

# Summer School on real-world crypto and privacy

2023



## Practical Side-Channel Attacks on Real-World ECDSA Implementations

Ján Jančár  [j08ny@mail.muni.cz](mailto:j08ny@mail.muni.cz)

Łukasz Chmielewski  [chmiel@fi.muni.cz](mailto:chmiel@fi.muni.cz)

Centre for Research on Cryptography and Security, Masaryk University

**CRoCS**  
Centre for Research on  
Cryptography and Security

## About us

- CRoCS, Masaryk University
- Ján Jančár
  - PhD Candidate @ CRoCS
  - <https://neuromancer.sk/>
- Łukasz Chmielewski
  - Assistant Professor @ CRoCS
  - Interested in practical aspects of SCA and FI



Thank you to Milan Šorf for help!

# Main Goals

- Give you an impression of the attack surface on the Elliptic Curve Digital Signature (ECDSA) algorithm with respect to Side-Channel Analysis and Fault Injection (FI)
- Give you practical experience on nonce-related attacks on ECDSA.
- Describe the Minerva vulnerability and show how to exploit
  - Exercises!
- **Disclaimer:** there will be a lot of technical details so it OK not to grasp everything
  - This is NOT a tutorial on ECC, ChipWhisperer, generic SCA...

# Outline

- A (very) brief introduction to:
  - Side-Channel Analysis (SCA) and Fault Injection (FI)
  - Elliptic Curve Cryptography (ECC)
- ECDSA & (Selected) Attacks on ECDSA
- Practical Exercises
  - Did everyone install the tooling / VM?
- Countermeasures & Conclusions

# (VERY BRIEF) INTRO TO SIDE-CHANNEL ANALYSIS AND FAULT INJECTION

# Why Is Hardware Security Important?

## Card / Money Theft



## Identity Theft



## • Premium Content



## Phone / Money Theft



## Impersonation



# Recent Practical Attacks

November 13, 2019



May 28, 2020

LadderLeak: Side-channel security flaws exploited to break ECDSA cryptography



SCA Titan: January 7, 2021



October 3, 2019

Researchers Discover ECDSA Key Recovery Method

October 3, 2019 • Add Comment • by Emma Davis



December 12, 2019

PLUNDERVAULT — Intel's SGX coughs up crypto keys when scientists tweak CPU voltage

Install fixes when they become available. Until then, don't sweat it.

DAN GOODIN • 12/10/2019, 11:41 PM





# Introduction: Side Channel Analysis



# Side Channels

- Time
- Power
- Electro Magnetic Emanations
- Light
- Sound
- Temperature
- ...

# Passive vs Active Side Channels

**Passive:** analyze device behavior

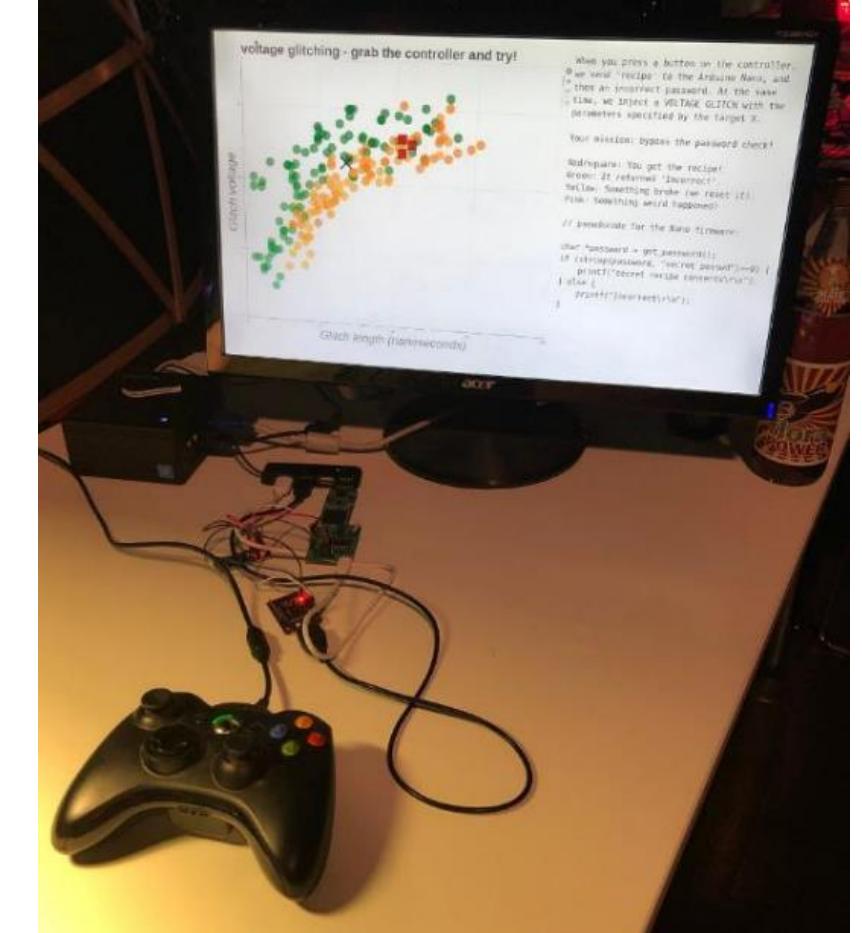


**Active:** change device behavior



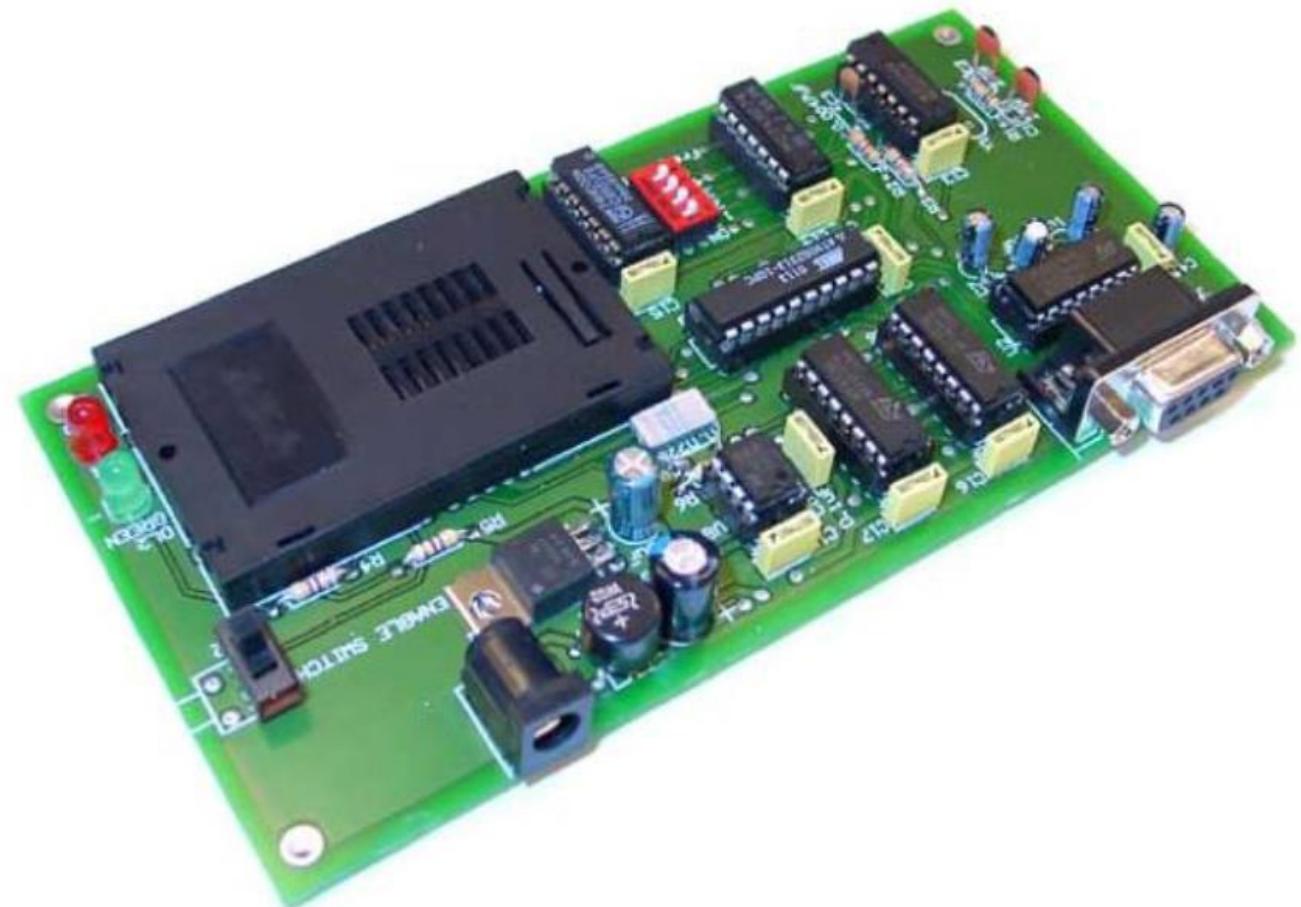
<https://escoptics.com/blogs/news/world-space-week-02-lasers>

