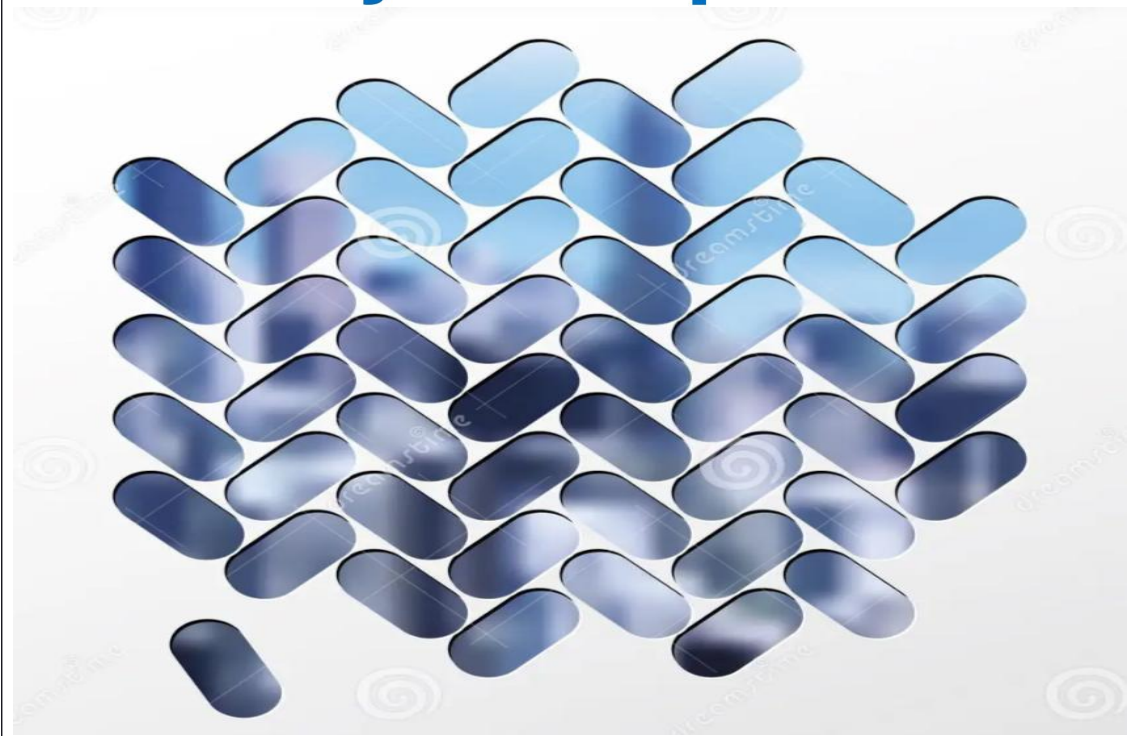


# **Inferential Statistics Project Report**



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# **1.Introduction:Inferential statistics**

## **1.1. What is inferential statistics?**

- A method of making decision about the parameters of population based on sample.
- Also means making probability judgement concerning population based on samples.
- Inferential statistics includes hypothesis, testing, confidence intervals and regression analysis among other techniques.
- These methods help researchers determine whether their findings are statistically significant and whether they can generalize their results to the larger population.
- Important technique under inferential statistics is Hypothesis Testing.

### **Hypothesis Testing:**

- It is a fundamental technique in inferential statistics. It involves testing a hypothesis about a population parameter, such as a mean ,proportion variance,frequency using sample data.
- The process typically involves setting up null and alternative hypothesis and conducting a statistical test to determine whether there is an enough evidence to reject the null hypothesis in favor of the alternative hypothesis.

## **1.2.Role of inferential statistics in business decision making?**

### **Informed Decision Making:**

In business, decisions based on statistically significant findings are more likely to lead to successful outcomes. For example, if we are testing a new marketing strategy, statistical significance can help us to determine whether an observed increase in sales is due to the new strategy or just random chance. This ensures that the decisions are grounded in reliable data.

### **Risk Management:**

Understanding statistical significance also helps in managing risks. By distinguishing between real effects and random noise, businesses can avoid costly mistakes. For instance, launching a new product based on insignificant data might lead to poor market performance. Conversely, statistically significant results can provide the confidence needed to proceed with strategic initiatives.

### **Resource Allocation:**

Businesses often operate with limited resources. Statistical significance helps prioritize initiatives by identifying those most likely to succeed. This means that resources—time, money, and manpower—can be allocated more efficiently, maximizing return on investment.

### **Market Research:**

In market research, statistical significance is used to validate findings from surveys and experiments. For example, a company might survey customers to understand their preferences and satisfaction levels.



Statistical analysis can then identify which findings are significant, guiding product development and marketing strategies.

#### Quality Control:

In manufacturing, maintaining high quality is essential. Statistical significance plays a role in quality control by analyzing production data to detect defects or variations. This ensures that products meet quality standards and reduces the risk of defects reaching the customer.

#### Moving Forward with Confidence:

Embracing the concept of statistical significance allows businesses to move forward with confidence. It transforms data into actionable insights, supporting strategic decision-making and fostering a culture of data-driven thinking. In a competitive landscape, the ability to discern meaningful patterns from data can be the difference between success and failure.



## 2.Problem 1 : Mens Football Team

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

|                           | Striker   | Forward   | Attacking<br>Midfielder | Winger    | Total      |
|---------------------------|-----------|-----------|-------------------------|-----------|------------|
| Players<br>Injured        | 45        | 56        | 24                      | 20        | <b>145</b> |
| Players<br>Not<br>Injured | 32        | 38        | 11                      | 9         | <b>90</b>  |
| <b>Total</b>              | <b>77</b> | <b>94</b> | <b>35</b>               | <b>29</b> | <b>235</b> |

### 2.1. What is the probability that a randomly chosen player would suffer an injury?

- Total players injured = 145
- Total number of players = 235
- Probability of a randomly chosen player would suffer an injury = Total players injured/ Total number of players =  $145/235$

Probability of a randomly chosen player would suffer an injury is **0.617(61.7%)**.

### 2.2. What is the probability that a player is a forward or a winger?

- Player is a forward or a winger = Forward+Winger=123
- Total number of players = 235

- Probability that a player is a forward or a winger =  $\frac{\text{Player is a forward or a winger}}{\text{Total number of players}} = \frac{123}{235}$

Probability that a palyer is a forward or a winger is **0.5234(52.34%)**.

### **2.3. What is the probability that a randomly chosen player plays in a striker position and has a foot injury?**

- Player plays in a striker position and has a foot injury = 45
- Total number of players = 235
- Probability that a randomly chosen player plays in a striker position and has a foot injury =  $\frac{\text{Player plays in a striker position and has a foot injury}}{\text{Total number of players}} = \frac{45}{235}$

Probability that a randomly chosen player plays in a striker position and has a foot injury is **0.1915(19.15%)**

### **2.4. What is the probability that a randomly chosen injured player is a striker?**

- Total players injured = 145
- Total number of strikers = 45
- Probability that a randomly chosen injured player is a striker =  $\frac{\text{Total number of strikers}}{\text{Total players injured}} = \frac{45}{145}$

Probability that a randomly chosen injured player is a striker is **0.3103(31.03%)**.

### 3.Problem 2 : Gunny Bags Strength

The breaking strength of gunny bags used for packing cement is normally distributed with a mean of 5 Kg per sq.centimeter and a standard deviation of 1.5 Kg per sq.centimeter. The quality team of the cement company wants to know the packing material to better understand wastage or pilferage within the supply chain.

