To turn the 20% vol above (we refer to 0.20 as 20% vol or simply "20 vol") into a 1-day s.d. move by: 20*sqrt (1/252) = 1.26% move is the 1 s.d. for 20 vol.

However, this is **NOT** the same as saying that, on any given day, the expected move of that product is 1.26%. This is somewhat counter-intuitive. For those who want to take our word for it, the expected daily move formula is simply:

🔧 Key Concepts in the Image

🔷 1. Volatility ≠ Expected Move

Even if a stock has 20% annualized volatility, that doesn't mean it "moves 20%" in a year. Instead:

- Volatility is the **standard deviation** of returns.
- Most daily changes are **less than the full σ**, and the **average daily move** is **not** σ.

$$\text{Expected Daily Move} = 0.8 * \sigma \sqrt{\frac{t}{252}}$$

And for the more mathematically inclined, see the equation below for a derivation. We'll leave it to the reader to note where the 0.8 approximation term above comes from by examining the equations below.

Let the daily move be a normally distributed random variable denoted by X.

$$X \sim N\left(0, \frac{\sigma^2}{256}\right),$$

where σ denotes the volatility of the product. For clarity purposes, we define $\sigma_d := \sigma/16$ as the daily volatility (not to be confused with expected daily move). To find the expected daily move, we have to determine $E(|X|)$. We apply the definition of expected value for a continuous distribution and evaluate the resulting integral.

$$\begin{aligned} E(|X|) &= \int_{-\infty}^{\infty} |x| \frac{1}{\sqrt{2\pi}\sigma_d} e^{-\frac{x^2}{2\sigma_d^2}} dx \\ &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{x}{\sigma_d} e^{-\frac{x^2}{2\sigma_d^2}} dx \\ &= \sigma_d \sqrt{\frac{2}{\pi}} \left[e^{-\frac{x^2}{2\sigma_d^2}} \right]_0^{\infty} \\ &= \frac{\sigma}{16} \sqrt{\frac{2}{\pi}}. \end{aligned}$$

For $\sigma = 16$, this evaluates to an expected move of 0.8%. For $\sigma = 32$, the expected move is 1.6%. Both of these predictions line up with what is observed in the above simulations.

Therefore, for a 16 vol product, we have a 1 s.d. move of 1% but an expected move of 0.8%.

The primary takeaway from this is: volatility is not the same as an expected move. They are related, but are not equal.

Noting the expected move is helpful when thinking about the moves we need to break even from gamma vs theta.

✅ Why it's wrong:

The formula shown is:

$$\text{Expected Move} = 0.8 \cdot \sigma \cdot \sqrt{\frac{t}{252}}$$

But this gives the **expected move over t trading days**, not just one. So unless $t = 1$, calling it the "Expected Daily Move" is incorrect.

🧠 Why the $\sqrt{t/252}$ Term?

This comes from the fact that **variance (not volatility) is additive over time** under the assumption of independent, normally distributed returns.

In detail:

If daily returns are modeled as:

$$X_i \sim \mathcal{N}(0, \sigma_d^2) \quad (\text{where } \sigma_d = \frac{\sigma}{\sqrt{252}})$$

then the total return over t trading days:

$$S_t = \sum_{i=1}^{t} X_i \sim \mathcal{N}(0, t \cdot \sigma_d^2) = \mathcal{N}\left(0, \sigma^2 \cdot \frac{t}{252}\right)$$

Thus, the **standard deviation** of returns over t days is:

$$\text{StdDev} = \sigma \cdot \sqrt{\frac{t}{252}}$$

🔍 Breakdown:

Symbol	Meaning
σ	**Annualized volatility** — the standard deviation of returns over a full trading year (typically 252 trading days)
σ_d	**Daily volatility** — the standard deviation of returns over a single trading day

🎯 **Goal**

Given:

$$X \sim \mathcal{N}(0, \sigma_d^2)$$

We want to compute:

$$\mathbb{E}[|X|]$$

🧠 Step 1: Recall the Definition of Expected Value for a Continuous Random Variable

For a continuous random variable X with PDF $f_X(x)$, we define:

$$\mathbb{E}[|X|] = \int_{-\infty}^{\infty} |x| f_X(x)\, dx$$

Since the normal distribution is **symmetric about 0**, we can simplify this:

$$\mathbb{E}[|X|] = 2 \int_0^{\infty} x f_X(x)\, dx$$

🧠 Step 2: Plug in the Normal PDF

If $X \sim \mathcal{N}(0, \sigma_d^2)$, then:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\ \sigma_d} \exp\left(-\frac{x^2}{2\sigma_d^2}\right)$$

So:

$$\mathbb{E}[|X|] = 2\int_0^\infty x \cdot \frac{1}{\sqrt{2\pi}\ \sigma_d} \exp\left(-\frac{x^2}{2\sigma_d^2}\right)\, dx$$

🧠 Step 3: Change of Variables

Let:

$$u = \frac{x}{\sigma_d} \quad \Rightarrow \quad x = \sigma_d u \quad \text{and} \quad dx = \sigma_d \, du$$

Then:

$$\mathbb{E}[|X|] = 2 \cdot \int_0^\infty (\sigma_d u) \cdot \frac{1}{\sqrt{2\pi}\ \sigma_d} \exp\left(-\frac{(\sigma_d u)^2}{2\sigma_d^2}\right) \cdot \sigma_d \, du$$

Simplify:

- $(\sigma_d u)^2/(2\sigma_d^2) = u^2/2$
- Cancel the σ_d's:

$$\mathbb{E}[|X|] = 2 \cdot \int_0^\infty u \cdot \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du \cdot \sigma_d$$

🧠 Step 4: Recognize a Known Integral

The integral:

$$\int_0^\infty u \cdot \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du = \frac{1}{\sqrt{2\pi}} \int_0^\infty u e^{-u^2/2}\, du$$

This is a standard result from probability theory:

$$\int_0^\infty ue^{-u^2/2}\,du = 1$$

So:

$$\frac{1}{\sqrt{2\pi}}\int_0^\infty ue^{-u^2/2}\,du = \frac{1}{\sqrt{2\pi}}\cdot 1 = \frac{1}{\sqrt{2\pi}}$$

But more accurately:

$$\int_0^\infty u\cdot\frac{1}{\sqrt{2\pi}}e^{-u^2/2}\,du = \sqrt{\frac{1}{2\pi}}$$

Then:

$$\mathbb{E}[|X|] = 2\cdot\sigma_d\cdot\sqrt{\frac{1}{2\pi}} = \sigma_d\cdot\sqrt{\frac{2}{\pi}}$$

🔢 Step 5: Final Result and Approximation

$$\mathbb{E}[|X|] = \sigma_d\cdot\sqrt{\frac{2}{\pi}} \approx \sigma_d\cdot 0.79788 \approx \sigma_d\cdot 0.8$$

🔧 You have:

$$x = \sigma_d u$$

Take the **derivative of both sides** with respect to u:

$$\frac{dx}{du} = \sigma_d \quad \Rightarrow \quad dx = \sigma_d \, du$$

✅ So:

That step comes directly from **differential calculus**:
If $x = \sigma_d u$, then the differential $dx = \sigma_d \, du$.

Done.

Historical Volatility Calculation Example

There are a few things to note before calculating historical volatilities that differ from the usual calculation of standard deviations. Assuming we are calculating volatility from daily closing prices, we:

- Calculate log returns, not absolute returns
- Assume the mean of the returns is always zero.

So, the formula of n-day annualized volatility looks like (using 252 trading days in a year):

$$vol = \sqrt{\frac{252}{n} \sum_{j=0}^{n} \left(\log \frac{S_j}{S_{j-1}} \right)^2}$$

The sum adds up the daily variances, dividing by n takes the average, and multiplying by 252 annualizes the average variance. This gives you the volatility.

Example: The rolling (annualized) 5-day volatility would look like:

	B	C	D	E	
3	Day	Closing Price	Log Returns^2	5-day vol	
4	30-Apr-14	586.81			
5	1-May-14	588.19	5.51751E-06		=LN(C5/C4)^2
6	2-May-14	589.29	3.49091E-06		=LN(C6/C5)^2
7	5-May-14	597.62	0.000197028		
8	6-May-14	591.11	0.000119968		
9	7-May-14	589.04	1.23063E-05	0.13058	=SQRT(252/5*SUM(D5:D9))
10	8-May-14	587.99	3.1832E-06	0.13013	=SQRT(252/5*SUM(D6:D10))
11	9-May-14	585.54	1.74343E-05	0.13280	
12	12-May-14	592.83	0.000153096	0.12418	
13	13-May-14	593.76	2.45711E-06	0.09746	
14	14-May-14	593.87	3.43149E-08	0.09424	
15	15-May-14	588.82	7.293E-05	0.11134	
16	16-May-14	597.51	0.000214637	0.14945	
17	19-May-14	604.59	0.000138757	0.14701	
18	20-May-14	604.71	3.93871E-08	0.14660	
19	21-May-14	606.31	6.98229E-06	0.14779	
20	22-May-14	607.27	2.50303E-06	0.13524	
21	23-May-14	614.13	0.000126183	0.11761	
22	27-May-14	625.63	0.000344196	0.15552	

We'd say, "on May 23, AAPL was realizing an 11.76 volatility" (on a rolling 5-day basis).

Note: Time-Variance is additive: If we know the (annualized) vol over a 10-day period and the (annualized) vol over the next 10 days, the 20-day vol is NOT the average. Rather it is the square root of the average of the squares.

Explanation:

✅ **What it measures:**

- This formula estimates the **annualized volatility** of a stock's returns over the past **n days**.
- Volatility here means the **standard deviation of daily log returns**, scaled to an annual level.

Term-by-term breakdown:

Term	Meaning
S_j	Price on day j
$\log\frac{S_j}{S_{j-1}}$	Log return from day $j-1$ to j. This is preferred over absolute return for time-additivity and normality assumptions.
$\left(\log\frac{S_j}{S_{j-1}}\right)^2$	Daily squared log return (i.e. daily variance assuming zero mean).
$\sum_{j=0}^{n}(\cdots)$	Sum of squared daily log returns over n days.
$\frac{1}{n}\sum_{j=0}^{n}(\cdots)$	**Average** of those squared log returns, i.e. empirical variance.
$\frac{252}{n}\sum(\cdots)$	Multiplies the daily variance by 252 to get **annualized variance** (assuming 252 trading days).
$\sqrt{\cdot}$	Square root of annualized variance = **annualized standard deviation**, or **volatility**.