# Some Practical Side-Channel Setups



# “Commercial” Example: the “unlooper” device

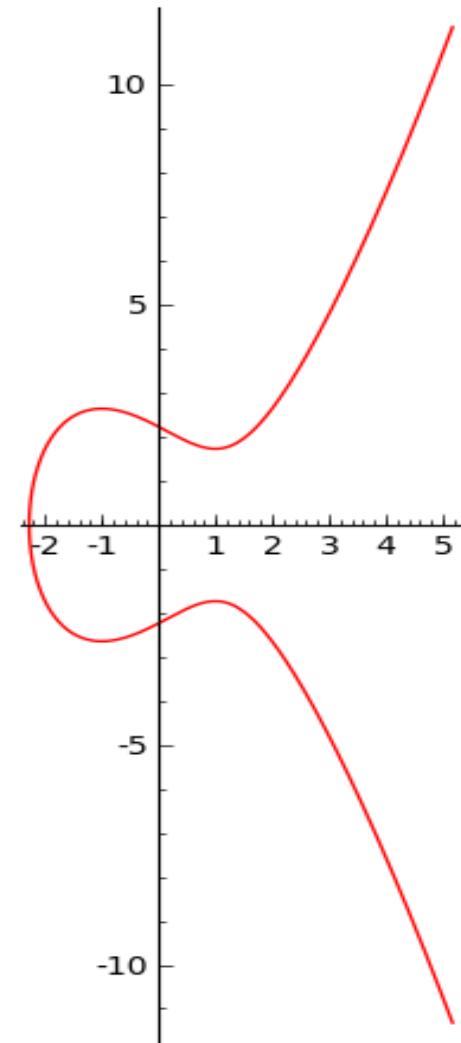
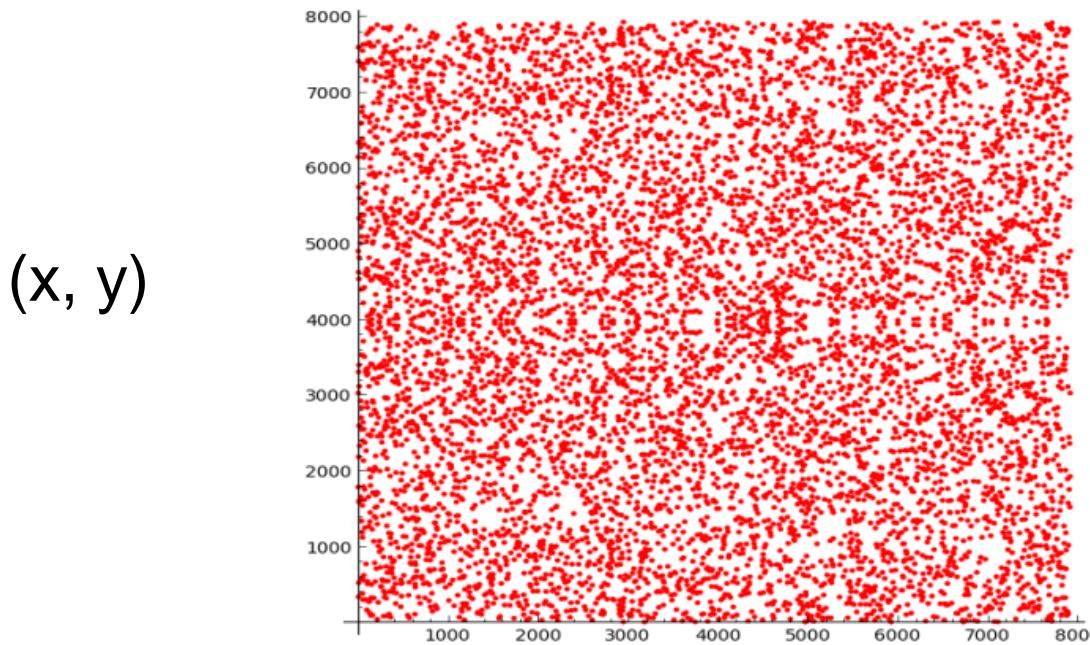
```
1 void entry() {
2     void* start = 0x80000000;
3     void* length = 0x00400000;
4
5     serial_puts("Start Secure Boot...\n");
6
7     loadOSFromHardDrive(start);
8
9     if (!authenticateOS(start,length) )
10        do {} while(1);
11
12    serial_puts("Run OS\n");
13
14    boot_next_stage(start);
15    //starts executing at the address start
16 }
```



# (VERY BRIEF) INTRO TO ELLIPTIC CURVE CRYPTOGRAPHY

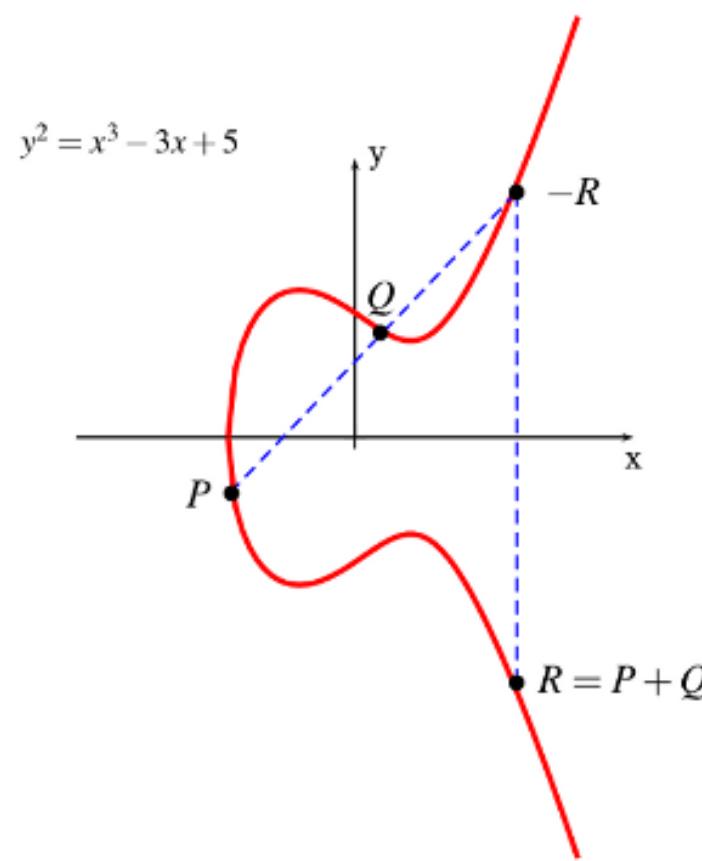
# Elliptic curves

- Example
  - $y^2 = x^3 - 3x^2 + 5$
- How would it look:  $\text{mod } 7919$ ?

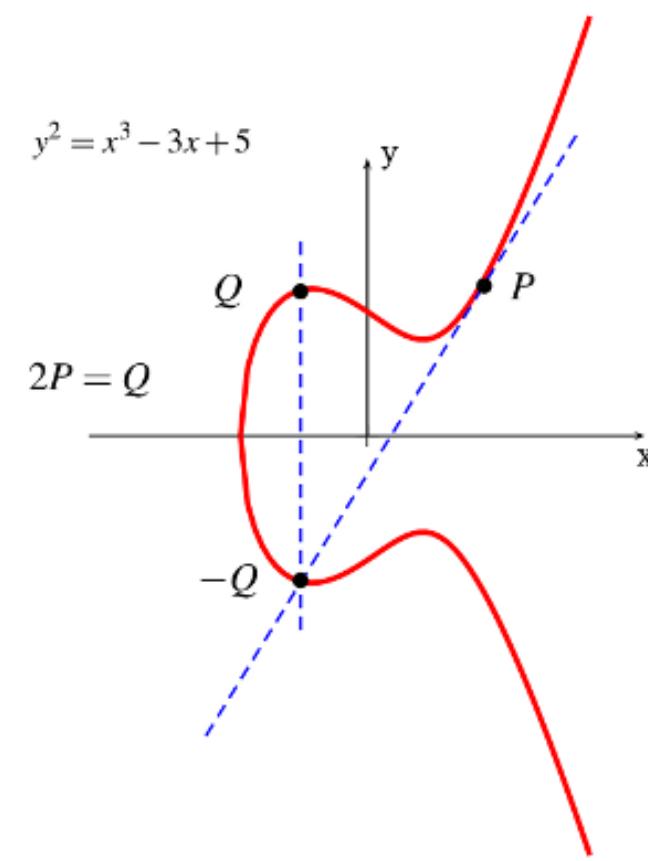


# Elliptic curve arithmetic - an abelian group

Addition



Doubling



# Scalar multiplication by $k$

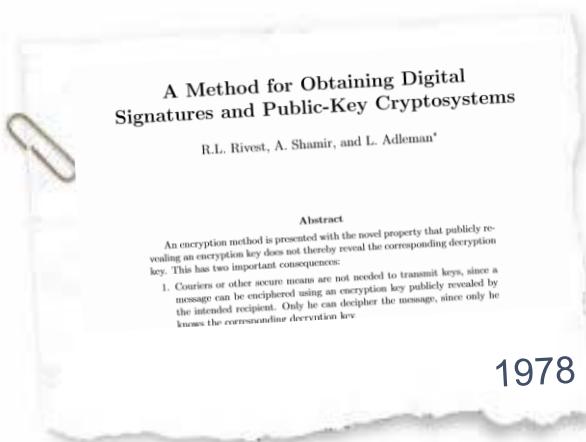
- Group = points on the curve + special point  $\infty$
- Add a point to itself  $k$  times:  $Q = [k]P$
- Analogous to modular exponentiation
  - square-and-multiply = double-and-add
- Many (equivalent) algorithms
  - LTR, RTL, Window, Comb, Ladders, ...

```
x=∞
for j=|k|-1 to 0 {
    x=DBL(x)
    if kj=1
        x=ADD(x,P)
return x0
```

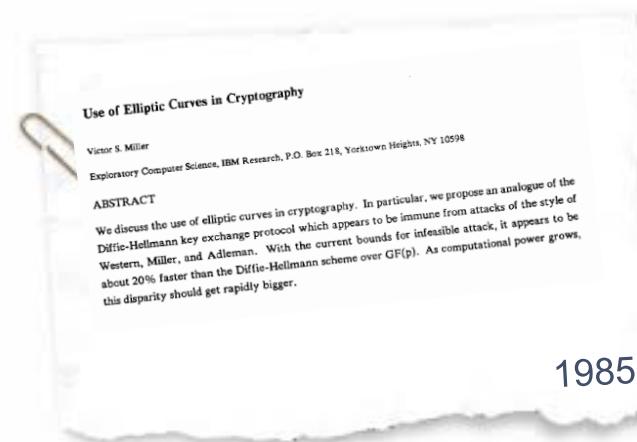
```
(0xb1011) = 11
x=∞
j=3
    x=DBL(∞)
    x=ADD(∞,P)
j=2
    x=DBL(P)
j=1
    x=DBL(2P)
    x=ADD(4P,P)
j=0
    x=DBL(5P)
    x=ADD(10P,P)=11P
```

# RSA ≈ ECC

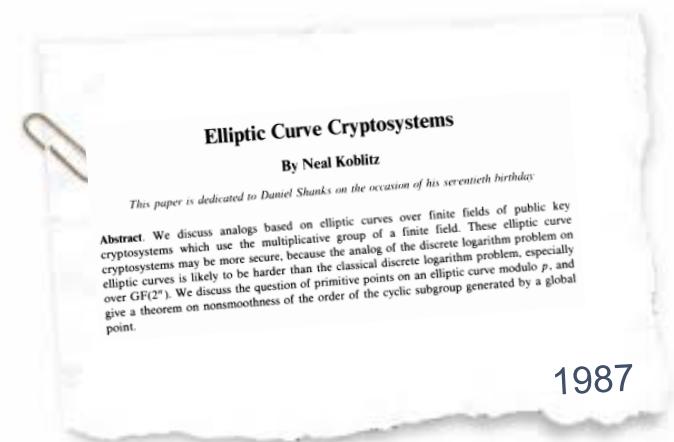
- RSA is based on modular exponentiation
- exponentiation ≈ scalar multiplication
  - $m = c^d \bmod n \approx Q = [k]P$
- multiplication ≈ points addition
- squaring ≈ point doubling



