PART - A

Q 1. Rayleigh - Ritz method :-

* It is Intergral approach method which is useful for Solving complex Structural Problem, encountered in finite element analysis,

* This method is possible only if a suitable function is available.

Q 2. Primary variable :-

Primary data refers to The first hand data gathered by The researcher himself.

Secondary variable :-

Secondary data means data collected by Someone else earlier

Example :-

* Surveys,
* observations
* Experiments.
* Personal Interview.

Q 3. TRANSVERSE VIBRATION :-

a vibration in which The element moves to and fro in a direction Perpendicular to The direction of The advance of The wave.

Q 4. Shape function :-

* The Shape function is The function which Interpolates The Solution between The discrete Value obtained at The mesh nodes

* In This work linear Shape functions are used.

Q 5. TWO DIMENSIONAL SCALAR VARIABLE :-

* In Structural Problems, The unknowns (displacements) are represented by The Components of vector field.

* for Example, In a two dimensional plate, The unknown quality is The vector Field $u(x,y)$, where u is a (2×1) displacement vector.

Q 6. ISOPARAMETRIC FORMULATION OF THE BAR ELEMENT :-

* When a particular coordinates is Substituted into [N] yields The displacement of a point on The bar in terms of The nodal degree of Freedom u_1 and u_2

* Since u and x are defined by The Same Shape functions at The Same nodes, The elements is called isoparametric

Q7. Shell Elements :-

* 4- to 8- node isoparametric quadrilaterals

* 3- to 6 - node triangular Elements in any 3-D orientation.

Q8. AXISYMMETRIC FORMULATION :-

* Axisymmetric elements are triangular tori such that each element is symmetric with respect to geometry and loading about an axis such as the z axis.

* Hence the z axis is called . the axis of symmetry or the axis of Revolution

Q 9. NEWTON COTES QUADRATURE METHOD:-

IN Numerical analysis, The newton - Cotes formulas, also called Newton - Cotes quadrature rules or simply Newton - cotes rules, are a group of formulas for numerical Integration based on evaluating the Intergrand at equally spaced points.

Q 10. JACOBIAN TRANSFORMATION:-

The Jacobian transformation is an algebraic method for determind the Probability distribution of a variable y that is a function of just one other variable x when we know the Probability distribution for x.

Part - B

11.(B)

Finite Element Analysis:-

* In this method, a body or a structure in which the analysis to be carried out is subdivided into smaller element of finite dimensions called finite element.

* Then the body is considered as an assemblage of these elements connected at a finite number of joints called "nodes" or nodal points.

* Based on application, the finite element problem are classified as follows:-

(i) Structural Problems:-

The displacement at each nodal point is obtained. By using these displacement solution, stress and strain in each element can be calculated.

Non-Structural Problem:-

The temperature or fluid Pressure at each nodal point is obtained. for each element can be calculated.

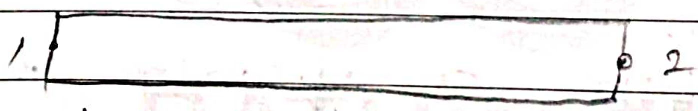
Steps :-

* Forced method
* Displacement or Stiffness method.

Step 1:-

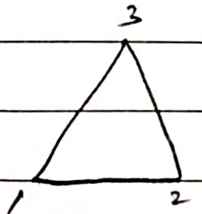
Discretization.

1 D Element :-

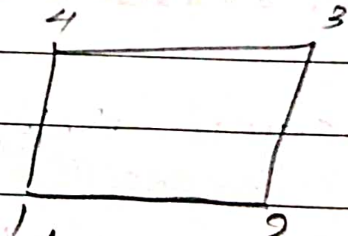


(Bar Element)

2D Element:-

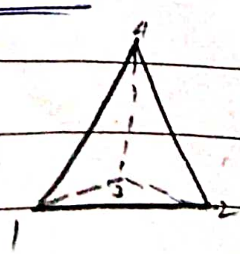


(Triangle Element)