1. Definition of variance

In general, the **variance** of a set of values $x_1, x_2, \dots, x_n$ is:

$$\mathrm{Var}(x) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

Where:

- x_i are your data points (e.g., log returns)
- μ is the **mean** of the data

2. What if the mean is 0?

In this historical volatility formula, the **mean is assumed to be zero**, so:

$$\mathrm{Var}(x) = \frac{1}{n}\sum_{i=1}^{n}x_i^2$$

In our context:

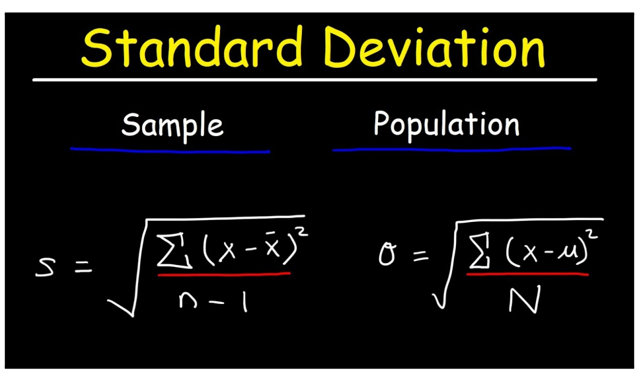
- Each $x_i = \log\left(\frac{S_i}{S_{i-1}}\right)$ is a **daily log return**
- So x_i^2 is just the square of that day's log return

👉 **Squaring the log returns directly gives you the squared deviation from the mean** — because the mean is zero.

3. So the formula:

$$\frac{1}{n}\sum_{j=0}^{n}\left(\log\frac{S_j}{S_{j-1}}\right)^2$$

...is just the **daily variance** of log returns, under the assumption that $\mu = 0$.



Hello and welcome to this video presented by Akuna Capital. Today we'll be discussing and defining historical and implied volatility. We'll start off by defining volatility and how it relates to asset distributions. We'll then define historical, historical implied, and current implied volatilities. And finally, I'll lay on some basic volatility equations that are used. Let's first talk about why volatility is so important to an options market maker, and then we'll define what it is. Unlike a stock or futures trader, who's only interested in the direction of the market, an options trader is interested not only in the direction, but also in the speed at which the asset moves. Options prices are directly related, and in fact, derived from the speed at which the underlying has moved or is forecast to move in the future. So we can look at the distribution of an underlying asset's returns over some interval to begin to quantify volatility. An asset that has a wide distribution of prices, or potential prices in the future, would be described as having a high volatility, while one that has a tighter distribution is said to have a low volatility. Once we compile our data on the underlying price distributions, we can define a volatility number for our asset. This will allow us to describe it in terms of standard deviation price changes over a period of time. We'll discuss this more at the end of the video. Now, I'm sure everyone is familiar with a normal distribution. This has a symmetrical standard deviation around the mean. However, because of the continuous compounding of price moves over time, we find that a log-normal curve better represents the distributions of an underlying instrument. Just as we compound interest rates continuously, leading to an appreciation and compounding effect over time, the volatility distribution will also show a larger tail to the right side of the underlying distribution, as well as a more compact distribution to the downside. Additionally, the fact that a log-normal distribution can't go below zero provides a well-modeled floor on stocks and futures, since their value can also never be less than zero. There are also some secondary factors in the way that stocks and futures tend to behave that fit well with a log-normal distribution model. For all of these reasons, a log-normal model is used to represent asset price calculations. Now, in this next part of the video, we'll lay out the differences between a few of the main volatility terms. There are several of these, and they are related, but far from the same. To add to the complexity, traders will often use the term volatility for each of these terms, and experienced traders will use context clues to understand which volatility term they're talking about. The first volatility we discuss is historical volatility. This is the volatility of an underlying asset over some time period in the past with a known start and end date. We can take that data and turn it into a volatility number. Since we know the start and end dates of this data, it's backward looking and therefore historical volatility. So as we touched on earlier, you can think of a historical volatility number as a measure of the dispersion or standard deviation between returns of an instrument. The higher the volatility, the more the underlying has moved, so the greater the dispersion of returns. So, if we compare our initial product in blue to the returns of a second product in orange, we can see that the orange product has a lower historical volatility over this time period. The second term is historical implied volatility. This term is less commonly discussed. This is the implied volatility priced by option markets during that time. I'll represent this with a few buildings, which stand for option marketplaces. This is separate from underlying futures or stock markets. The prices of the options in this venue are attempting to model the volatility over this time period as well. So while the future was moving, the options were implying that the future would have a volatility of 20 in this example. This was ever-changing each moment, and each day, based on the future movement and option orders in this marketplace. In a perfect world, these two values should be very close or equal, but that seldom occurs. The next term is called implied volatility. This is the implied volatility of the current underlying price for a future time period in the current options market. Let's take a second to walk through this one in more detail. If we draw a line to delineate the current time now, then the current option markets are still trying to predict the volatility of the future movement of the underlying. So we'll say that the current implied volatility is 19 in this example for a given time period. As the future keeps moving along, the current time continues moving as well, and the implied volatility in the options market continues to update based on this fact. The current implied volatility is the one talked about and interacted with most by options market makers. This is the volatility that sets the current option prices in the marketplace. Notice that our rolling historic volatility and historical implied volatility also change slightly as we move forward in time. A good rule of thumb is that the implied vol is always trying to measure future volatility, while the realized vol over some period is the realized, hence the name, calculated volatility. Now, let's lay out some of the basic mathematics for calculating volatility. Notice that I use that blanket term volatility here as a market maker would, but it should be clear to you which of the volatilities we just discussed is used for this analysis. Volatility is usually expressed as an annualized number. We can generalize annualized volatility, where T is years, as follows. I'll note that usually we're looking at volatility over a time period of less than a year. Given the daily returns for an instrument, we can find the standard deviation of those log-normal returns and find our annualized number again by multiplying our standard deviation of daily returns by the square root of trading days. So given a 1% daily standard deviation return for each day for 252 trading days, which is the average number of trading days in a year, we'll arrive at 0.587 volatility. Typically, we'll discuss and express the number 100 times greater in magnitude, and so we'll say that this has a 15.87 vol. If we want to find the expected one standard deviation move over T days, we'd use the general formula seen here. This will allow us to paint a portion of the picture of the distribution of the underlying given the data. Note that this isn't the expected daily move, which is given instead by this formula. These formulas will become more intuitive as you start applying and practicing with them. Hopefully, you were able to piece together the type of volatility used for these calculations. These would all use data on historical numbers, and therefore we're calculating historical volatilities. I hope that this video helped describe the three types of volatilities most commonly used on a trading desk. We'll have future videos that dive deeper into the mathematics and equations that I laid out above. That's all for this video. Thanks for watching and get in touch at AkunaCapital.com.

Vega

Vega

The vega of an option is the change in options value with a change in implied volatility. Specifically, vega is defined as the option value price change per 1-point change in implied volatility.

For example, let's look at the outright options screen below. We've highlighted the 66.5 line, which has a vega of 0.081 (second column from right) and is running a volatility of 23.52 which produces a call theo of 1.875 and a put theo of 1.929. If we raised implied volatility 1 point to 24.52, we'd see a change in each option price on that line of 0.081 (8.1 cents). The call would be worth 1.956 and the put 2.010.

Bid Q	Bid Px	Theo	Ask Px	Ask Q	SplD	St	T DT	Po	PBid	PBid P	PTheo	PAsk P	PAsk Q	Vega	Vol
3	3.72	**3.764**	3.80	4	74	63.5		-9	69	0.81	**0.824**	0.83	240	0.066	24.69
2	3.36	**3.401**	3.44	4	71	64.0	7	8	109	0.94	**0.960**	0.97	264	0.070	24.47
11	3.01	**3.055**	3.09	12	67	64.5			18	1.10	**1.114**	1.12	18	0.073	24.24
2	2.69	**2.729**	2.76	2	63	65.0	-7	-26	100	1.27	**1.286**	1.30	210	0.076	24.03
2	2.39	**2.422**	2.46	28	59	65.5		-6	100	1.46	**1.479**	1.49	158	0.079	23.84
37	2.11	**2.137**	2.17	64	55	66.0	-2	1	93	1.67	**1.693**	1.70	18	0.080	23.66
18	1.86	**1.875**	1.89	153	50	66.5		0	53	1.91	**1.929**	1.94	53	0.081	23.52
53	1.62	**1.635**	1.65	53	46	67.0	9	-3	172	2.16	**2.189**	2.21	158	0.081	23.40
97	1.40	**1.418**	1.43	58	41	67.5		16	26	2.43	**2.471**	2.50	27	0.079	23.30
18	1.21	**1.224**	1.23	18	37	68.0		-23	2	2.74	**2.776**	2.81	12	0.077	23.23

Vega is perhaps the most talked about greek for an options market making firm. Market makers can translate option prices to volatilities. Therefore, options traders are vega traders.

🧠 Why Do People Say "Options Traders Are Vega Traders"?

Because:

1. **Options don't trade on price — they trade on volatility.**
   - When a market maker sees an option quote (e.g., $2.50 for a call), they **invert the Black-Scholes formula** to back out the **implied volatility**.
   - That implied volatility is what they care about — it's the "language" they use.
   - Hence: **option prices ↔ implied volatilities**.
2. **Traders make or lose money based on changes in volatility**, not just underlying price.
   - If you're long Vega, and implied vol goes up → your options get more valuable.
   - If you're short Vega, and implied vol drops → you profit.

🏦 Market Makers & Vega

- Market makers quote prices for options all day.
- But they don't quote price based on opinion of the stock — they quote based on **volatility expectations**.
- They manage books of options, and those books have **net Vega exposure**.

So market makers care deeply about:

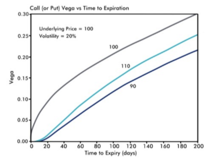
- What volatility they bought vs. what they sold
- Whether volatility is going up or down
- How to **hedge their Vega risk**

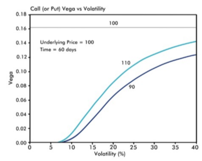
Hence the phrase:

> **"Options traders are Vega traders."**

It emphasizes that **volatility is the true traded quantity**, not the dollar price of the option.

Like other greeks, factors that govern an option's vega are 1) the time to expiry, 2) the distance of the option from ATM, and 3) the implied volatility of the option. The three figures below show some of these relationships. Options that are closer to ATM have the highest vega because their prices are the most sensitive to changes in volatility.





Outright options have a positive vega, so a buyer of an outright option (call or put) will make money if the implied volatility goes higher and lose money if implied volatility goes lower. Using the example above, the buyer of the call or put would profit as the price of the options increases with increasing implied volatility. As noted in our example above, calls and puts on the same strike (line) have the same vega. This also ties back into PCP where a call and a put are essentially synthetic.

🔍 What is Vega again?

Vega measures how sensitive an **option's price** is to changes in **implied volatility**.
If Vega = 0.10, a 1% increase in implied volatility means the option price increases by 0.10.

📊 Top Chart: Vega vs. Time to Expiry

- **x-axis**: Time to expiry (days)
- **y-axis**: Vega
- Each line = a different **strike price** (90, 100, 110), while underlying = 100.

Key Takeaways:

1. **Vega increases as time to expiry increases**
→ Longer-dated options are **more sensitive** to volatility changes.
Intuition: If you own a long-dated option and volatility rises, that extra time gives more opportunity to move profitably.
2. **At-the-money (ATM) options have the highest Vega**
→ The black curve (strike = 100) is highest at all expiries.
3. **Out-of-the-money (OTM) and in-the-money (ITM) options have lower Vega**
→ The 90 and 110 lines are below the ATM curve, showing less sensitivity to volatility.