1978



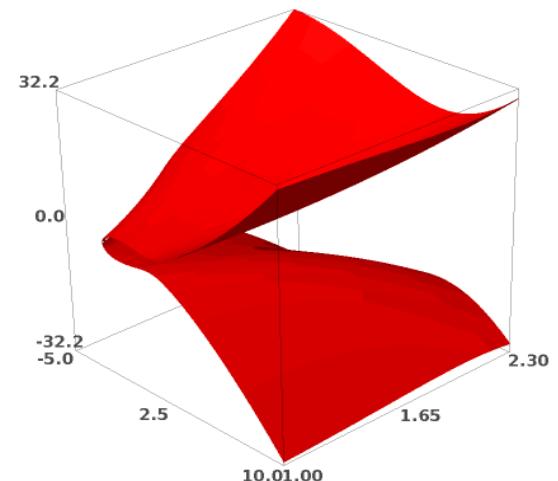
1985



1987

# Implementation Considerations

- Some curves are more common than others:
  - NIST curves (p256), Curve25519, or secp256k1
- Many security properties
  - Curves classification wrt. security: <http://safecurves.cr.yp.to/>
- Are there powerful fault attacks against ECDSA?
  - Invalid point attacks
  - Attacks against deterministic ECDSA
- The classical (x,y) representation is rarely used
  - For transport points compression is often used (note that x defines y)
  - For efficiency reasons it is better to represent a point in more dimensions (e.g., (x,y,z))



# What are the applications of ECC?

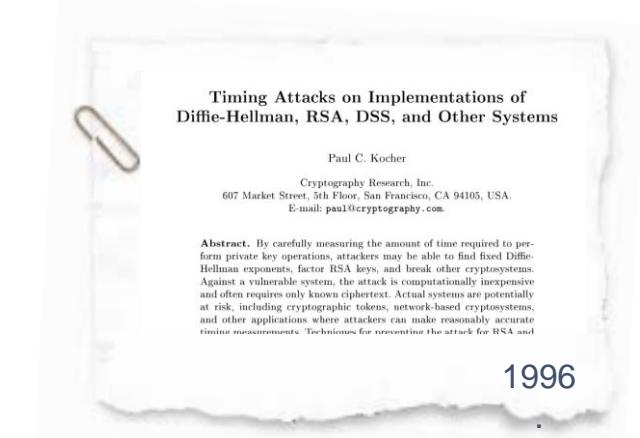
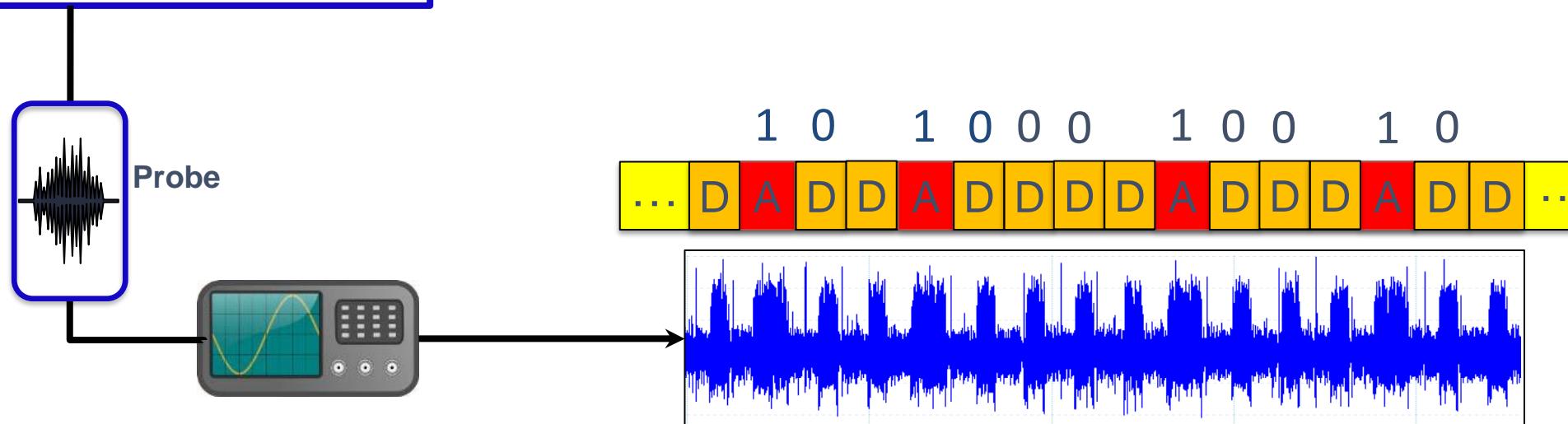
- Cryptocurrencies 
- Signatures: ECDSA and deterministic variants
- Key Exchange: ECDH
- Encryption: ECIES
- In this tutorial we concentrate on ECDSA

# “CLASSICAL” SCA ON SCALAR MULTIPLICATION

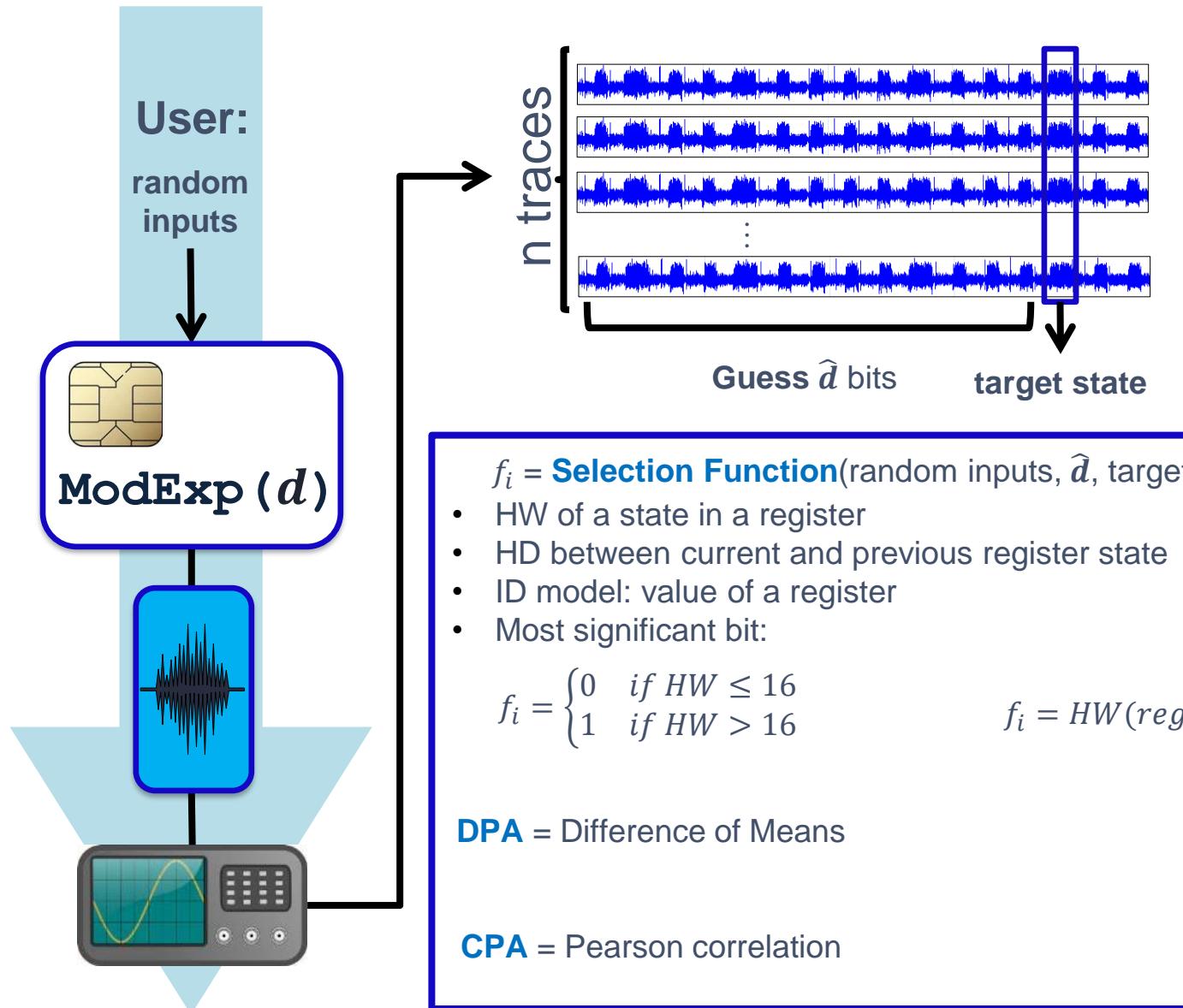
# Simple Power Analysis (SPA) on ECC (scalarmult)

```
ScalarMult(P) {
```

```
    A =  $\infty$ 
    for (i = n-1; i > 0; i)
        A = DOUBLE(A)
        if ( $k_i == 1$ )
            A = ADD(A,P)
        end if
    end for
    Return A = [k]P
}
```

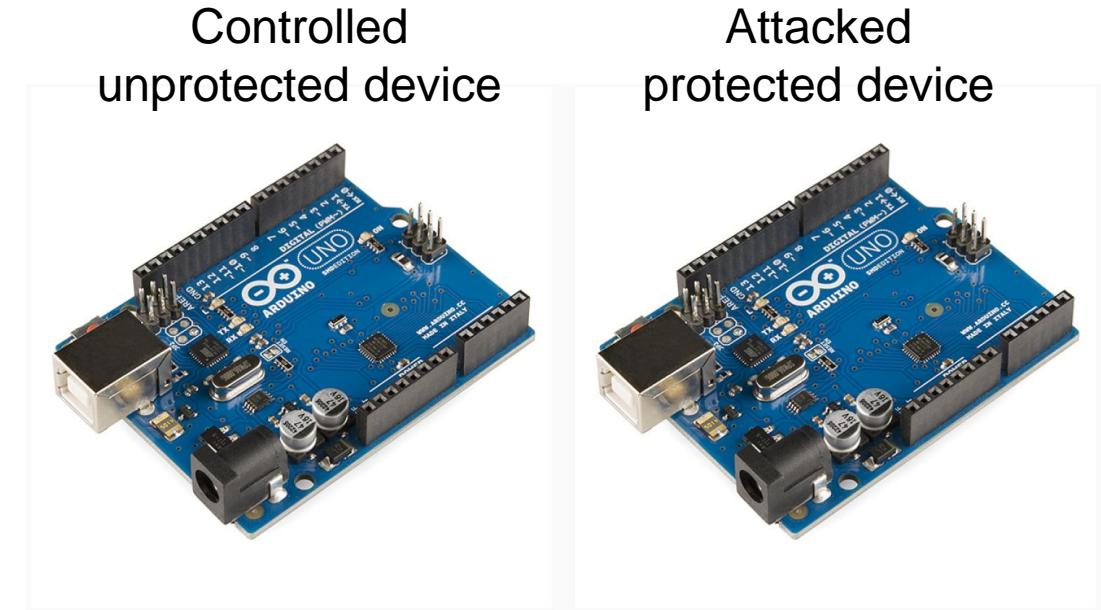


# Correlation Power Analysis on RSA/ECC



# Profiled Attacks

- Problems with the above approaches:
  - can we attack the key directly?
  - we often do not get many traces with the same secret
  - can we use an unprotected device of the same model?
- (Possible) Solution:
  - We profile, i.e. template the unprotected device
  - We use the profile to break the protected device
- Procedure:
  1. Choose a model that describes the power consumption
  2. Profile the unprotected device to create the template (Template Building)
  3. Use the template to break the protected device (Template Matching)
- The same steps are always performed but the model can be different.
  - So often we will not learn the secret but the hamming weight of the secret.
- Neural Networks can be used instead of Template Attacks



