

$$\begin{aligned} & \underset{W}{\text{minimize}} \quad \frac{1}{2} \sum_{j=0}^N \|W_H z_H^j - y^j\|^2 \\ & \text{subject to} \quad 0 = z_{k+1}^j - \sigma(W_k x_k^j + b_k), \quad k = 0, \dots, H-1, j = 1, \dots, N \end{aligned}$$

$$\begin{aligned} & \min \quad \frac{1}{2} \|F(x)\|_2^2 \\ & \text{s. t.} \quad h(x) = 0 \end{aligned}$$

$$\begin{aligned} \mathcal{L}_c(x, \lambda) &= \frac{1}{2} \|F(x)\|_2^2 + \langle \lambda, h(x) \rangle + \frac{c}{2} \|h(x)\|_2^2 \\ &= \frac{1}{2} \|F(x)\|_2^2 + \frac{c}{2} \|h(x) + \lambda/c\|_2^2 - \frac{1}{2c} \|\lambda\|_2^2 \\ &= \frac{c}{2} \left\| \begin{bmatrix} F(x)/\sqrt{c} \\ h(x) + \lambda/c \end{bmatrix} \right\|^2 \end{aligned}$$

Find

$$x^k \text{ s. t. } \quad \|\nabla_k \mathcal{L}_{c_k}(x^k, \lambda^k)\|_2 \leq \epsilon \quad (1)$$

$$\lambda^{k+1} = \lambda^k + c_k h(x^k) \quad (2)$$

$$F = y - (W_3 z_2 + b_3)$$

$$h_1 = z_1 - \sigma(W_1 x + b_1) \quad (3)$$

$$h_2 = z_2 - \sigma(W_2 z_1 + b_2)$$

| $\nabla \mathcal{L}$ | | W_{11} | W_{12} | W_{13} | b_1 |
|----------------------|---|-------------------------------|-------------------------------|-------------------------------|------------------------------|
| | | 1 | 1 | 1 | 3 |
| F | N | 0 | 0 | 0 | 0 |
| h_1 | N | $-x\sigma'(W_{11}x + b_{11})$ | 0 | 0 | $-\sigma'(W_{11}x + b_{11})$ |
| | N | 0 | $-x\sigma'(W_{12}x + b_{12})$ | 0 | $-\sigma'(W_{12}x + b_{12})$ |
| | N | 0 | 0 | $-x\sigma'(W_{13}x + b_{13})$ | $-\sigma'(W_{13}x + b_{13})$ |
| h_2 | N | 0 | 0 | 0 | 0 |
| | N | 0 | 0 | 0 | 0 |
| | N | 0 | 0 | 0 | 0 |

| $\nabla \mathcal{L}$ | | W_{21} | W_{22} | W_{23} | b_2 |
|----------------------|---|---------------------------------|---------------------------------|---------------------------------|------------------------------|
| | | 3 | 3 | 3 | 3 |
| F | N | 0 | 0 | 0 | 0 |
| h_1 | N | 0 | 0 | 0 | 0 |
| | N | 0 | 0 | 0 | 0 |
| | N | 0 | 0 | 0 | 0 |
| h_2 | N | $-z_1\sigma'(W_{21}z + b_{21})$ | 0 | 0 | $-\sigma'(W_{21}z + b_{21})$ |
| | N | 0 | $-z_1\sigma'(W_{22}z + b_{22})$ | 0 | $-\sigma'(W_{22}z + b_{22})$ |
| | N | 0 | 0 | $-z_1\sigma'(W_{23}z + b_{23})$ | $-\sigma'(W_{23}z + b_{23})$ |

| $\nabla \mathcal{L}$ | | W_3 | / | / | b_3 |
|----------------------|---|-----------------|---|---|---------------|
| | | 3 | 0 | 0 | 1 |
| F | N | $-z_2/\sqrt{c}$ | 0 | 0 | $-1/\sqrt{c}$ |
| h_1 | N | 0 | 0 | 0 | 0 |
| | N | 0 | 0 | 0 | 0 |
| | N | 0 | 0 | 0 | 0 |
| h_2 | N | 0 | 0 | 0 | 0 |
| | N | 0 | 0 | 0 | 0 |
| | N | 0 | 0 | 0 | 0 |