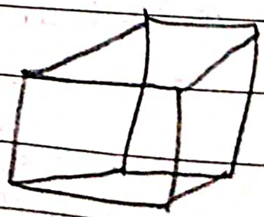


(Rectangular Element)

3D Element :-

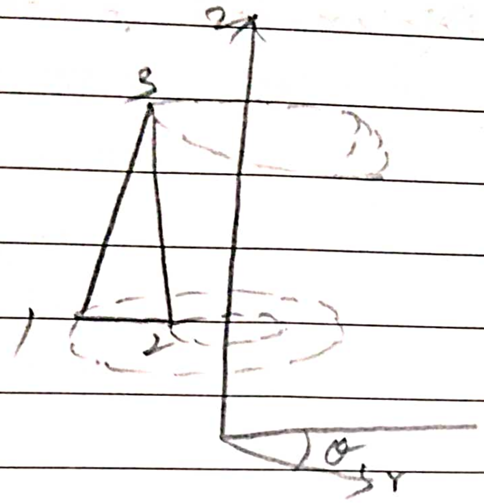


(Triangular Element)



(Hexahedral Element)

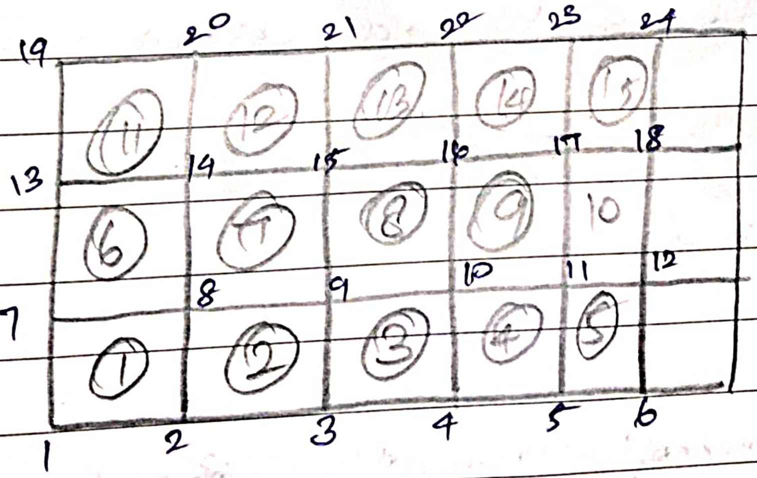
Axisymmetric Elements:



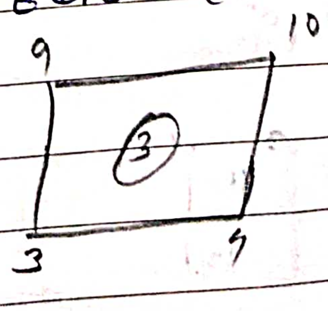
Step 2 :

Numbering of nodes & elements

Longer Side Numbering Process



Considering Element (3)



Maximum node Number = 10

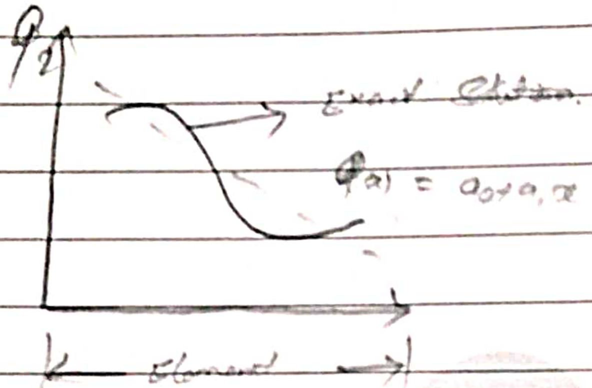
Minimum node Number = 3

∴

Difference = 7.

Step 3

Selection of a Displacement function or Interpolation function.



Linear approximation.

Step :- 4

Define the material behaviour by using Strain. Displacement and Strain - Stress Relationship.

$$e = \frac{du}{dx}$$

Step 5 :-

Derivation of Element Stiffness matrix and equations

$$\{F^e\} = [K^e] \{u^e\}$$

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

1) Direct Equilibrium method:-

It is used for 2D Element.