📊 Bottom Chart: Vega vs. Implied Volatility

- **x-axis**: Implied volatility (%)
- **y-axis**: Vega
- Time to expiry fixed at 60 days

Key Takeaways:

1. **Vega increases as implied volatility increases** — up to a point
    - For all strikes, Vega rises as vol increases.
    - Why? Because the option's price curve becomes more curved (convex) with higher vol → more sensitivity.
2. **ATM options (strike 100) again have the highest Vega**
3. **Vega plateaus** at high volatility
→ Beyond ~30–35% IV, Vega flattens — option prices become less responsive.

📌 Text Explanation Breakdown

> "Outright options have a positive Vega"
> ✅ True. Buying a call or put means you benefit if implied volatility increases.

> "So a buyer will make money if IV rises and lose money if IV falls"
> ✅ Because Vega > 0 for long options. Higher vol = more expensive options.

> "Price of options increases with increasing implied volatility"
> ✅ Because volatility increases the chance the option finishes in-the-money, which raises its value.

> "Calls and puts on the same strike have same Vega"
> ✅ This follows from **Put-Call Parity (PCP)** — they're synthetically equivalent. Vega is based on volatility, not direction.

$C - P = S - Ke^{-rT}$

Where:

- C = price of the call
- P = price of the put
- S = current underlying price
- K = strike price
- T = time to expiration
- r = risk-free rate

This tells us:

> A **call** and a **put** with the **same strike** and **same expiry** can be combined with a **stock** and **bond** to synthetically replicate each other.

So, **anything that changes the value of one (like volatility) must affect the other in the same way**, to preserve the arbitrage-free relationship.

2. Extrinsic Value Is Driven by Volatility

- **Vega only measures sensitivity to volatility** (not price direction).
- **Implied volatility only affects the extrinsic value** (time value) of an option — not intrinsic value.

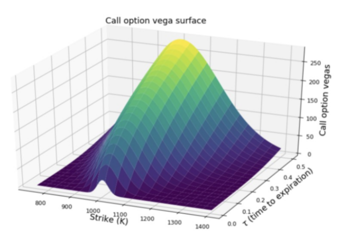
For **ATM options**, calls and puts have:

- **Same extrinsic value**
- **Same distance to ITM/OTM status**
- So **same exposure to volatility**

Therefore:

> 💡 Vega is the **same** for puts and calls **at the same strike and maturity**, because their extrinsic values are equally sensitive to changes in implied volatility.

The graph below shows option vegas across different strikes and expiries (through time). Notice the higher vega near the 1000, ATM, strike relative to the other strikes on the same time plane. Similarly, as we move out in time vega increases for each strike.



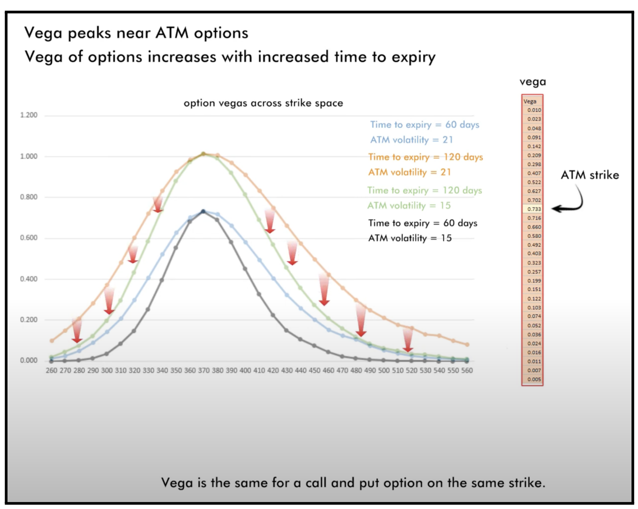
Market makers will use the word **vega** a few different ways. The first is the vega of an option. For example, "The 66.5 line has a 0.081 vega". The second is the vega of an options position or portfolio.

For example, if we buy 1000 of these options our vega position would be 0.081*1000*50 (the multiplier in this example) = 4050. This means that for every 1 point increase/decrease in implied volatility we would make or lose $4050.

Calculating the vega of a portfolio of options = (number of options) x (multiplier) x (option vega)

In this example traders might say "I'm long 4050 vegas". This refers to the position and not the vega of an option. Traders use context clues to differentiate between the two uses of the word vega. This is oftentimes confusing for those learning about greeks.

Hello, and welcome to this video. In it, we'll be talking about the Option Greek Vega. We'll start with the definition of the term. We'll then walk through a Vega calculation example. We'll then talk about how Vega changes across strike space and with changes in implied volatility. And finally, I'll leave you with a few practical tips about how traders use the term Vega interchangeably in a few different scenarios. We'll start this video by once again taking a look at our options board. Just to review, we have our call theoretical values and markets on the left of the black column with strikes, and our put theoreticals are to the right of this. Just to the right of the put theos, we've placed our option Vegas for that strike. So what is Vega? Let's start with the definition and some of the main talking points of this video. Vega is a measure of an option's price sensitivity to changes in the volatility of the underlying asset. Since market makers are essentially traders of options, and since option prices are sensitive to changes in implied volatility through this Vega parameter, Vega is probably the most talked about and watched Greek by market makers. Every option has a positive Vega. This means that when implied volatility goes up, all option prices go up, and if implied volatility goes down, all option prices go down. As you saw from the option board we just displayed, Vega is the same for a call and a put option on the same strike. Finally, the Vega of options has certain relationships across strike space, with changes in implied volatility, and with the passage of time. We'll talk about each of these topics in more detail. Let's first walk through an example of how the Vega of an option affects its price. Let's take a call option with a price of $1, a delta of 0.3, and a Vega of 0.04. This has an implied volatility of 15. Remember, the implied volatility is one of the inputs into our Black-Scholes equation that gives us the $1 price and the Greeks of that option. Now, let's see what happens when we change the implied volatility from 15 to 16. We see that the price of the option goes from $1 to $1.04. This is because the Vega of the option, 0.04, increased the price four cents for every 1-point increase in implied volatility. In this case, from 15 to 16. This is a linear relationship. So if we move the vol up by 0.1 to 15.1, the price of the option would increase four-tenths of a cent, and the total price of the option would be \$1.004. Therefore, Vega is expressed as the change in option price per one-point change in implied volatility. Now, let's talk about how the word Vega is used. Similar to the word "delta," Vega can be used in a few different contexts by traders. The first, which we just described, is the Vega sensitivity of an option to changes in implied volatility. So, for our example again, a trader might say the Vega of this option is 4. Many traders will say "4" instead of 0.04. But Vega can also refer to the Vega of a position or portfolio of options. In this case, a trader might say, "I'm long 4 Vegas," meaning they have a portfolio that has some long quantity of options times a multiplier. Four Vegas can sometimes also mean \$4,000, which means that the P&L of the portfolio will be impacted by \$4,000 times the change in implied volatility. We can back this out for our example. With a multiplier of 100 on each option, times 100 options, increasing four cents each, this gives us our \$4,000 P&L change when moving the vol from 15 to 16. To reiterate: options all have a positive Vega. So a buyer or owner of an option is positively exposed to Vega increases. The owner wants implied volatility to increase. Sellers of options will have a negative Vega position and will be negatively exposed to volatility increases. So, the seller of the 100 options would lose the \$4,000 if vol increased, or gain it if it decreased. Let's go back now and look at Vega per strike. Remember, we discussed that Vegas are the same for the call and the put on the same strike. Let's look at how Vegas change across strike space. I've highlighted the middle strike, which is the strike closest to at-the-money—the current future price. If we plot the Vegas across strike space, we can see that Vega is greatest near the at-the-money strikes and gradually decreases as we move further away. This means that the prices of options closest to at-the-money are the most sensitive to changes in implied volatility. It's worth mentioning—and you'll notice in the graph—that it's not symmetrical. The right side of the curve looks a bit higher. This is due to skew, or a higher implied volatility that I've plugged into this particular expiration. If we used a symmetric volatility surface, then the curve would be more symmetrical around the at-the-money strikes. Now, let's add another curve that is similar in every way, but just has twice the days until expiry. You can see that Vega increases with longer times to expiry. Therefore, we can see that prices of options for further expiries are more sensitive to changes in implied volatilities, since they have larger Vegas. Finally, we'll look at how the curves change if we lower our at-the-money volatility. The green and black curves only have a 15 vol instead of a 21. You can see that Vega decreases with decreased volatility. I won't go into why the at-the-money Vegas don't change here—that's a topic for a separate video. Hopefully, we've covered the basics on Vega. This quick summary of main points should serve as a review or refresher before we move on to the next video in the series.



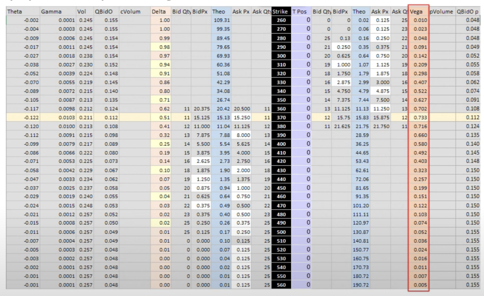
Definition: Vega is the change in option price with a change in implied volatility.

Vega is the most talked about greek since options market makers are volatility traders.

Vega changes across strike space, and with changes in implied vol, and time to expiry.

Every option has a positive vega.

vega



Vega peaks near ATM options

Vega of options increases with increased time to expiry

Vega of options increases with increased in volatility.

Vega Exercise

Answer the following questions without using any resources or materials.

Refer to the image below to answer some of the questions.

Assume interest rate = 0 and no dividends.