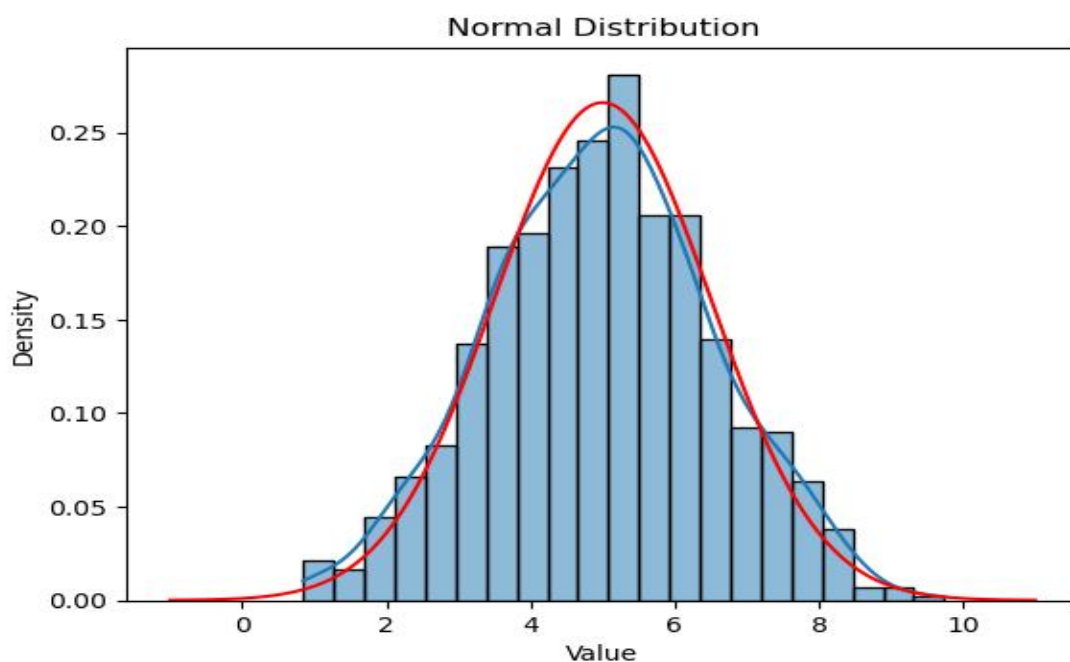
From the problem statement:

**$\mu=5$**

**$\sigma=1.5$**

#### 3.1.Plotting the distribution and importing neccassary libraries:

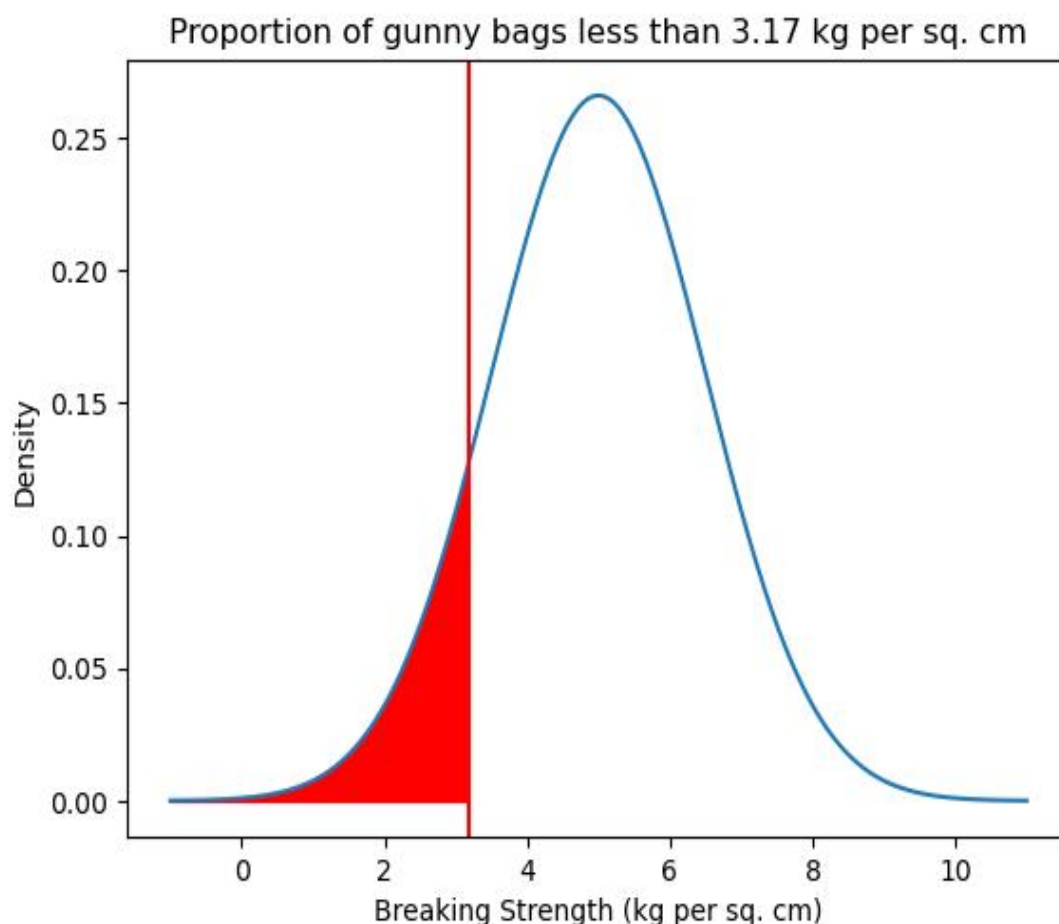
Plotting will help us to analyze the shape of data and visualize the PDF of normal distribution using the parameters mean and the standard deviation from the problem.



**Insight:** As we can see in the above plot, there are two curves red and blue. Blue curves represents the shape of the distribution and red curve represents the PDF(probability density function). This distribution is approximately normal. Thus, we can assume this distribution to be normal and perform our calculation based on normality assumption. With the assumption we import - **import scipy.stats as stats and from scipy.stats import norm.**

### **3.2. What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?**

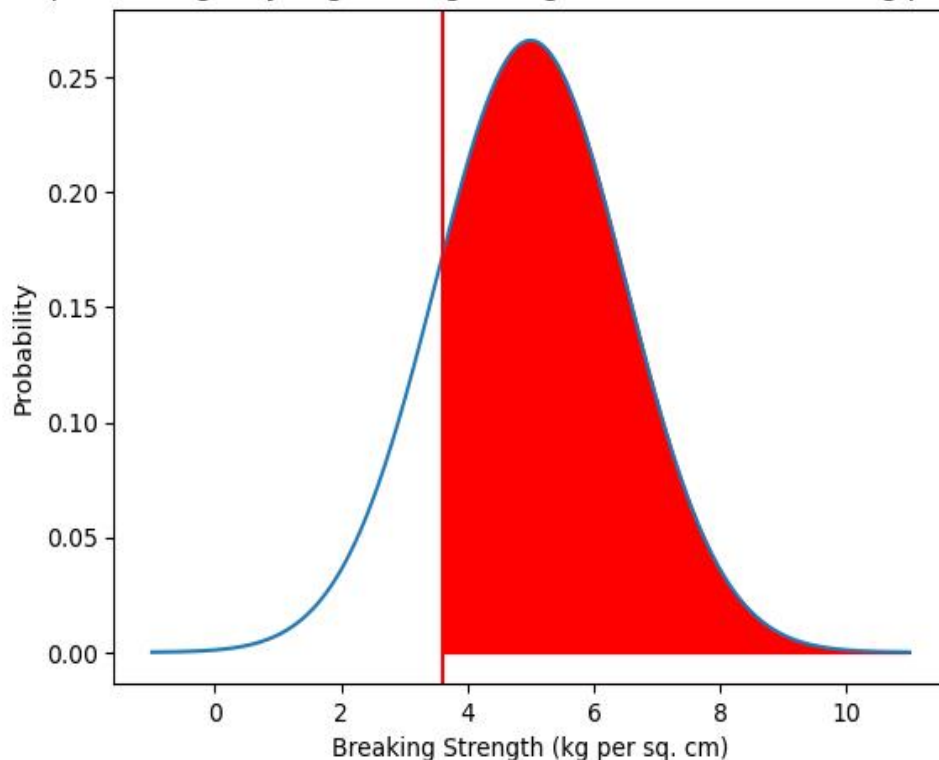
We use the CDF function to calculate the proportion of the gunny bags having a breaking strength less than 3.17 Kg per sq cm and the calculated proportion was **0.1112 (11.12%)**.



