Numerical exploration of Rossby waves

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1 Introduction

Rossby waves are essential in large-scale atmosphere-ocean dynamics as information transmitters and for distributing vorticity. In general, Rossby waves are generated from the planetary vorticity gradient, but they may also be generated from topography. Exploring these two types of Rossby waves mainly for the ocean, we will also consider the effect of a mean flow, as present in the atmosphere.

2 Theory

The dispersion relation for barotropic planetary waves can be derived from the linearized shallow water equations on a beta plane:

$$\omega = Uk - \frac{k\left(\beta - \frac{f_0}{D_0}\alpha\right)}{k^2 + l^2 + \frac{1}{R^2}}, \tag{1}$$

where ω is the angular frequency, k and l the zonal and meridional wavenumbers, respectively, β the linear planetary vorticity slope, f_0 the planetary vorticity at a given latitude, D_0 the depth of the fluid, α a topographic slope, and $R \equiv (gD_0)^{1/2} f_0^{-1}$ the Rossby radius of deformation.

The zonal phase and group velocities assuming flat topography ($\alpha = 0$) may be derived from eq. 1, yielding.

$$c_x = \frac{\omega}{k} = U - \frac{\beta}{k^2 + l^2 + \frac{1}{R^2}} \tag{2}$$

$$c_{gx} = \partial_k \omega = U + \beta \frac{k^2 - l^2 - 1/R^2}{(k^2 + l^2 + 1/R^2)^2}. \tag{3}$$

3 Method

Building on a model first presented by Kallmyr (2021), we have implemented a zonal mean flow and constant slope. We will use periodic boundaries in the east-west direction, and open boundaries with a sponge meridionally. The common model parameters for every run is listed in Table 1. Additionally, the time-space grid is categorized in Table 2.

Parameter	**Value**
g	$9.81\,\mathrm{ms}^{-2}$
f_0	$10^{-4}\,\mathrm{s}^{-1}$
β	$1.66 \cdot 10^{-11}\ (\mathrm{sm})^{-1}$
D_0	$4000\,\mathrm{m}$

Table 1: Model physical parameters.

For estimating the group velocities, we constructed a semi-manual algorithm for identifying the two first maxima, and calculated the slope between them. For estimating the phase velocities, we zoomed in on the first ridge and calculated the maximum along the western "border" slightly after initial conditions. The phase velocity was then obtained by calculat-

Parameter	**Value**
Nx	300
Ny	300
L_x	$7 \cdot 10^6$ m
L_y	$7 \cdot 10^6$ m
runtime	30 days

Table 2: Model grid parameters.

ing the slope between this maximum and the initial maximum.

Both methods are demonstrated on github. [1]

4 Results

Figure 1 shows a Hovmöller-diagram of a planetary Rossby wave in the northern hemisphere mid-latitudes. We can see that the wave propagates westward with a eastward group velocity, consistent with eqs. (2) and (3) without a zonal mean flow.

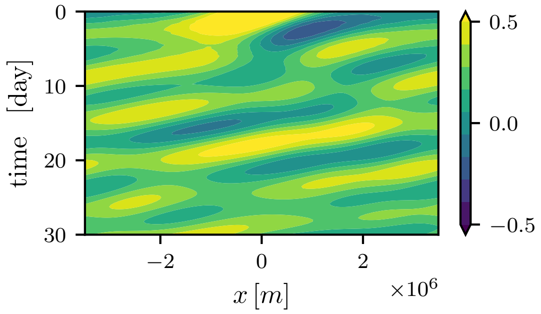

Figure 1: Hovmöller-diagram showing wave propagation at $y = 0$ for 30 days at $45°$ N with a planetary vorticity gradient $\beta \equiv 1.66 \cdot 10^{-11}$ s.

Exploring the effect of topography, Figure 2 shows the effect of sloping topography. Opposite to what is shown in Figure 2 the wave propagates eastward with a westward group velocity.

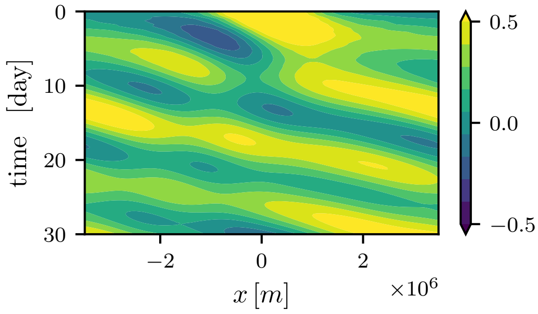

Figure 2: Hovmöller-diagram showing wave propagation at $y = 0$ for 30 days at $45°$ N with a planetary vorticity gradient $\beta \equiv 0$ s and topography $\alpha = -4.46 \cdot 10^{-4}$.

