Cairo University

Faculty of Computers & Artificial Intelligence

Operations Research & Decision Support Dept.

Watt's UP

Optimization of Energy Consumption & User Comfort in Smart Homes

The Graduation Project Submitted to
The Faculty of Computers and Artificial Intelligence, Cairo University
In Partial Fulfillment of the Requirements for the bachelor's degree

In

Operations Research and Decision Support

Under Supervision of:

Assoc. Prof Ayman Sabry Gohneim

CAIRO UNIVERSITY

JULY/2025

Acknowledgements

We express our deepest gratitude to our supervisor, Dr. Ayman Sabry Ghoneim, for his invaluable guidance, expertise, and unwavering support throughout this project. His insightful feedback and encouragement were instrumental in shaping the development and success of this work. Dr. Ghoneim's dedication to fostering our understanding of complex concepts and his commitment to academic excellence greatly contributed to the quality of this research. We are truly grateful for his mentorship and the opportunity to work under his supervision.

ABSTRACT

The rapid advancement of smart home technologies has emphasized the importance of developing intelligent energy management systems that can reduce electricity costs, lower overall energy consumption, and ensure user comfort. Conventional Home Energy Management Systems (HEMS) typically focus on optimizing the operation schedule of controllable electrical loads (CELs) to reduce grid dependence and improve energy efficiency. However, these systems often overlook the potential to minimize total energy usage, provide flexibility to users, and capitalize on the opportunity to sell surplus energy back to the grid—limiting both economic benefits and the user experience.

In this project, an enhanced multi-objective optimization model for smart homes has been developed. The model integrates renewable energy sources such as solar and wind, in addition to home battery storage and electric vehicle (EV) charging systems.
During the first phase, the model was designed to minimize energy cost, maximize renewable energy utilization, and preserve user comfort through optimized appliance scheduling. Comfort is addressed by considering each user's preferred operation time window for appliances, ensuring that schedules align closely with household routines and expectations, thereby minimizing disruption and increasing the practicality of automated control systems.

In the second phase, two new objectives were introduced: reducing total electrical energy consumption and enabling profitable energy selling to the grid. Given the increased complexity and inherent conflicts among these objectives—including cost vs. comfort, and consumption vs. revenue, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) was applied to produce Pareto-optimal solutions. The model's formulation carefully integrates constraints such as appliance precedence, energy balance, battery and EV limitations, and smart grid interaction policies, all while prioritizing user convenience and scheduling fairness.

The optimization system was implemented using Python, utilizing the DEAP library to execute the NSGA-II algorithm, and IBM CPLEX for solving linear sub-problems. Comprehensive testing covered real-life scenarios, including time-of-use pricing, fluctuating renewable generation, and limited appliance control. Results show that the proposed model effectively reduces electricity bills and total consumption while providing flexible, user-centered scheduling and enabling energy trading when profitable. This work demonstrates a practical and sustainable solution that balances economic performance, environmental responsibility, and everyday usability for future smart homes.

DECLARATION

We hereby declare that our dissertation is entirely our work and genuine / original. We understand that in the case of the discovery of any PLAGIARISM at any stage, our group will be assigned an F (FAIL) grade, and it may result in withdrawal of our bachelor's degree.

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PLAIGRISM CERTIFICATE

This is to certify that the project entitled "**Optimization of Energy Consumption & User Comfort in Smart Homes"**, which is being submitted here with for the award of the "**Bachelor of Computer and Artificial Intelligence Degree" in "Operations Research and Decision Support"**. This is the result of the original work by **Hazem Medhat, Mohamed Ashraf, Mariam Mohamed, Jana Raafat** and **Salma Ashraf** under my supervision and guidance. The work embodied in this project has not been done earlier for the basis of the award of any degree or compatible certificate or similar tile of this for any other diploma/examining body or university to the best of my knowledge and belief.

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CHAPTER 1
INTRODUCTION

The increasing demand for electrical energy, coupled with the depletion of fossil fuel resources and environmental concerns, has driven the need for intelligent energy management systems in modern smart homes. In Egypt, where residential buildings account for nearly 48% of total electricity consumption, optimizing energy usage is particularly important to reduce dependency on the grid, cut energy costs, and promote the use of renewable resources such as solar and wind energy. However, existing Home Energy Management Systems (HEMS) focus primarily on reducing electricity bills without adequately addressing total energy conservation, user comfort, or the economic benefits of selling surplus energy back to the grid. For users to adopt such systems in their daily lives, it is critical that energy-saving decisions respect household routines and preferences, offering not only technical efficiency but also a seamless and non-intrusive experience.

The primary objective of this project is to develop an optimization model capable of scheduling controllable electrical loads (CELs) in a smart home environment while simultaneously considering multiple goals. These objectives include minimizing electricity costs, lowering total energy consumption, ensuring user comfort, reducing grid dependency, and generating profit by selling excess energy when feasible. User comfort is specifically preserved by assigning preferred operation windows for appliances, ensuring that essential household tasks occur at appropriate times without causing inconvenience or disruption. The model also accounts for practical constraints such as the maximum number of appliances that can run concurrently and the precedence relationships between certain devices (e.g., running a dryer only after the washing machine has completed).

To achieve these objectives, various resources were utilized. Renewable energy sources, including photovoltaic solar panels and wind turbines, were considered in the model, along with home battery storage and electric vehicle (EV) batteries. The computational tools used include Python for algorithm development, the DEAP library for implementing the Non-dominated Sorting Genetic Algorithm II (NSGA-II), and IBM CPLEX for solving sub-problems related to energy flow and scheduling optimization.

The development methodology followed a structured approach. In the first phase, a mathematical model was built to manage energy cost, user comfort, and grid reliance. User preferences were incorporated through time-window constraints, allowing the system to schedule appliances within acceptable timeframes that align with lifestyle patterns. In the second phase, the model was expanded to include two additional goals: minimizing total energy consumption and enabling energy selling to the grid when profitable. NSGA-II was used to handle the multi-objective optimization problem, generating Pareto-optimal solutions that balance the trade-offs between energy cost, consumption, revenue, and user convenience. Multiple scenarios were tested to evaluate the model's performance under diverse household conditions, renewable availability, and electricity pricing schemes.

This report is organized as follows:

- Chapter 1 presents an overview of the project, including the problem statement, objectives, required resources, and methodology.
- Chapter 2 reviews the background and related literature on smart home energy management systems.
- Chapter 3 explains the mathematical formulation of the optimization model, including the objective functions, constraints, and variables.
- Chapter 4 describes the implementation details, software tools, and optimization algorithms used.
- Chapter 5 presents the experimental setup, testing scenarios, results, and analysis.
- Chapter 6 concludes the project and provides suggestions for future work and potential enhancements.

CHAPTER 2
BACKGROUND/EXISTING WORK

The integration of renewable energy sources, battery storage systems, and intelligent control devices has greatly enhanced the concept of Home Energy Management Systems (HEMS) in smart homes. These systems aim to efficiently schedule controllable electrical loads (CELs) to reduce electricity costs, lower grid dependency, and maintain user comfort by exploiting renewable resources such as solar and wind energy.

One of the foundational studies in this field was presented in [1], where a mathematical model was developed for the optimal scheduling of CELs, incorporating renewable generation sources and home energy storage. Their research demonstrated the potential of HEMS to reduce grid reliance while maintaining user convenience, but it did not address energy trading or total consumption minimization. Similarly, [2] explored nature-inspired optimization algorithms to efficiently manage residential energy usage. Their work highlighted the capability of metaheuristic methods to solve non-linear and multi-objective optimization problems under uncertainty; however, energy selling and broader economic considerations were not part of their approach.

To improve practical energy management in homes, recent studies have proposed optimization models that also account for electric vehicle (EV) charging and discharging cycles. Shewale et al. [5] introduced a technique based on the multiple knapsack problem to enhance appliance scheduling by integrating EV battery management, leading to more efficient utilization of energy resources. Yet, their work did not fully exploit opportunities for energy trading with the grid or overall consumption reduction.

Moreover, market reports suggest that the global smart home sector is growing rapidly due to increased demand for comfort, energy savings, and automation [4], which underscores the importance of developing intelligent systems capable of both optimizing energy use and enabling grid interaction. The Egyptian market, in particular, faces pressing needs for such systems due to its growing population, rising energy demand, and significant investment in renewable energy projects [3].

Despite these advances, gaps remain in existing HEMS models, particularly regarding the inclusion of energy trading and overall energy consumption minimization as explicit objectives. This project addresses these limitations by incorporating both objectives into the optimization model, using the Non-dominated Sorting Genetic Algorithm II (NSGA-II), a proven method for handling complex multi-objective problems, as supported by previous optimization research [2].

Home energy management system is an important part in smart home, smart home is assumed to be equipped with smart meter which smart control of generators, storages, and demand response programs. This paper studies a versatile convex optimization framework for the automatic energy management of various household loads in a smart home. The scheduling algorithm determines how energy resources are available to the end-users considering several constraints [8].

Smart homes have the potential to achieve optimal energy consumption with appropriate scheduling. The control of smart appliances can be based on optimization models, which should be realistic and efficient. However, increased realism also implies an increase in solving time. Many of the optimization models in the literature have limitations on the types of appliances considered and/or their reliability. This paper proposes a home energy management scheduling model that is more realistic and efficient. We develop a mixed integer linear optimization model that minimizes the energy cost while maintaining a given level of user comfort [9].

In this paper, carried out a detailed literature review of the techniques used for the optimization of energy consumption and scheduling in smart homes. Detailed discussion has been carried out on different factors contributing towards thermal comfort, visual comfort, and air quality comfort. have also reviewed the fog and edge computing techniques used in smart homes [10].

Smart grid enables consumers to control and schedule the consumption pattern of their appliances, minimize energy cost, peak-to-average ratio (PAR) and peak load demand. In this paper, a general architecture of home energy management system (HEMS) is developed in smart grid scenario with novel restricted and multi-restricted scheduling methods for residential customers. The optimization problem is developed under the time of use pricing (TOUP) scheme. To optimize the formulated problem, a powerful meta-heuristic algorithm called grey wolf optimizer (GWO) is utilized, which is compared with particle swarm optimization (PSO) algorithm to show its effectiveness. A rooftop photovoltaic (PV) system is integrated with the system to show the cost effectiveness of the appliances. For analysis, eight different cases are considered under various time scheduling algorithms [11].

In this paper, propose to model the energy consumption of smart grid households with energy storage systems as an intertemporal trading economy. Intertemporal trade refers to transaction of goods across time when an agent, at any time, is faced with the option of consuming or saving with the aim of using the savings in the future or spending the savings from the past [12].

In this paper, proposed an improved optimization function to achieve maximum user comfort in the building environment with minimum energy consumption. Comprehensive formulation is done for energy optimization with detailed analysis [13].

Home Energy Management (HEM) controllers have been widely used for residential load management in a smart grid. Generally, residential load management aims to reduce the electricity bills and curtail the Peak-to-Average Ratio (PAR). In this paper, designed a HEM controller based on four heuristic algorithms: Bacterial Foraging Optimization Algorithm (BFOA), Genetic Algorithm (GA), Binary Particle Swarm Optimization (BPSO), and Wind Driven Optimization (WDO). Moreover, we proposed a hybrid algorithm which is Genetic BPSO (GBPSO) [14].

CHAPTER 3

Mathematical Model

3.1 Base Model Overview

The foundation of this project is built upon an existing mathematical model for the optimal scheduling of smart home electrical loads. This base model is designed to optimize the operation of Controllable Electrical Loads (CELs) while integrating renewable energy sources and storage components such as home batteries and electric vehicles (EVs). The main objective of the base model is to minimize electricity cost by efficiently scheduling appliance operation and managing energy flow from various sources.

The model formulation includes a comprehensive set of decision variables, both binary and continuous. Binary variables indicate the operational status of appliances (on/off), connection and disconnection times, and precedence conditions between appliances. Continuous variables represent power flows such as grid consumption, battery charging/discharging, and renewable energy utilization. The model also incorporates energy storage dynamics for both home and EV batteries.

Constraints are defined to ensure the feasibility of appliance operation, energy balance, precedence between devices (e.g., the dryer must only run after the washing machine has finished), and system limits on the number of active appliances or their combined power. Special attention is given to modeling realistic appliance behavior, time windows for operation, and energy availability from renewable sources (solar and wind).

The base model is solved using classical optimization techniques and focuses solely on cost minimization, without considering other important aspects like total energy consumption or the potential to sell surplus energy to the grid.

3.2 Mathematical Formulation of the Base Model

Sets:

Ωt: set of time from analyzed period

Ωcel: set of controllable electrical load

Pcelj: set of active power from controllable electrical loads

Constants (Parameters):

Δ: discretization time t.

Ti: initial time from period analyzed.

Tf: end time from period analyzed.

$\mathbf{P_t^{sun}}$**:** active power from photovoltaic system.

$\mathbf{P_t^{wind}}$**:** active power from wind turbines.

$\mathbf{P_t^{re}} = \mathbf{P_t^{sun}} + \mathbf{P_t^{wind}}$

$\mathbf{P_t^{ncl}}$**:** active power from non-controllable load.

$\mathbf{P_j^{cl}}$**:** active power from controllable load.

$\boldsymbol{c_t^{re}}$**:** cost of renewable energy at time t.

$\boldsymbol{c_t^{grid}}$**:** cost of grid electricity at time t.

Tcli, **Tclf:** Time window for running appliance i.

λ: number of CELs that can be connected during the same period.

γ: maximum load that can be connected via CELs.

$\boldsymbol{\phi_{ij}}$**:** Precedence matrix, ϕ_{ij}=1 if i must run before j, else 0.

Pevmax: maximum power charging from electric vehicle in Δt

Eevmax: maximum stored energy from electric vehicle

P: percentage of battery storage from electric vehicles.

cel1: first controllable electrical load

celn: last controllable electrical load

Binary Decision Variables:

$\mathbf{x_{i,t}}$: 1 if the appliance i is on at time t, 0 otherwise.

$\boldsymbol{\alpha_{i,t}}$: 1 if the appliance i is connected at time t, 0 otherwise.

$\boldsymbol{\beta_{i,t}}$: 1 if the appliance i is disconnected at time t, 0 otherwise.

$\boldsymbol{\delta_{i,t}}$: 1 if the appliance i is finished at time t, 0 otherwise.

Continuous Decision variables:

$\mathbf{P_t^{grid}}$: Power bought from the grid at time t.

$\mathbf{P_t^{clr}}$: Total power consumed by controllable loads at time t.