### 3.3. What proportion of the gunny bags have a breaking strength of atleast 3.6 kg per sq cm?

Since we want to find the proportion for a value greater than, we use 1-CDF for calculating the proportion of gunny bags having a breaking strength of atleast 3.6 Kg per sq cm. The calculated proportion was **0.8247(82.47%)**.

Proportion of gunny bags having strength of atleast than 3.6 kg per sq. cm

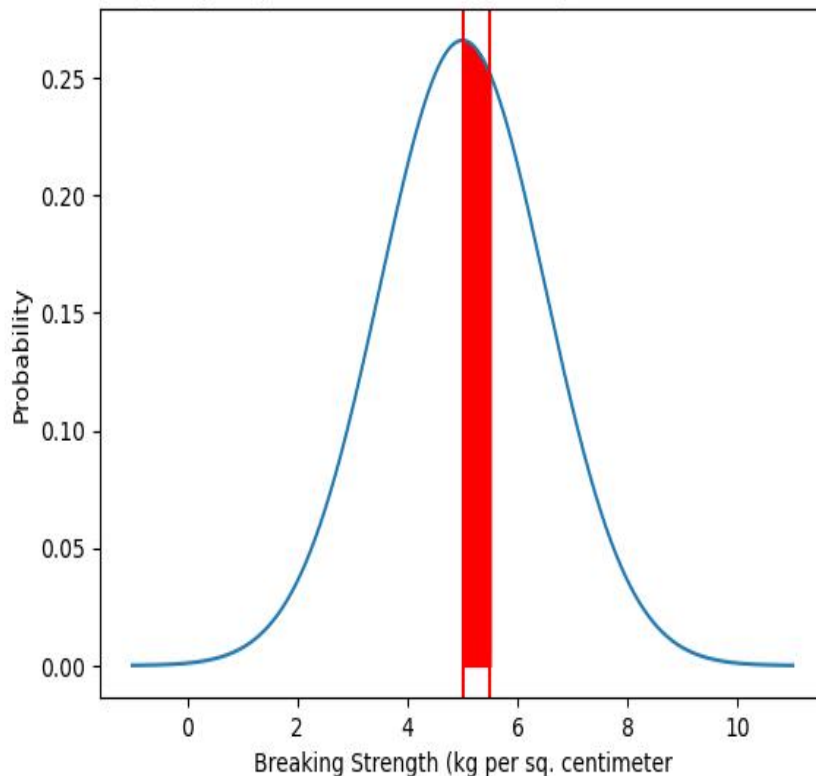


### 3.4. What proportion of gunny bags have a breaking strength between 5 and 5.5 kg per sq cm?

- To calculate the proportion of gunny bags having a breaking strength between 5 and 5.5 Kg per sq cm.
- We need to calculate proportion of breaking strength of 5 Kg per sq cm using CDF function and then calculate the proportion of breaking strength of 5.5 Kg per sq cm using CDF function.

- Finally, we subtract the proportion of breaking strength of 5.5 Kg per sq cm with the proportion of breaking strength of 5 Kg per sq cm which gives the proportion of gunny bags having a breaking strength between 5 and 5.5 Kg per sq cm.
- The calculated proportion of gunny bags having a breaking strength between 5 and 5.5 Kg per sq cm was **0.1306(13.06%)**.

Proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm

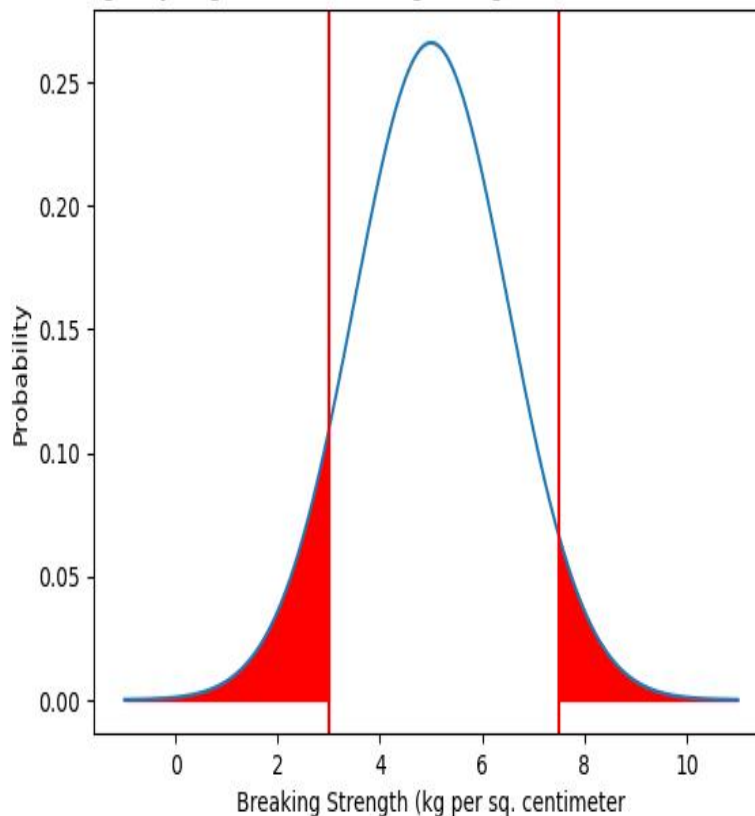


### 3.5. What proportion of gunny bags have a breaking strength Not between 3 and 7.5 kg per sq cm?

- To calculate the proportion of gunny bags having a breaking strength not between 3 and 7.5 Kg per sq cm.
- We need to calculate proportion of breaking strength of below 3 Kg per sq cm using CDF function and then calculate the proportion of breaking strength above 7.5 Kg per sq cm using 1-CDF.

- Finally, we add the proportion of breaking strength of below 3 Kg per sq cm with the proportion of breaking strength above 7.5 Kg per sq cm which gives the proportion of gunny bags having a breaking strength not between 3 and 7.5 kg per sq cm.
- The calculated of gunny bags having a breaking strength not between 3 and 7.5 kg per sq cm was **0.139(13.9%)**.

Proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm





## **4.Problem 3 : Zingaro Stone**

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. (assuming a 5% significance level).

### **4.1. Importing the required libraries and loading the dataset:**

The required libraries were imported and the dataset was loaded.

### **4.2. Structure,data description with data types:**

The loaded dataset has 75 rows and 2 columns with description as per below.

| Index | Column               | Non-Null | Count    | Dtype   |
|-------|----------------------|----------|----------|---------|
| 0     | Unpolished           | 75       | non-null | float64 |
| 1     | Treated and Polished | 75       | non-null | float64 |

dtypes: float64(2)  
memory usage: 1.3 KB

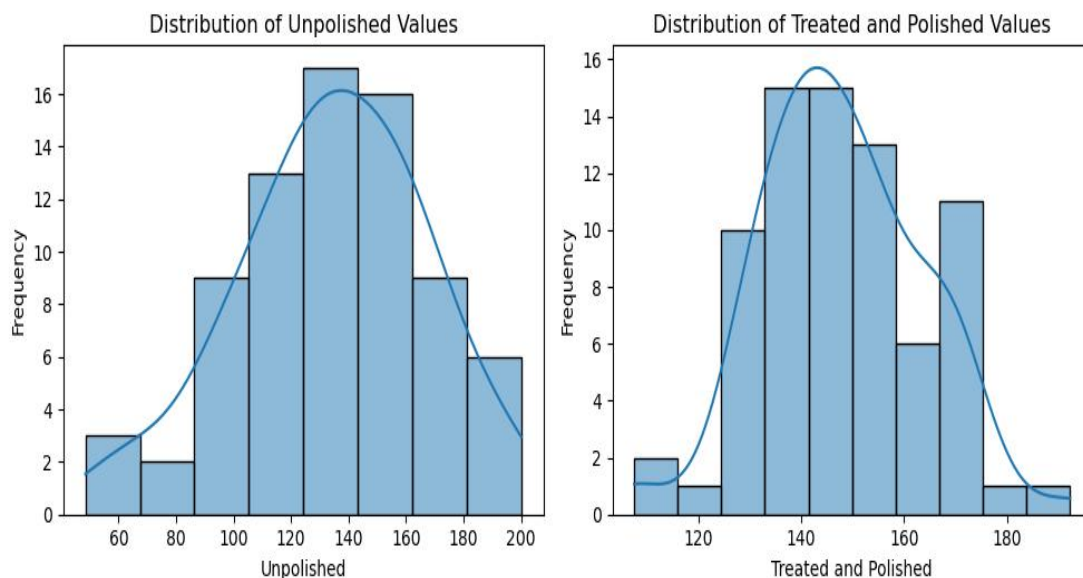
### 4.3.Statistical Summary:

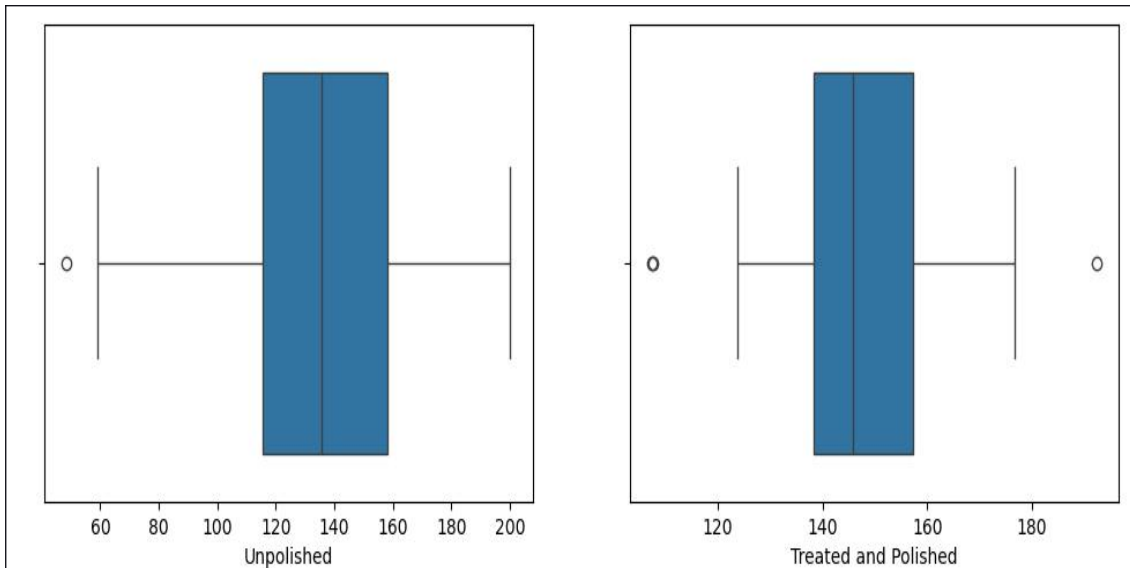
- From the statistical summary the variable 'Unpolished' shows the hardness of the stone is to range between 48.4 to 200.
- The variable 'Treated and Polished' shows the hardness of the stone is to range between 107.5 to 192.

### 4.4.Cleaning the dataset:

The column 'Unpolished' had whitespace issues, which resulted in problems reading the dataset. To address this, the dataset was processed with stripping to remove leading and trailing spaces from entries, ensuring it was suitable for proper use.

### 4.5. Visual representation of data:





- The above histogram shows the shape of the distribution which is almost being normally distributed.
- 50% of the hardness for Unpolished stone lies between 120 to 160 and for Treated and Polished stone 50% of hardness lies between 140 to 160.

**4.6. Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?**

**a. Appropriate test followed:**

From the problem statement we can understand **one sample T-test** is the appropriate test to be followed, which compares the sample mean to the population mean when standard deviation are unknown.

**b. Validating the assumption of the test followed:**

- Continuous data - Yes, hardness of the stone is measured on continuous scale.

- Normally distributed population and sample size > 30 - Yes.
- Observation are from a simple random sample - Yes.
- Population standard deviation is known - No.

### **c. Formulating Hypothesis:**

**Null Hypothesis (H<sub>0</sub>):** The unpolished stone surface of the received batch is suitable for printing.

$$H_0 : \mu \geq 150$$

**Alternative Hypothesis (H<sub>a</sub>):** The unpolished stone surface of the received batch is not suitable for printing.

$$H_a : \mu < 150$$

### **d. Considering significance level:**

$$\alpha = 0.05$$

### **e. Test Statistic:**

Using the function `ttest_1samp` the p-value obtained was 4.1712869974196533e-05 which is less than significance level(0.05).

### **f. Conclusion:**

- As the p-value is less than the level of significance(0.05), we reject the Null hypothesis.
- We do not have enough statistical evidence to say that the unpolished stone surface of the received batch is suitable for printing.

#### **4.7. Is the mean hardness of the polished and unpolished stones the same?**

##### **a. Appropriate test followed:**

From the problem statement we can understand **two sample independent T-test** is the appropriate test to be followed, which compares the sample means from 2 independent populations when standard deviation are unknown.

##### **b. Validating the assumption of the test followed:**

- Continuous data - Yes, hardness of the stone is measured on continuous scale.
- Normally distributed population and sample size  $> 30$  - Yes.
- Independent population - As we are provided with random samples for two different types of stones (i.e. polished and unpolished stones), the two samples are from two independent population.
- Observations are from a simple random sample - Yes.
- Population standard deviation is known - No.

##### **c. Formulating Hypothesis:**

**Null Hypothesis ( $H_0$ ):** The mean hardness of the polished and unpolished stone surface are same.

$$H_0 : \mu_1 = \mu_2$$

**Alternative Hypothesis ( $H_a$ ):** The mean hardness of the polished and unpolished stone surface are not same.

$$H_a : \mu_1 \neq \mu_2$$

**d.Considering significance level:**

$$\alpha=0.05$$

**e.Test Statistic:**

Using the function `ttest_ind` the p-value obtained was 0.0014655 which is less than significance level(0.05).

**f.Conclusion:**

- As the p-value is less than the level of significance(0.05), we reject the Null hypothesis.
- We do not have enough statistical evidence to say that the mean hardness of polished and unpolished stone surface are same.

## 5. Problem 4 : Dental Implant

The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

### 5.1. Importing the required libraries and loading the dataset:

The required libraries were imported and the dataset was loaded.

### 5.2. Structure,data description with data types:

The loaded dataset has 90 rows and 5 columns with description as per below.

| Index | Column   | Non-Null | Count    | Dtype |
|-------|----------|----------|----------|-------|
| 0     | Dentist  | 90       | non-null | int64 |
| 1     | Method   | 90       | non-null | int64 |
| 2     | Alloy    | 90       | non-null | int64 |
| 3     | Temp     | 90       | non-null | int64 |
| 4     | Response | 90       | non-null | int64 |

dtypes: int64(5)  
memory usage: 3.6 KB



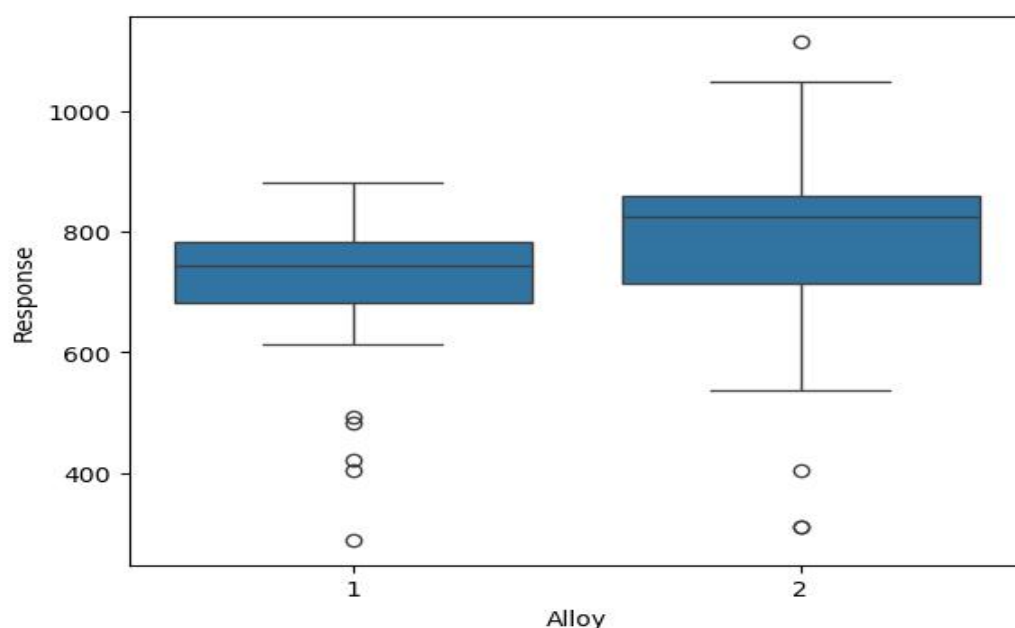
### 5.3. Cleaning the dataset:

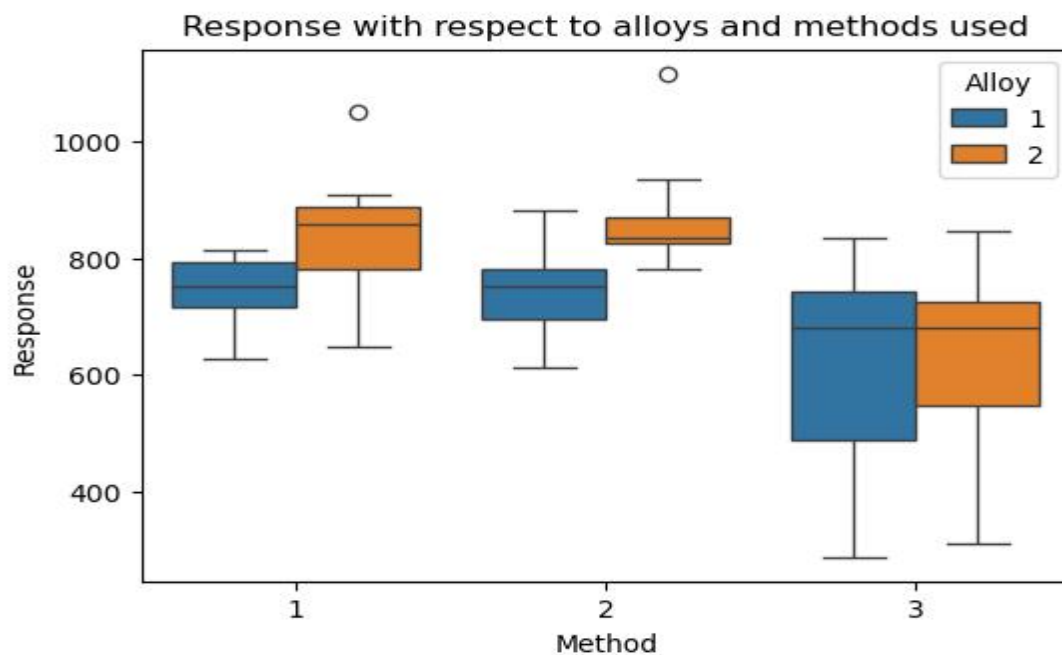
- The variable Dentist, Method and Alloy are with datatype int.
- Here integer is used to encode different levels of factors.
- Hence, these variables are converted as categorical which analysis to properly group and compare the data.
- This conversion ensures that the statistical analysis correctly interprets and compares the group providing meaningful and accurate results.

### 5.4. Creating subset:

- The dataset has been subsetting to include data for alloy 1 and alloy 2 for better analysis.
- This approach helps in determining which alloy in combination with other factor levels influences or effects the response variable of interest.

### 5.5. Visual representation of data:





- The 50% of the response are approximately between 700 to 800 for alloy 1.
- The 50% of the response are approximately between 750 to 850 for alloy 2.
- There are few outliers when using alloy 1.
- Method 3 followed by dentist seems to be low in response(hardness of implants) compared to method 2 and method 3 hardness response.
- Method 1 and method 2 combination with alloy 2 seems to have better response, compared to another treatment combination.

## 5.6. How does the hardness of implants vary depending on dentists?

### a. Appropriate test followed:

From the problem statement we can understand that One-way ANOVA test would be the appropriate test to be used. Where sample means are compared from two or more independent population with the involvement of single factor.

**b. Validating the assumptions of One way ANOVA TEST for alloy type 1 among dentists** - considering level of significance as 0.05

### **1. Shapiro Wilk's Test for Normality:**

**Null Hypothesis ( $H_0$ ):** The population from which sample is drawn forms a normal distribution.

**Alternative Hypothesis ( $H_a$ ):** The population from which sample is drawn does not form a normal distribution.

**P-value obtained:**

ShapiroResult(statistic=0.9113543033599854, pvalue=0.3254694640636444)

ShapiroResult(statistic=0.9642460942268372, pvalue=0.8415443301200867)

ShapiroResult(statistic=0.8721170425415039, pvalue=0.12953554093837738)

ShapiroResult(statistic=0.8368974328041077, pvalue=0.05333680287003517)

ShapiroResult(statistic=0.8534297943115234, pvalue=0.08127840608358383)

**Conclusion from test:**

As the p-value is greater than level of significance(0.05) for all five Dentist for alloy 1, we fail to reject the null hypothesis and conclude stating the population from which sample is drawn forms a normal distribution.

### **2. Levene's Test for equality of Variance:**

**Null Hypothesis ( $H_0$ ):** All the population variance are same.

**Alternative Hypothesis ( $H_a$ ):** Atleast one of the polulation variance is not the same.

**P-value obtained:**

LeveneResult(statistic=1.3847146992797106, pvalue=0.2565537418543795)

**Conclusion from test:**

As the p-value is greater than level of significance(0.05) for all five Dentist for alloy 1 , we fail to reject the null hypothesis and conclude stating all the population variance are same.

**c. Formulating Hypothesis for the problem statement for type 1 alloy:**

**Null Hypothesis ( $H_0$ ):** The mean hardness of the metal implants for alloy type 1 is same for all five dentists.

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

**Alternative Hypothesis ( $H_a$ ):** Atleast one of the mean hardness of metal implants for alloy type 1 is not the same among five dentists.

**c.i. Considering significance level:**

$$\alpha=0.05$$

**c.ii. Conducting Hypothesis test and computing p-value:**

The p-value obtained was 0.11656

**c.iii. Conclusion from the test for type 1 alloy:**

- As the p-value is greater than level of significance(0.05) we fail to reject the null hypothesis.
- We do not have enough statistical evidence to say that atleast one of the mean hardness of metal implants for alloy 1 is not the same among five dentists.

**d. Validating the assumptions of One way ANOVA TEST for alloy type 2 among dentists** - considering level of significance as 0.05

## 1. Shapiro Wilk's Test for Normality:

**Null Hypothesis( $H_0$ ):** The population from which sample is drawn forms a normal distribution.

**Alternative Hypothesis( $H_a$ ):** The population from which sample is drawn does not form a normal distribution.

**P-value obtained:**

ShapiroResult(statistic=0.9039730429649353, pvalue=0.27593860030174255)

ShapiroResult(statistic=0.9392004013061523, pvalue=0.5735077857971191)

ShapiroResult(statistic=0.9340972900390625, pvalue=0.5213102102279663)

ShapiroResult(statistic=0.7613219022750854, pvalue=0.007332688197493553)

ShapiroResult(statistic=0.9131585359573364, pvalue=0.33861199021339417)

**Conclusion from test:**

- As the p-value is greater than level of significance(0.05) for combination of Dentist 1 with alloy 2, we fail to reject the null hypothesis and conclude stating the population from which sample is drawn forms a normal distribution.
- As the p-value is greater than level of significance(0.05) for combination of Dentist 2 with alloy 2, we fail to reject the null hypothesis and conclude stating the population from which sample is drawn forms a normal distribution.
- As the p-value is greater than level of significance(0.05) for combination of Dentist 3 with alloy 2, we fail to reject the null hypothesis and conclude stating the population from which sample is drawn forms a normal distribution.
- As the p-value is less than level of significance(0.05) for combination of Dentist 4 with alloy 2, we reject the null hypothesis and conclude stating the population from which sample is drawn does not form a normal distribution.
- As the p-value is greater than level of significance(0.05) for combination of Dentist 5 with alloy 2, we fail to reject the null hypothesis and conclude stating the population from which sample is drawn forms a normal distribution.

hypothesis and conclude stating the population from which sample is drawn forms a normal distribution.