Spl	TO Qt	Bid Qt	Bid Px	Theo	Ask Px	Ask Q	Str	T Pos	PBid Q	PBid P	PTheo	PAsk P	PAsk Q	PTO C	Vega
100				33.64			77				0.00				0.000
100				32.64			78				0.00				0.000
100				31.64			79				0.00				0.000
100				30.64			80				0.00				0.000
100				29.64			81	8			0.00				0.000
100				28.64			82	28			0.01				0.000
100				27.64			83				0.01				0.000
100				26.64			84	56			0.01				0.001
100				25.64			85	1			0.02				0.001
100				24.64			86	1			0.02				0.002
100				23.64			87				0.03			1	0.004
100				22.64			88	183			0.03				0.006
99				21.65			89	-1			0.04				0.009
99				20.66			90	16			0.06				0.013
98				19.67			91	-2			0.07				0.018
98				18.69			92	-102			0.09			11	0.023
97				17.71			93	-18	3	0.075	0.12				0.029
97				16.74			94	-58	1	0.100	0.15			2	0.036
96				15.78			95	-32	1	0.125	0.19			20	0.043
95				14.83			96	-33	3	0.125	0.23	0.675	1	8	0.051
93				13.89			97	-10	6	0.050	0.29	1.000	3	4	0.061
92				12.96			98	-127	10	0.325	0.37			33	0.071
90				12.05			99	-97			0.45	1.000	3	5	0.082
88		1	3.000	11.15			100	292	190	0.500	0.55			67	0.094
86				10.28			101	180	30	0.475	0.68				0.106
83				9.44			102	169	15	0.650	0.83			28	0.119
81				8.62			103	130			1.01			15	0.132
77	275			7.83			104	91	6	1.000	1.21	3.200	6	665	0.144
74				7.07			105	148	10	1.100	1.45	2.000	1	113	0.156
70				6.35	10.000	1	106	191	5	1.400	1.73	9.200	1	325	0.168
66	2	1	2.250	5.67			107	-47			2.04			14	0.177
62	115	85	4.500	5.03			108	-332	2	2.000	2.40			31	0.185
58		3	2.500	4.42			109	-35			2.79			5	0.191
54	2			3.86	5.050	35	110	-76			3.22			122	0.195
50	62			3.34			111	-174			3.70			57	0.196
46	10			2.86	3.700	20	112	-378			4.22			2	0.195
41	3			2.42			113	-41			4.77				0.191
37	161	5	0.750	2.03	2.500	1	114	-62			5.38			10	0.184
32	102	3	0.600	1.68	2.000	2	115	183			6.02				0.175
28	70	4	0.800	1.38	1.850	4	116	13	1	4.500	6.72				0.165
24	2	1	0.700	1.12	1.900	2	117	31			7.46				0.152
20	53	2	0.700	0.90	1.300	4	118	102			8.24				0.138
16		1	0.125	0.73			119	8			9.06				0.124
13		17	0.350	0.58	1.000	1	120	12			9.91				0.111
11	30	12	0.075	0.46			121	4			10.79				0.097
8		47	0.250	0.36	1.500	20	122	157			11.69				0.085

We examine a call option on strike 120 with a vega of 0.3 and with the underlying stock at price $100. How does the vega of the option change if the underlying increases to $102?

- Decreases from 0.3 since the stock moves further from the strike
- Decreases from 0.3 since the stock moves closer to the strike
- Increases from 0.3 since the stock moves further from the strike

correct

- Increases from 0.3 since the stock moves closer to the strike ✔
- The vega of the option stays constant at 0.3
- can't tell. need more info

Continue ›

✅ **Correct Answer:**

Increases from 0.3 since the stock moves closer to the strike

📘 **Explanation:**

You're examining a **call option with a strike of 120** when the **underlying stock price is $100**. That means the option is initially **far out-of-the-money (OTM)**.

If the stock price rises to **$102**, the option is now **slightly closer to the strike price** — it becomes **less OTM** and closer to **at-the-money (ATM)**.

🔁 **Vega Behavior:**

- **Vega is highest for ATM options**
- As the **underlying moves closer to the strike**, the option's vega **increases**
- As the **underlying moves further away**, vega **decreases**

🔍 **Why not the others?**

- "Decreases from 0.3" → ❌ It's moving **closer**, not further
- "Stays constant" → ❌ Vega is **not constant**, it depends on moneyness
- "Can't tell" → ❌ We absolutely can tell given the info

🧠 **Intuition:**

Vega measures **sensitivity to changes in implied volatility**, and options closer to being ATM are more affected by volatility changes → **that's why vega increases** in this case.

Let me know if you want a graph of Vega vs. moneyness to lock this in.

Using the image above, what is the vega of the 112/114 call spread?

- 0.195
- 0.83
- 4.89
- 0.111

correct

- **0.011** ✔
- 0.279
- none of the above

‹ Back   Continue ›

Thanks — if **Vega(114 Call) = 0.184**, and **Vega(112 Call) = 0.195**, then:

🔷 **You are buying the 112 call and selling the 114 call:**

$$\text{Vegaspread} = \text{Vega112} - \text{Vega}_{114} = 0.195 - 0.184 = 0.011$$

✅ **Final Answer:**

0.011

This matches the multiple-choice option:
✅ **0.011** is correct.

What would be the price of the 110 strike call if we lowered the implied volatility by 0.1 points?

3.86

4.035

3.665

3.8795

4.055

0.4055

correct

3.8405 ✔

none of the above

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Continue ›

Thanks — now that you've clarified that the **Vega for the 110 strike call is 0.195**, let's recalculate accurately.

🔷 **Given:**

- **Strike = 110 call**
- **Original price** (from table) = **3.86**
- **Vega = 0.195**
- **Implied volatility drops by 0.1**

🔷 **Use the Vega formula:**

$$\text{New Price} = \text{Original Price} - (\text{Vega} \times \Delta\text{Vol})$$

$$= 3.86 - (0.195 \times 0.1) = 3.86 - 0.0195 = 3.8405$$

✅ **Correct Answer: 3.8405**

That matches one of the answer choices exactly.

If you buy 20 options with a vega of 0.8, how many options with a vega of 0.02 should you sell to make your portfolio "vega flat"?

16

20

200

1600

160

correct

none of the above ✔

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✅ **Goal:**

You want your total **vega to equal 0** (vega-flat).

🔢 **Given:**

- **Long 20 options** with **vega = 0.8**
→ Total vega = $20 \times 0.8 = 16$
- You want to **offset that** by **selling options** with **vega = 0.02**

🧮 **Calculation:**

Let x be the number of low-vega options to sell:

$$0.02x = 16 \Rightarrow x = \frac{16}{0.02} = 800$$

✅ **Final Answer:**

None of the above

(800 isn't listed, so "none of the above" is correct.)

Generally, a long gamma portfolio will make money when the underlying:

- out-realizes the historical volatility (measured close to close).

incorrect

- out-realizes the implied volatility (more than making up for theta paid). ✖
- under-realizes the implied volatility (more than making up for theta paid).
- under-realizes compared to the 30 and 10 day historical volatility.
- none of the above.

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✅ **The correct logic is:**

> A **long gamma portfolio profits** when **realized volatility exceeds implied volatility**, because:

- You're **long options**, paying **theta** daily
- You only recoup theta and make a profit if the **underlying moves more** than what was priced in (i.e., implied vol)
- That's exactly what "out-realizes implied volatility" means

---

🔍 **Why "out-realizes historical volatility" is misleading:**

- Historical volatility is **backward-looking** (e.g., 30-day close-to-close)
- You don't pay for historical vol — you pay for **implied vol**
- A long gamma portfolio doesn't care if realized > historical — only if **realized > implied**

✅ **The answer you chose —**

"out-realizes the implied volatility (more than making up for theta paid)"
—is the **correct one in practice** and in any trading desk context.

Vega is defined as:

- the sensitivity of an option prices to a change in time.
- the sensitivity of an option delta with a change in the underlying.
- the second derivative of gamma.
- the sensitivity of an option vega with a change in implied volatility.

correct

- the sensitivity of an option price with a change in implied volatility. ✔
- the change in an option delta with a change in implied volatility
- none of the above.

‹ Back Continue ›

We have a -30d put with a vega of 0.05. If we lower implied volatility 2 points, how is the vega of the put impacted?

it remains 0.05

it increases from 0.05

correct

it decreases from 0.05 ✔

it remains at 0.05

can't tell from information provided

‹ Back Continue ›

✅ **Correct Answer:**

It decreases from 0.05

📘 **Explanation:**

Vega measures how sensitive an option's price is to changes in implied volatility. It is:

- **Highest at-the-money**
- **Higher with more time to expiration**
- And it typically **decreases as implied volatility falls**

🔻 **Why does Vega decrease here?**

You're lowering **implied volatility by 2 points**, and you're told the put has **30 days to expiry**:

- Lowering implied volatility **flattens** the option price curve
- That makes the option **less sensitive to future volatility changes**
- So: **Vega decreases**

🔁 **Summary:**

Variable Changed	Vega Impact
Time ↓	Vega ↓
IV ↓	✅ Vega ↓
Moneyness ↑ or ↓ from ATM	Vega ↓

So yes — **Vega will drop below 0.05** in this case.

Break the call option given into it's total, intrinsic, and extrinsic values based on the following: Future = 20.43, strike = 19, put = 1.20, call = ??:

- Total: 1.63, Intrinsic: 1.43, Extrinsic: 1.20

correct

- Total: 2.63, Intrinsic: 1.43, Extrinsic: 1.20 ✔
- Total: 2.63, Intrinsic: 1.20, Extrinsic: 1.43
- Total: 1.23, Intrinsic: 0.43, Extrinsic: 0
- Total: 2.63, Intrinsic: 0, Extrinsic: 1.43

‹ Back Continue ›

Let's solve this step-by-step using **Put-Call Parity** and break the call option into:

- **Total value**
- **Intrinsic value**
- **Extrinsic value**

📘 **Given:**

- Future price $F = 20.43$
- Strike $K = 19$
- Put price $P = 1.20$
- Risk-free rate = 0
- No dividends

🧮 **Step 1: Use Put-Call Parity**

$C - P = F - K \Rightarrow C = F - K + P$

$C = 20.43 - 19 + 1.20 = 2.63$

✅ **Call Total Value = 2.63**

🧮 **Step 2: Intrinsic Value (Call)**

$$\text{Intrinsic} = \max(F - K, 0) = 20.43 - 19 = 1.43$$

🧮 **Step 3: Extrinsic Value**

$$\text{Extrinsic} = \text{Total} - \text{Intrinsic} = 2.63 - 1.43 = 1.20$$

✅ **Final Answer:**

Total: 2.63, Intrinsic: 1.43, Extrinsic: 1.20

✔ So the correct option is:
Total: 2.63, Intrinsic: 1.43, Extrinsic: 1.20 (second option from top)

If we have a -30 delta put and we raise implied volatility, what happens to the delta and vega of this option?

- Delta closer to -50, Vega decreases
- Delta closer to 0, Vega decreases
- Delta closer to -100, Vega decreases

correct

- Delta closer to -50, Vega increases ✔
- Delta closer to 0, Vega increases
- Delta closer to -100, Vega increases
- Delta closer to 0, Vega unchanged
- Delta closer to -50, Vega unchanged
- Delta closer to 0, Vega unchanged
- Can't tell. need more info.

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🔷 **Step 1: What does it mean to have a Delta of –30?**

You're dealing with a **put option**, and its **Delta is –30**, or –0.30 in decimal terms.

- **Put Deltas are negative**: deep in-the-money puts approach **–100** (i.e. –1.00)
- At-the-money (ATM) puts are around **–50**
- Deep out-of-the-money (OTM) puts approach **0**

So a **–30 Delta** (i.e. –0.30) is **in between**:

- –50 (ATM)
- and 0 (far OTM)

💡 That means this put is **slightly out-of-the-money**.

🔷 **Step 2: What happens when you raise implied volatility (IV)?**

1. Delta moves closer to –50

Implied volatility spreads out the probability distribution — wider distribution = more uncertainty.

For OTM puts, that **increased uncertainty** means:

- Higher chance they **end up in-the-money**
- That **boosts the magnitude** of Delta (more negative)
- But not to –100 — it pushes toward –50

So the Delta goes from –30 → closer to –50.

