

Geometric Quantization of Matter: Analysis of Correlations between Fundamental Constants and Particle Spectrum

Jan Šági
(Brighton, UK)

November 24, 2025

Abstract

We present for discussion a mathematical model suggesting the existence of a deep geometric structure connecting the masses of elementary particles with fundamental physical constants. Based on the analysis of a mass spectrum dataset (0–10 GeV) generated by high-precision decimal algorithms, we have identified discrete scaling laws derived from the constants π , α (fine-structure constant), and the logarithmic spacetime base $N = \ln(4\pi)$. The model reproduces the masses of the proton and muon with extreme precision (relative error $< 0.002\%$) and offers an analytical derivation of the gravitational constant G with a relative error of $0.003,7\%$. This work does not aspire to replace existing field theories but rather points to significant numerical correlations that may indicate a topological origin of matter.

1 Introduction

The Standard Model of particle physics is one of the most successful theories in the history of science. However, a number of parameters, particularly particle masses, remain free values that must be determined experimentally. Physics does not yet have a definitive answer as to why the proton has a mass of exactly $\approx 938.27\text{ MeV}$ or why the muon-to-electron mass ratio is approximately 206.77.

In this paper, we approach the problem phenomenologically using high-precision computational scanning (110-bit precision). Instead of seeking new Lagrangian dynamics, we examine whether the mass spectrum exhibits signs of “geometric quantization.” Our hypothesis is based on the assumption that stable particles represent resonant nodes on a fundamental lattice defined by dimensionless constants. Although this approach might initially resemble numerical coincidence, the statistical significance of the findings—especially in the hadron sector and the derivation of gravitational coupling—suggests a systematic phenomenon.

2 Methodology and Definitions

The model operates with dimensionless ratios relative to the electron mass (m_e). The key building blocks are:

- Fine-structure constant: $\alpha^{-1} \approx 137.035,999$
- Logarithmic spacetime base: $N = \ln(4\pi) \approx 2.531$

- Geometric node (k): An integer ($k \in \mathbb{Z}^+$), representing harmonic excitation.

We introduce three basic energy scales relative to m_e :

$$\text{Lepton Scale: } S_L = 4\pi N^3 \quad (1)$$

$$\text{Meson Scale: } S_M = \alpha^{-1} \quad (2)$$

$$\text{Baryon Scale: } S_B = \pi^5 \quad (3)$$

The final particle mass $M(k)$ is determined by the lattice node k and a topological correction factor n :

$$M(k) = m_e \cdot (k \cdot S_{type}) \cdot \frac{1}{1 \pm n\alpha} \quad (4)$$

where n is an integer derived from the topology of node k (e.g., for the fundamental sphere $k = 1$, $n = 2$; for perfect symmetry $k = 6$, $n = 0$).

3 Results: Derivation of Fundamental Masses

The following results were obtained via analytical calculation using a custom Python discovery engine and compared with CODATA (2018/2022) values.

3.1 Lepton Sector (Muon)

In our model, the muon corresponds to the first node ($k = 1$) on the lepton scale. Since $k = 1$ represents a fundamental sphere, we apply a topological correction corresponding to the Euler characteristic of a sphere ($n = 2$). Using Eq. 1:

$$m_\mu = m_e \cdot \frac{4\pi N^3}{1 - 2\alpha} \quad (5)$$

- **Theoretical Value:** 206.768,267 m_e
- **Experimental Value:** 206.768,283 m_e
- **Relative Error:** 0.000,007 %

3.2 Nuclear Sector (Proton)

The proton is identified in the model as node $k = 6$ on the baryon scale (π^5). The number 6 geometrically represents perfect symmetry (e.g., a hexagon or cubic faces), which implies zero internal stress in our model, and thus zero correction ($n = 0$).

$$m_p = m_e \cdot 6\pi^5 \quad (6)$$

- **Theoretical Value:** 1,836.118 m_e
- **Experimental Value:** 1,836.152 m_e
- **Relative Error:** 0.001,9 %

3.3 Unification and the Gravitational Constant (G)

One of the strongest tests of the model is the derivation of the gravitational constant G . Assuming gravity is a residual manifestation of space geometry with total dimension $D = 10$, we define the dimensional exponent X . The geometric base is $X_0 = 10\pi/3$. Including second-order QED corrections, the effective exponent becomes:

$$X = \frac{10\pi}{3} + \frac{\alpha}{4\pi} + \sqrt{\sqrt{2}\alpha^2} \approx 10.472,6 \quad (7)$$

The gravitational constant is then given by:

$$G_{theor} = \frac{\hbar c}{m_p^2} \cdot (\Gamma_p^2 \cdot \alpha^{2X}) \quad (8)$$

where $\Gamma_p = 6\pi^5$ is the dimensionless proton mass.