## **2. Levene's Test for equality of Variance**

**Null Hypothesis (H<sub>0</sub>):** All the population variance are equal.

**Alternative Hypothesis(H<sub>a</sub>):** Atleast one of the polulation variance is not the same.

**P-value obtained:**

LeveneResult(statistic=1.4456166464566966, pvalue=0.23686777576324952)

**Conclusion from test:**

As the p-value is greater than level of significance(0.05) for all five Dentist for alloy 2 , we fail to reject the null hypothesis and conclude stating all the population variance are same.

Note:

For one of the Dentist level distribution is not normal,however we continue further with the test.

### **e.Formulating Hypothesis for the problem statement for type 2 alloy:**

**Null Hypothesis (H<sub>0</sub>):** The mean hardness of the metal implants for alloy type 2 is same for all five dentists.

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

**Alternative Hypothesis (H<sub>a</sub>):** Atleast one of the mean hardness of metal implants for alloy type 2 is not the same among five dentists.

**e.i. Considering significance level:**

$$\alpha=0.05$$

**e.ii. Conducting Hypothesis test and computing p-value:**

The p-value obtained was 0.7180

**e.iii. Conclusion from the test for type 2 alloy:**

- As the p-value is greater than level of significance(0.05) we fail to reject the null hypothesis.
- We do not have enough statistical evidence to say that atleast one of the mean hardness of metal implants for alloy 2 is not the same among five dentists.

**5.7. How does the hardness of implants vary depending on methods?**

**a.Appropriate test followed:**

From the problem statement we can understand that One-way ANOVA test would be the appropriate test to be used. Where sample means are compared from two or more independent population with the involvement of single factor.

**b.Validating the assumptions of One way ANOVA TEST for alloy type 1 among methods- level of significance as 0.05**

**1.Shapiro Wilk's Test for Normality**

**Null Hypothesis ( $H_0$ ):** The population from which sample is drawn forms a normal distribution.

**Alternative Hypothesis( $H_a$ ):** The population from which sample is drawn does not form a normal distribution.



**P-value obtained:**

ShapiroResult(statistic=0.9183822870254517, pvalue=0.1819855272769928)

ShapiroResult(statistic=0.9732585549354553, pvalue=0.9030336737632751)

ShapiroResult(statistic=0.9114550352096558, pvalue=0.142548069357872)

**Conclusion from test:**

As the p-value is greater than level of significance(0.05) for all three methods for alloy 1 we fail to reject the null hypothesis and conclude stating the population from which sample is drawn forms a normal distribution.

**2.Levene's Test for equality of Variance**

**Null Hypothesis ( $H_0$ ):** All the population variance are equal.

**Alternative Hypothesis( $H_a$ ):** Atleast one of the polulation variance is not the same.

**P-value obtained:**

LeveneResult(statistic=6.52140454403598, pvalue=0.0034160381460233975)

**Conclusion from test:**

As the p-value is less than level of significance(0.05) for all three methods for alloy 1 , we reject the null hypothesis and conclude stating atleast one of the polulation variance is not the same.

**Note:**

Assumption of the Levene's test failed,but still we continue with the test.

**c. Formulating Hypothesis for the problem statement for type 1 alloy:**

**Null Hypothesis ( $H_0$ ):** The mean hardness of the metal implants for alloy type 1 is same for all 3 methods.

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

**Alternative Hypothesis ( $H_a$ ):** Atleast one of the mean hardness of metal implants for alloy type 1 is not the same among 3 methods.

**c.i. Considering significance level:**

$$\alpha=0.05$$

**c.ii. Conducting Hypothesis test and computing p-value:**

The p-value obtained was 0.00416

**c.iii. Conclusion from the test for type 1 alloy:**

- As the p-value is less than level of significance(0.05) we reject the null hypothesis.
- We do not have enough statistical evidence to say that the mean hardness of metal implants for alloy type 1 is same for all 3 methods.

**d. Validating the assumptions of One way ANOVA TEST for alloy type 2 among methods- level of significance as 0.05**

**1. Shapiro Wilk's Test for Normality:**

**Null Hypothesis ( $H_0$ ):** The population from which sample is drawn forms a normal distribution.

**Alternative Hypothesis( $H_a$ ):** The population from which sample is drawn does not form a normal distribution.

**P-value obtained:**

ShapiroResult(statistic=0.9638105630874634, pvalue=0.7582396864891052)

ShapiroResult(statistic=0.7557930946350098, pvalue=0.0010511117288842797)

ShapiroResult(statistic=0.9021322131156921, pvalue=0.1025901660323143)

**Conclusion from test:**

- As the p-value is greater than level of significance(0.05) for combination of Method 1 with alloy 2, we fail to reject the null hypothesis and conclude stating the population from which sample is drawn forms a normal distribution.
- As the p-value is less than level of significance(0.05) for combination of Method 2 with alloy 2, we reject the null hypothesis and conclude stating the population from which sample is drawn does not form a normal distribution.
- As the p-value is greater than level of significance(0.05) for combination of Method 3 with alloy 2, we fail to reject the null hypothesis and conclude stating the population from which sample is drawn forms a normal distribution.

## **2. Levene's Test for equality of Variance**

**Null Hypothesis ( $H_0$ ):** All the population variance are equal.

**Alternative Hypothesis( $H_a$ ):** At least one of the population variance is not the same.

**P-value obtained:**

LeveneResult(statistic=3.349707184158617, pvalue=0.04469269939158668)

**Conclusion from test:**

As the p-value is less than level of significance(0.05) for all three methods for alloy 2, we reject the null hypothesis and conclude stating at least one of the population variance is not the same.

**e. Formulating Hypothesis for the problem statement for type 2 alloy:**

**Null Hypothesis ( $H_0$ ):** The mean hardness of the metal implants for alloy type 2 is same for all 3 methods.

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

**Alternative Hypothesis ( $H_a$ ):** Atleast one of the mean hardness of metal implants for alloy type 2 is not the same among 3 methods.

**e.i. Considering significance level:**

$$\alpha=0.05$$

**e.ii. Conducting Hypothesis test and computing p-value:**

The p-value obtained was 5.415871051443187e-06

**e.iii. Conclusion from the test for type 2 alloy:**

- As the p-value is less than level of significance(0.05) we reject the null hypothesis.
- We do not have enough statistical evidence to say that the mean hardness of metal implants for alloy type 2 is same for all 3 methods.

**Since the mean hardness of metal implants for alloy 1 and alloy 2 differ among the 3 methods, further test is conducted to check for which pair of methods the hardness differs.**

#### **f. Performing Tukey's HSD test for methods under type 1 alloy**

Multiple Comparison of Means - Tukey HSD, FWER=0.05

```
=====
```

| group1 | group2 | meandiff  | p-adj  | lower     | upper    | reject |
|--------|--------|-----------|--------|-----------|----------|--------|
| -----  |        |           |        |           |          |        |
| 1      | 2      | -6.1333   | 0.987  | -102.714  | 90.4473  | False  |
| 1      | 3      | -124.8    | 0.0085 | -221.3807 | -28.2193 | True   |
| 2      | 3      | -118.6667 | 0.0128 | -215.2473 | -22.086  | True   |

#### **f.i. Conclusion from the Tukey's HSD test:**

- When method 2 is compared to method 1 there is no significance difference.
- When method 3 is compared to method 1 there is a significance difference.
- When method 3 is compared to method 2 there is a significance difference.
- We can come to the conclusion that the mean of method 3 is low compared to method 1 and method 2.