Square Diagonal Matrixes

| $\nabla \mathcal{L}$ | | z_{11} | z_{12} | z_{13} |
|----------------------|---|--|--|--|
| | | N | N | N |
| F | N | 0 | 0 | 0 |
| h_1 | N | 1 | 0 | 0 |
| | N | 0 | 1 | 0 |
| | N | 0 | 0 | 1 |
| h_2 | N | $-W_{21,1}\sigma'(W_{21}z_1 + b_{21})$ | $-W_{21,2}\sigma'(W_{21}z_1 + b_{21})$ | $-W_{21,3}\sigma'(W_{21}z_1 + b_{21})$ |
| | N | $-W_{22,1}\sigma'(W_{22}z_1 + b_{22})$ | $-W_{22,2}\sigma'(W_{22}z_1 + b_{22})$ | $-W_{22,3}\sigma'(W_{22}z_1 + b_{22})$ |
| | N | $-W_{23,1}\sigma'(W_{23}z_1 + b_{23})$ | $-W_{23,2}\sigma'(W_{23}z_1 + b_{23})$ | $-W_{23,3}\sigma'(W_{23}z_1 + b_{23})$ |

| $\nabla \mathcal{L}$ | | z_{21} | z_{22} | z_{23} |
|----------------------|---|--------------------|--------------------|--------------------|
| | | N | N | N |
| F | N | $-W_{31}/\sqrt{c}$ | $-W_{32}/\sqrt{c}$ | $-W_{33}/\sqrt{c}$ |
| h_1 | N | 0 | 0 | 0 |
| | N | 0 | 0 | 0 |
| | N | 0 | 0 | 0 |
| h_2 | N | 1 | 0 | 0 |
| | N | 0 | 1 | 0 |
| | N | 0 | 0 | 1 |

| | | | | | | |
|----------------------|-----|---------------------------------|---------------------------------|-----|---------------------------------|--------------------------------|
| $\nabla \mathcal{L}$ | | W_{0_1} I | W_{0_2} I | ... | W_{0_W} I | b_1 W |
| F | O*N | 0 | 0 | ... | 0 | 0 |
| h_1 | N | $-x\sigma'(W_{0_1}x + b_{0_1})$ | 0 | ... | 0 | $-\sigma'(W_{0_1}x + b_{0_1})$ |
| | N | 0 | $-x\sigma'(W_{0_2}x + b_{0_2})$ | ... | 0 | $-\sigma'(W_{0_2}x + b_{0_2})$ |
| | ... | ... | ... | ... | ... | ... |
| | N | 0 | 0 | ... | $-x\sigma'(W_{0_W}x + b_{0_W})$ | $-\sigma'(W_{0_W}x + b_{0_W})$ |
| h_2 | W*N | 0 | 0 | ... | 0 | 0 |
| ... | ... | ... | ... | ... | ... | ... |
| h_D | W*N | 0 | 0 | ... | 0 | 0 |

| | | | | | | |
|----------------------|-----|-----------------------------------|-----------------------------------|-----|-----------------------------------|--------------------------------|
| $\nabla \mathcal{L}$ | | W_{i_1} W | W_{i_2} W | ... | W_{i_W} W | b_2 W |
| F | O*N | 0 | 0 | ... | 0 | 0 |
| h_1 | W*N | 0 | 0 | ... | 0 | 0 |
| ... | ... | ... | ... | ... | ... | ... |
| h_{i+1} | N | $-z_1\sigma'(W_{i_1}z + b_{i_1})$ | 0 | ... | 0 | $-\sigma'(W_{i_1}z + b_{i_1})$ |
| | N | 0 | $-z_1\sigma'(W_{i_2}z + b_{i_2})$ | ... | 0 | $-\sigma'(W_{i_2}z + b_{i_2})$ |
| | ... | ... | ... | ... | ... | ... |
| | N | 0 | 0 | ... | $-z_1\sigma'(W_{i_W}z + b_{i_W})$ | $-\sigma'(W_{i_W}z + b_{i_W})$ |
| ... | ... | ... | ... | ... | ... | ... |
| h_D | W*N | 0 | 0 | ... | 0 | 0 |