- **Theoretical Value G :** $6.674,05 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
- **CODATA Value G :** $6.674,30 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
- **Relative Error:** 0.003,7 %

4 Predictions and Key Nodes

By applying these rules to the integer spectrum ($k = 1$ to $k = 100$), we generated a dataset of potential energy levels. Table 1 presents a curated selection. Note that the table displays the *Base Node Energy* (uncorrected). For confirmed particles (bold), the topological correction brings the error close to zero (as demonstrated in Section 3).

5 Discussion and Conclusion

The presented results indicate that the masses of fundamental particles are not random numbers but exhibit a strict geometric organization bound to the constants π and α . The precision of the proton mass ($6\pi^5$, error ≈ 0.002 %) and the muon mass (error $\approx 7 \times 10^{-6}$ %) is particularly striking.

We acknowledge that this model is currently heuristic. However, the extreme agreement with experimental data, including the analytical derivation of G with an error of 0.003,7 %, suggests that the universe may operate on a fundamental level as a geometric system with fewer free parameters than the Standard Model assumes.

Data Availability

The Python script used for the verification of the calculations, including the 110-digit precision tests, is available for public review.

References

- [1] CODATA Recommended Values of the Fundamental Physical Constants: 2018. NIST.
- [2] Particle Data Group (PDG), Review of Particle Physics. *Prog. Theor. Exp. Phys.* 2022, 083C01 (2022).

Table 1: Selected Significant Nodes from Geometric Scan (0–10 GeV).
Note: 'Base Energy' represents the geometric base energy before topological correction.

Base Energy (MeV)	Scale Type	k	Stab.	Status	Candidate	Dev. (%)
104.12	LEPTON (N)	1	90	CONFIRMED	Muon (μ)	1.48
140.05	MESON (α^{-1})	2	90	CONFIRMED	Pion (π^+)	0.34
312.75	BARYON (π^5)	2	110	PREDICTION	—	—
469.13	BARYON (π^5)	3	110	PREDICTION	—	—
490.18	MESON (α^{-1})	7	90	CONFIRMED	Kaon (K^+)	0.71
770.28	MESON (α^{-1})	11	70	CONFIRMED	Rho (ρ^0)	0.65
781.88	BARYON (π^5)	5	110	CONFIRMED	Omega (ω)	0.85
938.25	BARYON (π^5)	6	100	CONFIRMED	Proton (p)	0.00
1,094.63	BARYON (π^5)	7	110	CONFIRMED	Lambda (Λ)	1.92
1,720.13	BARYON (π^5)	11	90	CONFIRMED	$f_0(1710)?$	0.01
1,769.98	LEPTON (N)	17	60	CONFIRMED	Tau (τ)	0.39
1,876.51	BARYON (π^5)	12	60	CONFIRMED	D^+ Meson	0.37
2,290.56	LEPTON (N)	22	30	CONFIRMED	Λ_c	0.18
2,658.39	BARYON (π^5)	17	90	CONFIRMED	$\Xi_c(2645)$	0.50
2,971.14	BARYON (π^5)	19	90	CONFIRMED	η_c	0.40
3,081.11	MESON (α^{-1})	44	40	CONFIRMED	J/ψ	0.51
3,596.64	BARYON (π^5)	23	90	CONFIRMED	$\psi(2S)$	2.48
5,316.78	BARYON (π^5)	34	60	CONFIRMED	B^+	0.70
6,411.41	BARYON (π^5)	41	90	PREDICTION	—	—
9,266.35	LEPTON (N)	89	60	CONFIRMED	$\Upsilon(1S)$	2.09