# ECDSA

# Introduction

- A variant of the Digital Signature Algorithm (DSA)
- Signature Algorithm based on ECC used, among others, for:
  - Cryptocurrencies,
  - Secure boot
- Problems:
  - Backdoors (?), technical issues (hard to implement), quantum computers
- Libraries:
  - Bouncy Castle, mbed TLS, Microsoft CryptoAPI, OpenSSL, wolfCrypt, and many more...
- We omit analysis of key generation:
  - Alice creates a key pair:
    - a private key: random integer  $x$  from  $[1, q-1]$ , where  $q$  is the curve order and
    - a public key curve point  $Q=[d]P$ , where  $P$  is a group generator (constant).

# ECDSA Algorithm

- Signature Generation (secret:  $\alpha$ , input:  $m$ ; output:  $r, s$ )
  1. A random number  $k$ ,  $1 \leq k \leq q-1$  is chosen
  2. Compute  $[k]P = (x_1, y_1)$  is computed.
  3. Next,  $r = x_1 \bmod q$  is computed.
  4. We then compute  $k^{-1} \bmod q$
  5.  $s = k^{-1}(H(m) + d^*r) \bmod n$
- Signature Verification (input:  $m, r, s$ ; output: True/False)
  1. Verify  $r$  and  $s$  are in  $[1, q-1]$  for the signature to be valid.
  2. Compute  $u_1 = H(m)^*s^{-1} \bmod q$  and  $u_2 = r*s^{-1} \bmod q$ .
  3. Compute  $(x_1, y_1) = [u_1]G + [u_2]Q$ .
  4. The signature is valid if  $r = x_1 \bmod q$ , invalid otherwise.

But what to attack?

Functional:  
Randomness needs to be good!!!

Lattice attacks on the nonces.  
More about that later.

# ECDSA Algorithm – Passive SCA

- Signature Generation (secret:  $x$ ; input:  $m$ ; output:  $r, s$ )
  1. A random number  $k, 1 \leq k \leq q-1$  is chosen
  2. Compute  $[k]P = (x_1, y_1)$  is computed.
  3. Next,  $r = x_1 \bmod q$  is computed.
  4. We then compute  $k^{-1} \bmod q$
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  3. Compute  $(x_1, y_1) = [u_1]G + [u_2]Q$ .
  4. The signature is valid if  $r = x_1 \bmod q$ , invalid otherwise.

+ Lattice attacks on the nonces.

# ECDSA Algorithm – Fault Injection

- Signature Generation (secret:  $\alpha$ , input:  $m$ ; output:  $r, s$ )
  1. A random number  $k$ ,  $1 \leq k \leq q-1$  is chosen
  2. Compute  $[k]P = (x_1, y_1)$  is computed.
  3. Next,  $r = x_1 \bmod q$  is computed.
  4. We then compute  $s = k^{-1}(H(m) + \alpha r) \bmod q$ .
- Signature Verification
  - 1. Verify  $r$  and  $s$  are valid
  - 2. Compute  $u_1 = H(m)$
  - 3. Compute  $(x_1, y_1)$
  - 4. The signature is valid if  $(x_1, y_1) = [s]P$

To avoid randomness issues,  
can ECDSA be done without  
randomness?

Yes, but it has its own risks...

+ Lattice  
attacks on  
the nonces.

# ECDSA AND NONCES MINERVA EXERCISES

# COUNTERMEASURES/MITIGATIONS

## SUMMARY

# Generic protection techniques

1. Do not leak
  - Constant-time crypto, bitslicing...
2. Shielding - preventing leakage outside
  - Acoustic shielding, noisy environment
3. Creating additional “noise”
  - Parallel software load, noisy power consumption circuits
4. Compensating for leakage
  - Perform inverse computation/storage
5. Prevent leaking exploitability
  - Ciphertext and key blinding, key regeneration, masking of the operations

# Specific protection techniques

$$\mathbf{M} = [s]\mathbf{P} = [s](X, Y) = [s](x, y, 1)$$

1.  $\mathbf{M} = [s](x, z, y, z, z)$  coordinate blinding

2.  $s_r = s + r * |E|$  scalar blinding

3.  $\mathbf{M}_r = [s_r](x, z, y, z, z)$  blinded scalar mult.

4. no unblinding

The sequence of operations (D, A) is related to the scalar bits.

However:

- If s is random: the sequence of scalar bits changes for every scalar multiplication execution
- If P is random: Intermediate data is random (masked) -> hardly predicted!

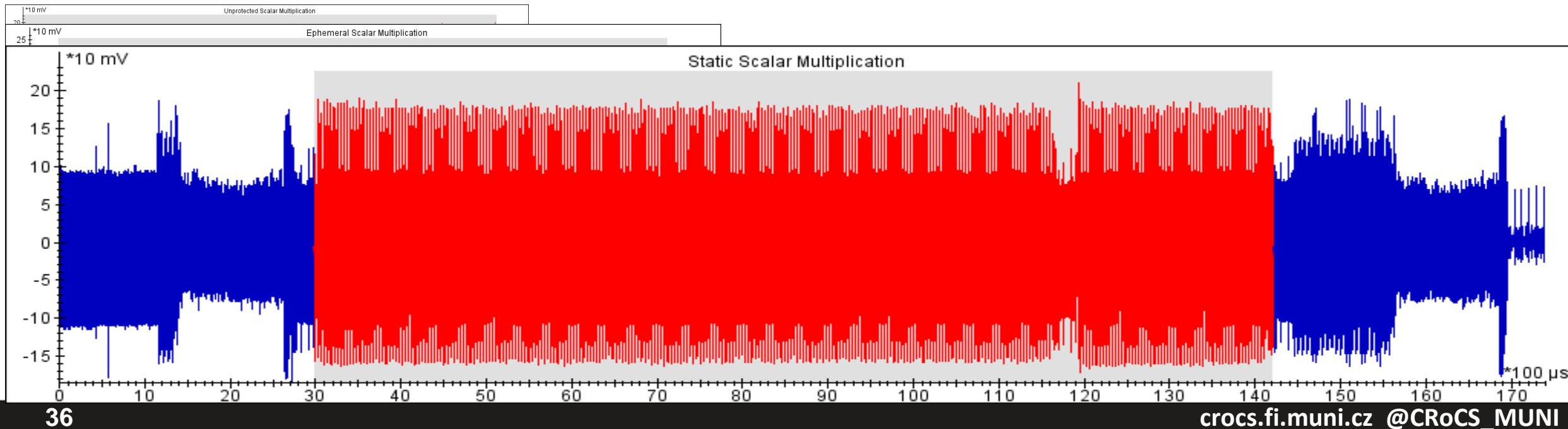
DPA is based on the prediction of intermediate data. We hope that: any side-channel attack requiring multiple traces is repelled by message and exponent blinding.

Point blinding is also possible to prevent zero-point attacks but not presented above.

There are many more similar countermeasures, e.g. scalar splitting.

# Scalar Multiplication Countermeasures

- No public (open source) protected library for ECDSA
  - There are partially protected libraries
- For ECDH situation is a bit better, for example:
  - [BCHSP2023] L. Batina, Ł. Chmielewski, B. Haase, N. Samwel, and P. Schwabe: SoK: SCA-secure ECC in software – mission impossible? CHES 2023



# Differences between ECDH and ECDSA scalar mult.

- Can secure ECDH  
Implementation be used for  
ECDSA scalar multiplication?  
What could be the problem?

```
379 int crypto_scalarmult_curve25519(uint8_t *r, const uint8_t *s,  
380                                     const uint8_t *p) {  
381     ST_curve25519ladderstepWorkingState state;  
382     uint8_t i;  
383     volatile uint8_t retval = -1;  
384     // Fault injection detection counter ### alg. step 1 ###  
385     volatile uint32_t fid_counter = 0;  
386  
387     // Initialize return value with random bits ### alg. step 2 ###  
388     randombytes(r, 32);  
389  
390     // Prepare the scalar within the working state buffer.  
391     for (i = 0; i < 32; i++) {  
392         state.s.as_uint8_t[i] = s[i];  
393     }  
394  
395     // Copy the affine x-coordinate of the base point to the state.  
396     fe25519_unpack(&state.x0, p);  
397  
398     fe25519_setone(&state.xq);  
399     fe25519_setzero(&state.zq);  
400     fe25519_cpy(&state.xp, &state.x0);  
401     fe25519_setone(&state.zp);  
402  
403     // Clamp scalar ### alg. step 3 ###  
404     state.s.as_uint8_t[31] &= 127;  
405     state.s.as_uint8_t[31] |= 64;  
406  
407     // ### alg. step 4 ###  
408     shiftRightOne(&state.s);  
409     shiftRightOne(&state.s);
```



# Additional ECDSA countermeasures

1. Blind modular multiplication  $x * r$  (in  $s = k^{-1} * (H(m) + x * r) \bmod q$ )
  - Multiplicative blinding
2. Protect Inversion
  - Use the Bernstein-Yang inversion constant-time inversion algorithm
  - Multiplicative blinding
3. Protect most significant bits:
  - Scalar randomization or deterministic way:
    - Conditionally adding the group order  $q$  or  $2q$  to the scalar  $k$  until the resulting scalar has a fixed length.
4. Point blinding
5. Still blind the nonce!
6. Add generic protections against FI:
  - Redundancy, control flow protections ...

And much more...

# CONCLUSIONS

# Conclusions

- ECDSA is vulnerable to attacks on nonces, in particular lattice attacks.
  - Minerva was discovered on certified products.
- Protecting ECDSA against SCA and FI is hard
- Good randomness is crucial! Protect the nonces!

