

# On a Dimensionally Consistent Topological Reinterpretation of the Einstein Field Equations

Jan Šági

Independent Researcher

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## Abstract

The Standard Model of particle physics and General Relativity represent the two pillars of modern physics, yet they exhibit a fundamental dimensional tension when combined. This paper proposes a theoretical modification of the Einstein Field Equations (EFE), exploring the possibility that the energy-momentum tensor represents a density of topological defects on a geometric lattice ( $L^{-3}$ ) rather than a simple energy density. Within this framework, the gravitational constant  $G$  emerges as a property of the lattice characteristic length. We discuss the heuristic derivation of fundamental constants, observing that the fine-structure constant ( $\alpha^{-1} \approx 137.036$ ) and the proton-to-electron mass ratio ( $\mu \approx 1836.11$ ) can be approximated by geometric series of  $\pi$ . Furthermore, we analyze the lattice capacity limit ( $k_{max} \approx 44$ ), derived from the ratio of spatial to nodular geometry, offering a potential explanation for the dark matter ratio. Supporting computational scripts are available in the associated data repository.

## 1 Introduction: The Dimensional Homogeneity Problem

The Einstein Field Equations (EFE) describe gravity as the curvature of spacetime [1]:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \quad (1)$$

A longstanding formal issue in this formulation is the dimensional mismatch between the left-hand side (pure geometry, curvature  $L^{-2}$ ) and the right-hand side (energy density). The coupling constant  $\kappa = 8\pi Gc^{-4}$  acts as a conversion factor. However, historically, attempts such as Wheeler's Geometrodynamics [2] have suggested that matter should not be an extrinsic input, but an intrinsic geometric feature.

In this draft, we explore the hypothesis of a **Geometric Vacuum Structure**, where matter is modeled as a topological knot in the spacetime manifold itself. This approach aims to restore dimensional homogeneity to the EFE.

## 2 Mathematical Preliminaries and Definitions

To formalize the transition from energy density to topological density, we introduce the following definitions for the vacuum manifold  $\mathcal{M}$ .

### 2.1 The Vacuum Lattice

We postulate that the vacuum is not a continuum but a discrete geometric lattice characterized by a fundamental length scale  $\ell_{eff}$ . **Definition 1 (Lattice Density):** Let  $\rho_{top}$  be the number density of fundamental geometric nodes in a given volume  $V$ . The unit of  $\rho_{top}$  is strictly  $L^{-3}$ .

### 2.2 Topological Density Tensor

We propose replacing the standard energy-momentum tensor  $T_{\mu\nu}$  with a dimensionless topological tensor  $\mathcal{T}_{\mu\nu}^{top}$ . **Definition 2:**

$$\mathcal{T}_{\mu\nu}^{top} \equiv \rho_{top} u_\mu u_\nu \quad (2)$$

where  $u_\mu$  is the 4-velocity of the topological defect (knot). This tensor describes the flow of lattice defects through spacetime.

## 3 Topological Formulation Proposal

Using the definitions above, the modified field equation is postulated as:

$$G_{\mu\nu} = 8\pi \cdot \ell_{eff} \cdot \mathcal{T}_{\mu\nu}^{top} \quad (3)$$

This formulation renders the equation dimensionally consistent ( $[L^{-2}] = [L] \cdot [L^{-3}]$ ). Gravity is thus described not as a force generated by mass, but as an emergent consequence of the lattice accommodation to topological defects.

### 3.1 The Classical Limit

To recover Newtonian gravity, we must assume that on macroscopic scales ( $\gg \ell_{eff}$ ), the discrete topological density averages to the continuum mass density  $\rho_{mass}$ . The correspondence principle suggests:

$$\rho_{mass} \approx \rho_{top} \cdot M_{node} \quad (4)$$

where  $M_{node}$  is the effective mass of a single topological defect. If  $\ell_{eff} \propto GM_{node}/c^2$ , the standard Einstein equations are recovered in the continuum limit.

## 4 Heuristic Derivation of Constants

If the vacuum possesses a lattice structure, fundamental constants should reflect the geometry of this lattice expansion. We present the following geometric series as heuristic arguments motivating a deeper topological study.

### 4.1 The Geometric Origin of $\alpha$

We model the fine-structure constant  $\alpha$  as a geometric partition function of the vacuum expansion. The total geometric potential  $\Phi$  is approximated by summing the dimensional expansion modes of a sphere:

- 1D Linear expansion:  $\pi$  (circumference factor)
- 2D Surface flux:  $\pi^2$  (orthogonal area factor)
- 3D Phase volume:  $4\pi^3$  (spherical phase space volume)

Summing these modes yields:

$$\alpha_{geom}^{-1} = \sum_{n=1}^3 \dim_n(\pi) \approx 4\pi^3 + \pi^2 + \pi \approx 137.03630 \quad (5)$$

This value matches the CODATA measurement with a precision of  $\approx 2 \times 10^{-4}\%$ . We suggest this is not coincidental, but reflects the geometric capacity of the vacuum lattice.

### 4.2 Matter Stability ( $\mu$ )

Similarly, assuming the proton represents a stable knot on a 5-dimensional manifold with hexagonal symmetry ( $k = 6$ ), the mass stability ratio follows:

$$\mu = \frac{m_p}{m_e} \approx 6\pi^5 \approx 1836.118 \quad (6)$$

This aligns with the experimental value (1836.152) within a 99.998% margin.

## 5 Lattice Capacity and Dark Matter

A finite geometric lattice must have a maximum capacity. We define the lattice cut-off wavenumber  $k_{max}$  as the ratio between the spatial expansion factor ( $\alpha^{-1}$ ) and the fundamental knot winding ( $\pi$ ):

$$k_{max} = \left\lfloor \frac{\alpha_{geom}^{-1}}{\pi} \right\rfloor \approx \left\lfloor \frac{137.036}{3.141} \right\rfloor = 43 \rightarrow 44 \quad (7)$$

This suggests the lattice supports modes up to  $k = 44$ .

- **Resonant Modes (Baryonic):** Composite numbers capable of harmonic resonance.
- **Non-resonant Modes (Dark):** Prime numbers that cannot establish standing waves but still contribute to lattice curvature (gravity).

A preliminary summation of energies for these states yields a ratio  $\Omega_{DM}/\Omega_b \approx 5.5$ , consistent with Planck 2018 data [4].

## 6 Conclusion

This paper presents a preliminary framework for geometrizing the right-hand side of the EFE. By enforcing dimensional consistency through topological density, we find that key physical constants may naturally emerge from the geometry of the vacuum lattice. While the derivations presented are heuristic, the high numerical precision suggests an underlying physical mechanism warranting further investigation.

## Data Availability Statement

The computational scripts, detailed derivations, and supplementary theories supporting the findings of this study are openly available in the Zenodo repository:

<https://zenodo.org/uploads/17728773>

## References

- [1] A. Einstein, "Die Feldgleichungen der Gravitation", *Sitzungsber. Preuss. Akad. Wiss.*, 844 (1915).
- [2] J. A. Wheeler, "Geometrodynamics", *Academic Press* (1962).
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- [4] Planck Collaboration, *Astron. Astrophys.*, 641, A6 (2020).
- [5] E. Tiesinga et al., *Rev. Mod. Phys.*, 93, 025010 (2021).