### **g.Performing Tukey's HSD test for methods under type 2 alloy:**

Multiple Comparison of Means - Tukey HSD, FWER=0.05

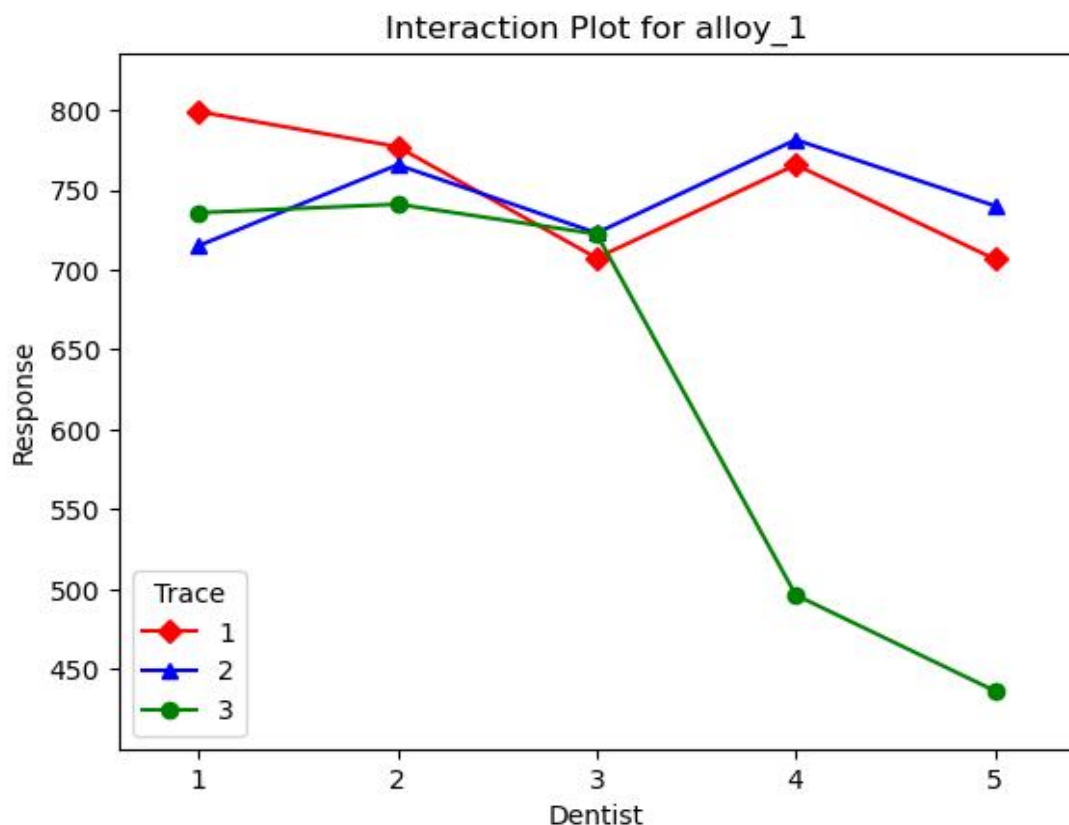
```
=====
=====
group1 group2 meandiff p-adj      lower  upper  reject
-----
  1     2       27.0  0.8212 -82.4546 136.4546  False
  1     3      -208.8 0.0001 -318.2546 -99.3454  True
  2     3      -235.8  0.0   -345.2546 -126.3454  True
```

### **g.i.Conclusion from the Tukey's HSD test:**

- When method 2 is compared to method 1 there is no significance difference.
- When method 3 is compared to method 1 there is a significance difference.
- When method 3 is compared to method 2 there is a significance difference.
- We can come to the conclusion that the mean of method 3 is low compared to method 1 and method 2.

## 5.8. What is the interaction effect between the dentist and method on the hardness of dental implants for type of alloy?

a. Interaction effect between the dentist and method for type 1 alloy and its inferences:

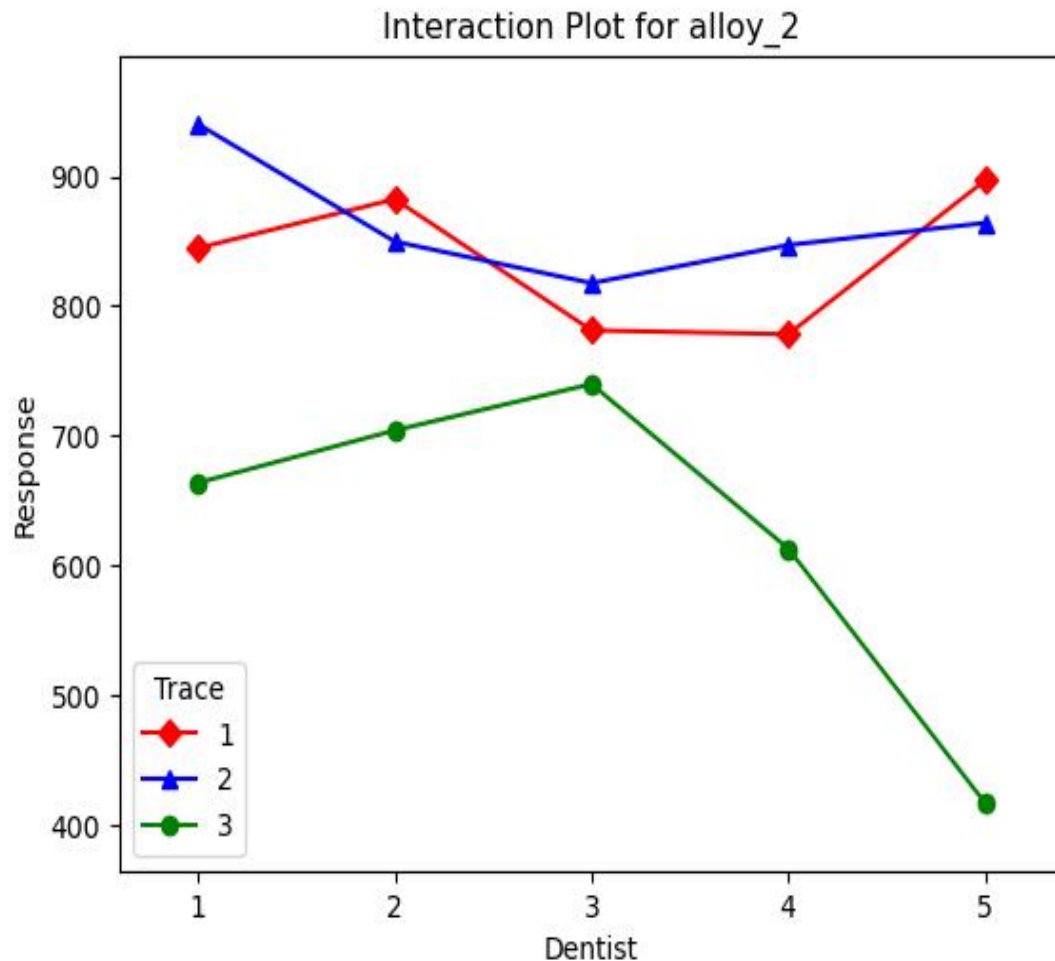


### Inferences:

- The above plot shows there is an interaction effect between the independent variables 'Dentist' and 'Methods' for type 1 alloy on hardness of implant.
- The effect of one independent variable on response variable is not consistent across the levels of other independent variable.



**b.Interaction effect between the dentist and method for type 2 alloy and its inferences:**



**Inferences:**

- The above plot shows there is an interaction effect between the independent variables 'Dentist' and 'Methods' for type 2 alloy on hardness of implant. But we can see method 3 used by the Dentists has no interaction effect between other methods used by the Dentists.
- The effect of one independent variable on response variable is not consistent across the levels of other independent variable.