$\mathbf{P_t^{br}}$: Battery charging power at time t.

$\mathbf{P_t^{bi}}$: Battery discharging power at time t.

$\mathbf{P_t^{evb}}$: EV battery charging power at time t.

$\mathbf{E_{tf}^{ev}}$: Final energy in the EV battery.

Objective Function:

Energy Cost Objective

$$\textit{Minimize } z_{cost} = \sum_{t \epsilon T} C_t^{grid} * P_t^{grid} - \sum_{t \epsilon T} C_t^{re} * P_t^{re}$$

Constraints:

1- Energy Balance

$$P_t^{sun} + P_t^{wind} + P_t^{grid} + P_t^{bi} = P_t^{br} + P_t^{evb} + P_t^{clr} + P_t^{ncl} + \sum_{j \epsilon cels} P_{j,t}^{cl} \quad \forall t \in T$$

2- Appliance Operation

a. Status Transition = Connection – Disconnection

$$X_{j,t} - X_{j,t-1} = \alpha_{j,t} - \beta_{j,t} \qquad \forall j \in CEL, t = 1, \dots, Tf$$

b. Initial Condition

$$\boldsymbol{X_{j,1} - X_{j,0} = \alpha_{j,t} - \beta_{j,1}} \ \ \forall \boldsymbol{j} \in \boldsymbol{CEL}$$

c. Connect Exactly Once in Allowed Window

$$\sum_{\boldsymbol{t=Tcli}[\boldsymbol{j}]}^{\boldsymbol{t=Tclf}[\boldsymbol{j}]} \boldsymbol{\alpha_{j,t} = 1} \ \ \forall \boldsymbol{j} \in \boldsymbol{CEL}$$

d. Disconnect Exactly Once in Allowed Window

$$\sum_{\boldsymbol{t=Tcli}[\boldsymbol{j}]}^{\boldsymbol{t=Tclf}[\boldsymbol{j}]} \boldsymbol{\beta_{j,t} = 1} \ \ \forall \boldsymbol{j} \in \boldsymbol{CEL}$$

e. Enforce Operation Duration

$$\sum_{\boldsymbol{t=Tcli}[\boldsymbol{j}]}^{\boldsymbol{t=Tclf}[\boldsymbol{j}]} \boldsymbol{X_{j,t} = \Delta t_j} \ \ \forall \boldsymbol{j} \in \boldsymbol{CEL}$$

f. Prohibit Operation Outside Allowed Window

$$\sum_{\boldsymbol{t<\Delta ti \text{ or } t>\Delta tf}} \boldsymbol{X_{j,t} = 0} \ \forall \boldsymbol{j} \in \boldsymbol{CEL}$$

$$\sum_{\boldsymbol{t=Tcli}}^{\boldsymbol{t=Tclf}} |\, \boldsymbol{X_{j,t} - X_{j,t-1}}| = \boldsymbol{2}$$

3- Device Completion and Precedence

a. Completion Status Implies Device Is Off

$$\boldsymbol{\delta_{j,t} \leq 1 - X_{j,t}}$$

b. Completion Propagates Upon Disconnection

$$\boldsymbol{\delta_{j,t} \geq \delta_{j,t-1} - \beta_{j,t}}$$

c. Completion Persists After Disconnection

$$\boldsymbol{\delta_{j,t+1} \leq \delta_{j,t} - \left(1 - \beta_{j,t}\right)}$$

d. Precedence Between Appliances

$$\boldsymbol{\alpha_{j,t} \leq \delta_{i,t} + \left(1 - \phi_{i,j}\right)}$$

4- Simultaneous Operation

a. Maximum Number of Simultaneous CELs

$$\boldsymbol{\sum_{j \in cels} X_{j,t} \leq \lambda}$$

b. Maximum Simultaneous CEL Load

$$\boldsymbol{\sum_{j \in cels} X_{j,t} * P_j^{cl} \leq \gamma}$$

5- Electric Vehicle (EV) Battery

a. EV Charging Power Limit

$$\boldsymbol{P_t^{evb} \leq P_{max}^{ev}}$$

b. Final Battery Target Level

$$\boldsymbol{E_{tf}^{ev} = \rho * E_{max}^{ev}}$$

3.3 Limitations of the Base Model

The base model provides a strong foundation for optimizing appliance scheduling and minimizing electricity costs in smart homes. It introduces essential components such as controllable electrical load (CEL) scheduling, renewable energy integration, and battery storage, all structured within a mathematically sound and practically applicable framework. However, as smart home systems evolve and energy flexibility becomes more critical, additional capabilities are needed to address new priorities and use cases.

1. Energy trading not included:
   The base model was designed with a focus on internal household efficiency. While this approach is effective for cost reduction, it does not incorporate mechanisms for selling surplus energy to the grid, a valuable opportunity in homes equipped with renewable generation and storage.

2. Single-objective focus:
   The model effectively minimizes energy cost but does not explicitly consider other objectives such as user comfort or energy consumption reduction. A multi-objective framework is now desirable to reflect the diverse goals of smart energy systems.

3. Basic treatment of home battery dynamics:
   Battery charging and discharging are modeled correctly at a high level, but more refined controls such as preventing simultaneous charge/discharge or enforcing minimum reserve levels, can improve system reliability and battery health in real implementations.

4. Limited EV battery scheduling considerations:
   The base model includes EV battery charging but does not fully incorporate flexible departure times, tariff-based charging strategies, or real-world constraints like availability windows. Enhancing the EV logic allows for more efficient use of energy resources and better alignment with user mobility needs.

These observations are not weaknesses but rather opportunities for expansion. Building on the strength and structure of the original model, this project introduces enhancements to make the system more comprehensive, adaptable, and aligned with modern smart home expectations.

3.4 New Model and Improvements

To overcome these limitations, we developed an enhanced multi-objective mathematical model that extends the original framework. The improvements include:

1. Introducing new objectives: In addition to minimizing energy costs, minimizing discomfort for the user to schedule appliances.

2. Incorporating energy trading mechanisms: The new model allows smart homes to sell energy to the grid when prices are favorable, subject to constraints such as battery capacity and profitability thresholds.

3. Enhancing home battery management: The new model includes precise control over home battery charging and discharging, enforces mutual exclusivity (charging or discharging at one time), and prevents overcharging or depletion, ensuring long-term battery health and optimal utilization.

4. Updating electric vehicle (EV) battery scheduling: The model accounts for the EV's availability at home, charging limits, and required state of charge by a specified departure time. It also aligns the EV charging schedule with low-tariff periods and renewable generation availability to reduce costs and support energy balance.

5. Applying NSGA-II for multi-objective optimization: To handle the conflicting nature of cost, energy usage, comfort, and profit, we employed the Non-dominated Sorting Genetic Algorithm II (NSGA-II), implemented using the DEAP library in Python.

These improvements result in a more comprehensive and practical Home Energy Management System (HEMS) that supports sustainability, economic benefit, and user flexibility.

3.5 Mathematical Formulation of The New Model

Sets:

Ωt: set of time from analyzed period

Ωcel: set of controllable electrical load

Pcelj: set of active power from controllable electrical loads

Constants (Parameters):

Δ: discretization time t.

Ti: initial time from period analyzed.

Tf: end time from period analyzed.

$\mathbf{P_t^{sun}}$**:** active power from photovoltaic system.

$\mathbf{P_t^{wind}}$**:** active power from wind turbines.

$\mathbf{P_t^{re}} = \mathbf{P_t^{sun}} + \mathbf{P_t^{wind}}$

$\mathbf{P_t^{ncl}}$**:** active power from non-controllable load.

$\mathbf{P_j^{cl}}$**:** active power from controllable load.

c_t^{re}**:** cost of renewable energy at time t.

c_t^{grid}**:** cost of grid electricity at time t.

Tcli, **Tclf:** Time window for running appliance i.

$\mathbf{d_i}$**:** Duration of appliance i.

λ number of CELs that can be connected during the same period Δt.

γ maximum load that can be connected via CELs.

$\mathbf{\phi_{ij}}$**:** Precedence matrix, $\phi_{ij}=1$ if i must run before j, else 0.

$\mathbf{P_{max}^{grid}}$**:** Maximum grid power allowed.

$\mathbf{P_{max}^{sell}}$**:** Maximum power that can be sold to grid.

$w1, w2$: Weights for cost and discomfort objectives.

$Z_{max}^{cost}, \mathbf{D_{max}}$**:** Normalization constants for objective function.

Home Battery:

E_{init}^{b}**:** Initial Home battery energy.

E_{max}^{b}**:** Maximum battery capacity.

$\boldsymbol{P_{max}^{br}}$**:** Maximum charging power.

$\boldsymbol{P_{max}^{bi}}$**:** Maximum discharging power.

Electrical Vehicle:

$\boldsymbol{E_{max}^{ev}}$**:** Maximum stored energy from electric vehicle.

$\boldsymbol{P_{max}^{ev}}$**:** Maximum power charging from electric vehicle in Δt.

$\boldsymbol{\rho}$**:** Percentage of battery storage from electric vehicles.

Binary Decision Variables:

$\mathbf{x_{i,t}}$**:** 1 if the appliance i is on at time t, 0 otherwise.

$\boldsymbol{\alpha_{i,t}}$**:** 1 if the appliance i is connected at time t, 0 otherwise.

$\boldsymbol{\beta_{i,t}}$**:** 1 if the appliance i is disconnected at time t, 0 otherwise.

$\boldsymbol{\delta_{i,t}}$**:** 1 if the appliance i is finished at time t, 0 otherwise.

$\mathbf{mode_t^{grid}}$**:** 1 if the grid buying at time t, 0 otherwise.

$\mathbf{mode_t^{pr}}$**:** 1 if changing home battery at time t, 0 otherwise.

Continuous Decision variables:

$\mathbf{P_t^{grid}}$**:** Power bought from the grid at time t.

$\mathbf{P_t^{clr}}$**:** Total power consumed by controllable loads at time t.

$\mathbf{P_t^{sell}}$**:** Power sold to the grid at time t.

$\mathbf{P_t^{br}}$**:** Battery charging power at time t.

$\mathbf{P_t^{bi}}$**:** Battery discharging power at time t.

$\mathbf{P_t^{evb}}$**:** EV battery charging power at time t.

$\mathbf{E_{tf}^{ev}}$**:** Final energy in the EV battery.

$\mathbf{E_t^{b}}$**:** Energy stored in the home battery at time t.

$\boldsymbol{\eta_i}$**:** Actual start time of appliance i.

Objective Function:

1- Energy Cost Objective

$$\boldsymbol{Minimize\ z_{cost} = \sum_{t\epsilon T} C_t^{grid} * P_t^{grid} - \sum_{t\epsilon T} R_t^{sell} * P_t^{sell} + \sum_{t\epsilon T} C_t^{re} * P_t^{re}}$$

This equation minimizes the total energy cost by considering the price of purchased electricity from the grid, deducting revenue from selling surplus energy, and adding the cost of using renewable energy sources.

2- Discomfort Objective

$$\boldsymbol{Minimize\ z_{discomfort} = \sum_{j\in cels} \frac{\eta[j] - T_{cli}[j]}{T_{clf}[j] - T_{cli}[j]}}$$

This equation minimizes user discomfort by penalizing deviations between the preferred operation time window and the actual start time of each appliance.

Constraints:

1- Strat Time

$$\boldsymbol{\eta_j = t * \alpha_{j,t}\ \forall j \in CEL}$$

This equation calculate starts connection time for each controllable appliance.

2- Energy Balance

$$\boldsymbol{P_t^{sun} + P_t^{wind} + P_t^{grid} + P_t^{bi} = P_t^{br} + P_t^{evb} + P_t^{clr} + P_t^{ncl} + \sum_{j\epsilon cels} P_{j,t}^{cl} \quad \forall t \in T}$$

This equation guarantees that, at every time step, the total supplied energy (from renewables, grid, batteries) matches the total energy consumed by loads, batteries, and EVs.

3- Grid Interaction and Selling

a. Grid Buy/Sell Exclusivity

$$\boldsymbol{P_t^{grid} \le mode_t^{grid} * P_{max}^{grid} \quad \forall t \in T}$$

$$\boldsymbol{P_t^{sell} \le (1 - mode_t^{grid}) * P_{max}^{sell} \quad \forall t \in T}$$

The system can either buy energy from the grid or sell energy to it at a given time but not both simultaneously.

b. Selling Limited by Battery Level

$$\boldsymbol{P_t^{sell} \le E_t^b \quad \forall t \in T}$$

The amount of energy sold cannot exceed the energy stored in the home battery.

c. Selling Only If Profitable

$$\boldsymbol{P_t^{sell} \le sell\_allowed_t * P_{max}^{sell} \quad \forall t \in T}$$

$$\boldsymbol{sell\_allowed_t = \begin{cases} 1 & if\ R_t^{sell} > C_t^{grid} + 0.08 \\ 0 & otherwise \end{cases}}$$

Energy is only sold to the grid if the selling price is greater than the grid's buying price plus a defined threshold.

4- Appliance Operation

a. Status Transition = Connection – Disconnection

$$\boldsymbol{X_{j,t} - X_{j,t-1} = \alpha_{j,t} - \beta_{j,t} \qquad \forall j \in CEL, t = 1, \dots, Tf}$$

Appliance status changes are determined by its connection and disconnection actions.

b. Initial Condition

$$\boldsymbol{X_{j,0} = \alpha_{j,0} - \beta_{j,0} \quad \forall j \in CEL}$$

Appliance status at the initial time is set by its initial connection and disconnection states.

c. Connect Exactly Once in Allowed Window

$$\sum_{t=Tcli[j]}^{t=Tclf[j]} \boldsymbol{\alpha_{j,t} = 1} \ \ \forall \boldsymbol{j} \in \boldsymbol{CEL}$$

Each appliance can only connect once within its allowed operating window.

d. Disconnect Exactly Once in Allowed Window

$$\sum_{t=Tcli[j]}^{t=Tclf[j]} \boldsymbol{\beta_{j,t} = 1} \ \ \forall \boldsymbol{j} \in \boldsymbol{CEL}$$

Each appliance can only disconnect once within its allowed operating window.

e. Enforce Operation Duration

$$\sum_{t=Tcli[j]}^{t=Tclf[j]} \boldsymbol{X_{j,t} = D_j} \ \ \forall \boldsymbol{j} \in \boldsymbol{CEL}$$

Each appliance must operate for exactly its required duration within the allowed time window.

f. Prohibit Operation Outside Allowed Window

$$\sum_{t \notin [Tcli[j], Tclf[j]]} \boldsymbol{X_{j,t} = 0} \ \forall \boldsymbol{j} \in \boldsymbol{CEL}$$

Appliances cannot operate outside of their predefined operational time windows.

