DISCRETE MATHEMATICES

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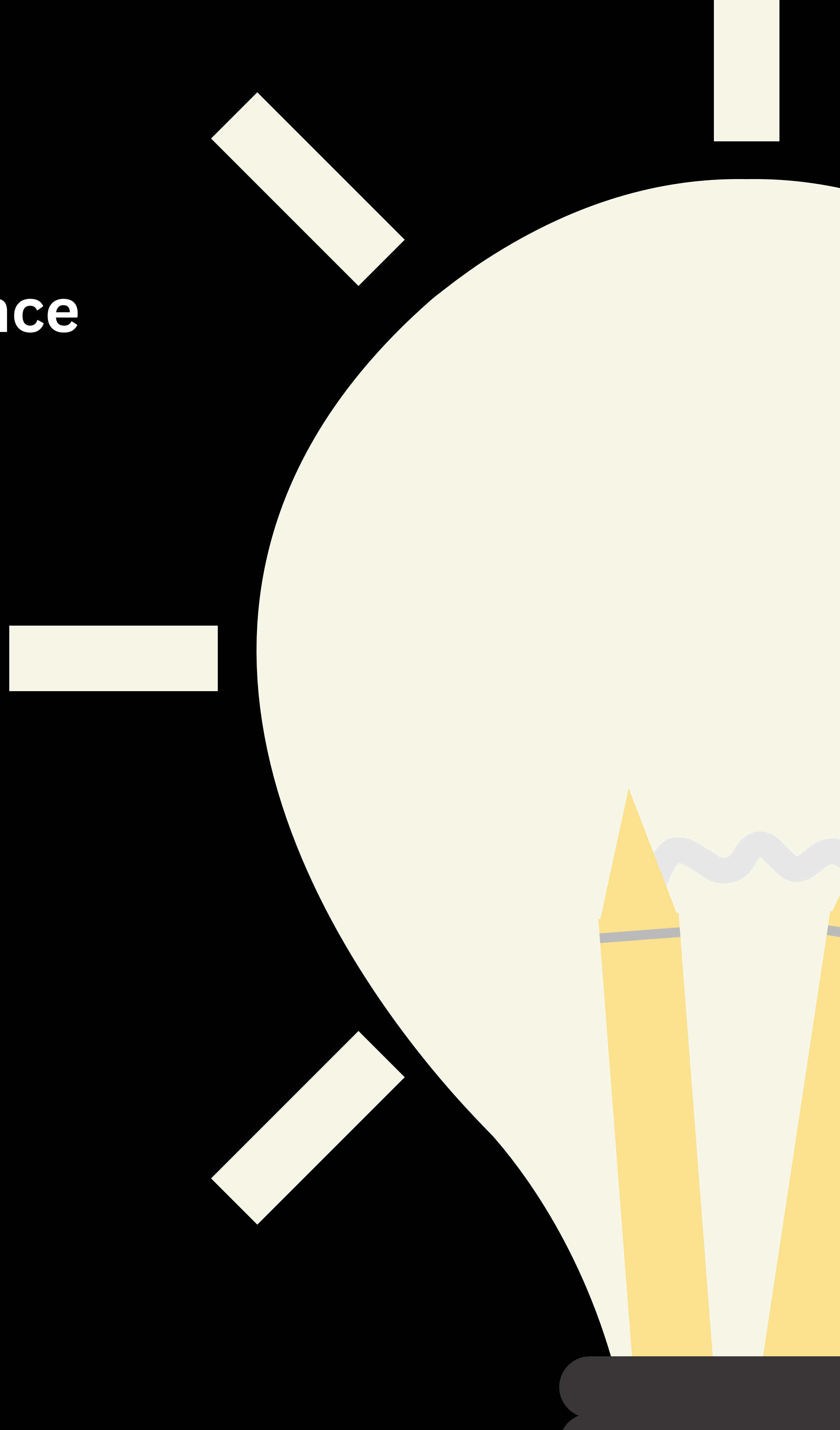
propositions

A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.



Example

cairo is the capital of egypt

1+3=4

4+3=2

Not proposition

x+2=5

what is the time?

COMPOUND PROPOSITONS

compound propositions, are
formed from existing propositions using logical operators.

LOGICAL OPERATORS

1-NOT

2-AND

3-OR

4-XOR

NOT

The proposition ¬p is read "not p." The truth value of the negation of p, ¬p, is the opposite of the truth value of p.

Example

let p : icecream is healthy ,

then ¬p : icecream is not healthy

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

AND

Let p and q be propositions. The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Example

if you do all worksheets and get A in final you will get A

The answer is true if the two statments is true otherwise it will be false

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

OR

Let p and q be propositions. The disjunction p ∨ q is false when both p and q are false and is true otherwise.