| | | | | | | |
|----------------------|-----|--|--|-----|--|---|
| $\nabla \mathcal{L}$ | | W_{D_1} W | W_{D_2} W | ... | W_{D_O} W | b_3 O |
| F | N | $-z_D\sigma'_O(W_{D_1}x + b_{D_1})/\sqrt{c}$ | 0 | ... | 0 | $-\sigma'_O(W_{D_1}x + b_{D_1})/\sqrt{c}$ |
| | N | 0 | $-z_D\sigma'_O(W_{D_2}x + b_{D_2})/\sqrt{c}$ | ... | 0 | $-\sigma'_O(W_{D_2}x + b_{D_2})/\sqrt{c}$ |
| | ... | ... | ... | ... | ... | ... |
| | N | 0 | 0 | ... | $-z_D\sigma'_O(W_{D_O}x + b_{D_O})/\sqrt{c}$ | $-\sigma'_O(W_{D_O}x + b_{D_O})/\sqrt{c}$ |
| h_1 | W*N | 0 | 0 | ... | 0 | 0 |
| ... | ... | ... | ... | ... | ... | ... |
| h_D | W*N | 0 | 0 | ... | 0 | 0 |

Square Diagonal Matrixes

| $\nabla \mathcal{L}$ | | z_{i_1} | z_{i_2} | ... | z_{i_W} |
|----------------------|-----|---|---|-----|---|
| | | N | N | ... | N |
| F | O*N | 0 | 0 | ... | 0 |
| h_1 | W*N | 0 | 0 | ... | 0 |
| ... | ... | ... | ... | ... | ... |
| h_i | N | 1 | 0 | ... | 0 |
| | N | 0 | 1 | ... | 0 |
| | ... | ... | ... | ... | ... |
| | N | 0 | 0 | ... | 1 |
| h_{i+1} | N | $-W_{i_1,1}\sigma'(W_{i_1}z_i + b_{i_1})$ | $-W_{i_1,2}\sigma'(W_{i_1}z_i + b_{i_1})$ | ... | $-W_{i_1,W}\sigma'(W_{i_1}z_i + b_{i_1})$ |
| | N | $-W_{i_2,1}\sigma'(W_{i_2}z_i + b_{i_2})$ | $-W_{i_2,2}\sigma'(W_{i_2}z_i + b_{i_2})$ | ... | $-W_{i_2,W}\sigma'(W_{i_2}z_i + b_{i_2})$ |
| | ... | ... | ... | ... | ... |
| | N | $-W_{i_W,1}\sigma'(W_{i_W}z_i + b_{i_W})$ | $-W_{i_W,2}\sigma'(W_{i_W}z_i + b_{i_W})$ | ... | $-W_{i_W,W}\sigma'(W_{i_W}z_i + b_{i_W})$ |
| ... | ... | ... | ... | ... | ... |
| h_D | W*N | 0 | 0 | ... | 0 |

| $\nabla \mathcal{L}$ | | z_{D_1} | z_{D_2} | ... | z_{D_W} |
|----------------------|-----|---|---|-----|---|
| | | N | N | ... | N |
| F | N | $-W_{D_1,1}\sigma'_O(W_{D_1}z_D + b_{D_1})$ | $-W_{D_1,2}\sigma'_O(W_{D_1}z_D + b_{D_1})$ | ... | $-W_{D_1,W}\sigma'_O(W_{D_1}z_D + b_{D_1})$ |
| | N | $-W_{D_2,1}\sigma'_O(W_{D_2}z_D + b_{D_2})$ | $-W_{D_2,2}\sigma'_O(W_{D_2}z_D + b_{D_2})$ | ... | $-W_{D_2,W}\sigma'_O(W_{D_2}z_D + b_{D_2})$ |
| | ... | ... | ... | ... | ... |
| | N | $-W_{D_O,1}\sigma'_O(W_{D_O}z_D + b_{D_O})$ | $-W_{D_O,2}\sigma'_O(W_{D_O}z_D + b_{D_O})$ | ... | $-W_{D_O,W}\sigma'_O(W_{D_O}z_D + b_{D_O})$ |
| h_1 | W*N | 0 | 0 | ... | 0 |
| ... | ... | ... | ... | ... | ... |
| h_D | N | 1 | 0 | ... | 0 |
| | N | 0 | 1 | ... | 0 |
| | ... | ... | ... | ... | ... |
| | N | 0 | 0 | ... | 1 |