Questions 

MUNI  
FI

CR<sup>CS</sup>

Centre for Research on  
Cryptography and Security

June 9, 2023

# Practical part: ECDSA and nonces

Jan Jancar  
Łukasz Chmielewski

# ECDSA nonce $k$

- Needs to be unpredictable & unknown to an attacker
- ? What happens if it is not?
  - Nonce (full) leak
  - Nonce reuse
  - Nonce bias
  - Nonce (partial) leak

$$k \xleftarrow{\$} \mathbb{Z}_n^*$$

# ECDSA nonce $k$

- Needs to be unpredictable & unknown to an attacker
- ? What happens if it is not?
  - Nonce (full) leak
  - Nonce reuse
  - Nonce bias
  - Nonce (partial) leak
- ! Answer: Private key recovery
  - Number of signatures required  $2 \dots 2^{32}$

$$k \xleftarrow{\$} \mathbb{Z}_n^*$$

# Nonce leak

- ( $r, s$ ) is the signature,  $m$  is the message
- Knowing  $k$ , we can compute  $x$
- Why would this happen?

1 signature

$$r \equiv ([k]G)_x \pmod{n}$$

$$s \equiv k^{-1}(H(m) + rx) \pmod{n}$$

$$x \equiv (ks - H(m))r^{-1}$$

# Nonce leak

- $(r, s)$  is the signature,  $m$  is the message
- Knowing  $k$ , we can compute  $x$
- Why would this happen?
  - Clueless developer?

1 signature

$$r \equiv ([k]G)_x \mod n$$

$$s \equiv k^{-1}(H(m) + rx) \mod n$$

$$x \equiv (ks - H(m))r^{-1}$$

# Nonce reuse

- Subtract  $s_1 - s_2$ , obtain  $k$
- From there get  $x$  as before
- Why would this happen?

2 signatures

$$r \equiv ([k]G)_x$$

$$s_1 \equiv k^{-1}(H(m_1) + rx)$$

$$s_2 \equiv k^{-1}(H(m_2) + rx)$$

$$s_1 - s_2 \equiv k^{-1}(H(m_1) - H(m_2))$$

$$k \equiv \frac{H(m_1) - H(m_2)}{s_1 - s_2}$$

# Nonce reuse

- Subtract  $s_1 - s_2$ , obtain  $k$
- From there get  $x$  as before
- Why would this happen?
  - Clueless developer?
  - Static nonce? (PlayStation 3) [1]
  - Bad RNG?

2 signatures

$$r \equiv ([k]G)_x$$

$$s_1 \equiv k^{-1}(H(m_1) + rx)$$

$$s_2 \equiv k^{-1}(H(m_2) + rx)$$

$$s_1 - s_2 \equiv k^{-1}(H(m_1) - H(m_2))$$

$$k \equiv \frac{H(m_1) - H(m_2)}{s_1 - s_2}$$

# Nonce bias $\approx$ partial leak

- Some bias is like an information leak  $2 \dots 2^{32}$  signatures
- 2 attacks
  - Lattice reduction (HNP)
  - “Bleichenbacher” $k < n/2^l$
- Why would this happen?  $k \equiv 5 \pmod{8}$

# Nonce bias $\approx$ partial leak

- Some bias is like an information leak  $2 \dots 2^{32}$  signatures
- 2 attacks
  - Lattice reduction (HNP)
  - “Bleichenbacher”
- Why would this happen?
  - Bad RNG? [2]
  - Bad constant-timeness fix?
  - Side-channel leak?

$$k < n/2^l$$

$$k \equiv 5 \pmod{8}$$

# Nonce bias $\approx$ partial leak

## Lattice reduction (HNP)

- Hidden Number Problem
- $\geq 2$  bits of info
- Small number of signatures
- Transforms into CVP or SVP
- [3,4,5]

## Bleichenbacher

- Bias amplification + search
- Even 1 bit of info
- Large number of signatures
- [6]

# Minerva

Leaky loops

Ingredients: Athena iOPmect,  
Elioglypt, Buric, wolfSSL,  
MatsuSSL. May contain traces  
of bees.



Ján Janičák  
Vladimir Sedláček  
Petr Švenda  
Marek Sýs



9 780201 379624  
**CROCS**

Centre for Research in  
Cryptography and Security

Leaky loops



## Discovery: ECDSA testing

- ASN.1 parsing 

## Discovery: ECDSA testing

- ASN.1 parsing 
- Signature malleability 

## Discovery: ECDSA testing

- ASN.1 parsing 
- Signature malleability 
- Test-vectors 

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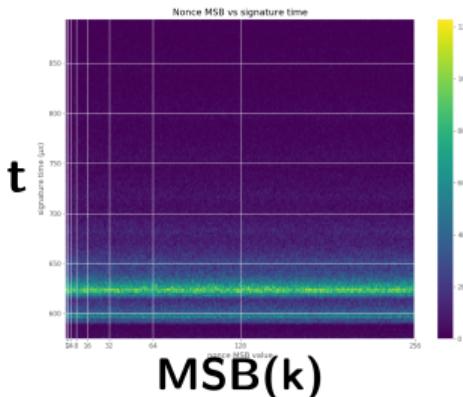
Let's test timing as well!

# Minerva

## Discovery: ECDSA testing

- ASN.1 parsing ✓
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- Test-vectors ✗
- Nonce randomness ✓

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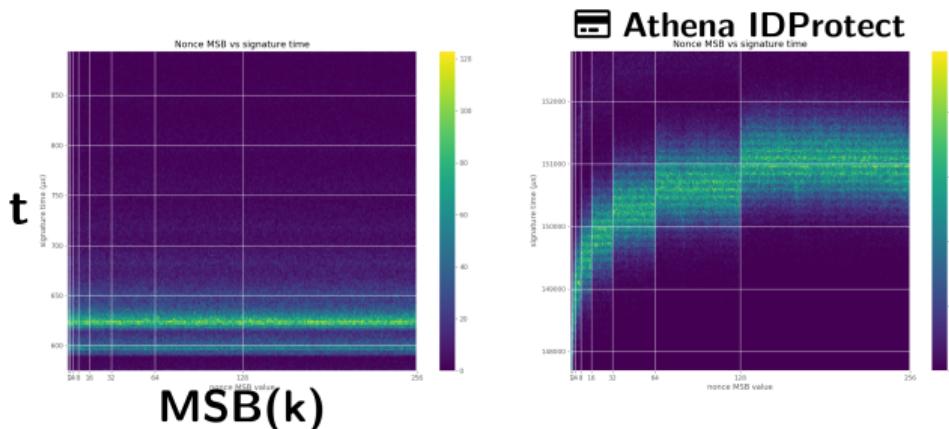


# Minerva

## Discovery: ECDSA testing

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- Test-vectors ~
- Nonce randomness ✓

Let's test timing as well!

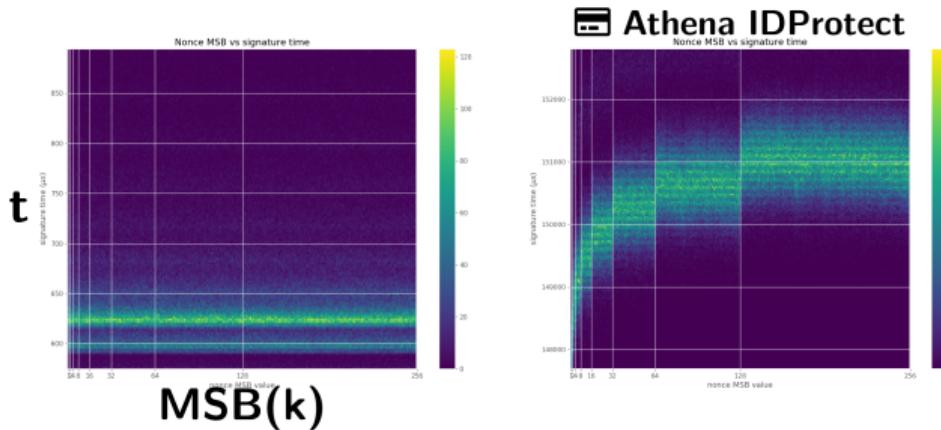


# Minerva

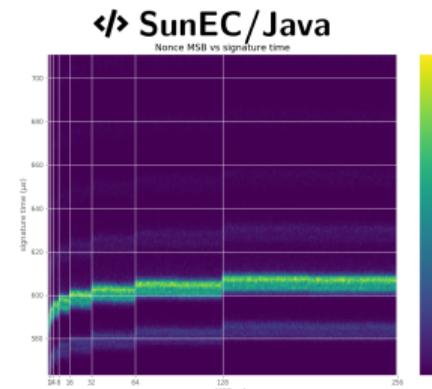
## Discovery: ECDSA testing

- ASN.1 parsing ✓
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- Test-vectors ~
- Nonce randomness ✓

Let's test timing as well!



Athena IDProtect



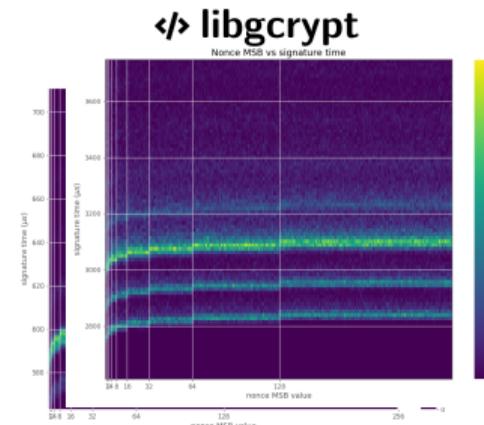
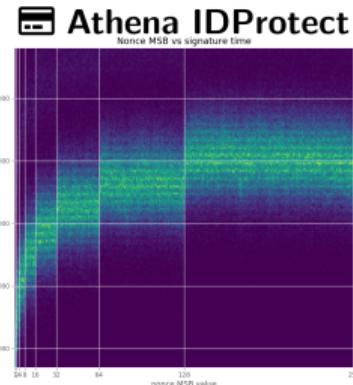
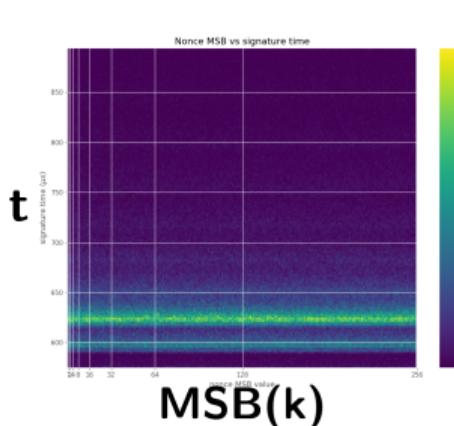
Practical part: ECDSA and nonces

# Minerva

## Discovery: ECDSA testing

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Let's test timing as well!