Example

if you do all worksheets or get more than B in exam you will pass the course

note: in language or is exclusive not inclusive

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

XOR

Let p and q be propositions. The exclusive or of p and q, is $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

Example

you can only buy an apple or an orange

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

CONDITIONAL STATMENTS

In the conditional statement $p \rightarrow q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion (or consequence).

Sufficiency
"if p, then q"
"p is sufficient for q"

necessity
"p only if q"
"q is necessary for p"

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example

if you do all your homework you will get A+

BICONDITIONALS

The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-mplications.

"p is necessary and sufficient for q"

Example

you can get A if only you make the assignments and get A in final exam

TABLE 6 The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

PRECEDENCE TABLE

This displays the precedence levels of the logical operators, ¬, ∧, ∨, →, and ↔.

TABLE 8
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

TABLE 7 **The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.**

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Logical Transformations

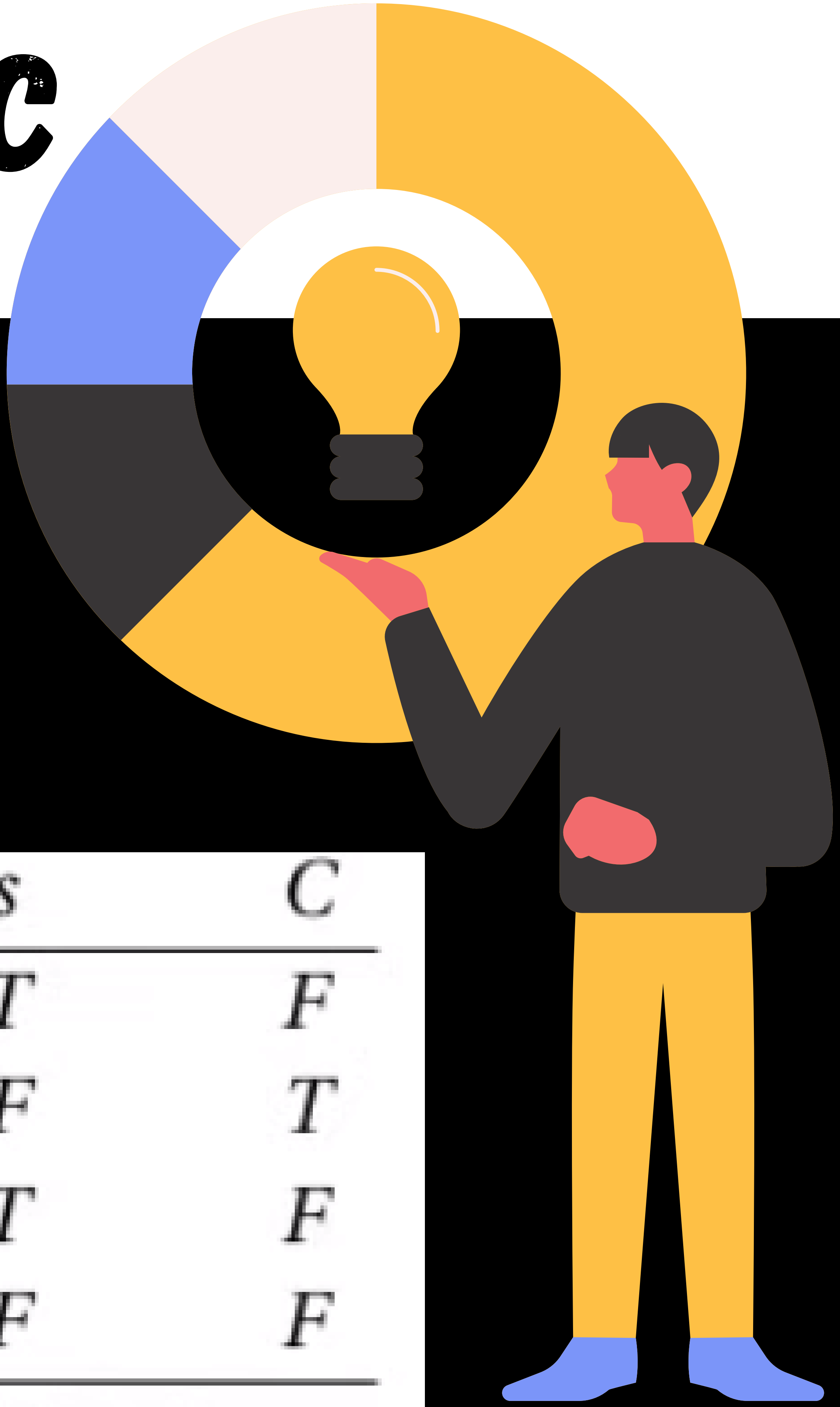
Logical transforms are operations performed on Conditional statements.
They are Contrapositive, Converse, and Inverse.