2) Variation method:-

It is to the determination of element equations for complicated element.

3) Weighted Residual Method:-

(galerkin's method)

It is used for Thermal analysis Problem.

Step 6:-

Assemble the Ecrers equation.

$\{F\} = [K]\{u\}$

Step 7

Applying boundary conditions [K] is a singular matrix to zero.

Step 8

Solution for the unknown displacement gaussian elimination method.

Step 9

Computation of the element strains and

Stresses from the nodal displacement (a)

$$e = \frac{du}{dx}$$

$$= \frac{u_2 - u_1}{x_2 - x_1}$$

u_1 & u_2 → displacement node.

12. (B)

given

$l = 800mm$

$F = 3kN = 3\times10^3 N$

$E = 2\times10^5 N/mm^2$

$A = 300mm^2$

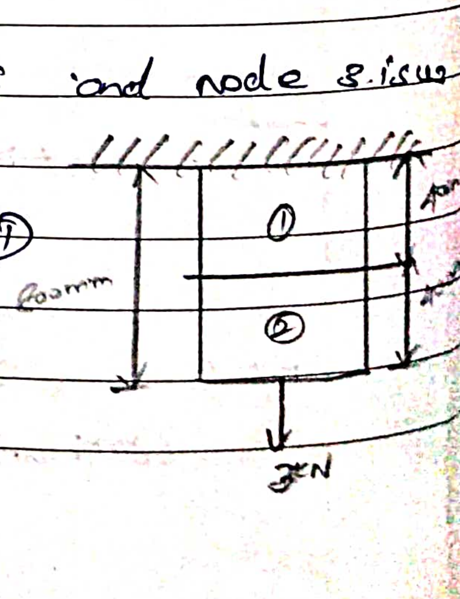
to find

Elongation, u

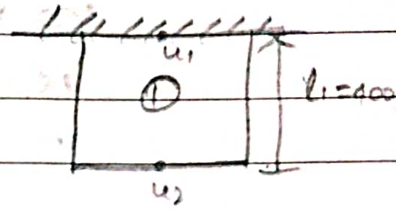
Soln

Displacement at node 1 is u_1, node 2 is and node 3 is u_2

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \frac{AE}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad \text{—(1)}$$



for element 1 (node 1,2) :-

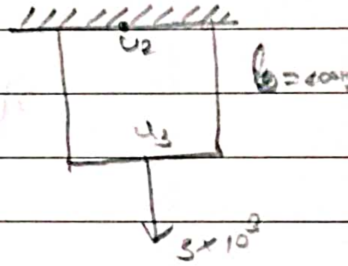


$$\frac{A_1 E}{l_1}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

$$\frac{300\times 2\times 10^5}{400}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix}$$

$$150\times 10^3\begin{bmatrix}\overset{a_{11}}{1} & \overset{a_{12}}{-1}\\ \overset{a_{21}}{-1} & \overset{a_{22}}{1}\end{bmatrix}\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = \begin{Bmatrix}F_1\\ F_2\end{Bmatrix} \quad — (2)$$

for Element 2 (nodes 2,3) :-



$$\frac{A_2 E}{l_2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix} = \begin{Bmatrix}F_2\\ F_3\end{Bmatrix}$$

$$\frac{200\times 2\times 10^5}{400}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix} = \begin{Bmatrix}F_2\\ F_3\end{Bmatrix}$$

$$150\times 10^3\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix} = \begin{Bmatrix}F_2\\ F_3\end{Bmatrix} \quad — (3)$$