2. Vega increases

Vega is highest for ATM options and rises with:

- More uncertainty (higher IV)
- More time to expiration

If you increase IV, you're **increasing uncertainty** → Vega increases.

✅ **Final Answer:**

Delta closer to –50, Vega increases

This is the only answer that reflects:

- Movement toward ATM delta (–50)
- Increased Vega from greater implied volatility

Let me know if you want the payoff shape or probability curve visualization to go with it.

If we take a 20 delta call with a vega of 0.05, and raise implied volatility 2 points (2.0), what is the vega of the options after the change?

- 0.01
- 0.02
- 0.03
- 0.05

incorrect

- 0.10 ✖
- 0.15
- (0.05)
- (0.10)
- can't tell, need more info

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❌ **Why 0.10 Is Wrong**

You probably thought:

> "0.05 vega × 2-point IV move = 0.10" → so maybe the new vega is 0.10.

But that's **not** how vega works.

That equation tells you how much **option price changes**:
$\Delta \text{Price} = \text{Vega} \times \Delta \text{IV}$

It does **not** tell you what **new vega** is.

Vega is not additive. It **doesn't double** just because IV went up 2 points. You don't calculate "new vega" like that. Vega is a **function**, not a **counter**.

📉 **Vega Depends On More Than Just IV**

You **can't get the new vega** without knowing:

- the **strike** of the option
- the **underlying price**
- how far OTM it is **after** IV changes (because delta will change!)
- the **new delta** after the IV change
- time to expiry (not given!)

All of these affect how **vega changes**.

Rho & Boxes

Rho

Rho is the sensitivity of an option's price to a change in interest rates. Mathematically, it is the first derivative of option value with respect to interest rates. Unlike other greeks, calls and puts on the same line can have different rho values. Rho is expressed as a change in option price for a 1 point (1%) change in interest rates. Therefore, an option with a theoretical price of 0.93 and a rho of 0.02, with an interest rate of 3.5% would increase in price to 0.95 if we increased rates to 4.5%.

Usually rho risk isn't managed on a per-option basis, but instead on a portfolio of options. A small investor won't worry much about rho risk, but because of the large options positions market making firms monitor rho risk carefully. We sum up our total option position exposure and think about rho as the risk or exposure of an entire options position (book) to changes in interest rates. Therefore, the rho of an options position is expressed as P&L per 1% increase in an interest rate. For example, if we have a rho risk of $2,000 we will make $2000 if interest rates move from 3.5% to 4.5% and lose $2000 if rates move from 3.5% to 2.5% instead.

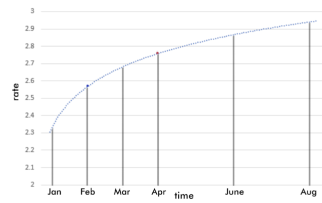
Rho can be aggregated across products and managed on a firm level. Therefore, coordination and communication between trading departments is key to managing rho risk.

Interest rate curves

There isn't simply one interest rate number that's used for options. Instead, there is an interest rate curve that's used by traders to model rates.

The chart below is an example of an interest rate curve across time. This curve depicts the expected interest rates across time. Since market makers trade many different expiry months, will have different rates based on expected rate at that expiry. Changes in both the underlying economic rate set by the Federal Reserve Bank and the rates the clearing house charges Akuna influence the rates used to model option prices.

As with all other inputs into our model we can change the shape of this curve. For example, if we're long (positive exposure) rho on the Feb expiry (blue dot) and short rho in April (red dot) we could adjust the the curve against us (lower blue, raise red) that position to change prices in our model and close out this risk. We leave the details of this for another time, as it's outside the scope of this 101 class. But the fundamental concept of an interest rate curve that models rates across time should be understood by the reader.



🔢 Setup

You're promised **$100 in 1 year**, and we want to know what that's worth **today** depending on the interest rate r.

We use the **present value formula**:

$$\mathrm{PV} = \frac{100}{(1+r)^T}$$

or equivalently:

$$\mathrm{PV} = 100 \cdot e^{-rT} \quad \text{(continuous compounding)}$$

We'll use **continuous compounding**, which is what the Black-Scholes model uses.

📉 Case 1: Interest rate = 0%

$$\mathrm{PV} = 100 \cdot e^{-0 \cdot 1} = 100 \cdot 1 = 100$$

So $100 in 1 year is worth **$100 today** if rates are 0%.

📈 Case 2: Interest rate = 10% (i.e., $r = 0.10$)

$$\mathrm{PV} = 100 \cdot e^{-0.10 \cdot 1} = 100 \cdot e^{-0.10} \approx 100 \cdot 0.9048 = 90.48$$

So $100 in 1 year is worth only **$90.48 today** if rates are 10%.

💡 Where it comes from:

If you compound interest **n times per year**, the formula is:

$$\mathrm{FV} = \mathrm{PV} \cdot \left(1 + \frac{r}{n}\right)^{nt}$$

As $n \to \infty$ (i.e., compound infinitely often), that limit becomes:

$$\lim_{n \to \infty} \left(1 + \frac{r}{n}\right)^{nt} = e^{rt}$$

💡 Now apply this to calls vs puts:

🟦 Call Option

You're buying the stock **at strike price** K in the future.
When r goes up:

- Ke^{-rT} **decreases**
- You're paying **less in present value terms**
- ✅ **Call becomes more valuable**

🟥 Put Option

You're selling the stock **at strike price** K in the future.
When r goes up:

- The cash you'll receive (Ke^{-rT}) is **worth less**
- ✅ **Put becomes less valuable**

🧠 Conclusion

The math confirms:

Interest Rate ↑	Present Value of Strike ↓	Call Value ↑	Put Value ↓
Ke^{-rT} drops	You pay less	Calls get cheaper to execute	Future payoff is worth less

Let me know if you want me to plug this into actual Black-Scholes call/put pricing formulas side-by-side.

🔷 Call Example (You Pay $100 Later)

If you're buying a call, you're agreeing to **pay $100 in the future**.

- At $r = 0\%$, $100 in 1 year is worth $100 now.
- At $r = 10\%$, $100 in 1 year is worth only $90.48 now.

👉 So that **future payment is now cheaper** in today's terms.

➡️ **The call is more valuable**, because you're paying less in present value to get the stock.

🔴 Put Example (You Receive $100 Later)

If you're buying a put, you're hoping to **sell the stock and receive $100 in the future**.

- That $100 is worth less today when interest rates go up.

👉 So the **cash you're receiving is discounted more heavily**, which **hurts you** as a put holder.

➡️ **Put is less valuable**

🧠 It's Not About Strike Changing — It's About When You Pay or Get It

You're **always paying/receiving $100** at expiration.
But the **value of that $100 in today's dollars** changes with interest rates.

✅ Summary Table

Role	What Happens at Expiry	Why Rates Matter
Call buyer	You **pay $100** if exercised	Higher rates = cheaper present value of payment = **better for you**
Put buyer	You **receive $100** if exercised	Higher rates = lower present value of receipt = **worse for you**

🧮 Textbook vs. Reality

Textbook Black-Scholes	Akuna's Real-World Environment
Uses "risk-free rate" (e.g. 5%)	Uses **effective trading rate** (maybe 4.6%)
Assumes frictionless borrowing/lending	Accounts for **actual financing and clearing costs**
Ignores margin/collateral impact	Adjusts for **capital charges, haircuts, carry risk**

🎯 Why This Affects Option Pricing

Interest rates affect the **discounting of the strike** in the Black-Scholes model:

- **Call price** increases as rates go up
- **Put price** decreases as rates go up

If Akuna uses a curve that's **too high or too low compared to their real costs**, they will:

- Misprice options
- Mis-hedge risk
- Lose edge

So they **custom build a curve** to reflect what it actually **costs them** to hedge and carry positions.

✅ Bottom Line

> Akuna prices options using **their own interest rate curve** because that curve reflects **their real cost of capital, financing risk, and trading friction** — not what's printed in a textbook.

Let me know if you want to see how this curve is constructed step by step or used in quotes.

① Delta shifts

Delta = how much the option price changes per $1 move in the underlying

- In Black-Scholes, delta depends on d_1, which **depends on the interest rate** r:

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

- So a **higher** r increases d_1 → **higher call delta**, **lower put delta**
- This changes your hedge ratio — how many shares you need to buy or sell to hedge exposure

> 🔁 Change rate → Change delta → Change hedge size

🔧 Precise Framing:

> **Akuna's internal "risk-free rate"** (i.e. the interest rate they use in their options pricing models)
> = **Fed Funds Rate** (or similar benchmark)
> **– the actual cost they incur to finance and hedge positions**

💸 What Are These "Costs to Hedge"?

When Akuna hedges an option (e.g., buying/selling stock, futures, or other options), they incur:

1. **Financing charges**
   - Borrowing money to hold delta-hedge positions (repo cost, overnight interest)
2. **Clearinghouse & margin costs**
   - Capital tied up in margin doesn't earn the full risk-free rate
3. **Collateral haircuts**
   - Cash or securities posted as margin may be discounted in value
4. **Internal capital charges**
   - Akuna allocates risk capital per trade — this capital has a cost

---

⚖️ So Their "Effective Rate" = What They Actually Earn or Lose

- In theory, you earn 5.00% risk-free.
- In practice, if you pay 0.40% in frictions → your **real earnings are 4.60%**.
- So that's the rate you should use when **discounting future cashflows** in option models.

🧠 Why This Matters in Pricing

Using 5.00% would **overvalue calls and undervalue puts** relative to what you can **actually hedge and carry**.

Using 4.60% = **realistic**, **defensible**, **profitable**.

✅ One-line summary:

> **Internal capital charges are a cost to hedge because the act of hedging ties up Akuna's own capital — and they assign a cost to using that capital.**

Let me know if you want to see this diagrammed as a full capital flow for a hedged position.

Akuna uses interest rates
→ so they can **accurately price options**
→ which lets them **confidently post quotes** close to theoretical value
→ which helps them **quote as wide a spread as competition allows**
→ so they can **maximize profit per trade** as a market maker
→ while avoiding being **picked off by informed traders**

This structure captures exactly what's happening:

- **Interest rate accuracy → better theoretical pricing**
- **Better pricing → more control over spread width**
- **Spread width → direct profit**, bounded by market competition

Want a diagram or slide-version of this for a deck or interview prep?

Boxes provide an interesting way to capture interest rate mechanics through options. We provide a brief example below.

In this example we provide an options expiry 4 months from today. The 700-1200 box is a combination of 4 options (+700c-700p+1200p-1200c).

This box will be worth 500.00 at expiry, since that's the difference between the strike prices. No matter where the future finishes in 4 months this box (+700c-700p+1200p-1200c) will be worth 500.00. However, at this moment in time the model shows the box is worth 499.20 because there is some cost to tying up capital for 4 months to purchase an asset today that will be worth 500.00 in 4 months. You can discount the final price of 500.00 using the interest rate and using a simple NPV (net present value) equation. This represents the interest one could earn by putting this capital to work by instead buying a bond (or some other low risk investment) that matures in 4 months at the prevailing rate. The interest rate currently used in this model gives the "fair value" of 499.20. Since this is our fair interest rate we would be able to make a market around it. Perhaps we'd make a market 499.10 at 499.30. We can then back out what these prices (bid/offer) imply in interest rate terms.