5- Device Completion and Precedence

a. Completion Status Implies Device Is Off

$$\boldsymbol{\delta_{j,t} \leq 1 - X_{j,0}} \ \ \forall \boldsymbol{j} \in \boldsymbol{CEL}, \forall \boldsymbol{t} \in \boldsymbol{T}$$

Once an appliance finishes, it cannot run.

b. Completion Propagates Upon Disconnection

$$\boldsymbol{\delta_{j,t} \geq \delta_{j,t-1} - \beta_{j,t} \; \forall j \in CEL, t = 1, \dots, T}$$

The completion status of an appliance updates when it is disconnected.

c. Completion Persists After Disconnection

$$\boldsymbol{\delta_{j,t+1} \leq \delta_{j,t} - \left(1 - \beta_{j,t}\right) \quad \forall j \in CEL, t = 0, \dots, T_f - 1}$$

Once completed, an appliance remains in the complete state.

d. Precedence Between Appliances

$$\boldsymbol{\alpha_{j,t} \leq \delta_{j,t} + \left(1 - \phi_{i,j}\right) \quad \forall j, i \in CEL, \forall t \in T}$$

An appliance cannot start until another required appliance has finished if defined by the precedence matrix.

6- Simultaneous Operation

a. Maximum Number of Simultaneous CELs

$$\boldsymbol{\sum_{j \in cels} X_{j,t} \;\leq\; \lambda \;\; \forall t \in T}$$

Limits the number of appliances that can operate simultaneously.

b. Maximum Simultaneous CEL Load

$$\boldsymbol{\sum_{j \in cels} X_{j,t} \; * \; P_j^{cl} \leq \gamma \;\; \forall t \in T}$$

Limits the total electrical power drawn by appliances to avoid exceeding system capacity.

7- Electric Vehicle (EV) Battery

a. EV Charging Power Limit

$$\boldsymbol{P_t^{evb} \;\leq\; P_{max}^{ev} \;\; \forall t \in T}$$

The EV charging power cannot exceed its maximum allowable limit.

b. EV Final Battery Energy Equals Accumulated Charge

$$\boldsymbol{E_{tf}^{ev}} = \sum_{t \in T} \boldsymbol{P_t^{evb}}$$

The final stored energy in the EV battery equals the total charged energy.

c. Final Battery Target Level

$$\boldsymbol{E_{tf}^{ev}} = \boldsymbol{\rho} * \boldsymbol{E_{max}^{ev}}$$

Ensures that the EV battery reaches its required charge level by the end of the time period.

8- Home Battery

a. Initial Battery Energy Balance

$$\boldsymbol{E_0^b} = \boldsymbol{E_{init}^b} - \boldsymbol{P_0^{bi}}$$

Sets the home battery's initial energy level based on starting energy and initial discharge.

b. Battery Energy Update

$$\boldsymbol{E_t^b} = \boldsymbol{E_{t-1}^b} + \boldsymbol{P_t^{br}} - \boldsymbol{P_t^{bi}} - \boldsymbol{P_t^{sell}} \quad \forall \boldsymbol{t} \in \boldsymbol{T}, \boldsymbol{t} \geq \boldsymbol{1}$$

Updates the home battery’s energy considering charging, discharging, and energy sold to the grid.

c. Discharge Cannot Exceed Stored Energy

$$\boldsymbol{P_t^{bi}} \leq \boldsymbol{E_{t-1}^b} \quad \forall \boldsymbol{t} \in \boldsymbol{T}, \boldsymbol{t} \geq \boldsymbol{1}$$

$$\boldsymbol{P_0^{bi}} \leq \boldsymbol{E_{init}^b}$$

The battery cannot discharge more energy than it currently holds.

d. Prevent Overcharging Beyond Battery Capacity

$$\boldsymbol{P_t^{br}} \leq \boldsymbol{E_{max}^b} - \boldsymbol{E_{t-1}^b} \quad \forall \boldsymbol{t} \in \boldsymbol{T}, \boldsymbol{t} \geq \boldsymbol{1}$$

The battery cannot be charged beyond its maximum capacity.

e. Mutual Exclusivity: Charging or Discharging

$$\boldsymbol{P_t^{br} \leq P_{max}^{br} * mode_t^{pr} \quad \forall t \in T}$$

$$\boldsymbol{P_t^{bi} \leq P_{max}^{bi} * (1 - mode_t^{pr}) \quad \forall t \in T}$$

The battery cannot charge and discharge simultaneously.

Chapter 4
Model Implementation

4.1 Implementation Setup

To implement the smart home energy optimization model, we used a combination of software tools input data and model configurations. This section explains the setup in detail.

Software and Tools:

The project was first implemented by CPLEX, a powerful optimization solver, to find the best schedule for appliances and energy usage. CPLEX is ideal for solving linear and mixed-integer programming problems efficiently. Then, we programmed the model by genetic algorithm called NSGA-II in Python with key libraries like:

1. DEAP
   a. We used the DEAP library in Python to efficiently implement the NSGA-II genetic algorithm for our optimization model. It provided built-in evolutionary computation tools that simplified handling multi-objective trade-offs (energy cost vs. discomfort), custom constraints and variable types (binary appliance schedules, continuous power flows).
2. Matplotlib for dynamic visualization of Pareto fronts and energy schedules (e.g., energy usage graphs).
3. Pandas for handling and organizing input data (e.g., appliance profiles, energy prices).
4. NumPy for numerical computations (e.g., calculating energy balances).

Data Inputs:

The model required several types of input data to function:

1. Time Horizon: We analyzed a 24-hour day divided into 1-hour intervals ($\Delta t = 1$ hour). This allowed us to schedule appliances and energy usage in manageable time slots.
2. Appliance Profiles:
3. Controllable loads (e.g., washing machine, electric vehicle charger) were defined by their power ratings, allowed operating time windows, and how long they needed to run.
4. Non-controllable loads (e.g., lights, refrigerator) had fixed energy consumption patterns based on historical data.
5. Renewable Generation: We used solar and wind power forecasts to predict how much renewable energy would be available at each hour.
6. Cost Parameters: Electricity prices varied by time of day (time-of-use tariffs), and we also

considered how much the grid would pay for excess energy sold back (sell-back rates).

7. Home Battery Parameters: Initial Home battery energy (E^{b}_{init}), Maximum battery capacity (E^{b}_{max}), Maximum charging power (P^{br}_{max}), Maximum discharging power (P^{bi}_{max}).
8. Electrical Vehicle: Maximum stored energy from electric vehicle/Max EV capacity (E^{ev}_{max}), Maximum power charging from electric vehicle in Δt (P^{ev}_{max}), Percentage of battery storage from electric vehicles (ρ).

4.2 CPLEX Implementation for Smart Home Energy Optimization

4.2.1 Overview of CPLEX

IBM CPLEX is a high-performance optimization solver designed to solve complex decision-making problems in operations research and industrial applications. It solves linear programming (LP), mixed-integer programming (MIP), and quadratic programming (QP) problems. With its powerful algorithms and efficient constraint handling, CPLEX is widely used in logistics, energy management, and financial modeling. It guarantees optimal solutions and is fast for small or medium problems. Its limitations in handling non-linearities, fuzzy and large-scale uncertainties led us to complement it with metaheuristic approaches like NSGA-II for our smart home energy optimization project. This hybrid strategy combines CPLEX with the flexibility of evolutionary algorithms to address real world complexities.

4.2.2 Key Strengths of CPLEX

1- Exact Optimization: Guarantees globally optimal solutions for linear and convex problems.
2- Mixed-Integer Linear Programming (MILP): Combines discrete (appliance scheduling) and continuous (power flows) variables.
3- Single-Objective Support: Weighted-sum approach is used to convert multiple-objective problems into single-objective problems.
4- Constraint Handling: Prevents infeasible or unrealistic schedules and ensures considering user preferences.

4.2.3 Model Initialization and implementation

Proper initialization ensures CPLEX interprets the problem correctly.

Missing variables or mis formulated constraints can lead to infeasible or suboptimal results.

Model Creation:

The optimization model begins by creating a structured framework using IBM's CPLEX solver to organize variables and constraints logically. We name the model, it helps identify the model and serves as a container for all variables, constraints and objectives (e.g., "Smart_Home_Energy_Scheduling").

Defining sets:

Time Horizon: we define the scheduling period as a 24-hour day, (Δt = 1 hour).

Appliance Profiles: a list of controllable appliances that can be scheduled. These are devices whose operation can be shifted (e.g., CELs = ["Washing Machine", "Dryer", ...]). The model optimizes their power demand, duration and time windows, affect the solution.

Defining Parameters:

Parameters are fixed inputs that define the problem's conditions. There are three categories:

Energy System Parameters: Renewable Generation (P_sun,P_wind) and energy Costs (c_grid, c_re)

Appliance Parameters: Power Demand (P_j^{cl}) and time windows (T_j^{cli},T_j^{clf})

Battery/EV Parameters: Home Battery (E_{init}^{b}, E_{max}^{b},P_{max}^{br},P_{max}^{bi}) and Electric Vehicle (E_{max}^{ev},P_{max}^{ev},ρ)

Objective Function:

Our model is a multi-Objective Optimization model. We seek to achieve 2 objectives and find the best trade-offs between them: Minimize energy cost (grid + renewable costs - revenue from selling power to grid) and minimize user discomfort (delays in appliance start times).
We set weights *w1 and w2* for each objective representing how important each objective is to the decision maker and minimize the weighted sum of normalized energy cost (w1*E_cost_norm) and normalized user discomfort (w2*D_norm).

Constraints:

Energy Balance Constraint: The total power available (from solar, wind, grid, and battery discharge) must exactly match the power used (by appliances, EV charging, battery charging, and non-controllable loads) at every hour. This ensures no energy shortages or surpluses.

Appliance Scheduling Constraints:

1. Operation Window: Each appliance can only run within its allowed time range T_j^{cli},T_j^{clf}. This reflects user preferences.
2. Duration Enforcement: An appliance must run for its exact required duration (e.g., the dishwasher for 2 hours)
3. Precedence Rules: Some appliances must finish before others start (e.g., the dryer runs after the washing machine finishes). This models real world dependencies.

Battery Management Constraints: The home battery system had limits on its initial energy, maximum capacity, charging rates and discharging rates. These constraints ensured the battery operated safely and efficiently without overcharging or draining completely.

1. Capacity Limits: The battery can't store more than its maximum capacity (e.g., 30 kWh) or discharge below zero
2. Charge/Discharge Rates: The battery can't charge or discharge more than its rated power (e.g., 5 kW).
3. No Simultaneous Charge/Discharge: The battery can't charge and discharge at the same time. This prevents illogical operations.

EV Charging Constraints: The electric vehicle must charge to at least the required level (e.g., 100% of its capacity) by the end of the schedule.

Grid Interaction:

1. Selling Only When Profitable: power can only be sold back to the grid if the selling price is higher than the grid cost.
2. Buy/Sell Exclusivity: The system can't buy and sell electricity at the same time.

4.2.4 Limitations of CPLEX

1. Struggles with Complex, Non-Linear Problems: CPLEX works best for linear problems. Our smart home model had non-linear parts (e.g., battery efficiency, dynamic pricing), making CPLEX slow or unable to find a solution.

2. Limited Handling of Multiple Conflicting Goals: CPLEX optimizes a single combined objective (e.g., w1*cost + w2*discomfort). Changing weights w1, w2 requires resolving repeatedly to explore trade-offs. So, it needs manual weight tuning. NSGA-II helped in finding many optimal solutions at once (Pareto front), showing how cost and discomfort trade off.

3. Inflexible with Uncertainty: CPLEX assumes perfect knowledge (e.g., exact solar power output) and can't adapt uncertainty. Real world energy data is unpredictable. NSGA-II helped in incorporating randomness (e.g., varying renewable energy) by evaluating solutions under different scenarios. NSGA-II gave more flexible, realistic solutions for our smart home problem.

4.3 NSGA-II Implementation for Multi-Objective Optimization

4.3.1 Overview of NSGA-II

The Non-dominated Sorting Genetic Algorithm (NSGA) proposed by Srinivas and Deb in 1994 has been applied to various problems. However there have been a number of criticisms of the NSGA as shown in [7] such as its highest computational complexity, lack of elitism and need for specifying the sharing parameter. It has been modified to the NSGA-II approach in order to overcome all the difficulties.

NSGA-II (Non-dominated Sorting Genetic Algorithm II) is a evolutionary algorithm used to solve multi-objective optimization problems such as balancing cost savings and user comfort in smart homes. Unlike traditional methods that give a single "best" solution, NSGA-II finds a set of optimal trade-offs (called the Pareto front), where neither objective can be improved without worsening the other, letting decision makers choose between solutions in this set (user preference). It uses the crowding distance operator for the mechanism of diversity preservation. It is computing fast. [8] showed the basic structure of NSGA-II. It is based on five main principles, which are: Non-Dominated Sorting, Elite Preserving Operator, Crowding Distance, Selection Operator and Genetic Operations such as Cross Over and Mutation. Procedure of NSGA-II was represented in [9] which illustrates how the algorithm works.

4.3.2 Key Strengths of NSGA-II

1. Competing Goals: Our problem has two conflicting objectives: minimizing energy costs (run appliances when electricity is cheapest) and discomfort (avoid delaying appliances too much). NSGA-II optimizes both simultaneously.

2. Works with Complex Constraints: Appliances have specific time windows and precedence rules

3. NSGA-II handles these gracefully using penalty functions and repair mechanisms.

4. Finds Diverse Solutions: Unlike CPLEX (which gives one solution), NSGA-II outputs multiple options along the Pareto front, empowering users to pick their preferred balance.

