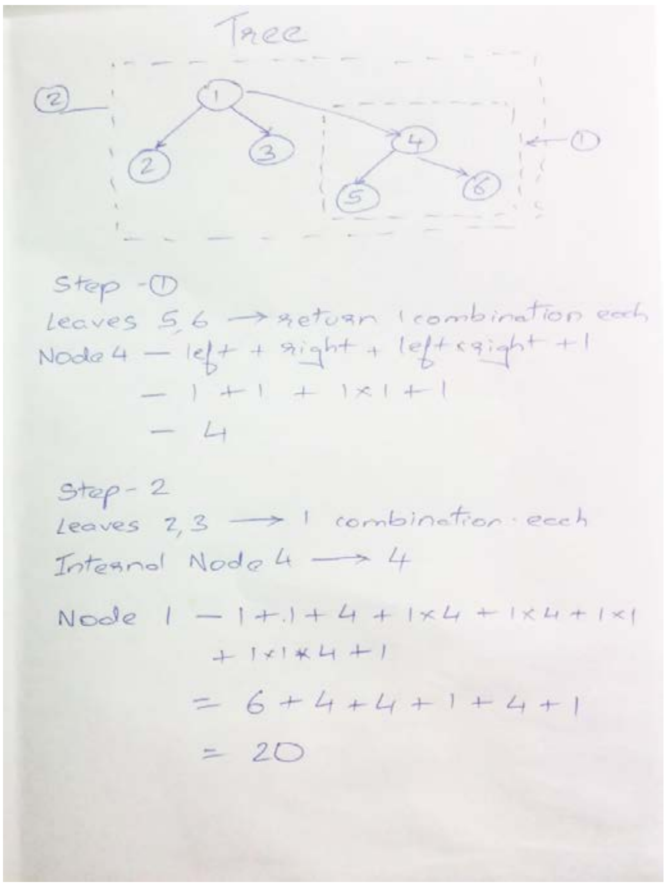
B 503 Mini-Project Report

We have proposed two algorithms, one which efficiently enumerates consistent sub-DAGs in a tree while the other enumerates for a graph, too.

Description

Algorithm 1(works for tree only):

We follow a bottom up approach(DFS), that is, we start from the leaf node(u) and consider a sub-tree up to one level(which includes u, all its siblings and its parent v). For that sub-tree, we find the number of consistent sub-DAGs. This sub-tree(which would be represented by a count of consistent sub-DAGs) forms a left subtree or a right subtree for some parent and we keep calculating the consistent sub-DAGs for the above levels as shown in the example below:

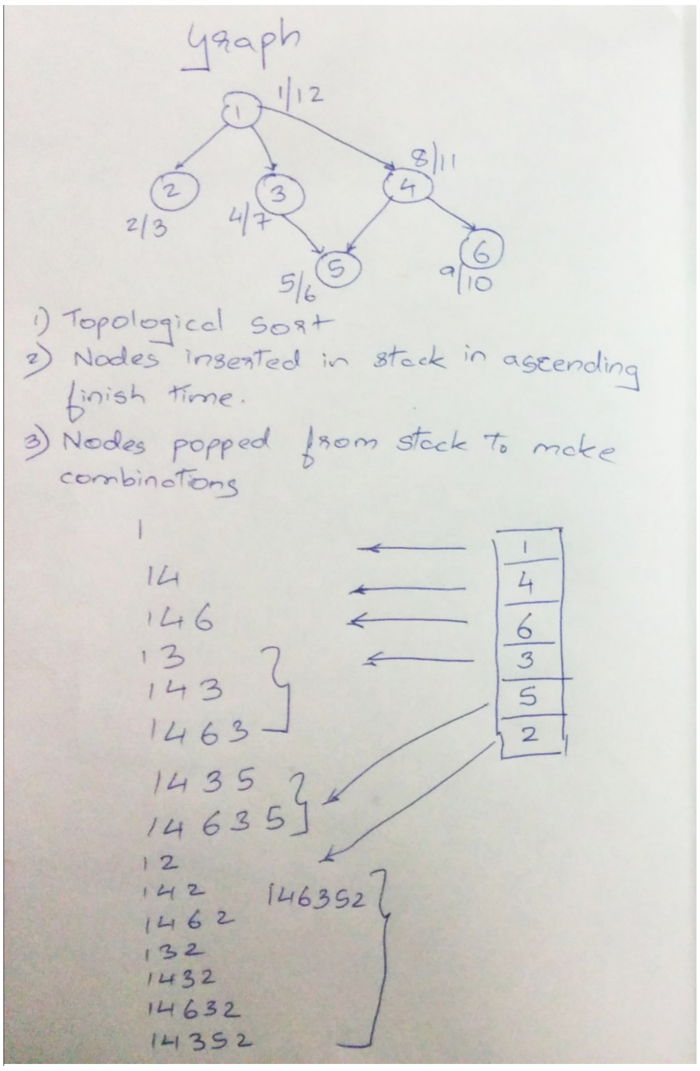


Step -1
Leaves 5,6 → return 1 combination each
Node 4 — left + right + left×right + 1
— 1 + 1 + 1×1 + 1
— 4

Step - 2
Leaves 2,3 → 1 combination each
Internal Node 4 → 4
Node 1 — 1+1+4 + 1×4 + 1×4 + 1×1
+ 1×1×4 + 1
= 6 + 4 + 4 + 1 + 4 + 1
= 20

Algorithm 2(works for trees and graphs as well):

We first call topologicalSorting() that traverses nodes in DFS and pushes them into a stack when finish time is assigned to a node. Next, we pop the nodes from the stack and check whether for each node (v) popped, all of its parent(s) from revadjacency list (that stores child->parent relations) is/are present or not in the set of consistent sub-DAGs generated. If the parent(s) is/are absent, remove all those sub-DAGs from consideration. For all the other sub-DAGs left, add them into the list of consistent sub-DAGs by appending 'v' to them.



1) Topological Sort
2) Nodes inserted in stack in ascending finish time.
3) Nodes popped from stack to make combinations

Correctness

Algorithm 1:

The DP approach maintains a count of consistent sub-DAG from left sub-tree and a count of consistent sub-DAGs from right sub-tree and a combination of consistent sub-DAG from left sub-tree and right sub-tree. Say sub-tree rooted at T has children $v_i, v_{i+1} \ldots v_j$. Then, the total numbers of combinations for that sub-tree are:

for k=i to j-1

$$T(v_k)= T(v_k)+ N(v_k)+N(v_{k+1})+N(v_k)*N(v_{k+1})$$

where $N(v_k)$ represents the number of combination a v_k makes with its parent T and $T(v_k)$ accumulates the count of all the combination amongst the children and parent. This technique is applied to all the sub-trees from the bottom to the top at all the levels.

Algorithm 2:

We need to make sure that before we visit any node, all of its parent must be visited. This can be done by using a topological sort. Say for 2 nodes u,v ∊ V, if there is an edge from u to v, then the finish time for v will be less than u which means that node u(parent) was visited before node v(child). This algorithm will list consistent sub-DAGs because every new consistent sub-DAGs are created by scanning for its parent(s) in the set of preceded consistent sub-DAGs. For lookup, we have used KMP algorithm to reduce the computation time as much as possible.

Complexity

Analysis of Algorithm 1:

The algorithm follows a Depth-First Search which would take O (|V|+|E|) time. The cost of calculating the count of consistent sub-DAGs(T(v_k)) will be constant.

T(n)=O(|V|+|E|)

Analysis of Algorithm 2:

Here the topologicalSorting() internally calls dfsVisit() that pushes a node by their finish time. This would take O (|V|+|E|). Next, we pop every node v from stack, and scan the list of consistent sub-DAGs already. For each node v, pre-processing will take O (M) where m is length of node 'v' and the matcher will take O(C) where 'C' is a consistent sub-DAG previously formed. This search is done for 2^{v-1} consistent sub-DAGs for every v.

T(n)=O(|V|+|E|+(v*M)(2^{v-1}*C))

Counts of consistent sub-DAGs for the data-sets:

mini-mfo2: 692168

tree_25: 89960

tree_100: 185699666138602844715676

Contributions

Algorithm 1 was proposed and implemented by Amish Shah.

Algorithm 2 was proposed and implemented by Janak Bhalla and Karan Shah.