Abstract

The classical converse theorem of Gauss sums asserts that certain identities among twisted Gauss sums determine multiplicative characters uniquely over $\mathbb{C}$. However, this uniqueness can fail in modular settings. In this paper, we investigate the failure of the converse theorem when Gauss sums are reduced modulo a prime ℓ. We implement a computational framework in SageMath to construct Gauss sum tables over both $\mathbb{C}$ and $\mathbb{F}_\ell$, identify known counterexamples, and discover new ones. Our results disprove a recent conjecture that characterized when the converse theorem fails and provide evidence for deeper, unexplored structure in the modular behavior of Gauss sums.