4.3.3 Implementation of model

4.3.3.1 System Architecture Overview

In our project, we used an NSGA-II algorithm to handle both the deterministic and stochastic smart home energy scheduling models. The core algorithm structure, including population initialization, non-dominated sorting, crowding distance calculation, selection, crossover, and mutation, remains identical across both models. The key difference lies in the evaluation of objective functions, where the deterministic model uses a single scenario for evaluation, while the stochastic model evaluates solutions over multiple randomly generated scenarios to capture uncertainty in renewable generation and user behavior. The implementation follows a modular, object-oriented design with a clear separation of concerns:

Core Classes:

Device: Represents individual appliances with operational constraints

Scenario: Manages device collections and precedence relationships

Battery: Models the home battery storage system

ElectricVehicle: Models the EV charging requirements

SmartHomeScheduler: Core optimization engine

StochasticEnergySystem: Handles uncertain energy parameters and sampling

StochasticSmartHomeScheduler: Implements the main NSGA-II optimization engine

StochasticAnalyzer: Doing multiple optimizations runs and statistical analysis

DEAP Framework Integration:

Multi-objective fitness with weights (-1.0, -1.0) for minimization

Custom crossover and mutation operators tailored to problem structure

NSGA-II selection mechanism for Pareto-optimal solutions

4.3.3.2 Solution Representation

The solution encoding represents the scheduling decisions for all appliances across the optimization horizon. Each individual (chromosome) in the population is encoded as a vector for use within NSGA-II, contains:

Device start times:

Stored as integers (0–23) indicating the hour each controllable device starts.

Battery operation modes:

Encoded as integers for each hour:

0 = idle

1 = charge

2 = discharge

EV operation modes:

Encoded similarly to the battery for each hour.

4.3.3.3 Algorithm Parameters

The NSGA-II algorithm relies on several hyperparameters that control its exploration, exploitation, and convergence toward the Pareto front:

Population Size: Number of individuals in each generation.

Number of Generations: Number of iterations for evolving the population (typically 50–500).

Crossover Probability: Probability of applying crossover between two individuals to exchange parts of their encoded schedules (typically 0.7).

Mutation Probability: Probability of mutating each part of an individual's encoded schedule (device start time, battery mode, EV start).

4.3.3.4 Objective Function Implementation

Each chromosome is decoded into actionable schedules before evaluation, ensuring that objectives reflect realistic appliance operations, battery usage, and EV charging patterns under system constraints.

The objective function implementation in our NSGA-II smart home energy scheduling system is designed to evaluate each candidate's schedule based on two objectives:

Objective 1: Minimize Energy Cost

Energy cost is calculated as the total cost of energy provided minus the revenue from selling excess energy, ensuring economic efficiency.

Objective 2: Minimize User Discomfort

User discomfort quantifies the delay between the user's earliest preferred start time for a device and its scheduled start time.

4.3.3.5 Constraint Handling Strategy

The implementation adopts a strategy to ensure solution feasibility during optimization:

Constraints Handled:

1- Device operational windows and durations.
2- Device precedence constraints (e.g., Dryer starts after Washing Machine).
3- Limits on simultaneous device operation (lambda - λ) and maximum demand (gamma - γ).
4- Battery and EV operational constraints.
5- Grid import/export limits.

If violations persist after repair attempts, scaled penalties are applied to the objective values, guiding the optimizer toward feasible, high-quality solutions.

4.3.3.6 Stochastic Model Integration

For handling uncertainty in the stochastic model, the implementation incorporates:

Scenario Generation:

Randomized Renewable Generation

In the EnergySystem class, Parameters Made Stochastic: [Solar Generation (P_sun), Wind Generation (P_wind), Grid Prices (C_grid), Renewable Prices (C_re), Selling Prices (R_sell), Non-Controllable Load (P_ncl)]

Distributions Used for Stochastic Modeling

1- Gaussian (Normal) Distribution:

Solar Generation with std = 0.03

Wind Generation with std = 0.03

Non-Controllable Load with std = 0.01

2- Uniform Distribution: Grid Prices, Renewable Prices, Selling Prices

Conversion Steps

Start from deterministic hourly base profiles (X_base).

Apply Gaussian noise or Uniform scaling as appropriate for each parameter.

Use np.clip to enforce realistic lower/upper bounds on values.

Result: For each run with USE_STOCHASTIC_INPUTS = True, a unique but realistic scenario is generated.

User Behavior Variability:

User demand and device start probabilities are indirectly varied by running the optimizer multiple times with different seeds across num_runs, generating diverse Pareto fronts under varied conditions.

Robust Solution Evaluation:

The NSGA-II algorithm is executed repeatedly (e.g., 30 runs) under different random seeds, enabling:

Generation of multiple Pareto fronts reflecting variability in the system.

Statistical Analysis:

After all runs, solutions are aggregated, and: Duplicate solutions are removed, non-dominated solutions across all runs are extracted

Visualization and Analysis:

Histograms and line charts show distribution of costs, discomforts, demand, Battery modes and device start times under variability.

4.3.3.7 Performance Optimization

To ensure the scalability and efficiency of the NSGA-II smart home scheduling implementation, several performance optimization strategies are incorporated within the code:

a. Parallelization of Fitness Evaluation:

   self.toolbox.register("map", map)

b. Decaying Mutation Probability schedule:

   indpb = 0.3 × (1 − (gen / generations)) + 0.05

c. Constraint-Aware Mutation and Repair

   Custom mutation and crossover respect time windows and precedence, with post-crossover repair ensuring feasibility and reducing wasted evaluations.

d. Duplicate Solution Filtering.

e. Controlled Scenario Complexity: By toggling the number of devices (4, 8, 12) and using scenario selection

Chapter 5
Validation, Testing, and Results

5.1 Chapter Overview

In this chapter, we present the validation and testing of the optimization models developed in this project, including a CPLEX model and two variants of the NSGA-II (Non-dominated Sorting Genetic Algorithm II) algorithm. The NSGA-II is a widely recognized multi-objective evolutionary algorithm designed to approximate the Pareto front by producing a diverse set of non-dominated solutions with good convergence properties.

Specifically, we implemented two NSGA-II models: one with all parameters treated deterministically, and another incorporating stochastic parameters to capture uncertainty in the problem setting. This dual approach allows us to compare the robustness and performance of NSGA-II under both fixed and variable conditions.

The validation focuses on assessing the algorithms' performance regarding convergence to the true Pareto front, diversity of solutions, and overall solution quality. To this end, we tested the models on three scenarios with varying numbers of devices 4, 8, and 12 devices to evaluate scalability and effectiveness across different problem sizes.

Benchmark functions and well-established performance metrics were employed to rigorously evaluate the results. The CPLEX model serves as a baseline for exact optimization, while the NSGA-II variants provide heuristic solutions that balance multiple objectives efficiently. This chapter discusses the comparative outcomes, highlighting how the deterministic and stochastic NSGA-II models perform relative to each other and to the CPLEX benchmark in terms of solution quality and computational effort.

Overall, this chapter demonstrates the successful implementation and validation of both deterministic and stochastic NSGA-II models alongside the CPLEX model, providing insights into their applicability for multi-objective optimization problems involving different device scales.

5.2 CPLEX Model

Appliances Scheduling Table: We performed the optimization model over 4 appliances (Washing Machine, Dryer, Dishwasher and Oven).

Cost and Comfort Metrics: we performed normalization over multi-objective functions (Energy Cost measured in dollars and Discomfort measured in scaling scores), Normalization is the process of scaling different objectives to a common range (typically [0, 1]) to ensure fair comparison and meaningful weighting, so that we can combine them in one single objective. It is important in avoiding unit bias and balancing weighted sums between trade-offs.

5.2.1 Test Case:

1. Table 5.1 shows the values of time and devices parameters.

Parameter	Example Value
T	range (24)
CELs	["Washing Machine", "Dryer", "Dishwasher", "Oven"]
T_i	0
T_f	23

1.Table 5.1(Time and Devices Parameters)

2. Table 5.2 shows the values of costs and renewable energy soures.

Parameter	Example Value
P^{sun}	[0.0, 0.0, 0.1, 0.3, 0.7, 1.0] *4
P^{wind}	[0.3, 0.4, 0.5, 0.6, 0.7, 0.8] *4
C^{re}	[0.15, 0.10, 0.08, 0.07, 0.09, 0.05] *4
C^{grid}	[0.25, 0.20, 0.18, 0.15, 0.12, 0.10, 0.09, 0.08, 0.07, 0.06, 0.05, 0.04, 0.07, 0.10, 0.13] *2
R^{sell}	[0.20, 0.15, 0.25, 0.05, 0.20, 0.06, 0.08, 0.10, 0.20, 0.10, 0.08, 0.20, 0.15, 0.12, 0.10] *2

2.Table 5.2(Renewable Energy and Costs)

3. Table 5.3 shows the values of non-controllable appliances.

Parameter	Example Value
P^{ncl}	[1.5, 1.2, 1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0.2, 0.1, 0.5] *2

3.Table 5.3(Non-Controllable Load)

4. Table 5.4 shows the values of preference time of each appliance.

Parameter	Example Value
Tcli	{"Washing Machine": 1, "Dryer": 5, "Dishwasher": 7, "Oven": 9}
Tclf	{"Washing Machine": 8, "Dryer": 11, "Dishwasher": 10, "Oven": 13}

4.Table 5.4(Device Scheduling Windows)

5. Table 5.5 shows the values of power load and duration of each appliance.

Parameter	Example Value
P^{cl}	{"Washing Machine": 1.0, "Dryer": 0.8, "Dishwasher": 0.6, "Oven": 1.2}
d_i	{"Washing Machine": 3, "Dryer": 1, "Dishwasher": 2, "Oven": 1}

5.Table 5.5(Device Load and Duration)

6. Table 5.6 shows the value of system constant.

Parameter	Example Value
λ	2
γ	20
P_grid_Max	20
P_sell_Max	4

6.Table 5.6(System Constants)

7. Table 5.7 shows the values of home battery parameters.

Parameter	Example Value
E_b_init	5.0
P_br_max	5.0
P_bi_max	5.0
E_b_max	30.0

7.Table 5.7(Home Battery)

8. Table 5.8 shows the values of EV battery parameters.

Parameter	Example Value
Pev_max	30.0
Eev_max	20.0
p	1.0

8.Table 5.8(EV Battery)

9. Table 5.9 shows the values of multi-objective parameters.

Parameter	Example Value
W1	0.7
W2	0.3
Z_cost_max	195.85
Z_discomfort_max	7.0

9.Table 5.9(multi-objective parameters)

5.2.2 Results:

Table 5.10 shows the results, considering weights w1=0.7, w2=0.3 reflect user priorities. Adjusting these weights allowed us to prioritize either saving money or minimizing delays in appliance operation.

	Total Energy Cost	Total Discomfort
Before Normalization	2.29	0.00
After Normalization	0.0117	0.00

10.Table 5.10(CPLEX Objectives Result)

Combined Objective Value (Weighted Sum): 0.0082

Table 5.11 summarizes the power flow (kW) between energy sources and loads for each slot. At each time step, Energy Balance ensures supply meets demand and Renewable Priority makes power taken from renewable resources (P^{re}) is fully utilized before consuming from the battery (P^{bi}) or the grid (P^{grid}). System switches between charging (storing) and discharging (suppling) the battery to optimize cost and renewable usage.

	Supply			Demand				
Time Period	P_re	Pgrid	P_bi	P_br	Pncl	P_evb	Pclr	Pcl_total
0	0.3	0	1.2	0	1.5	0	0	0
1	0.4	0	1.8	0	1.2	0	0	1
2	0.6	0	1.4	0	1	0	0	1
3	0.9	0.4	0.6	0	0.9	0	0	1
4	1.4	0	0	0.6	0.8	0	0	0
5	1.8	0	0	0.3	0.7	0	0	0.8
6	0.3	0	0.3	0	0.6	0	0	0

11.Table 5.11(Sample Power results)

Table 5.12 shows the washing machine's operation schedule, where it starts (connected) at hour 1, it remains ON for hours 1-3, it stops (disconnected) at hour 4 and is marked finished from hour 4 till the end of the day.

Time Period	X_j	α_j	β_j	δ_j
0	0	0	0	0
1	1	1	0	0
2	1	0	0	0
3	1	0	0	0
4	0	0	1	1
5	0	0	0	1
6	0	0	0	1

12.Table 5.12(Washing Machine schedule)

Table 5.13 shows the dryer's operation schedule, where it starts (connected) at hour 5, it remains ON for an hour, it stops (disconnected) at hour 6 and is marked finished from hour 6 till the end of the day.

Time Period	X_j	α_j	β_j	δ_j
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	1	1	0	0
6	0	0	1	1
7	0	0	0	1

13.Table 5.13(Dryer schedule)

Table 5.14 shows the dishwasher's operation schedule, where it starts (connected) at hour 7, it remains ON for 2 hours, it stops (disconnected) at hour 9 and is marked finished from hour 9 till the end of the day.

Time Period	X_j	α_j	β_j	δ_j
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
7	1	1	0	0
8	1	0	0	0
9	0	0	1	1
10	0	0	0	1

14.Table 5.14(Dishwasher schedule)

Table 5.15 shows the oven's operation schedule, where it starts (connected) at hour 9, it remains ON for an hour, it stops (disconnected) at hour 10 and is marked finished from hour 10 till the end of the day.

Time Period	X_j	α_j	β_j	δ_j
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	1	1	0	0
10	0	0	1	1
11	0	0	0	1

15.Table 5.15(Oven schedule)

5.3 NSGA-II Model

The NSGA-II algorithm was executed to identify optimal trade-offs between minimizing energy cost and user discomfort for a 24-hour smart home scheduling scenario. The results were analyzed with deterministic parameters across multiple independent runs to validate the robustness and consistency of the generated Pareto-optimal solutions. Each run of the algorithm produced a set of non-dominated solutions. These represent candidate configurations that balance energy cost and user comfort without being outperformed by other solutions in both objectives.

5.3.1 Test Cases

a. Pareto Front generation: NSGA-II runs 30 times (each with a population of 200 and 500 generations) to ensure robustness.

b. Post-Processing Pareto Fronts we do the following:
   i. Remove Duplicates: Within a single run, no identical solutions exist in a Pareto front and across all 30 runs, solutions that appear in multiple runs are removed.
   ii. Remove Dominated Solutions: filtering out solutions that are dominated by others across all runs, leaving only the global non-dominated set.

c. Scenario Testing: We have run the simulation under three different scenarios each with a population size of 200, 500 generations and 30 runs (each produces a pareto front).

5.3.2 Scenarios

Table 5.16 shows the difference between each scenario problem size.