SpIDlt	TO Qt	Bid Qt	Bid Px	Theo	Ask Px	Ask Q	Str ^	T Pos	PBid Q	PBid P	PTheo	PAsk Px	PAsk Q	PTO C	Vega
100				325.41			620				0.09				0.000
100				305.41			640				0.12				0.000
100				285.41			660				0.16				0.000
100				265.41			680				0.22				0.019
100				245.45			700				0.31				0.056
100				225.53			720	1			0.43				0.107
99				205.69			740				0.61			25	0.173
99				185.94			760				0.88				0.260
98				166.34			780	310			1.29				0.376
97				146.97			800	1			1.91				0.528
95				127.93			820	-18			2.86	4.500	3		0.723
93				109.42			840	-8	1	0.875	4.32			70	0.970
88				91.73			860	-239	1	1.625	6.59	11.500	10		1.265
82				75.23			880	-30	1	1.625	10.04	27.000	4	73	1.585
74	2			60.29			900	156	4	2.500	15.03	30.000	6	223	1.887
65	3			47.22			920	240			21.90	44.000	11	209	2.118
54	3			36.23			940	500	10	0.125	30.84			126	2.239
44	253			27.37			960	-611			41.91			127	2.235
34	120			20.46			980	-27	1	30.000	54.93				2.121
26	425	4	10.000	15.28			1000	-91			69.69			2	1.933
20	120	100	7.000	11.48			1020	-111	2	38.000	85.83				1.710
15	165			8.72	36.125	1	1040	104			103.01			15	1.484
11	194	5	2.000	6.72			1060	66			120.97				1.274
8	115	50	3.250	5.27			1080	158			139.47				1.088
6	65	100	2.625	4.20	4.625	1	1100	265			158.37				0.930
5	86	10	0.125	3.42			1120	-10			177.55				0.798
4	51			2.82			1140	-103			196.93				0.689
3				2.37			1160	-137			216.46				0.599
2		1	0.875	2.02			1180	-145			236.09				0.524
2	2	4	1.000	1.73	4.875	1	1200	-213			255.79				0.461
2		1	1.125	1.50			1220	6			275.55				0.406
2	27			1.29			1240	29			295.34				0.358
1	5			1.11			1260	-5			315.16				0.315
1				0.96			1280	26			335.01				0.277
1	4			0.82			1300	670			354.89				0.243
1				0.71			1320	188			374.79				0.214

Questions to consider: Is the buyer of the box borrowing or lending money? How will the price of the box change if rates go up/down unexpectedly? Who would profit? How can we back out the rate implied by this box from the price 499.20?

Example: Use the equation Pe^{-rt}, and the numbers provided above to find the interest rate used in the example, where the box is worth 499.20?

Answer: We solve $499.20 = Pe^{-(r*\frac{4}{12})}$, and find the rate of 0.004804 or 0.48%

Great — let's go through the **"Questions to Consider"** in the box spread example, one by one.

📦 Q1: Is the buyer of the box borrowing or lending money?

Answer: The **buyer of the box is lending money**.

- They pay ~499.20 now
- They receive **$500.00 in 4 months**
- This is equivalent to **buying a zero-coupon bond**: pay now, get more later

➡️ So the **buyer earns interest** → they are acting as the **lender**

> The **seller** is the **borrower** — they receive cash now and owe $500 later.

📈 Q2: How will the price of the box change if rates go up or down unexpectedly?

- The box price represents a **discounted value** of $500 using current interest rates.

🔺 If interest rates go up:

- The present value of $500 **decreases**
- So the **box price drops**

🔻 If interest rates go down:

- The present value of $500 **increases**
- So the **box price rises**

➡️ **Box price moves inversely with interest rates** — just like a bond.

💰 **Q3: Who would profit from a change in rates?**

If rates go up unexpectedly:

- **The seller of the box profits** (they bought low, locked in cheap funding)
- The buyer loses because they lent at a now-too-low rate

If rates go down unexpectedly:

- **The buyer of the box profits** (they locked in a high return)
- The seller loses because they're now overpaying interest

🔍 **Q4: How can we back out the rate implied by this box from the price 499.20?**

We use the formula for discounting under continuous compounding:

$$\text{Present Value} = \text{Future Value} \cdot e^{-rt}$$

Here:

- PV = 499.20 (box price)
- FV = 500 (at expiry)
- $t = \frac{4}{12} = \frac{1}{3}$ years

Solving:

$$499.20 = 500 \cdot e^{-r \cdot \frac{1}{3}} \Rightarrow \frac{499.20}{500} = e^{-r/3}$$

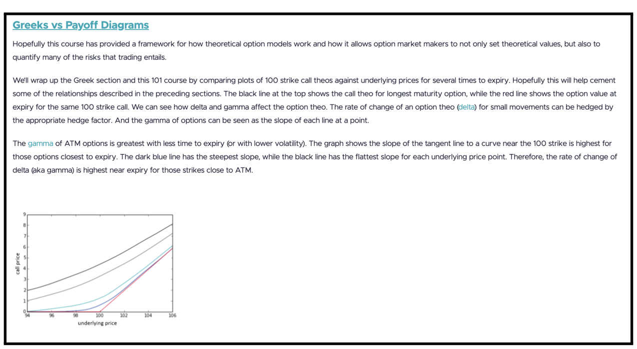
Take the natural log of both sides:

$$\ln\left(\frac{499.20}{500}\right) = -\frac{r}{3}$$

$$\ln(0.9984) \approx -0.0016 = -\frac{r}{3} \Rightarrow r \approx 0.0048 = 0.48\%$$

Hello and welcome to this video presented by Akuna Capital. Today we'll be discussing the Greek rho. We'll start by defining rho and talking about its link to benchmark interest rates. We'll then go over its importance, which is often overlooked. We'll walk through a few simple examples to clarify the definitions and mechanics of it as a Greek, and finally, we'll briefly touch on rho as it relates to a firm borrowing or lending money. Let's start with our definition of rho. Rho, as it pertains to a single option contract, is the sensitivity of an option's price to changes in interest rates. Mathematically, it's the first derivative of an option's value with respect to the rate. You'll see traders often use the term risk-free interest rates, since we use some benchmark rate, like the Fed Funds rate, as a starting point for the rates used in options models. The most common way that rho is discussed is in the case of a book or options position. In this case, it's the risk exposure of that book to interest rate changes, and we'll get into examples of each of these. I'll mention here that rho is often considered the least cared about or least important Greek. This is because interest rates have been so low for the past 15 years or so. As you can see in this chart of the Fed Funds rate, about a decade ago, rates were just 19 basis points, and about seven years later, we were still in about the same place. But in the last several years, as rates have started to move, rho has become a much more important and talked-about Greek for market makers. So let's break down the impact of rho on options and options books. For an option, rho is expressed on each option strike in terms of price per 1% change in the underlying rate. For example, if we're using a 2% rate in our options model, that will output an option price of \$1. The rho of that option line is seven cents. Therefore, if we change the interest rate in our model from 2% to 3%, we'll get a new option price of \$1.07. If the rate were to go down to 1%, the option would then be worth 93 cents instead. For a book or portfolio of options, the total rho is the summation of the product of the option quantity and the option multiplier. So, if we owned about 200 of these options with a seven-cent rho and a multiplier of 100, we'd have a rho position of 1,400 for this book. If we change the rate from 2% to 3%, we'd make \$1,400 of P&L because of the rate change. And if we move the rate down half a percent, we'd lose half that amount, or about \$700. Now, let's take a moment to think about these percents I've been using as an example. Remember, we don't just put a flat rate into our model, but instead a curve of our expectations of the rate over time across all our different expiries that we trade. So, we could be long rho exposure on one point of the curve, noted by a blue dot, and short some rho exposure on another expiry on the curve, noted by a red dot. So it's not just the shift of the curve up and down by some uniform point value, but also the shape of the curve that will dictate our exposure and our adjustments as we build up rho positions. These are more advanced topics, and we'll get into them in future videos. But for now, since rho is the change in rates that a company is using to model options, rho can be summed up across products that use the same rate curve, and it can be managed on a firm-wide level. Therefore, various products can have different rho exposures, but if Akuna can sum those exposures and manage the overall rho of the firm by adjusting the curve to flatten the exposure in any one or combination of the products traded, then the firm's total rate risk is properly managed. The final point we'll briefly touch on is how we think about rho risk in terms of lending or borrowing money. We'll have a separate in-depth video on this concept, so I'll just briefly discuss it here. If you're borrowing money from someone, you want interest rates to go up, assuming that you're using a floating rate that moves with the Fed Funds or some other benchmark. You've borrowed the money at some rate, so if rates go up, you've locked in your lower rate, and you're happy you did so. Similarly, if you're lending money, you lose if rates go up. You've lent someone money at a lower rate, and now you have less lending opportunity at a higher rate, and vice versa if rates go down. Therefore, a borrowing money scenario can be thought of as equivalent to a long rho position, since this makes money as rates go up, and a lender is equivalent to a market participant who is short rho risk. Therefore, we can say that market makers are either lending or borrowing based on their cumulative rho position. As a quick example, consider a box, which is a combination of four option contracts across two strikes. This combination has no other Greek exposure except for rho. We also know that the price of the box at expiration settles to the difference between the two strikes. Therefore, this box will be worth \$5 at expiration, which is the difference between \$255 and \$250. The current value or discounted price of this box can be used as a mechanism to lend or borrow money. The price of that box, at this moment, is \$4.80 with some time left to expiry, using a 2% rate. If two individuals or firms trade this box at a price of \$4.80, then we say that the buyer of the box is lending money at a 2% rate, while the seller of the box is borrowing money at that rate. Again, this warrants a more in-depth video, and we'll discuss the mechanics of clearing houses, interest rates, borrowing, and lending in that one. But I'll leave you with that tip of the iceberg for now. So, to review, rho is the sensitivity of an option, or book of options, to a change in interest rates. For options, it's expressed as a change in option price per 1% change in rates. For a book or portfolio of options, we sum those up line by line and express that exposure as a P&L change per 1% change in rates. Rho can be summed up across products and managed on a firm-wide level. Market makers are said to borrow or lend money depending on whether they're long or short rho exposure. I'll leave you with these summary points to think about for now. Hopefully, this video has explained the basics and risks of the often-ignored Greek rho.

Closing Thoughts



Greeks vs Payoff Diagrams

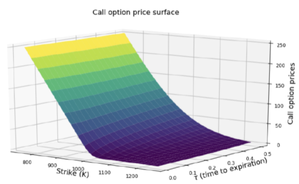
Hopefully this course has provided a framework for how theoretical option models work and how it allows option market makers to not only set theoretical values, but also to quantify many of the risks that trading entails.