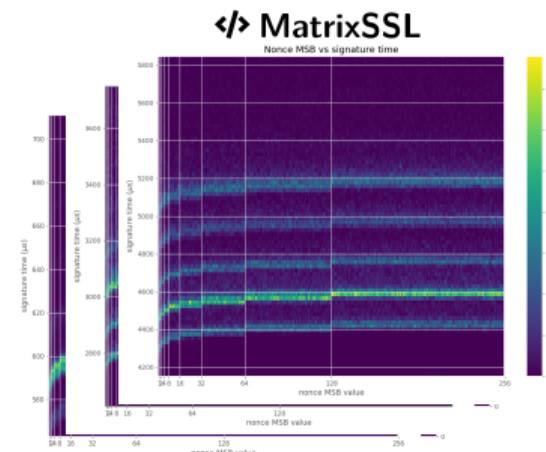
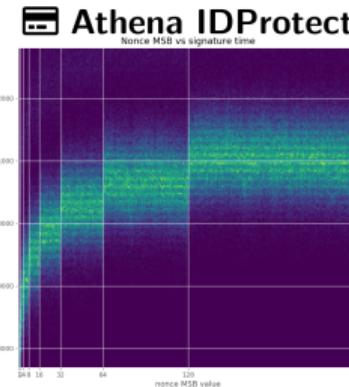
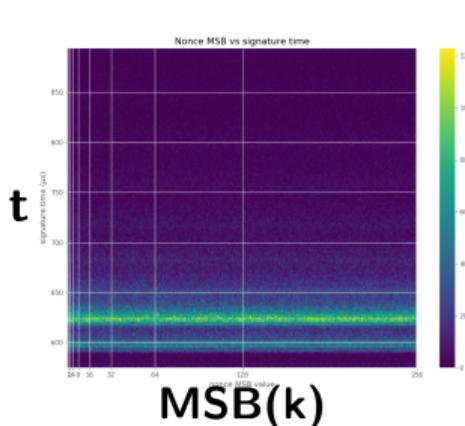


# Minerva

## Discovery: ECDSA testing

- ASN.1 parsing ✓
- Signature malleability ✓
- Test-vectors ~
- Nonce randomness ✓

Let's test timing as well!

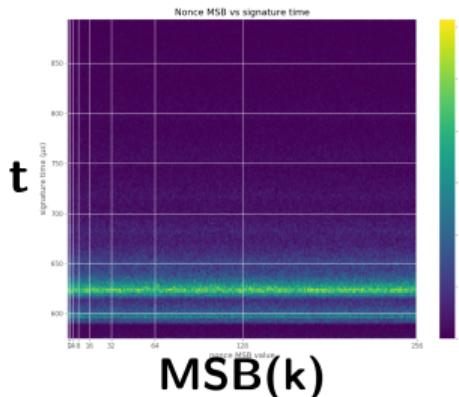


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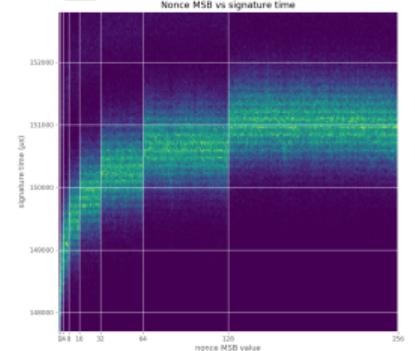
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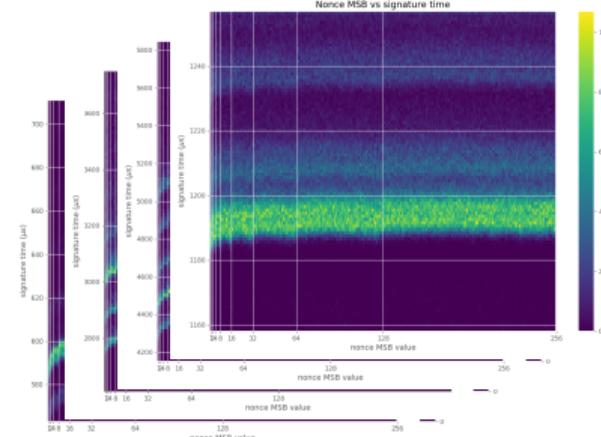
Let's test timing as well!



Athena IDProtect



WolfSSL

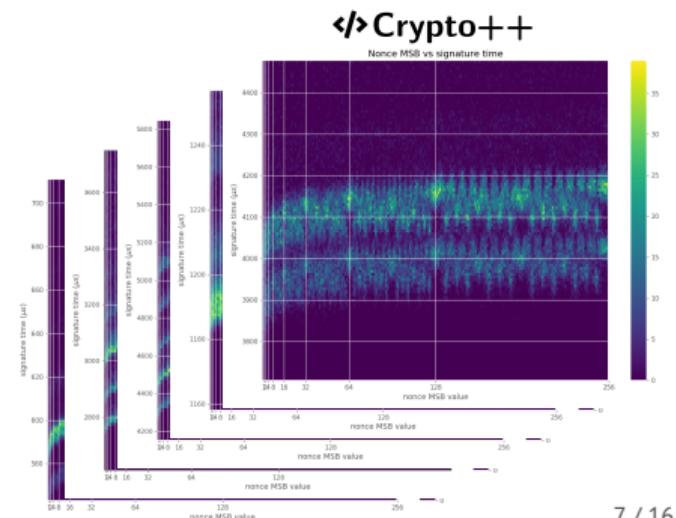
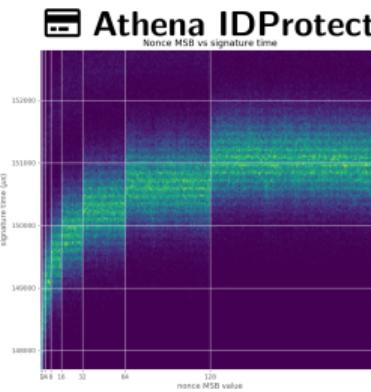
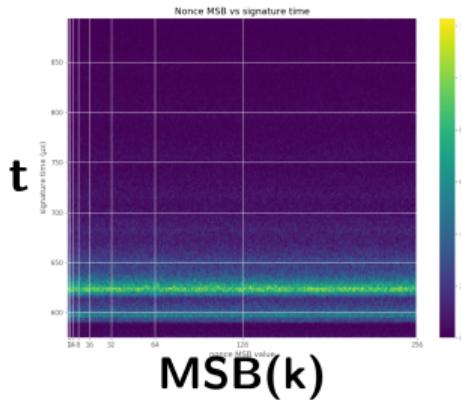


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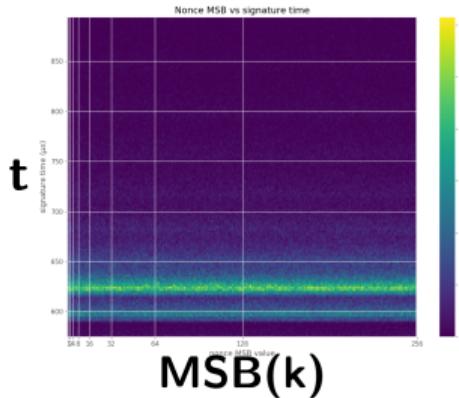
Let's test timing as well!



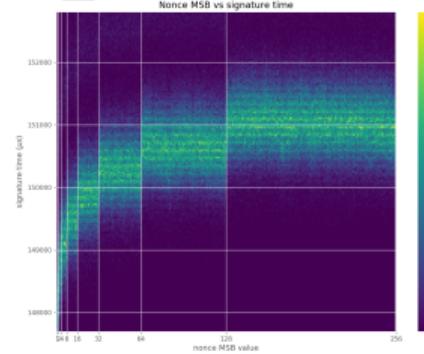
# Minerva

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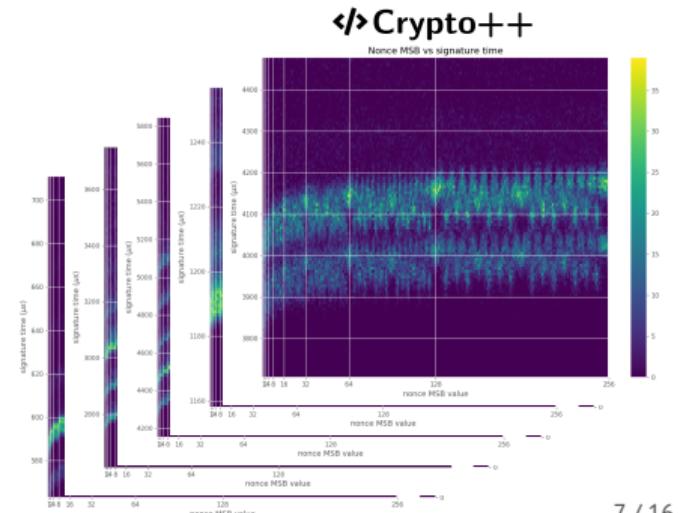
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Practical part: ECDSA and nonces



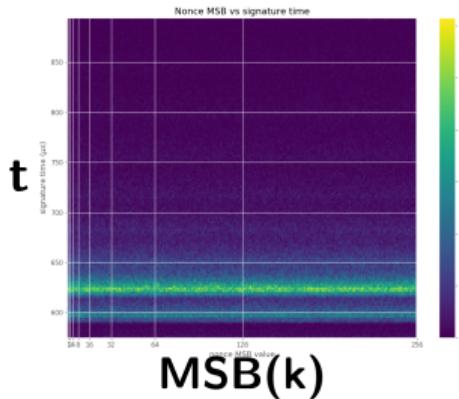
# Minerva

## Discovery: ECDSA testing

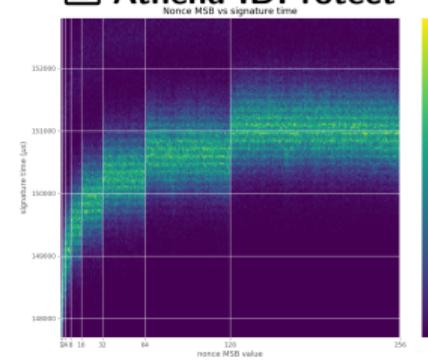
- ASN.1 parsing ✓
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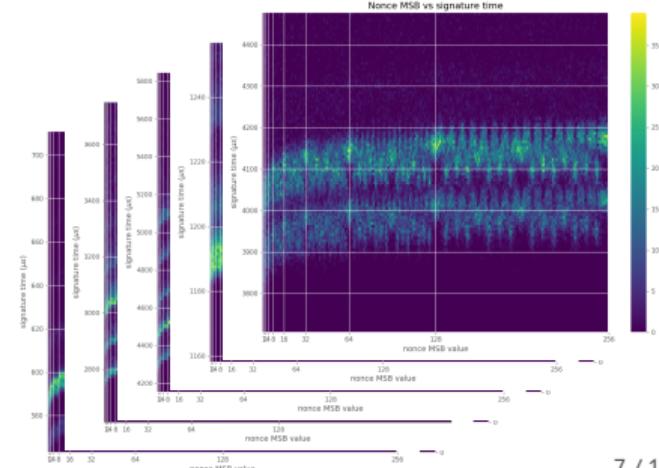
1  
5  
7