Scenario	Appliances	Precedence	Binary Vars	Continuous Vars	Total Decision Vars	Constraints
Scenario 1	4	1	432	173	605	~690
Scenario 2	8	2	816	177	993	~1,160
Scenario 3	12	4	1,200	181	1,381	~1,633

16.Table 5.16(NSGA-II Scenarios)

a. Appliances: {"Washing Machine", "Dryer", "Dishwasher", "Oven", "Rice Cooker", "Kettle", "Heater", "Air Conditioner", "Vacuum Cleaner", "Smart fan", "Pool Pump", "Water Heater"}.

b. Precedence: "Dryer": "Washing Machine", "Dishwasher": "Oven", "Oven": "Rice Cooker", "Smart fan": "Air Conditioner"}

5.3.3 Result of deterministic model

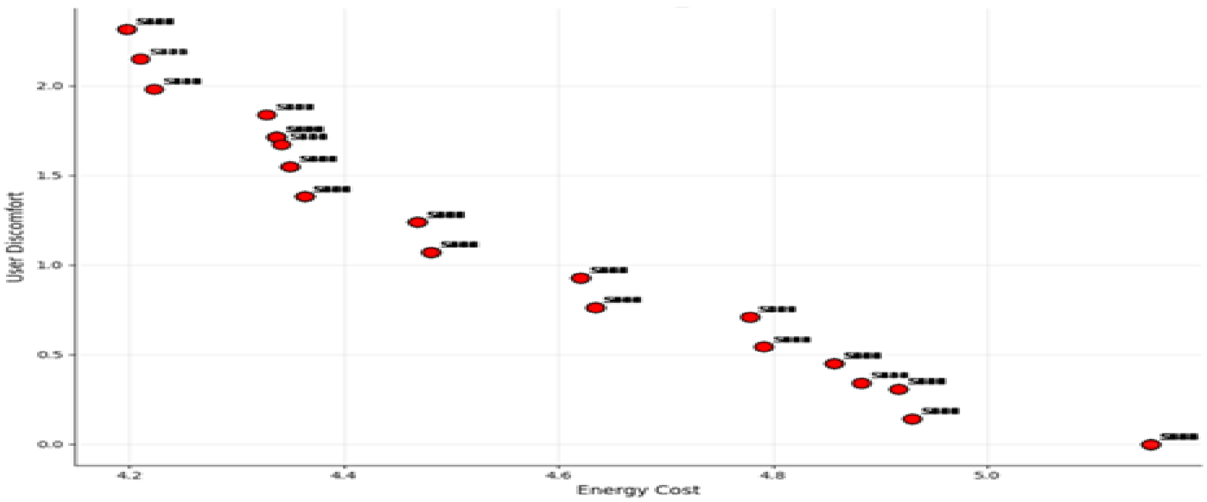
a) Scenario 1:

Number of Unique Pareto Fronts (after cross-run deduplication): 30
Each Pareto front corresponds to one execution of the NSGA-II algorithm.

Number of Non-Dominated Solutions Across All Runs: 825
These are raw, per-run non-dominated solutions, some of which may overlap or be weakly dominated across different runs

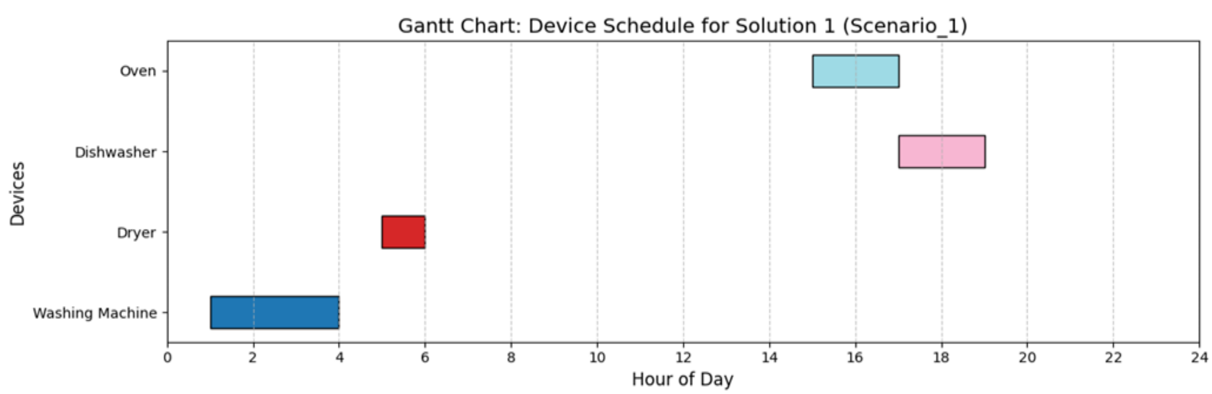
Each solution represents a trade between cost and comfort. Ex: Solution 1: Energy Cost = 5.15, Discomfort = 0.00

Figure 5.1 shows the distribution of Pareto-optimal solutions across energy cost (USD) and user discomfort (normalized scale) for Scenario 1 (4 devices). Each point represents a non-dominated solution from 30 NSGA-II runs, illustrating trade-offs between minimizing cost and discomfort.



1.Figure 5.1(Pareto Front of scenario 1)

Figure 5.2 shows the scheduling of each device over 24 slots for scenario 1. Bars indicate active time slots for each device, adhering to precedence constraints.



2.Figure 5.2 (Gantt Chart of scenario 1)

b) Scenario 2:

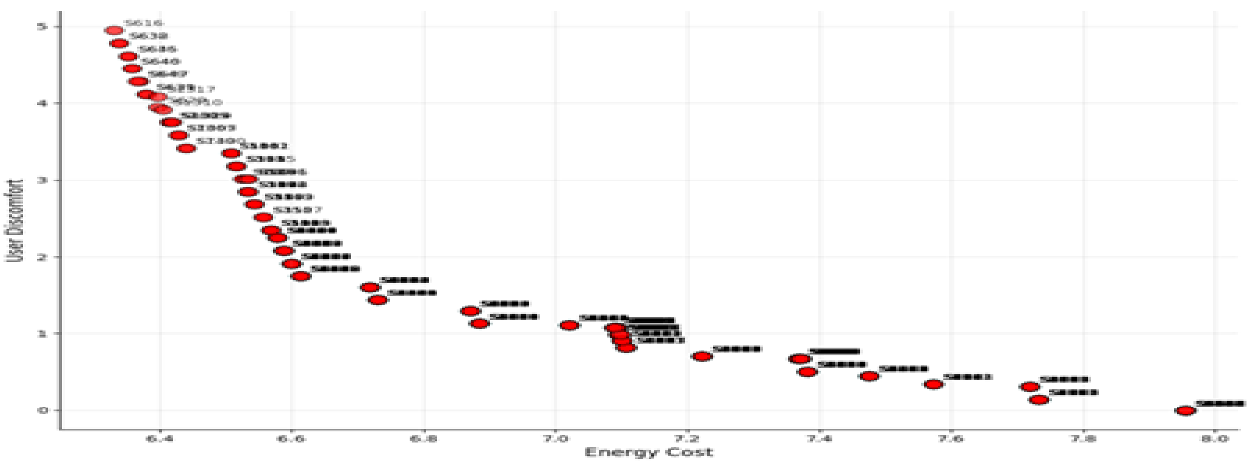
Number of Unique Pareto Fronts (after cross-run deduplication): 30
Each Pareto front corresponds to one execution of the NSGA-II algorithm.

Total Unique Solutions Across All Pareto Fronts: 1787
This includes all distinct solutions discovered, including intermediate dominated ones.

Number of Non-Dominated Solutions Across All Runs: 1360
These are raw, per-run non-dominated solutions, some of which may overlap or be weakly dominated across different runs

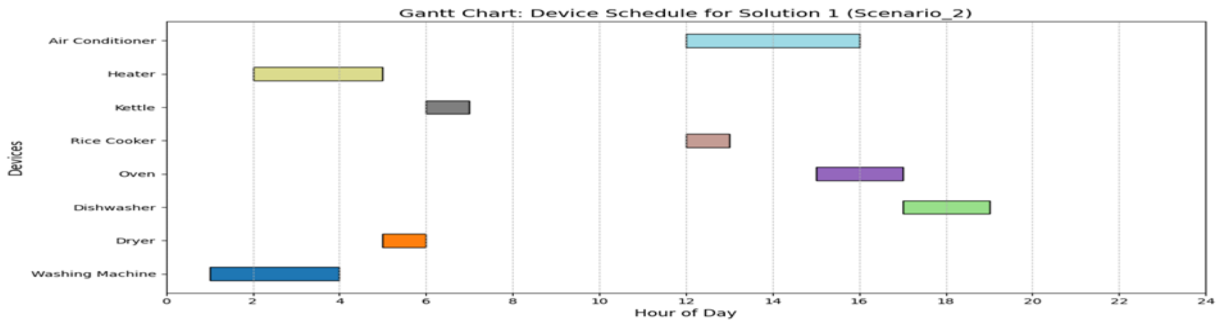
Each solution represents a trade of between cost and comfort. Ex: Solution 1: Energy Cost = 7.95, Discomfort = 0.00

Figure 5.3 shows the distribution of Pareto-optimal solutions across energy cost (USD) and user discomfort (normalized scale) for Scenario 2 (8 devices). Each point represents a non-dominated solution from 30 NSGA-II runs, illustrating trade-offs between minimizing cost and discomfort.



3.Figure 5.3 (Pareto Front of scenario 2)

Figure 5.4 shows the scheduling of each device over 24 slots for scenario 2. Bars indicate active time slots for each device, adhering to precedence constraints.



4.Figure 5.4 (Gantt Chart of scenario 2)

c) Scenario 3:

Number of Unique Pareto Fronts (after cross-run deduplication): 30

Each Pareto front corresponds to one execution of the NSGA-II algorithm.

Total Unique Solutions Across All Pareto Fronts: 2924

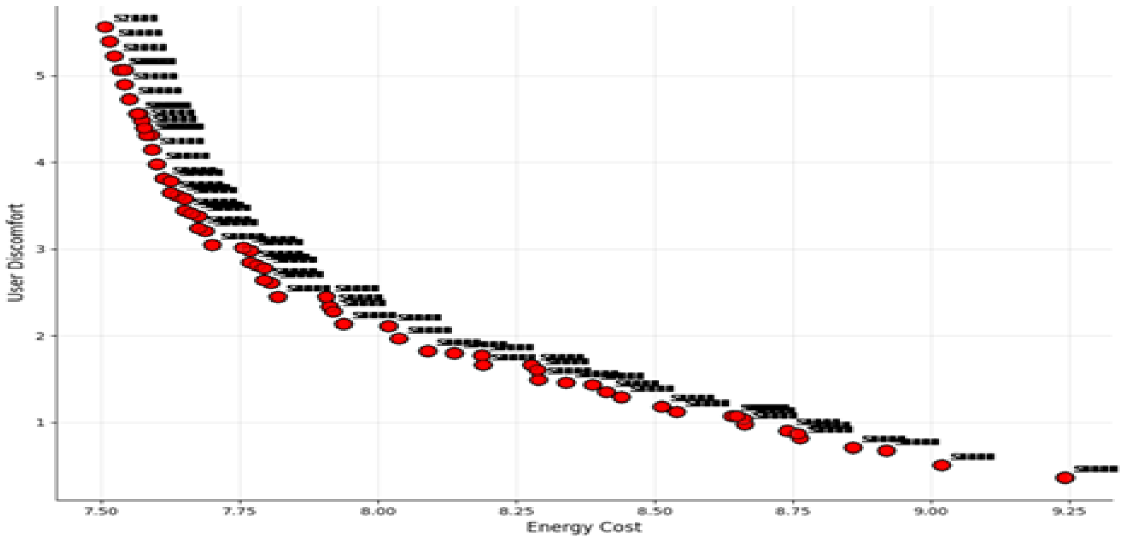
This includes all distinct solutions discovered, including intermediate dominated ones.

Number of Non-Dominated Solutions Across All Runs: 2792

These are raw, per-run non-dominated solutions, some of which may overlap or be weakly dominated across different runs

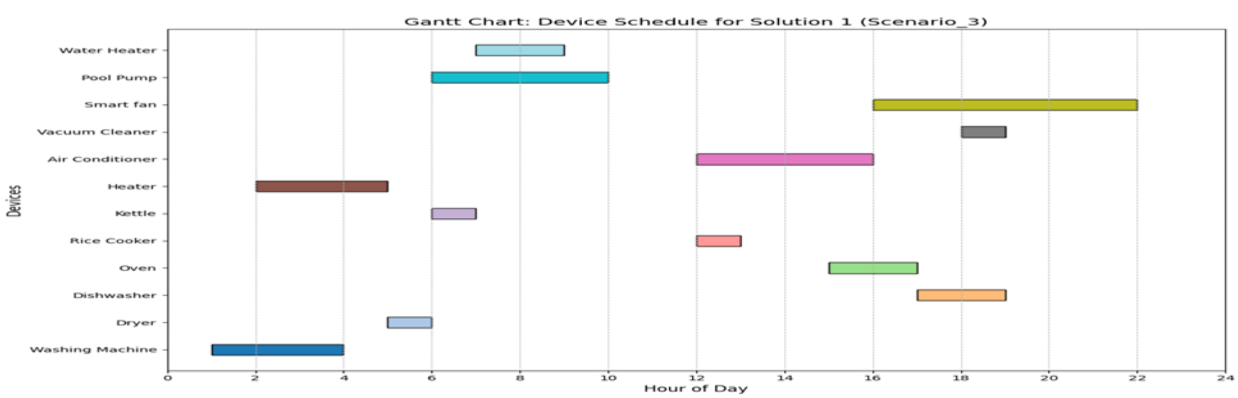
Each solution represents a trade of between cost and comfort. Ex: Solution 1: Energy Cost = 9.24, Discomfort = 0.36

Figure 5.5 shows the distribution of Pareto-optimal solutions across energy cost (USD) and user discomfort (normalized scale) for Scenario 3 (12 devices). Each point represents a non-dominated solution from 30 NSGA-II runs, illustrating trade-offs between minimizing cost and discomfort.



5.Figure 5.5 (Pareto Front of scenario 3)

Figure 5.6 shows the scheduling of each device over 24 slots for scenario 3. Bars indicate active time slots for each device, adhering to precedence constraints.