We'll wrap up the Greek section and this 101 course by comparing plots of 100 strike call theos against underlying prices for several times to expiry. Hopefully this will help cement some of the relationships described in the preceding sections. The black line at the top shows the call theo for longest maturity option, while the red line shows the option value at expiry for the same 100 strike call. We can see how delta and gamma affect the option theo. The rate of change of an option theo (delta) for small movements can be hedged by the appropriate hedge factor. And the gamma of options can be seen as the slope of each line at a point.

The gamma of ATM options is greatest with less time to expiry (or with lower volatility). The graph shows the slope of the tangent line to a curve near the 100 strike is highest for those options closest to expiry. The dark blue line has the steepest slope, while the black line has the flattest slope for each underlying price point. Therefore, the rate of change of delta (aka gamma) is highest near expiry for those strikes close to ATM.

Vega and theta are a bit more difficult to ascertain from the graph above. We'd have to plot various curves with uniform implied volatilities or times to expiry to better examine these relationships. But hopefully the previous sections of this 101 course presented the information clearly and you can see how these important Greeks influence option values.

Below is another representation showing call prices across strikes for various times to expiry. This surface would contain similar lines as those from the graph above, and presents the option price surface from another perspective.



Models and option prices

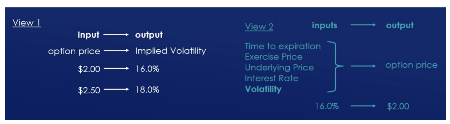
For market makers option theoretical values are used as a mid-point to provide liquidity to the marketplace. They allow Akuna to provide fair and orderly markets, capture the bid/ask spread and manage risk.

One of the main ways we link option values to volatilities is by translating option prices to implied volatility levels. There are two views on how this method allows us to function.

View 1 - If we know the option price at which we just transacted we can translate that into an implied volatility (IV) level. This lets us compare options in a way that is more useful than option prices because underlying price changes and other greeks can impact our theoretical values. If we translate price into IV it gives us a cleaner way to compare our trades.

View 2 - We input an implied volatility into our model. This is our expectation of future movement in the underlying instrument. Using this IV as well as other model inputs shown below, we derive an options value as an output. This value acts as our midpoint and allows us to make two-sided markets and provide valuable liquidity to other market participants.

Both views are correct and it really depends on the goals and objectives of the models and the questions one is trying to ask of it.



In this 101 course we've only started scratching the surface of option dynamics. Option markets are a complex mix of market forces, optimization problems, first and second order derivatives, interesting game theory, puzzles, and psychological and technological challenges. We hope that this has been an informative and challenging course. Feel free to reach out with thoughts/questions/suggestions. Thanks for participating!

Please take the final quiz in the next section to finish this course! Well done in completing it!

Quiz 3

Akuna Capital Options 101 Quizzes

Page 1 of 17

Welcome to Quiz 3 for the Akuna Capital Options 101 Course!

akuna capital

What to Expect During this Quiz:

This quiz consists of several multiple choice questions designed to test your understanding of the material presented in the Akuna Capital Options 101 Course up until this Quiz 3 section. You should only take this Quiz after you've reached the Quiz 3 prompt.

Each question is timed and you will be transitioned to the next question when time has expired if you have not already hit the "Next" button.

This quiz should take approximately 10 minutes.

***If you are not ready to complete the quiz at this time, you can always come back later.**

If you need to come back later, simply copy the link that was sent in your invitation email.

If you're ready, click "Next" to begin the quiz.

Note, you'll have a time limit (on the upper left corner of the screen) for each question.

- [x] * I'm ready for the quiz. Question 1 has a 45 second timer that begins on the next page

Save and Next

Question 1

Question: **Given the initial call price, find the current call price given the move in the underlying:**

Initial future price = 45.00
call theoretical price = 2.30
call delta = 0.40
(assume gamma = 0 for this question)

Current future price = 45.20
current call theoretical price = ??

A: 2.22
B: 2.26
C: 2.36
D: 2.38
E: 2.45
F: 2.50

1. Choose the answer from the choices below:

Select an Option

Question 2

Question: **For the previous question we assumed no gamma effects. If we had included the effects of gamma, how would our answer have differed?**

A: it would have been the same option price.
B: the call price would have been higher due to the gamma effects
C: the call price would have been lower due to the gamma effects
D: you'd need to know if the option gamma was positive or negative
E: you can't tell from the information given since it depends on where the strike is compared to underlying

2. Choose an answer from the choices below:

Select an Option

Question 3

Question: **If you sell 200 calls that have a 0.30 delta, what must you do to hedge your delta exposure from this trade?**

A: buy 600 futures
B: sell 600 futures
C: buy 200 futures
D: sell 200 futures
E: buy 60 futures
F: sell 60 futures
G: buy 30 futures
H: sell 30 futures
I: buy 3 futures
J: sell 3 futures

3. Choose the answer from the choices below:

Select an Option

Question 4

Question: **Which of the following choices best describes the delta of an option that is very far In-the-Money (ITM):**

A: 100
B: 100 for calls
C: 100 for puts and calls
D: 0 for both puts and calls
E: 100 for calls, 0 for puts
F: -100 for puts
G: 100 for calls, -100 for puts

4. Choose the answer from the choices below:

Select an Option

Question 5

Question: **If a call has a delta of 43 then the same strike put has a delta of approximately _______. The put option can best be described as _______.**

A: 57, ITM
B: 43, OTM
C: -43, ATM
D: -57, ITM
E: -57, OTM

5. Choose an answer from the choices below:

Select an Option

Question 6

Question: **Gamma is defined as the ___________________.**

A: the change in option delta with a change in time to expiry
B: the change in option delta with a change in implied volatility
C: the change in option delta with a change in underlying price
D: the change in option options price with a change in strikes
E: the change in option price with a change in underlying price

6. Choose an answer from the choices below:

Select an Option

Question 7

Question: **Gamma is greatest for options that are:**

A: Out of the money (OTM)
B: In the money (ITM)
C: At the money (ATM)
D: the lowest listed strike of the expiry
E: the highest listed strike of the expiry
F: ITM and OTM equally for every strike since calls = puts

7. Choose an answer from the choices below:

Select an Option

Question 8

Question: **Which of the choices below best describes our preference for the movement of the underlying if we have an options portfolio with greeks are long gamma and flat delta:**

A: move in a small range.
B: not move at all (stay at current price).
C: only move up.
D: only move down.
E: move in either direction (we're indifferent)

8. Choose an answer from the choices below:

Select an Option

Question 9

Question: **Use both delta and gamma to calculate the new option price if the underlying moves up by $0.50:**

Option price = $3.00
option delta = 0.40
option gamma = 0.08

A: 2.79
B: 2.80
C: 3.20
D: 3.21
E: 3.22
F: 3.24
G: 3.44
H: 3.48

9. Choose an answer from the choices below:

Select an Option

Question 10

Question: **What is the delta of the 65 strike call option?**

Bid Q	Bid	Theo	Ask	Ask Q	Delta	Strke			Bid Q	Bid	Theo	Ask	Ask Q	Vega	Vol	
3	7.17	**7.207**	7.23	2	99	**59.5**			956	0.01	**0.013**	0.02	3016	0.003	36.51	0.006
4	6.67	**6.710**	6.74	3	99	**60.0**			1960	0.01	**0.017**	0.02	22	0.003	35.39	0.006
3	6.18	**6.216**	6.25	2	99	**60.5**			98	0.02	**0.022**	0.03	3035	0.004	34.26	0.006
3	5.68	**5.722**	5.75	2	98	**61.0**	1	**3**	5	0.03	**0.029**	0.04	3127	0.005	33.13	0.006
3	5.19	**5.231**	5.27	3	98	**61.5**			1858	0.03	**0.038**	0.04	47	0.007	32.02	0.006
3	4.70	**4.744**	4.78	3	97	**62.0**			1936	0.04	**0.050**	0.06	2613	0.008	30.94	0.006
3	4.22	**4.261**	4.29	2	96	**62.5**			1163	0.06	**0.067**	0.08	2174	0.010	29.90	0.006
3	3.75	**3.785**	3.82	3	94	**63.0**		**-4**	5	0.09	**0.091**	0.10	1136	0.013	28.94	0.006
3	3.28	**3.319**	3.35	2	91	**63.5**			442	0.12	**0.125**	0.13	160	0.016	28.06	0.007
3	2.83	**2.867**	2.90	2	88	**64.0**			25	0.17	**0.172**	0.18	326	0.020	27.28	0.007
3	2.39	**2.434**	2.47	3	84	**64.5**	6	**6**	633	0.23	**0.239**	0.25	742	0.024	26.60	0.008
3	1.98	**2.024**	2.06	3	78	**65.0**		**-7**	208	0.32	**0.329**	0.34	375	0.028	25.96	0.008
3	1.61	**1.643**	1.68	3	72	**65.5**	4	**8**	55	0.44	**0.448**	0.46	342	0.032	25.34	0.009
3	1.26	**1.295**	1.30	5	65	**66.0**	-2	**-10**	83	0.59	**0.600**	0.61	28	0.035	24.70	0.009
108	0.97	**0.988**	1.00	9	56	**66.5**	-6	**3**	213	0.78	**0.793**	0.81	108	0.037	24.11	0.010
13	0.72	**0.729**	0.74	13	45	**67.0**		**4**	35	1.00	**1.034**	1.07	10	0.037	23.63	0.010
268	0.51	**0.522**	0.53	13	35	**67.5**		**3**	12	1.29	**1.326**	1.36	2	0.035	23.32	0.010
411	0.35	**0.363**	0.37	83	26	**68.0**	-3	**-6**	3	1.63	**1.667**	1.71	2	0.031	23.18	0.010
329	0.24	**0.248**	0.26	450	18	**68.5**	-2	**2**	2	2.02	**2.052**	2.09	3	0.026	23.22	0.010
153	0.16	**0.169**	0.17	188	12	**69.0**		**-4**	2	2.44	**2.472**	2.51	3	0.022	23.46	0.010
1365	0.10	**0.116**	0.12	880	8	**69.5**	6	**14**	3	2.88	**2.919**	2.95	3	0.017	23.93	0.010
1316	0.07	**0.082**	0.08	161	5	**70.0**	1	**-14**	2	3.35	**3.385**	3.42	3	0.014	24.57	0.010
638	0.05	0.058	0.06	895	4	**70.5**	1	4	2	3.83	3.861	3.90	3	0.011	25.30	0.010
8	0.04	**0.042**	0.05	2189	3	**71.0**	3	**-14**	3	4.31	**4.345**	4.38	3	0.008	26.04	0.010
9	0.03	**0.030**	0.04	2465	2	**71.5**			2	4.80	**4.833**	4.87	3	0.007	26.79	0.010
746	0.02	**0.022**	0.03	2463	1	**72.0**		**6**	3	5.29	**5.325**	5.37	3	0.005	27.58	0.010
2975	0.01	**0.017**	0.02	267	1	**72.5**		**-1**	2	5.79	**5.820**	5.86	3	0.004	28.46	0.010
1312	0.01	**0.013**	0.02	2109	1	**73.0**		**25**	3	6.28	**6.316**	6.36	3	0.003	29.42	0.010