■ Athena IDProtect



</> Crypto++



# Minerva

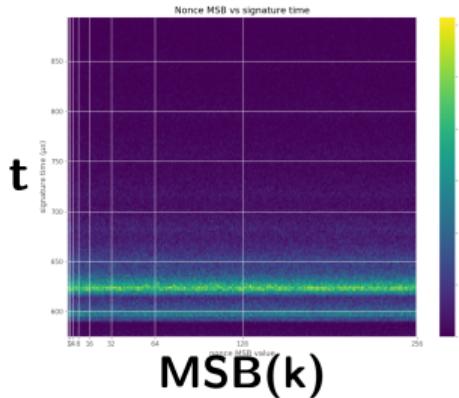
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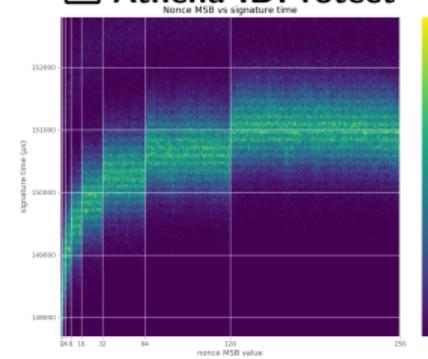


1 ✗

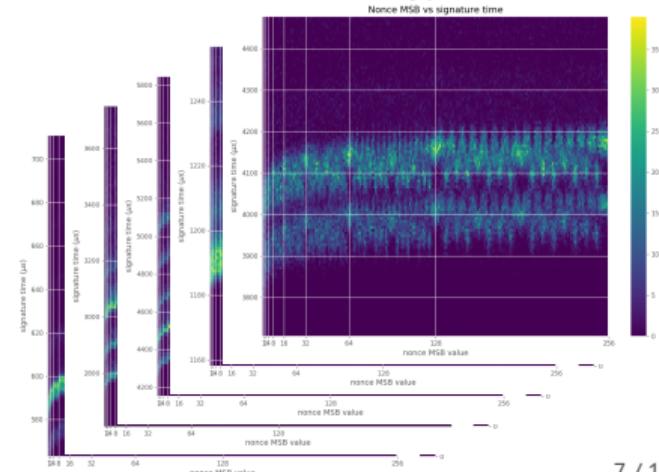
2



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Crypto++



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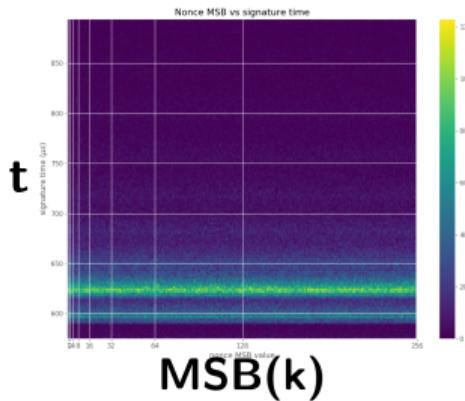
1 ✗

Déjà Vu

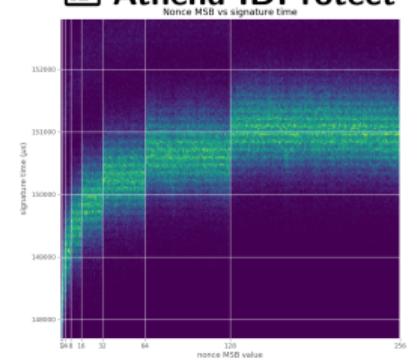
1

2

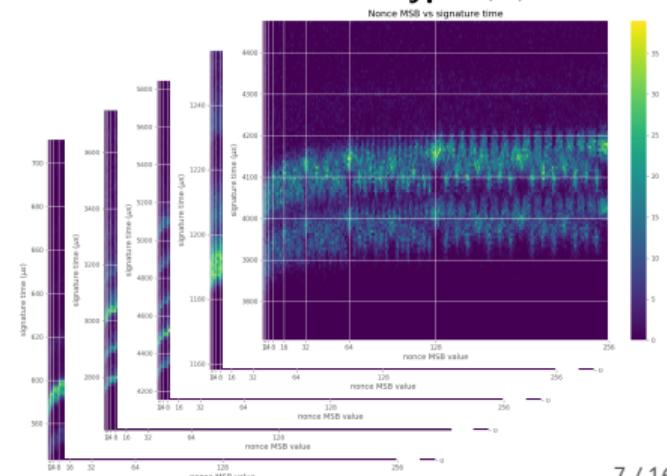
...



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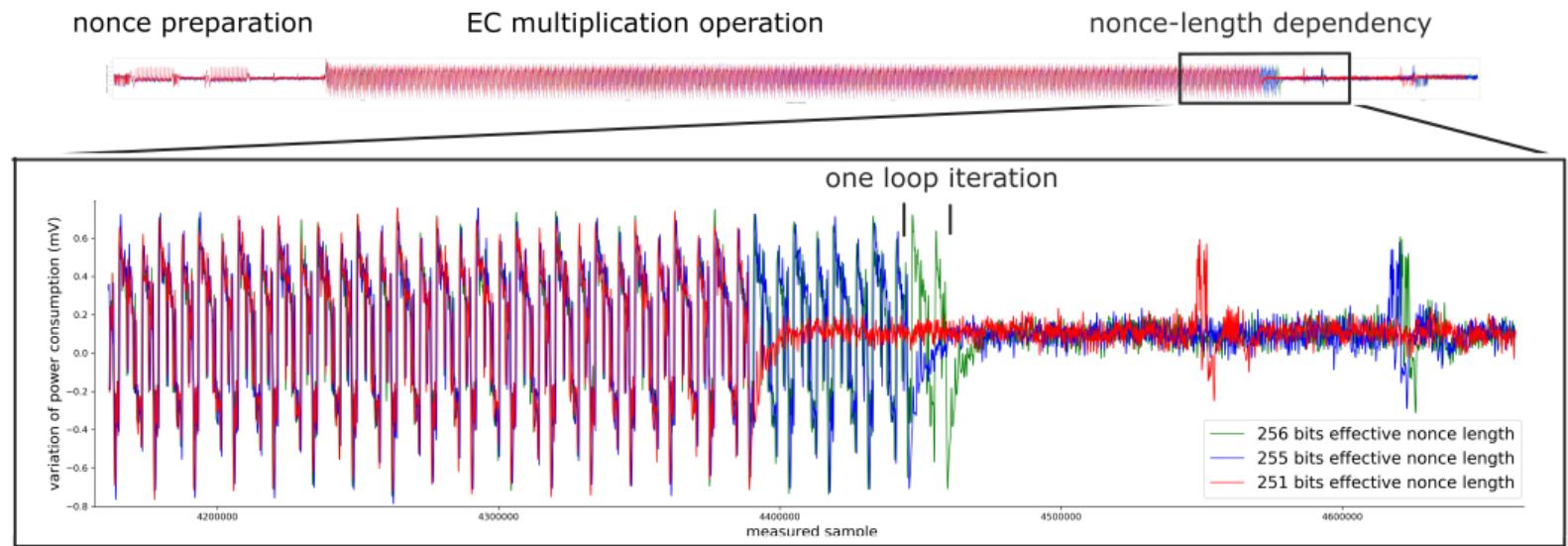
Crypto++



# Discovery

## Leak

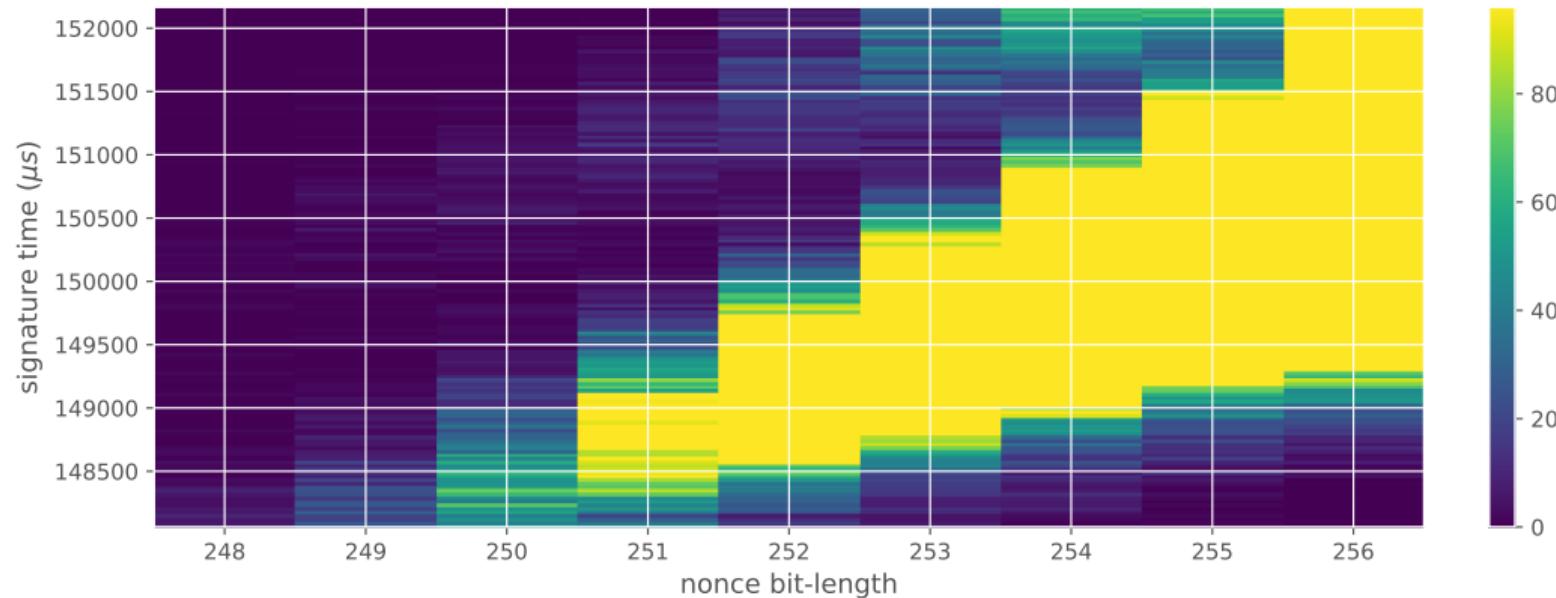
[k]G



# Discovery

## Leak

[ $k$ ]G



# Exploitation

## Hidden Number Problem

- Average 1 LZB per signature
- There is noise

# Exploitation

## Hidden Number Problem

- Average 1 L2B per signature
- There is noise

### Hardness of Computing the Most Significant Bits of Secret Keys in Diffie-Hellman and Related Schemes

(Extended Abstract)

[7]

DAN BONEH<sup>1</sup>  
Princeton University

RAMARATHNAM VENKATESAN<sup>2</sup>  
Bellcore

# Exploitation

## Hidden Number Problem

- Average 1 LZB per signature
- There is noise

### Hidden Number Problem (HNP) [1]

Given an oracle computing:

$$\mathcal{O}_{b,t}() = \text{MSB}_l(at + b \bmod n)$$

with  $t$  u.i.d. in  $\mathbb{Z}_n^*$ , find  $a$ .