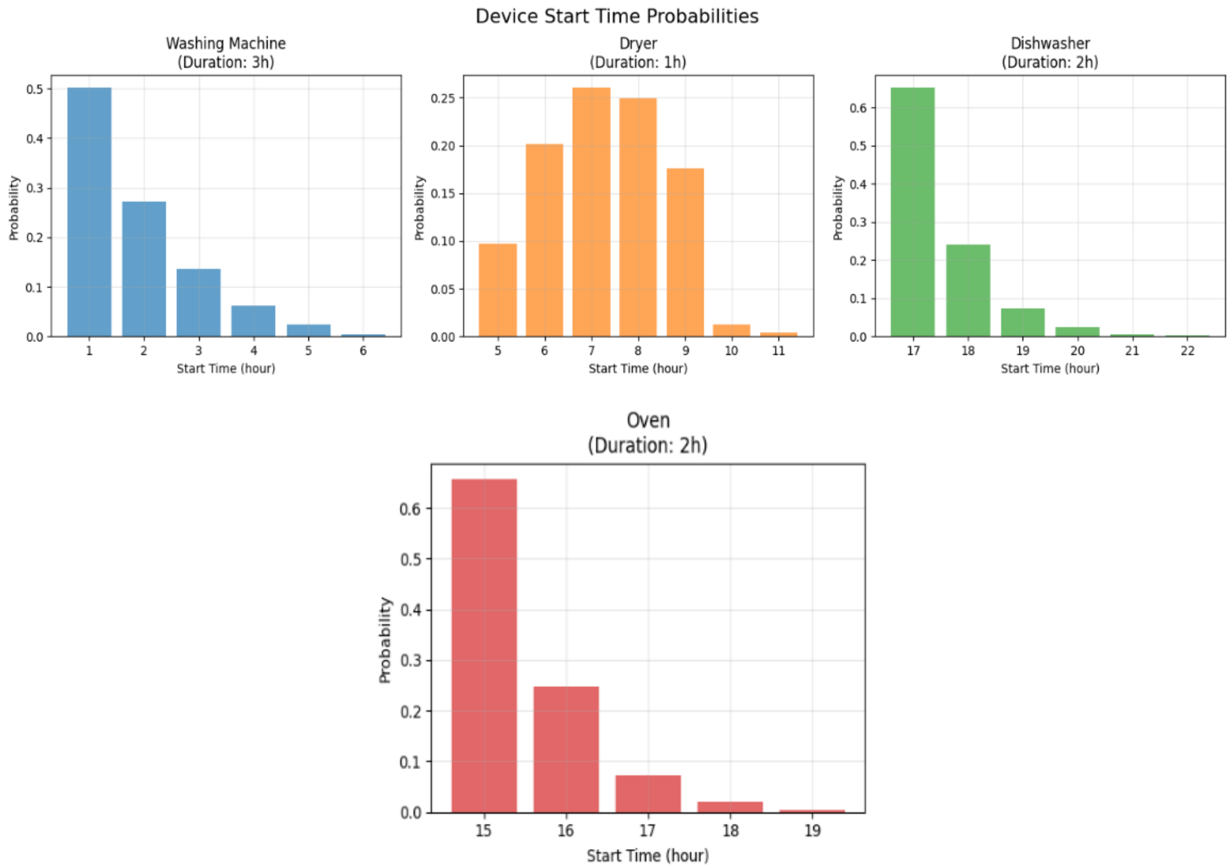


6.Figure 5.6 (Gantt Chart of scenario 3)

5.3.4 Result of Stochastic model

a) Scenario 1:

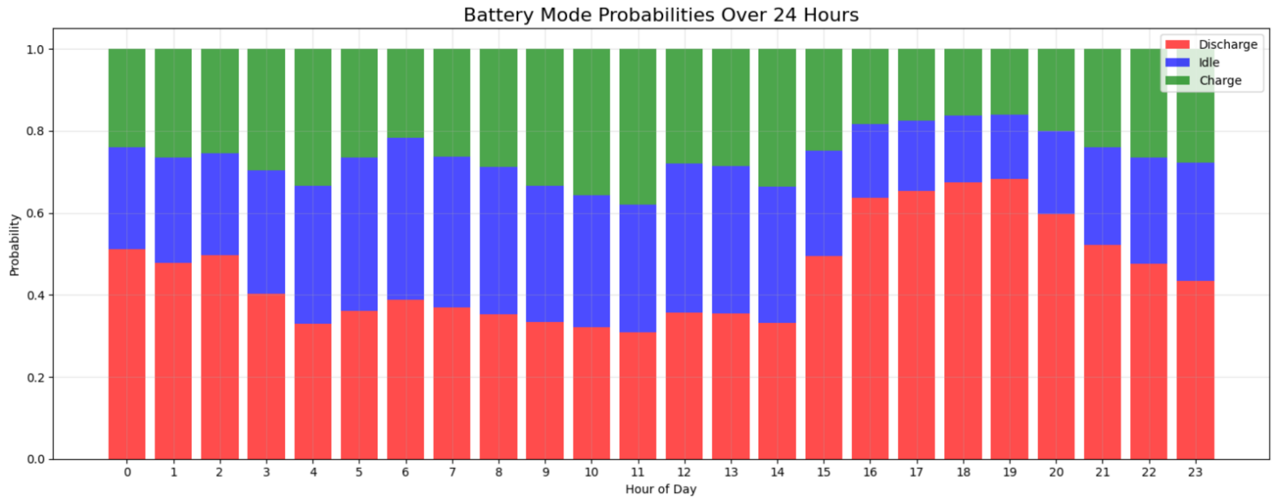
Figure 5.7 shows probability of distributions for optimal start times of 4 household appliances for scenario 1.



7.Figure 5.7 (Scenario 1 Device Start Time Probability Distributions)

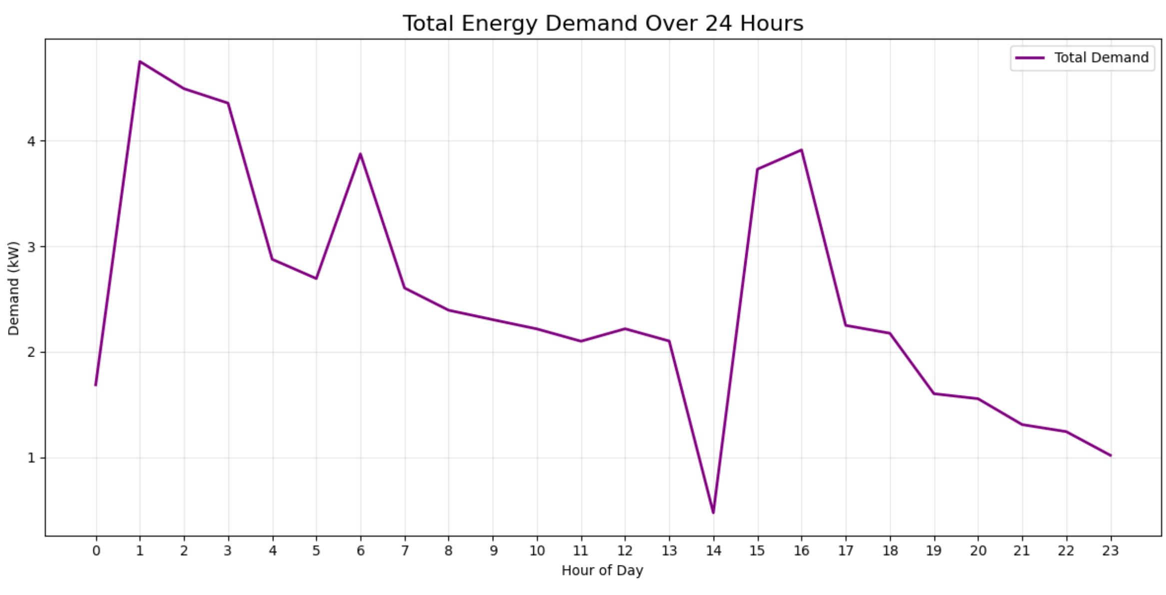
Figure 5.8 presents a probabilistic analysis of home battery operation modes across a 24-hour period for scenario 1, There are three operational modes:

- Charge: Occurs when surplus renewable energy is available
- Discharge: Activates during high-demand or expensive grid periods
- Idle: Neutral state during balanced conditions



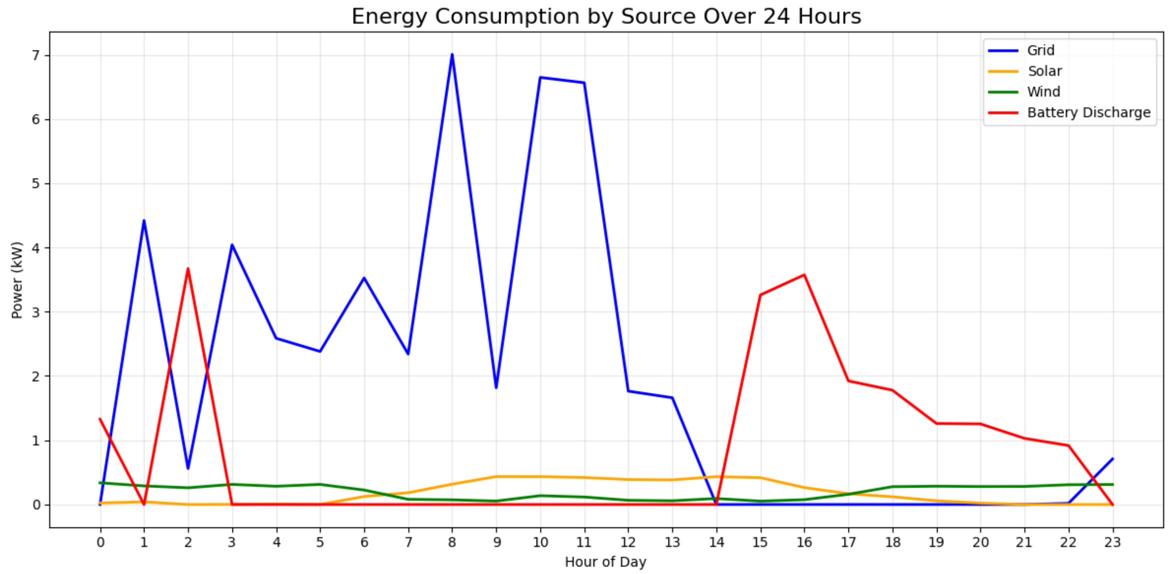
8.Figure 5.8 (Scenario 1 Battery Mode Probability Distribution Analysis)

Figure 5.9 visualizes the household's total energy demand (in kW) across a 24-hour period, showing consumption patterns and peak usage times. The x-axis represents hours of the day (0-23), while the y-axis shows power demand in kilowatts for scenario 1.



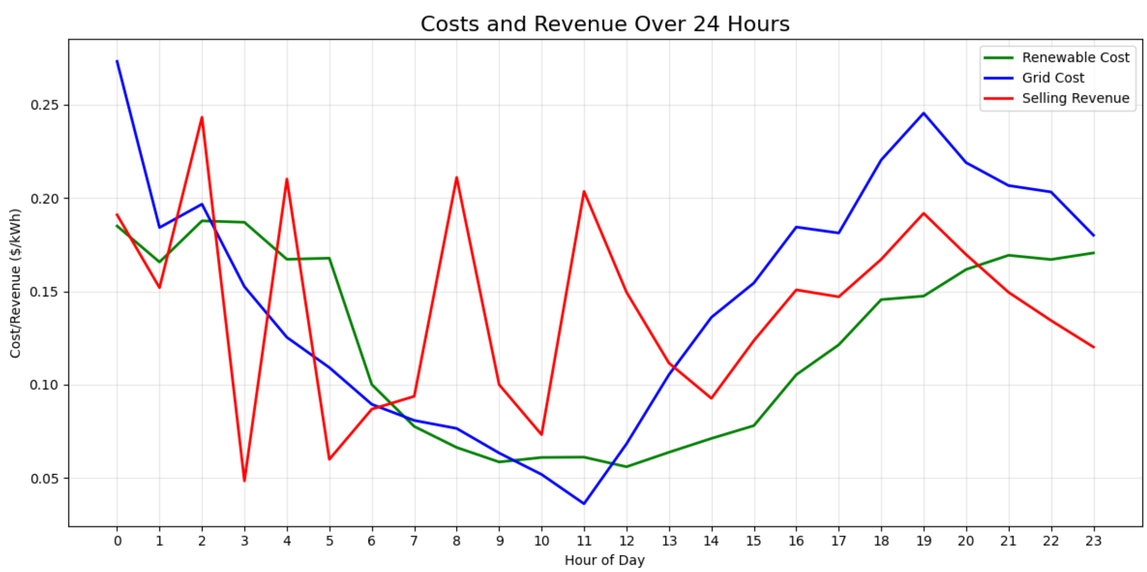
9.Figure 5.9 (Scenario 1 Total Energy Demand Profile)

Figure 5.10 illustrates the hourly power contribution from different energy sources across a 24-hour period, showing how the smart home system balances grid power, solar PV, wind generation, and battery discharge to meet household demand for scenario 1.