A: 66
B: 78
C: -22
D: -35
E: -66
F: -78
G: 65
H: None of the above

10. Choose an answer from the choices below:

Select an Option

Question 11

Question: **If we have a 50 delta call with a vega of 0.05 and we raise implied volatility 3 points, what is the new vega of the option?**

A: 0.15
B: -0.10
C: 0.10
D: 0.05
E: 0.25
F: 0.08
G: 0.025
H: 0.075

11. Choose an answer from the choices below:

Select an Option

Question 12

Question: **If we raise the implied volatility of this expiry 2 points, what is the new theoretical value of the 68 put?**

Bid Q	Bid	Theo	Ask	Ask Q	Delta	Strke			Bid Q	Bid	Theo	Ask	Ask Q	Vega	Vol	
3	7.17	**7.207**	7.23	2	99	**59.5**			956	0.01	**0.013**	0.02	3016	0.003	36.51	0.006
4	6.67	**6.710**	6.74	3	99	**60.0**			1960	0.01	**0.017**	0.02	22	0.003	35.39	0.006
3	6.18	**6.216**	6.25	2	99	**60.5**			98	0.02	**0.022**	0.03	3035	0.004	34.26	0.006
3	5.68	**5.722**	5.75	2	98	**61.0**	1	**3**	5	0.03	**0.029**	0.04	3127	0.005	33.13	0.006
3	5.19	**5.231**	5.27	3	98	**61.5**			1858	0.03	**0.038**	0.04	47	0.007	32.02	0.006
3	4.70	**4.744**	4.78	3	97	**62.0**			1936	0.04	**0.050**	0.06	2613	0.008	30.94	0.006
3	4.22	**4.261**	4.29	2	96	**62.5**			1163	0.06	**0.067**	0.08	2174	0.010	29.90	0.006
3	3.75	**3.785**	3.82	3	94	**63.0**		**-4**	5	0.09	**0.091**	0.10	1136	0.013	28.94	0.006
3	3.28	**3.319**	3.35	2	91	**63.5**			442	0.12	**0.125**	0.13	160	0.016	28.06	0.007
3	2.83	**2.867**	2.90	2	88	**64.0**			25	0.17	**0.172**	0.18	326	0.020	27.28	0.007
3	2.39	**2.434**	2.47	3	84	**64.5**	6	**6**	633	0.23	**0.239**	0.25	742	0.024	26.60	0.008
3	1.98	**2.024**	2.06	3	78	**65.0**		**-7**	208	0.32	**0.329**	0.34	375	0.028	25.96	0.008
3	1.61	**1.643**	1.68	3	72	**65.5**	4	**8**	55	0.44	**0.448**	0.46	342	0.032	25.34	0.009
3	1.26	**1.295**	1.30	5	65	**66.0**	-2	**-10**	83	0.59	**0.600**	0.61	28	0.035	24.70	0.009
108	0.97	**0.988**	1.00	9	56	**66.5**	-6	**3**	213	0.78	**0.793**	0.81	108	0.037	24.11	0.010
13	0.72	**0.729**	0.74	13	45	**67.0**		**4**	35	1.00	**1.034**	1.07	10	0.037	23.63	0.010
268	0.51	**0.522**	0.53	13	35	**67.5**		**3**	12	1.29	**1.326**	1.36	2	0.035	23.32	0.010
411	0.35	**0.363**	0.37	83	26	**68.0**	-3	**-6**	3	1.63	**1.667**	1.71	2	0.031	23.18	0.010
329	0.24	**0.248**	0.26	450	18	**68.5**	-2	**2**	2	2.02	**2.052**	2.09	3	0.026	23.22	0.010
153	0.16	**0.169**	0.17	188	12	**69.0**		**-4**	2	2.44	**2.472**	2.51	3	0.022	23.46	0.010
1365	0.10	**0.116**	0.12	880	8	**69.5**	6	**14**	3	2.88	**2.919**	2.95	3	0.017	23.93	0.010
1316	0.07	**0.082**	0.08	161	5	**70.0**	1	**-14**	2	3.35	**3.385**	3.42	3	0.014	24.57	0.010
638	0.05	0.058	0.06	895	4	**70.5**	1	4	2	3.83	3.861	3.90	3	0.011	25.30	0.010
8	0.04	**0.042**	0.05	2189	3	**71.0**	3	**-14**	3	4.31	**4.345**	4.38	3	0.008	26.04	0.010
9	0.03	**0.030**	0.04	2465	2	**71.5**			2	4.80	**4.833**	4.87	3	0.007	26.79	0.010
746	0.02	**0.022**	0.03	2463	1	**72.0**		**6**	3	5.29	**5.325**	5.37	3	0.005	27.58	0.010
2975	0.01	**0.017**	0.02	267	1	**72.5**		**-1**	2	5.79	**5.820**	5.86	3	0.004	28.46	0.010
1312	0.01	**0.013**	0.02	2109	1	**73.0**		**25**	3	6.28	**6.316**	6.36	3	0.003	29.42	0.010

A: 0.363
B: 0.934
C: 0.425
D: 1.606
E: 1.629
F: 1.698
G: 1.719
H: 1.729

12. Choose an answer from the choices below:

Select an Option

Question 13

Question: **Let's examine a put with a delta of -0.30 and with a vega of 0.05. If we lower implied volatility 2 points how is the vega of the put impacted?**

A: It increases from 0.05
B: It decreases from 0.05
C: It remains 0.05
D: It increases by the opposite but equal direction of the call. (put up, call down)
E: Can't tell. Not enough information given.

13. Choose an answer from the choices below:

Select an Option

Question 14

Question: **Let's look at the following option prices and volatilities:**

- **60 strike vega = 0.4**
- **70 strike vega = 0.5**
- **60 strike implied vol = 18.40**
- **70 strike implied vol = 17.80**
- **60/70 put spread (70p - 60p) price = 4.20**

If 60 strike implied volatility rises to 19.00 and 70 strike volatility rises to 18.20, what is the new price of the 60/70 put spread?

A: 4.28
B: 4.24
C: 4.20
D: 4.16
E: 4.12
F: 4.08
G: None of the above

14. Choose an answer from the choices below:

Select an Option

Question 15

Question: **The 330 strike call is worth 42.92. If the future is worth 370, what is the value of the 330 strike put? (assume no interest rate or dividend effects)**

A: 42.92
B: 82.92
C: 12.92
D: 4.92
E: 2.92
F: 0.92
G: can't tell from the information given

15. Choose an answer from the choices below:

Select an Option

Question 16

Question: **What is the delta of the 68/69 put spread?**

Bid Q	Bid	Theo	Ask	Ask Q	Delta	Strke			Bid Q	Bid	Theo	Ask	Ask Q	Vega	Vol	
3	7.17	**7.207**	7.23	2	99	**59.5**			956	0.01	**0.013**	0.02	3016	0.003	36.51	0.006
4	6.67	**6.710**	6.74	3	99	**60.0**			1960	0.01	**0.017**	0.02	22	0.003	35.39	0.006
3	6.18	**6.216**	6.25	2	99	**60.5**			98	0.02	**0.022**	0.03	3035	0.004	34.26	0.006
3	5.68	**5.722**	5.75	2	98	**61.0**	1	**3**	5	0.03	**0.029**	0.04	3127	0.005	33.13	0.006
3	5.19	**5.231**	5.27	3	98	**61.5**			1858	0.03	**0.038**	0.04	47	0.007	32.02	0.006
3	4.70	**4.744**	4.78	3	97	**62.0**			1936	0.04	**0.050**	0.06	2613	0.008	30.94	0.006
3	4.22	**4.261**	4.29	2	96	**62.5**			1163	0.06	**0.067**	0.08	2174	0.010	29.90	0.006
3	3.75	**3.785**	3.82	3	94	**63.0**		**-4**	5	0.09	**0.091**	0.10	1136	0.013	28.94	0.006
3	3.28	**3.319**	3.35	2	91	**63.5**			442	0.12	**0.125**	0.13	160	0.016	28.06	0.007
3	2.83	**2.867**	2.90	2	88	**64.0**			25	0.17	**0.172**	0.18	326	0.020	27.28	0.007
3	2.39	**2.434**	2.47	3	84	**64.5**	6	**6**	633	0.23	**0.239**	0.25	742	0.024	26.60	0.008
3	1.98	**2.024**	2.06	3	78	**65.0**		**-7**	208	0.32	**0.329**	0.34	375	0.028	25.96	0.008
3	1.61	**1.643**	1.68	3	72	**65.5**	4	**8**	55	0.44	**0.448**	0.46	342	0.032	25.34	0.009
3	1.26	**1.295**	1.30	5	65	**66.0**	-2	**-10**	83	0.59	**0.600**	0.61	28	0.035	24.70	0.009
108	0.97	**0.988**	1.00	9	56	**66.5**	-6	**3**	213	0.78	**0.793**	0.81	108	0.037	24.11	0.010
13	0.72	**0.729**	0.74	13	45	**67.0**		**4**	35	1.00	**1.034**	1.07	10	0.037	23.63	0.010
268	0.51	**0.522**	0.53	13	35	**67.5**		**3**	12	1.29	**1.326**	1.36	2	0.035	23.32	0.010
411	0.35	**0.363**	0.37	83	26	**68.0**	-3	**-6**	3	1.63	**1.667**	1.71	2	0.031	23.18	0.010
329	0.24	**0.248**	0.26	450	18	**68.5**	-2	**2**	2	2.02	**2.052**	2.09	3	0.026	23.22	0.010
153	0.16	**0.169**	0.17	188	12	**69.0**		**-4**	2	2.44	**2.472**	2.51	3	0.022	23.46	0.010
1365	0.10	**0.116**	0.12	880	8	**69.5**	6	**14**	3	2.88	**2.919**	2.95	3	0.017	23.93	0.010
1316	0.07	**0.082**	0.08	161	5	**70.0**	1	**-14**	2	3.35	**3.385**	3.42	3	0.014	24.57	0.010
638	0.05	0.058	0.06	895	4	**70.5**	1	4	2	3.83	3.861	3.90	3	0.011	25.30	0.010
8	0.04	**0.042**	0.05	2189	3	**71.0**	3	**-14**	3	4.31	**4.345**	4.38	3	0.008	26.04	0.010
9	0.03	**0.030**	0.04	2465	2	**71.5**			2	4.80	**4.833**	4.87	3	0.007	26.79	0.010
746	0.02	**0.022**	0.03	2463	1	**72.0**		**6**	3	5.29	**5.325**	5.37	3	0.005	27.58	0.010
2975	0.01	**0.017**	0.02	267	1	**72.5**		**-1**	2	5.79	**5.820**	5.86	3	0.004	28.46	0.010
1312	0.01	**0.013**	0.02	2109	1	**73.0**		**25**	3	6.28	**6.316**	6.36	3	0.003	29.42	0.010

A: -12
B: 14
C: -14
D: 38
E: 5
F: 12
G: 17

16. Choose the answer from the choices below:

Select an Option