Applying Boundary conditions

$u_1 = 0$; $F_3 = 3\times 10^3$ N ; $F_1 = F_2 = 0$

Sub u_1, F_1, F_2, F_3 equation - (3)

$$\Rightarrow 150\times 10^3\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{Bmatrix}0\\ u_2\\ u_3\end{Bmatrix} = \begin{Bmatrix}0\\ 0\\ 3\times 10^3\end{Bmatrix}$$

$$150\times 10^3\begin{bmatrix}2 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}u_2\\ u_3\end{Bmatrix} = (1)$$

$$150 \times 10^3 (2u_2 - u_3) = 0 \quad \text{---} (4)$$

$$150 \times 10^3 (-u_2 + u_3) = 3 \times 10^3 \quad \text{---} (5)$$

$$\overline{150 \times 10^3 (u_2) = 3 \times 10^3}$$

$$\boxed{u_2 = 0.02\,mm}$$

u_2 value in equation (4)

$$150 \times 10^3 (2 \times 0.02 - u_3) = 0$$

$$2 \times 0.02 - u_3 = 0$$

$$2 \times 0.02 = u_3$$

$$\boxed{u_3 = 0.04\,mm}$$

Total Elongation:

$$\delta L = \frac{PL}{AE}$$

$$= \frac{3 \times 10^3 \times 800}{300 \times 2 \times 10^5}$$

$$\boxed{\delta L = 0.04\,mm}$$

Result:

node 1, $u_1 = 0$

node 2, $u_2 = 0.02\,mm$

node 3, $u_3 = 0.04\,mm$.

13. (B)

given

$x_1 = 0$; $y_1 = 0$

$x_2 = 2$; $y_2 = 0$

$x_3 = 1$; $y_3 = 0$

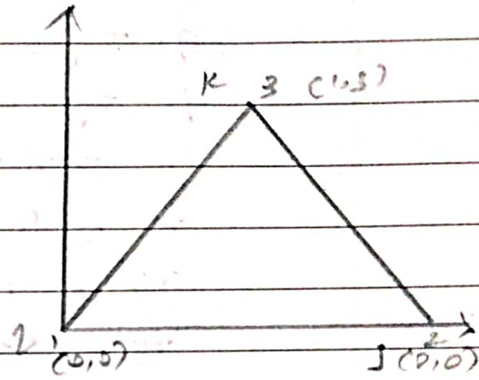
$E = 2\times10^5 \ N/mm^2$

$\nu = 0.25$

$t = 5mm$

$\Delta T = 10^\circ c$

$\alpha = 6\times10^{-6} \ ^\circ/c$



to Find

$[k]$, $\{F\}$.

Soln

$$[k] = [B]^T [D] [B] A t \quad \text{—— (1)}$$

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 3 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(6-0) - 0 + 0\right]$$

$$A = 3mm^2 \quad \text{—— (2)}$$

$$[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix} \quad \text{—— (3)}$$

$q_1 = y_2 - y_3 = 0 - 3 = -3$

$q_2 = y_3 - y_1 = 3 - 0 = 3$

$q_3 = y_1 - y_2 = 0 - 0 = 0$

$r_1 = x_3 - x_2 = 1 - 2 = -1$

$r_2 = x_1 - x_3 = 0 - 1 = -1$

$r_3 = x_2 - x_1 = 2 - 0 = 2$

$$[B] = \frac{1}{2A}\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$[B] = \frac{1}{2\times 3}\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$[B] = 0.1667\begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix} \quad \text{--- (A)}$$

$$[D] = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{bmatrix}$$

$$= \frac{2\times 10^5}{1-0.25^2}\begin{bmatrix} 1 & 0.25 & 0 \\ 0.25 & 1 & 0 \\ 0 & 0 & \frac{1-0.25}{2} \end{bmatrix}$$

$$= \frac{2\times 10^5 \times 0.25}{0.9375}\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$[B]^T = 0.1667 \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

$$[B]^T[D] = 0.1667 \begin{bmatrix} -3 & 0 & -1 \\ 0 & -1 & -3 \\ 3 & 0 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} \times 53.33\times10^3 \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

$$= 0.1667 \begin{bmatrix} -12 & -3 & -1.5 \\ -1 & -4 & -4.5 \\ 12 & 3 & -1.5 \\ -1 & -4 & 4.5 \\ 0 & 0 & 3 \\ 2 & 8 & 0 \end{bmatrix} \times 53.33\times10^3$$