10.Figure 5.10 (Scenario 1 Energy Source Contribution Analysis)

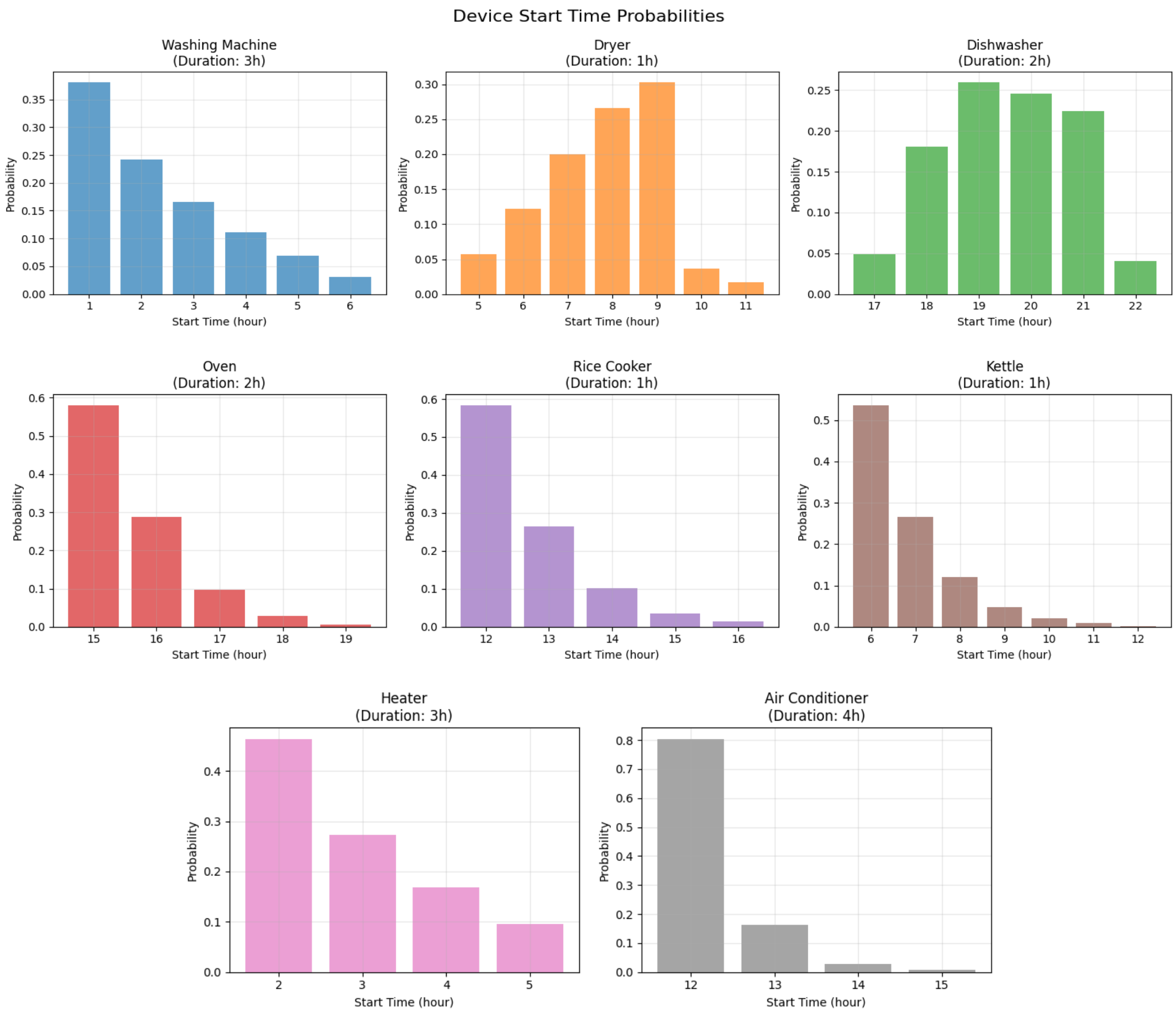
Figure 5.11 presents the hourly economic performance of the smart home energy system, comparing three key financial flows: Renewable Energy Cost, Grid Electricity Cost and Energy Selling Revenue for scenario 1.



11.Figure 5.11 (Scenario 1 Energy Cost and Revenue Analysis)

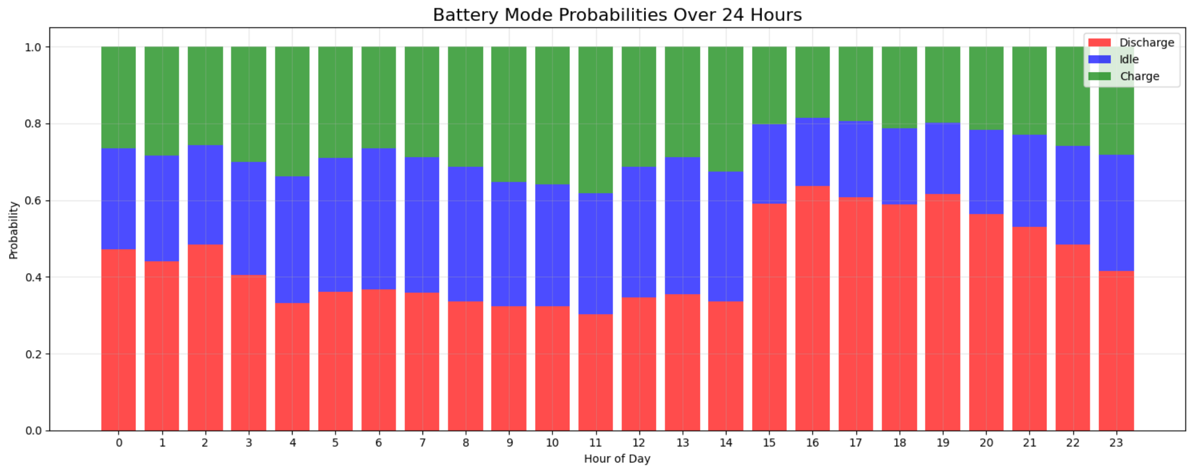
b) Scenario 2:

Figure 5.12 shows the probability of distributions for optimal start times of 8 household appliances for scenario 2.



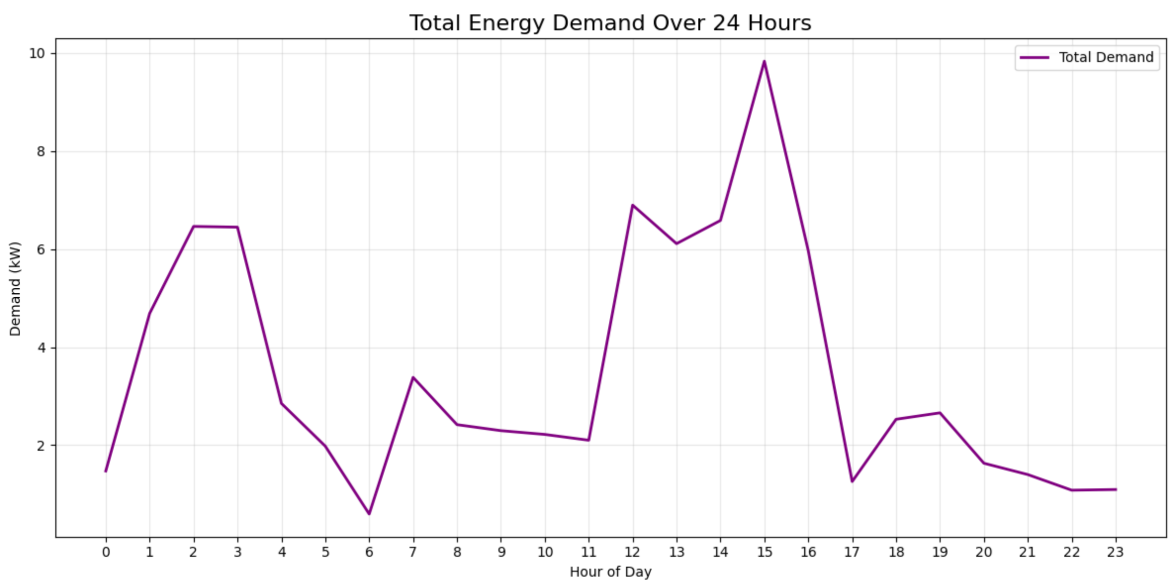
12.Figure 5.12 (Scenario 2 Device Start Time Probability Distributions)

Figure 5.13 presents a probabilistic analysis of home battery operation modes across a 24-hour period for scenario 2.



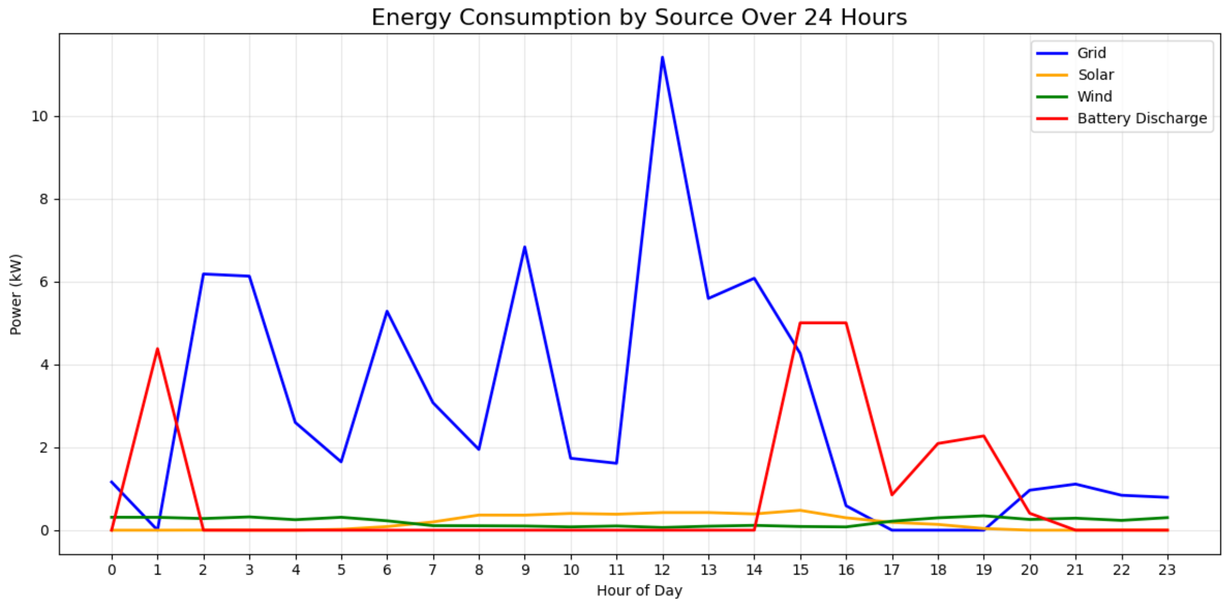
13.Figure 5.13 (Scenario 2 Battery Mode Probability Distribution Analysis)

Figure 5.14 visualizes the household's total energy demand (in kW) across a 24-hour period for scenario 2.



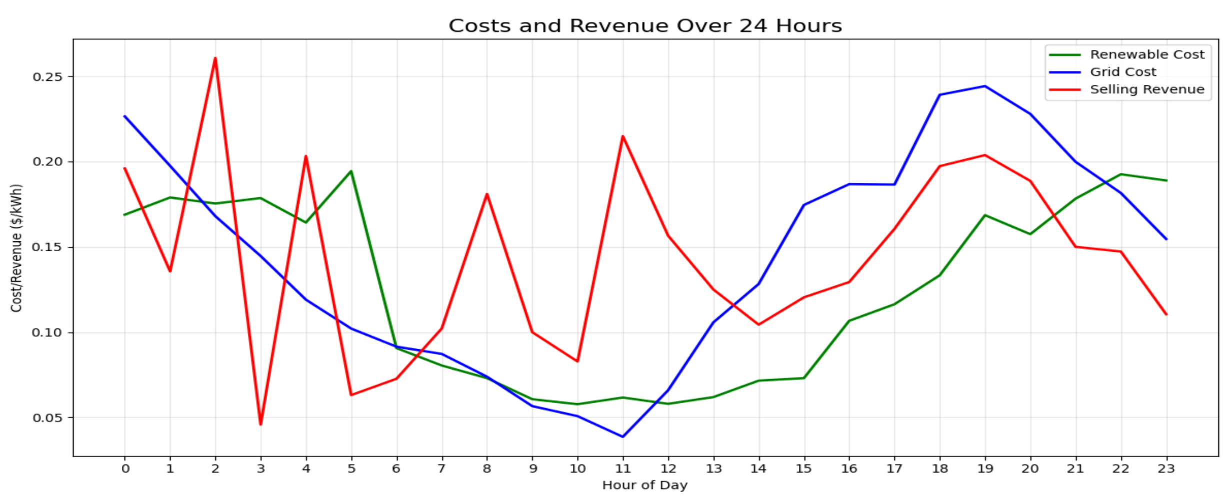
14.Figure 5.14 (Scenario 2 Total Energy Demand Profile)

Figure 5.15 illustrates the hourly power contribution from different energy sources across a 24-hour period for scenario 2.



15.Figure 5.15 (Scenario 2 Energy Source Contribution Analysis)

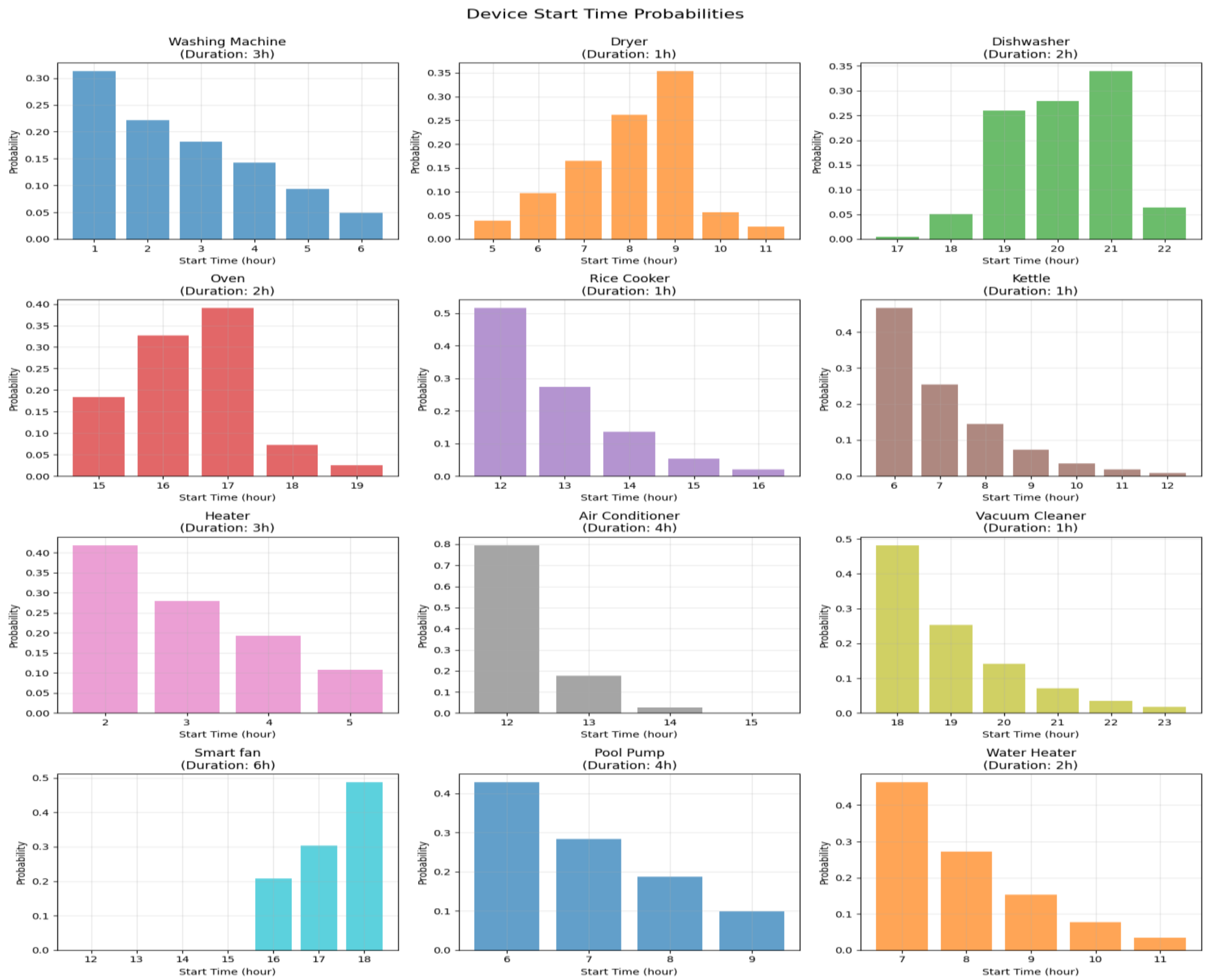
Figure 5.16 presents the hourly economic performance of the smart home energy system, comparing three key financial flows: Renewable Energy Cost, Grid Electricity Cost and Energy Selling Revenue for scenario 2.