---

# Exploitation

## Hidden Number Problem

- Average 1 LZB per signature
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### Hidden Number Problem (HNP) [1]

Given an oracle computing:

$$\mathcal{O}_{r,s}() = \text{MSB}_l(k \bmod n)$$

# Exploitation

## Hidden Number Problem

- Average 1 LZB per signature
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### Hidden Number Problem (HNP) [1]

Given an oracle computing:

$$\mathcal{O}_{r,s}() = \text{MSB}_l(xs^{-1}r + H(m)s^{-1} \bmod n)$$

find  $x$ .

---

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## Basic attack [8]

- Collect  $N$  signatures, take  $d$  of the fastest

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- Construct a lattice with basis **B** and reduce it:

$$\mathbf{B} = \begin{pmatrix} 2^{l_1}n & 0 & 0 & \dots & 0 & 0 \\ 0 & 2^{l_2}n & 0 & \dots & 0 & 0 \\ \vdots & & & & \vdots & \\ 0 & 0 & 0 & \dots & 2^{l_d}n & 0 \\ 2^{l_1}t_1 & 2^{l_2}t_2 & 2^{l_3}t_3 & \dots & 2^{l_d}t_d & 1 \end{pmatrix}$$

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- Construct a target  $\mathbf{u} = (2^{l_1}u_1, \dots, 2^{l_d}u_d, 0)$
- Solve  $\text{CVP}(\mathbf{B}, \mathbf{u})$ . The closest lattice point is often:  $\mathbf{v} = (2^{l_1}t_1x, \dots, 2^{l_d}t_dx, x)$

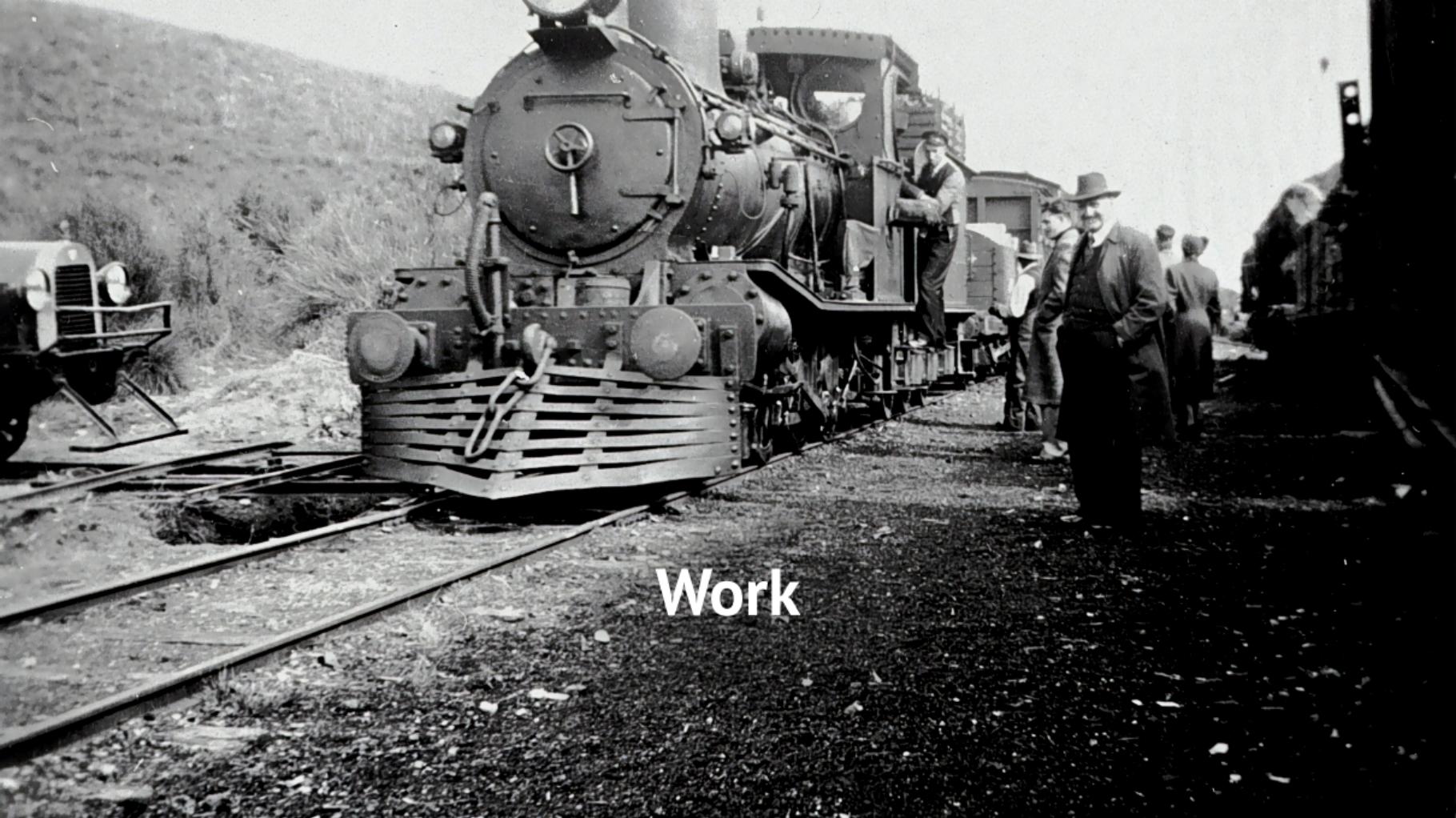
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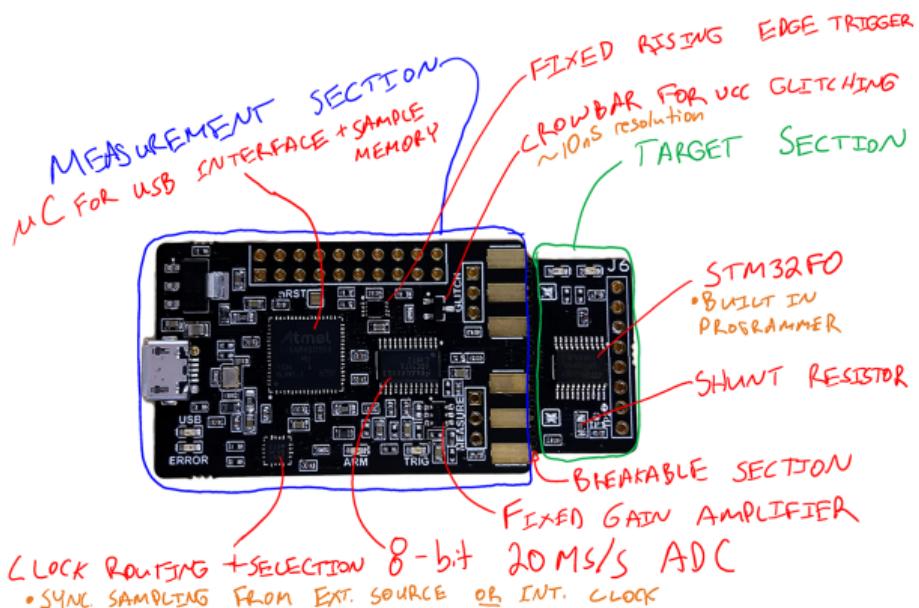
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- Because  $\forall i: (xt_i - u_i) \bmod n$  is small



Work

# Practical part

You will mount several side-channel attacks on a modified micro-ecc ECDSA implementation running on a ChipWhisperer-Nano board.



**pyecsca**

# Practical part

## Prerequisites

Clone the repository <https://github.com/J08nY/cwnano-micro-ecc>  
Have setup ready (see [installation.md](#))

- 1 Building the target implementation
- 2 Interacting with the target
- 3 Running the nonce-reuse attack
- 4 Running the nonce-bitlength-leak attack
  - 1 With timing
  - 2 With powertraces

# Practical part

- Work in groups of 2-3, only 19 devices
- Potentially share the devices
- Instructions in [tutorial.md](#)
- Consult [troubleshooting.md](#) in case of issues

# Practical part

## Hopeful fix

```
pip install -U ipython pyzmq  
And reboot VM
```

# Thanks!

 J08nY |  neuromancer.sk |  jan@neuromancer.sk  
[crocs.fi.muni.cz](http://crocs.fi.muni.cz)

Icons from  Noun Project &  Font Awesome

Photos from  Unsplash

## References

- 1 failOverflow; **Console Hacking 2010: PS3 Epic Fail**
- 2 Joachim Breitner, Nadia Heninger; **Biased Nonce Sense: Lattice Attacks against Weak ECDSA Signatures in Cryptocurrencies**
- 3 Jan Jancar, Vladimir Sedlacek, Petr Svenda, Marek Sys; **Minerva: The curse of ECDSA nonces (Systematic analysis of lattice attacks on noisy leakage of bit-length of ECDSA nonces)**
- 4 Martin R. Albrecht, Nadia Heninger; **On Bounded Distance Decoding with Predicate: Breaking the "Lattice Barrier" for the Hidden Number Problem**
- 5 Chao Sun, Thomas Espitau, Mehdi Tibouchi, Masayuki Abe; **Guessing Bits: Improved Lattice Attacks on (EC)DSA**
- 6 Daniel Bleichenbacher; **On the generation of one-time keys in DL signature schemes**
- 7 Dan Boneh, Ramarathnam Venkatesan; **Hardness of Computing the Most Significant Bits of Secret Keys in Diffie-Hellman and Related Schemes**
- 8 Billy Bob Brumley, Nicola Tuveri; **Remote Timing Attacks are Still Practical**