Converse : $q \rightarrow p$

contrapositive: $\neg q \rightarrow \neg p$

inverse: $\neg p \rightarrow \neg q$

CONEVRT ENGLISH TO LOGIC



Example 20 Translate to logical expression: "You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old"

q: "You can ride the roller coaster".

r: "You are under 4 feet tall".

s: "You are older than 16 years old".

The solution should be: C-q, but what is C

r	s	C
T	T	F
T	F	T
F	T	F
F	F	F

$$C = r \wedge \neg s$$

BOOLEAN SEARCH

Example

Search for universities in Cairo or Alexandria.

(Cairo OR Alexandria) AND Universities

LOGIC AND BIT OPERATIONS

BIT stands for binary

At the hardware level it is a wire with high voltage or zero voltage

TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

PROPOSITIONAL EQUIVALENCES

A compound proposition that is always true, no matter what the truth values of the proposi-tional variables that occur in it, is called a tautology. Acompound proposition that is always false is called a contradiction. A compound proposition that is neither a tautology nor acontradiction is called a contingency.

TABLE 1 Examples of a Tautology and a Contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

The compound propositions p and q are called logically equivalent if p ↔ q is a tautology. The notation p ≡ q denotes that p and q are logically equivalent.

TABLE 3 Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

1. Identity Laws: Real-life example: Adding 0 to a number. For instance, if you have 5 apples and you add 0 more apples to them, you still have 5 apples.
2. Domination Laws: Real-life example: If you have an empty box and you add any number of items to it, the contents of the box remain the same (empty). So, adding anything to nothing results in nothing.
3. Idempotent Laws: Real-life example: If you clean your room twice, it's the same as cleaning it once because the result is still a clean room.
4. Double Negation Law: Real-life example: Stating "It is not true that it is not raining" is the same as saying "It is raining." This emphasizes the idea that double negations cancel each other out.
5. Commutative Laws: Real-life example: Addition or multiplication of numbers doesn't change based on the order. For instance, 2 + 3 is the same as 3 + 2, and 2 * 3 is the same as 3 * 2.

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

1. Associative Laws: Real-life example: If you're having a party and need to invite three friends, it doesn't matter if you invite two friends first and then the third or if you invite one friend first and then the other two together. The end result is the same.
2. Distributive Laws: Real-life example: When distributing candies equally among friends, you can either distribute them one by one, or you can first group the candies together and then distribute each group. The total number of candies each friend gets remains the same.
3. De Morgan's Laws: Real-life example: Imagine a situation where you have a room with two doors. To say that "It is not true that both doors are closed" is equivalent to saying "At least one door is open." This is applying De Morgan's Law to the scenario of door states.
4. Absorption Laws: Real-life example: If you're wearing a black shirt and you put on a black jacket, it's like absorbing the jacket into the shirt; the overall appearance doesn't change.
5. Negation Laws: Real-life example: If someone says, "It's not raining outside," the negation of that statement would be, "It is raining outside." Similarly, if someone says, "I am not hungry," the negation would be, "I am hungry."

TABLE 6 Logical Equivalences.

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$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
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$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Predicates and Quantifiers

predicated: is a statment involving variables. The predicate P(x) takes a propositional value (True or False) after assigning the variable to x

“x = y + 3.”

set x = 1 and y = 2 , the statment is False

set x = 7 and y = 4 , the statment is True

Quantifiers

to scope a predicate over a range of values, "domain" or "universe of discourse", of the variable.

"universal quantification" defines a domain of all possible values.

"existential quantification" defines the existence of one or more values

The *universal quantification* of $P(x)$ is the statement

"$P(x)$ for all values of x in the domain."

The notation $\forall x P(x)$ denotes the universal quantification of $P(x)$. Here $\forall$ is called the **universal quantifier.** We read $\forall x P(x)$ as "for all $x P(x)$" or "for every $x P(x)$." An element for which $P(x)$ is false is called a **counterexample** of $\forall x P(x)$.

- P(x): "x+1>x" and the domain is all real numbers. Then the statement "∀xP(x)" is true.

- Q(x); "x<2" and the domain is all real numbers. Then "∀xQ(x)" is false by a counterexample x = 3.

Existential quantification

The existential quantification of P(x) is the statement: "There exists an element x in the domain such that P(x)"

• It is denoted by $\exists xP(x)$ and read as "there exists x such that P(x)"

• Other readings "for at least one”, “there is”, “for some", etc.

TABLE 1 Quantifiers.

Statement	*When True?*	*When False?*
$\forall x P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

P(x):x>3" and the domain is real numbers. Then, the statement $\exists xP(x)$" is true.

• Q(x):"x=x+1" and the domain is real numbers. Then, the statement $\exists xP(x)$ Q(x) is false.