Research to complete: Optimal Asset Allocation under HARA Utility with Regret Relative to Benchmarked Contributions

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Abstract

This paper studies optimal asset allocation in a continuous-time setting where the investor has hyperbolic absolute risk aversion (HARA) utility and experiences regret relative to a benchmark defined by cumulative net contributions. The model captures behavioral frictions in pension investing and derives the optimal portfolio strategy under regret aversion. Analytical results and comparative statics highlight how regret reshapes risk-taking relative to benchmark proximity.

1 Introduction

Why do some investors remain under-invested in equity markets, even with long horizons? How should optimal asset allocation reflect not only risk preferences but also psychological costs of underperforming a reference point, such as cumulative contributions? This paper tackles these questions by embedding regret relative to a benchmark into a continuous-time portfolio model with HARA utility.

We depart from classical Merton-style models that assume time-separable preferences and rational agents optimizing expected utility over terminal wealth Merton1969, Merton1971. While elegant, these models omit behavioral dynamics such as regret aversion, which plays a significant role in real-world investing, especially in defined contribution pension schemes. Behavioral finance has long recognized that investors derive utility not just from outcomes, but from how outcomes compare to reference points Bell1982, LoomesSugden1982, Barberis2013.

We introduce regret over terminal wealth relative to a benchmark defined by cumulative contributions, capturing the psychological loss investors feel when final wealth underperforms the capital they themselves invested. This form of regret is intuitive and policy-relevant: underperformance relative to one's own contributions is a common concern in pension planning.

Using HARA preferences provides flexibility in modeling varying risk aversion across wealth levels KaratzasShreve1998, Ingersoll1987. We embed the regret-adjusted utility into a dynamic optimization framework and solve for the optimal risky asset share using the Hamilton–Jacobi–Bellman (HJB) equation.

Main contributions:

- We formulate a continuous-time control problem where regret is triggered by the difference between terminal wealth and a benchmark based on net contributions.
- We derive closed-form expressions for optimal risky asset shares under HARA utility with regret aversion.
- Comparative statics show how benchmark proximity amplifies risk aversion, reducing equity exposure when wealth is close to or below benchmark.
- We outline implications for pension design and lifecycle investing.

2 Literature Review

Classical portfolio theory begins with Merton1969, Merton1971, where investors maximize expected utility of terminal wealth. Extensions include stochastic labor income DybvigLiu2010 and habit formation. HARA preferences generalize CRRA/constant relative risk aversion utility and permit analytical tractability Ingersoll1987.

Behavioral extensions to portfolio choice include regret aversion Bell1982, LoomesSugden1982, loss aversion, and probability weighting. Applications include regret over missing market rallies, timing errors, or underperformance relative to benchmarks. BasakMakarov2014 and GongYang2020 develop strategic allocation models incorporating regret or benchmarking, while WuZeng2022 provide continuous-time formulations.

Our paper builds on this by modeling regret relative to net contributions—a psychologically and contractually salient benchmark in pension systems.

3 Model Setup

3.1 Financial Market

We consider a standard market with a risk-free asset G_t and a risky asset S_t, evolving as:

$$\frac{dS_t}{S_t} = ...$$

$$\frac{dG_t}{G_t} = ...,$$

with $\mu_S > r > 0$ and $\sigma_S > 0$.

3.2 Contributions and Benchmark

Define the cumulative net contribution benchmark:

$$M_\tau = ...$$

3.3 Wealth Dynamics

The investor's wealth W_t evolves according to:

$$dW_t = ...$$

where $\alpha_t \in [0, 1]$ is the proportion invested in the risky asset.

3.4 Utility Function with Regret

We adopt a HARA utility function:

$$U(W_\tau - M_\tau) = \frac{1}{1-\gamma} \left(a(W_\tau - M_\tau) + b\right)^{1-\gamma}, \quad a > 0, b \geq 0,$$

with $\gamma \neq 1$. The benchmark M_τ induces regret aversion when $W_\tau is near M_\tau$.

4 Problem Formulation and HJB Equation

The objective is to:

$$\max_{\alpha_t} E_{t,x} \left[U(W_\tau - M_\tau)\right].$$

We formulate the corresponding HJB equation and guess a value function form using HARA homotheticity to derive closed-form solutions.

5 Solution and Optimal Strategy

We derive the optimal allocation:

$$\alpha_t^* = ...$$

adjusted for the regret term by modifying the effective risk aversion when $W_t \approx M_\tau$. The closer the investor is to the benchmark, the more conservative the strategy becomes.

6 Comparative Statics and Insights

7 Numerical Illustration

Simulate sample paths, plot optimal α_t^* as a function of $W_t - M_\tau$, perform sensitivity to γ, M_τ, and a.

8 Conclusion

We show that regret relative to contributions distorts optimal asset allocation,

References