With the demonstrated effect of topography, a balance between topography and the planetary vorticity gradient may be obtained, as shown in Figure 3 where the Rossby wave is arrested at the same location throughout the entire 30 day period. This is the case for $\alpha = -6.64 \cdot 10^{-4}$, which is the analytical value as calculated from eq. (1).

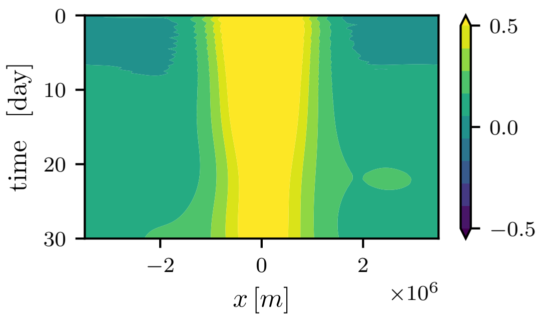

Figure 3: Hovmöller-diagram showing wave propagation at $y = 0$ for 30 days at $45°$ N with a planetary vorticity gradient $\beta \equiv 1.66 \cdot 10^{-11}$ s and topography $\alpha = -6.64 \cdot 10^{-4}$.

As a baseline comparison with theory (solid lines), the zonal group (blue) and phase (or-

[1] https://github.com/janadr/MO8004/tree/master/L3

ange) velocities of planetary waves were estimated numerically (dots) as shown in Figure 2. There is a large discrepancy in group velocity for $kR \approx 1$ and phase velocity for $kR \approx 2$.

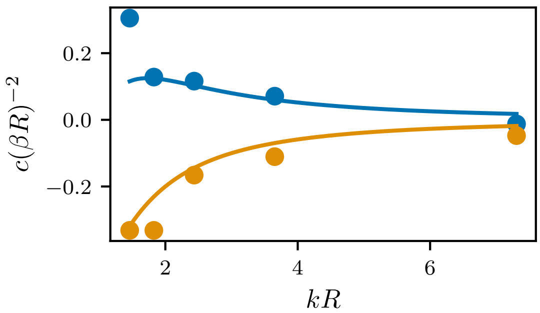

Figure 4: Group (blue) and phase (orange) velocities of planetary waves calculated from a Hovmöller diagram as a function of wavenumber. Points - numerical values and lines - analytical values.

The effect of a eastward mean flow is demonstrated in Figure 5 where the zonal group velocity is shown as a function of zonal mean flow. The black line shows the analytical dependence, while the grey line is fitted to the numerically calculated points. All numerical values slightly overestimate the analytical values. Extrapolating linear dependence yields zero group velocity for a westward mean flow $U_0 \approx -5\,\mathrm{ms}^{-1}$.

5 Discussion

The discrepancies in Figure 4 noted in Section 4 are most likely due to the estimation method failing in edge cases. I.e., in the case of the group velocity, the method may choose the wrong second maximum as multiple are present in the same time frame, and so the direction and speed is wrong. As for the phase velocity, the situation is more ambiguous, and requires a deeper analysis.

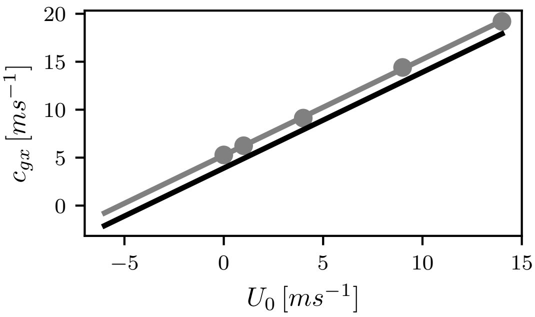

Figure 5: Group velocities of planetary waves as a function of zonal mean flow. Grey - numerical values with linear fit $c = 5.24 + 1.00x$ and black - analytical values.

References

Jan-Adrian Kallmyr. Numerical exploration of geostrophic adjustment. 2021.