$$[B]^T[D] = 8.890\times10^3 \begin{bmatrix} -12 & -3 & -1.5 \\ -1 & -4 & -4.5 \\ 12 & 3 & -1.5 \\ -1 & -4 & 4.5 \\ 0 & 0 & 3 \\ 2 & 8 & 0 \end{bmatrix} \quad \text{———} \quad 6$$

$$[B]^T[D][B] = 8.890\times10^3 \begin{bmatrix} -12 & -3 & -1.5 \\ -1 & -4 & 4.5 \\ 12 & 3 & -1.5 \\ -1 & -4 & 4.5 \\ 0 & 0 & 3 \\ 2 & 8 & 0 \end{bmatrix} \times 0.1667 \begin{bmatrix} -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 2 \\ -1 & -3 & -1 & 3 & 2 & 0 \end{bmatrix}$$

$$[B]^T[D][B] = 1.482\times10^3 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -24.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

Sub $[B]^T[D][B]$ And A, t equ — ①

$$[K] = 1.482 \times 10^3 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix} \times 3 \times 5$$

$$[K] = 22.23 \times 10^3 \begin{bmatrix} 37.5 & 7.5 & -34.5 & -1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -34.5 & 1.5 & 37.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -2 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & -8 & 0 & 16 \end{bmatrix}$$

$$\{e_0\} = \begin{Bmatrix} \alpha \Delta T \\ \alpha \Delta T \\ 0 \end{Bmatrix}$$

$$\{e_0\} = \begin{Bmatrix} 6 \times 10^{-6} \times 10 \\ 6 \times 10^{-6} \times 10 \\ 0 \end{Bmatrix} = 1 \times 10^{-6} \begin{Bmatrix} 60 \\ 60 \\ 0 \end{Bmatrix} \quad \text{— ⑧}$$

$$\{F\} = [B]^T [D] \{e_0\} A\, t$$

$$\{F\} = 8890 \times 10^3 \begin{bmatrix} -12 & -3 & -1.5 \\ -1 & -4 & -4.5 \\ 12 & 3 & -1.5 \\ -1 & -4 & 4.5 \\ 0 & 0 & 3 \\ 2 & 8 & 0 \end{bmatrix} \times 1 \times 10^{-6} \begin{Bmatrix} 60 \\ 60 \\ 0 \end{Bmatrix} \times A \times t$$

$$= 0.1235 \begin{Bmatrix} (-12 \times 60) + (-3 \times 60) + 0 \\ (-1 \times 60) + (-4 \times 60) + 0 \\ (12 \times 60) + (3 \times 60) + 0 \\ (-1 \times 60) + (-4 \times 60) + 0 \\ 0 + 0 + 0 \\ (2 \times 60) + (8 \times 60) + 0 \end{Bmatrix}$$

$$\{F\} = 0.1335 \begin{Bmatrix} -900 \\ -300 \\ 900 \\ -300 \\ 0 \\ 600 \end{Bmatrix} \Rightarrow \{F\} = \begin{Bmatrix} -120.15 \\ -40.05 \\ 120.16 \\ -40.05 \\ 0 \\ 80.10 \end{Bmatrix}$$

Result:

$$[K] = 22.83 \times 10^{3} \begin{bmatrix} 37.5 & 7.5 & -31.5 & 1.5 & -3 & -6 \\ 7.5 & 17.5 & 1.5 & -9.5 & -9 & -8 \\ -31.5 & 1.5 & 31.5 & -7.5 & -3 & 6 \\ -1.5 & -9.5 & -7.5 & 17.5 & 9 & -8 \\ -3 & -9 & -3 & 9 & 6 & 0 \\ -6 & -8 & 6 & \cancel{8}\ -8 & 0 & 16 \end{bmatrix}$$

$$\{F\} = \begin{Bmatrix} -120.75 \\ -40.05 \\ 120.15 \\ -40.05 \\ 0 \\ 80.10 \end{Bmatrix}$$