## **5.9.How does the hardness of implants vary depending on dentists and methods together?**

### **a.Appropriate test followed:**

This is a problem,concerning the effect of two independent variables on a dependent variable.**Two-way ANOVA test** is an appropriate test.

### **b.Validating the assumptions of the test followed:**

- The populations from which the samples are obtained must be normally distributed - No,from the Shapiro Wilk's test followed.
- Sampling is done correctly.Observation for within and between groups must be independent- Yes.
- Variances among populations must be equal -No, from the levene's test conducted.
- The dependent data must be measured at an interval scale - Yes,Consistent.

### **c.Formulating Hypothesis for the problem statement for type 1 alloy:**

**Null Hypothesis ( $H_0$ ):** The interaction effect between dentists and methods using alloy type 1 on hardness of implants is not significant.

**Alternative Hypothesis ( $H_a$ ):**The interaction effect between dentists and methods using alloy type 1 on hardness of implants is significant.

**c.i. Considering significance level:**

$$\alpha=0.05$$

**b. ii. Conducting Hypothesis test and computing p-value:**

|                      | df   | sum_sq        | mean_sq      | F         | PR(>F)   |
|----------------------|------|---------------|--------------|-----------|----------|
| C(Dentist)           | 4.0  | 106683.688889 | 26670.922222 | 3.899638  | 0.011484 |
| C(Method)            | 2.0  | 148472.177778 | 74236.088889 | 10.854287 | 0.000284 |
| C(Dentist):C(Method) | 8.0  | 185941.377778 | 23242.672222 | 3.398383  | 0.006793 |
| Residual             | 30.0 | 205180.000000 | 6839.333333  | NaN       | NaN      |

**c.iii. Conclusion from the test for type 1 alloy:**

- As the p-value (0.006793) which is less than level of significance(0.05) we reject the null hypothesis.
- We do not have enough statistical evidence to say that the interaction effect between dentists and methods using alloy type 1 on hardness of implants is not significant.

**d. Formulating Hypothesis for the problem statement for type 2 alloy:**

**Null Hypothesis ( $H_0$ ):** The interaction effect between dentists and methods using alloy type 2 on hardness of implants is not significant.

**Alternative Hypothesis ( $H_a$ ):** The interaction effect between dentists and methods using alloy type 2 on hardness of implants is significant.

**d.i. Considering significance level:**

$$\alpha=0.05$$

## d.ii. Conducting Hypothesis test and computing p-value:

|                      | df   | sum_sq        | mean_sq       | F         | PR(>F)   |
|----------------------|------|---------------|---------------|-----------|----------|
| C(Dentist)           | 4.0  | 56797.911111  | 14199.477778  | 1.106152  | 0.371833 |
| C(Method)            | 2.0  | 499640.400000 | 249820.200000 | 19.461218 | 0.000004 |
| C(Dentist):C(Method) | 8.0  | 197459.822222 | 24682.477778  | 1.922787  | 0.093234 |
| Residual             | 30.0 | 385104.666667 | 12836.822222  | NaN       | NaN      |

## d.iii. Conclusion from the test for type 1 alloy:

- As the p-value (0.093234) which is greater than level of significance(0.05) we fail to reject the null hypothesis.
- We do not have enough statistical evidence to say that the interaction effect between dentists and methods using alloy type 2 on hardness of implants is significant.

## e. Performing Tukey's HSD Test for methods and dentist under type 1 alloy:

Multiple Comparison of Means - Tukey HSD, FWER=0.05

| group1 | group2 | meandiff  | p-adj  | lower     | upper    | reject |
|--------|--------|-----------|--------|-----------|----------|--------|
| 1      | 2      | 11.3333   | 0.9996 | -145.0423 | 167.709  | False  |
| 1      | 3      | -32.3333  | 0.9757 | -188.709  | 124.0423 | False  |
| 1      | 4      | -68.7778  | 0.7189 | -225.1535 | 87.5979  | False  |
| 1      | 5      | -122.2222 | 0.1889 | -278.5979 | 34.1535  | False  |
| 2      | 3      | -43.6667  | 0.9298 | -200.0423 | 112.709  | False  |
| 2      | 4      | -80.1111  | 0.5916 | -236.4868 | 76.2646  | False  |
| 2      | 5      | -133.5556 | 0.1258 | -289.9312 | 22.8201  | False  |
| 3      | 4      | -36.4444  | 0.9626 | -192.8201 | 119.9312 | False  |
| 3      | 5      | -89.8889  | 0.4805 | -246.2646 | 66.4868  | False  |
| 4      | 5      | -53.4444  | 0.8643 | -209.8201 | 102.9312 | False  |

#### Multiple Comparison of Means - Tukey HSD, FWER=0.05

| group1 | group2 | meandiff  | p-adj  | lower     | upper    | reject |
|--------|--------|-----------|--------|-----------|----------|--------|
| 1      | 2      | -6.1333   | 0.987  | -102.714  | 90.4473  | False  |
| 1      | 3      | -124.8    | 0.0085 | -221.3807 | -28.2193 | True   |
| 2      | 3      | -118.6667 | 0.0128 | -215.2473 | -22.086  | True   |

#### e.i.Conclusion from the Tukey's HSD Test:

- When the five dentist using type 1 alloy are compared to each other there is no significant difference.
- When method 2 is compared to method 1 there is no significance difference.
- When method 3 is compared to method 1 there is a significance difference.
- When method 3 is compared to method 2 there is a significance difference.
- We can come to the conclusion that the mean of method 3 is low compared to method 1 and method 2 for type 1 alloy.

#### f.Performing Tukey's HSD Test for methods and dentist under type 2 alloy:

#### Multiple Comparison of Means - Tukey HSD, FWER=0.05

| group1 | group2 | meandiff | p-adj  | lower     | upper    | reject |
|--------|--------|----------|--------|-----------|----------|--------|
| 1      | 2      | -4.1111  | 1.0    | -225.5687 | 217.3465 | False  |
| 1      | 3      | -36.5556 | 0.9895 | -258.0131 | 184.902  | False  |
| 1      | 4      | -70.0    | 0.8941 | -291.4576 | 151.4576 | False  |
| 1      | 5      | -90.1111 | 0.7724 | -311.5687 | 131.3465 | False  |
| 2      | 3      | -32.4444 | 0.9933 | -253.902  | 189.0131 | False  |
| 2      | 4      | -65.8889 | 0.9132 | -287.3465 | 155.5687 | False  |

|   |   |          |        |           |          |       |
|---|---|----------|--------|-----------|----------|-------|
| 2 | 5 | -86.0    | 0.8008 | -307.4576 | 135.4576 | False |
| 3 | 4 | -33.4444 | 0.9925 | -254.902  | 188.0131 | False |
| 3 | 5 | -53.5556 | 0.9574 | -275.0131 | 167.902  | False |
| 4 | 5 | -20.1111 | 0.999  | -241.5687 | 201.3465 | False |

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Multiple Comparison of Means - Tukey HSD, FWER=0.05  
=====

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| group1 | group2 | meandiff | p-adj | lower | upper | reject |
|--------|--------|----------|-------|-------|-------|--------|
|--------|--------|----------|-------|-------|-------|--------|

-----

|   |   |        |        |           |           |       |
|---|---|--------|--------|-----------|-----------|-------|
| 1 | 2 | 27.0   | 0.8212 | -82.4546  | 136.4546  | False |
| 1 | 3 | -208.8 | 0.0001 | -318.2546 | -99.3454  | True  |
| 2 | 3 | -235.8 | 0.0    | -345.2546 | -126.3454 | True  |

### **f.i.Conclusion from the Tukey's HSD Test:**

- When the five dentist using type 2 alloy are compared to each other there is no significant difference.
- When method 2 is compared to method 1 there is no significance difference.
- When method 3 is compared to method 1 there is a significance difference.
- When method 3 is compared to method 2 there is a significance difference.
- We can come to the conclusion that the mean of method 3 is low compared to method 1 and method 2 for type 2 alloy.