16.Figure 5.16 (Scenario 2 Energy Cost and Revenue Analysis)

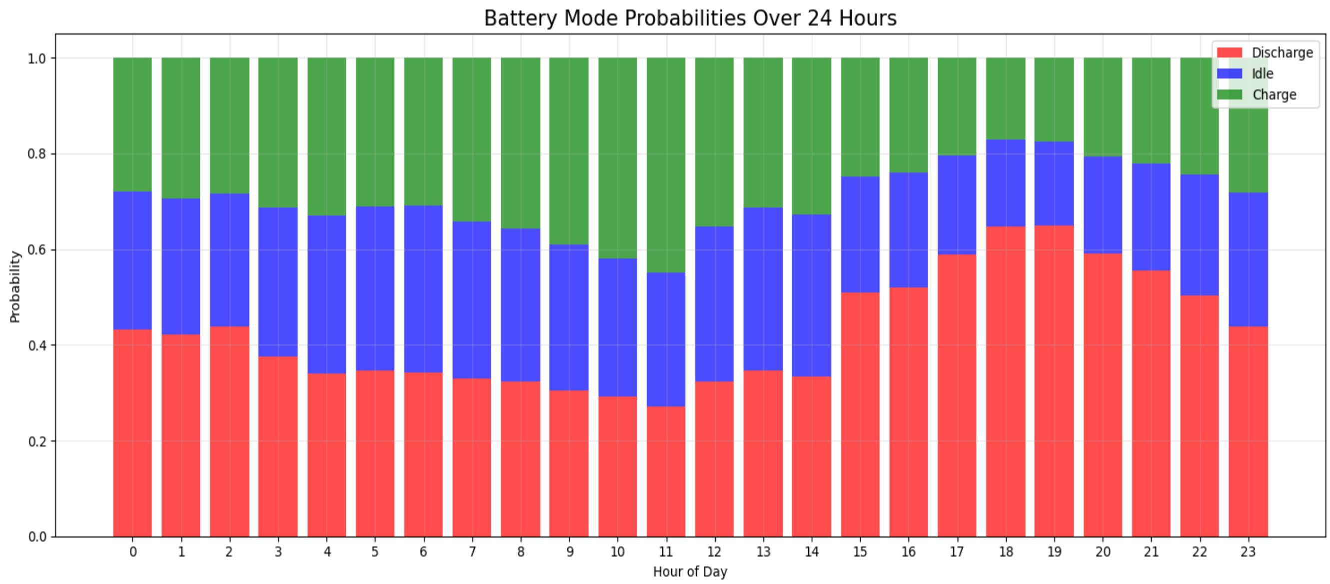
c) Scenario 3:

Figure 5.17 shows the probability of distributions for optimal start times of 12 household appliances for scenario 3.



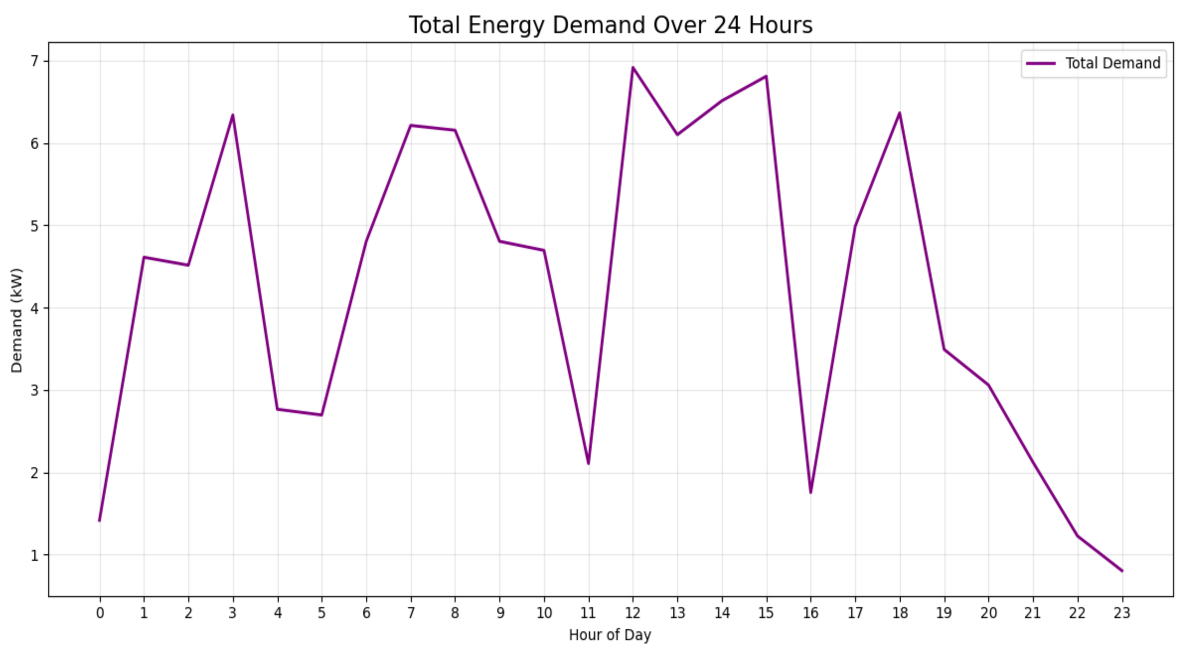
17.Figure 5.17(Scenario 3 Device Start Time Probability Distributions)

Figure 5.18 presents a probabilistic analysis of home battery operation modes across a 24-hour period for scenario 3.



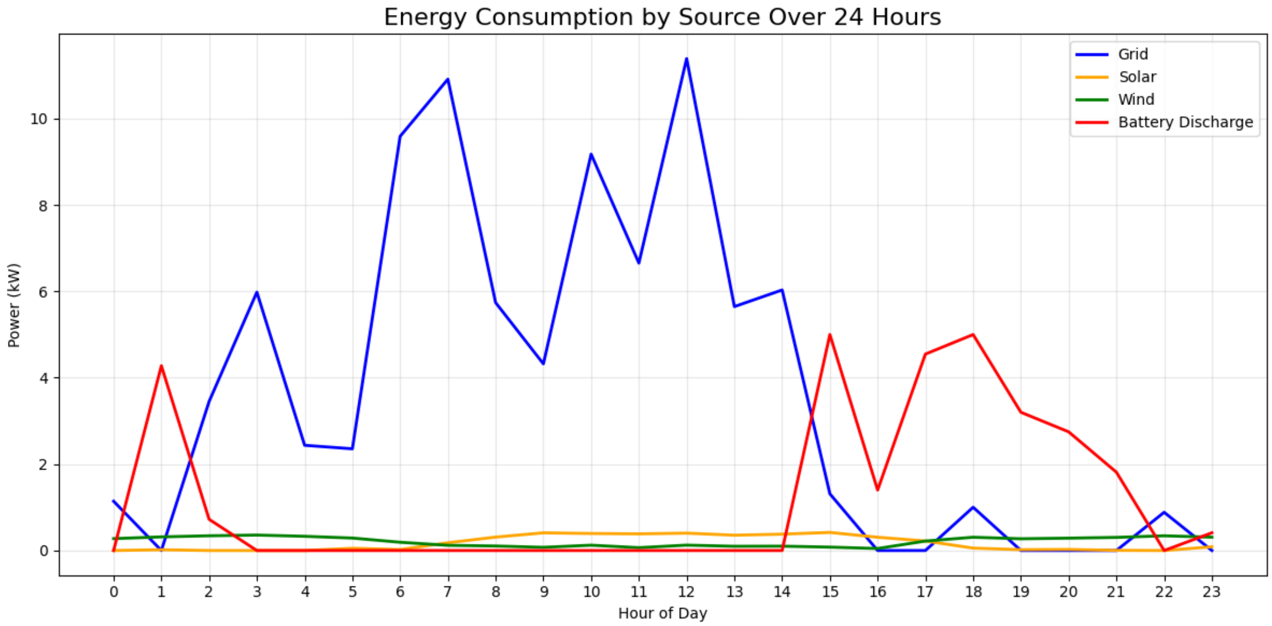
18.Figure 5.18 (Scenario 3 Battery Mode Probability Distribution Analysis)

Figure 5.19 visualizes the household's total energy demand (in kW) across a 24-hour period for scenario 3.



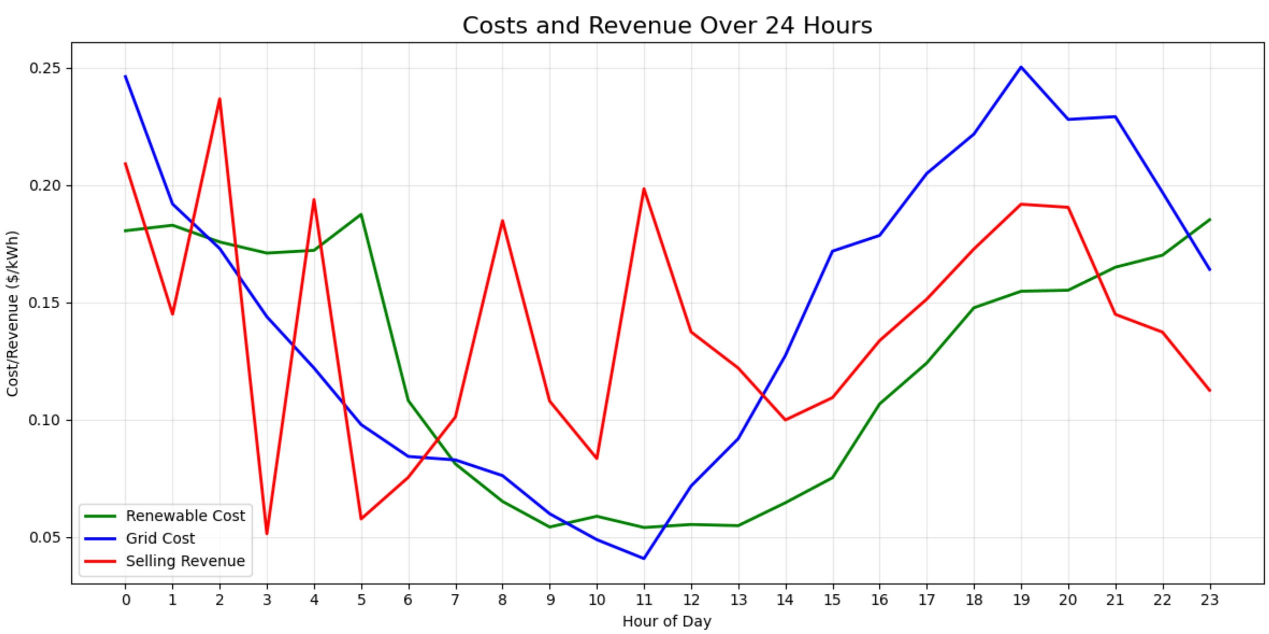
19.Figure 5.19 (Scenario 3 Total Energy Demand Profile)

Figure 5.20 illustrates the hourly power contribution from different energy sources across a 24-hour period for scenario 3.



20.Figure 5.20 (Scenario 3 Energy Source Contribution Analysis)

Figure 5.21 presents the hourly economic performance of the smart home energy system, comparing three key financial flows: Renewable Energy Cost, Grid Electricity Cost and Energy Selling Revenue for scenario 3.



21.Figure 5.21 (Scenario 3 Energy Cost and Revenue Analysis)

Chapter 6
Conclusion & Future Work

6.1 Conclusion

This project successfully developed and implemented an advanced mathematical model for the optimal scheduling of smart home appliances, integrating renewable energy sources, home battery storage, and electric vehicle (EV) charging requirements. The work was conducted through a systematic three-step approach: reviewing a basic mathematical model, updating it with new objectives and constraints and implementing and verifying the model.

In the first step, a foundational mathematical model was thoroughly reviewed to understand the core components, including sets, parameters, decision variables, objectives, and constraints. This provided a robust starting point for further enhancements.

The second step involved updating the model to incorporate multi-objective optimization, balancing energy cost minimization and user discomfort. New constraints were added to account for realistic operational scenarios, such as precedence relationships between appliances, simultaneous operation limits, and EV charging targets. The inclusion of home battery dynamics and grid interaction (buying and selling power) further enriched the model's applicability to modern smart homes.

In the final step, the updated model was implemented using Python, leveraging the DEAP library for multi-objective optimization via the NSGA-II algorithm. The implementation was tested across multiple scenarios with varying numbers of devices and precedence constraints, ensuring robustness and scalability.

Verification was achieved through extensive analysis of the Pareto front, which provided a set of non-dominated solutions balancing energy cost and discomfort.

Visualizations, including Gantt charts and distribution plots, confirmed the model's ability to produce feasible and practical schedules. The results demonstrated that the model was effective. optimizes energy usage while adhering to operational constraints and user preferences.

In conclusion, this project advanced the state-of-the-art in smart home energy management by delivering a comprehensive, flexible, and computationally efficient scheduling model. The successful integration of renewable energy, battery storage, and EV charging, combined with rigorous testing, positions this model as a valuable tool for enhancing energy efficiency and user satisfaction in smart homes. Future work could explore real-time adaptation of the model and integration with IoT platforms for practical deployment.

6.2 Future Work

While the proposed model has achieved promising results in optimizing energy cost, user comfort, and energy consumption, several directions remain open for future enhancement and real-world deployment:

1. Real-Time Data Integration,
   Incorporate live data feeds from smart meters, weather APIs, and market prices to transition from offline scheduling to real-time, adaptive control.

2. User Behavior Prediction,
   Apply machine learning techniques to model and predict user habits, device usage patterns, and load demands, enabling more personalized and anticipatory scheduling.

3. Price Prediction,
   Use time-series forecasting techniques to predict grid electricity prices and selling prices in real-time based on inputs. This would enable the system to make more informed decisions about when to buy from or sell to the grid, further reducing costs.

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