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ABSTRACT

Portfolio Optimization is performed here using the FTSE 100 companies.

1 MARKOWITZ EFFICIENT FRONTIER

Here a portfolio with two assets is considered – A and B. The return on Asset A R(A) is 10% and the return on asset B R(B) is 10%. The variance on Asset A is 0.5% and the variance on asset B is 0.5%.

The volatility of Asset A is 0.07 and volatility of Asset B is 0.07.

Eq. 1(a) Variance = Var A * (Weight A)^2 + Var B * (Weight B)^2 + 2*S.D(A)*S.D(B)*Weight A*Weight B*Corr(AB)

Eq. 1(b) SD = sqrt(Var)

Eq. 1(c) Mean Return = Weight A * R(A) + Weight B * R(B)

Sum of the weight of both assets is assumed as 1

Eq. 1(d) Weight A + Weight B = 1

Since correlation coefficient is not known, the efficient frontier is calculated for two different values of p = -0.5,0,0.5

Using the above formulas and substituting the values, the Volatility and Mean Return were calculated and displayed in Table 1.1 and Table 1.2 respectively.

Weight A	**Weight B**	**Volatility**	**Mean Return**
0.1	0.9	6.04%	10.00%
0.2	0.8	5.10%	10.00%
0.3	0.7	4.30%	10.00%
0.4	0.6	3.74%	10.00%
0.5	0.5	3.54%	10.00%
0.6	0.4	3.74%	10.00%
0.7	0.3	4.30%	10.00%
0.8	0.2	5.10%	10.00%
0.9	0.1	6.04%	10.00%

Table 1.1: Calculations with P = - 0.5

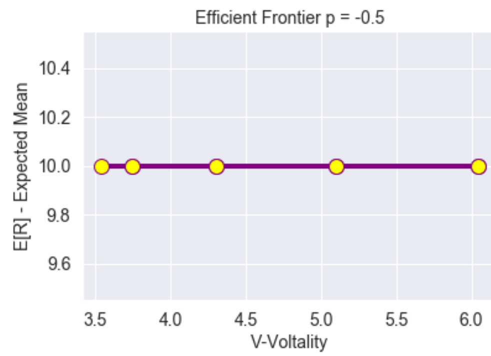

Fig 1.1: Efficient Frontier with Correlation Coefficient = - 0.5

Weight A	**Weight B**	**Volatility**	**Mean Return**
0.1	0.9	6.74%	10.00%
0.2	0.8	6.48%	10.00%
0.3	0.7	6.28%	10.00%
0.4	0.6	6.16%	10.00%
0.5	0.5	6.12%	10.00%
0.6	0.4	6.16%	10.00%
0.7	0.3	6.28%	10.00%
0.8	0.2	6.48%	10.00%
0.9	0.1	6.74%	10.00%

Table 1.2: Calculations with P = + 0.5

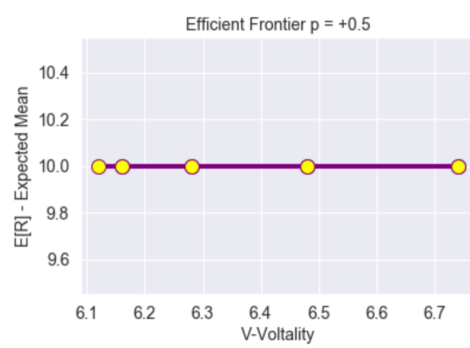

Fig 1.2: Efficient Frontier with Correlation Coefficient = + 0.5

2 RANDOM PORTFOLIO GENERATION

Let us consider three securities (A, B, C) with mean M and Co-variance matrix C.

mean M= [0.10 0.20 0.15] and

covariance matrix C = [[0.005, -0.010, 0.004],
[-0.010, 0.040, -0.002],
[0.004, -0.002, 0.023]]

100 Random Portfolios for the above mean and covariance were generated and their efficient frontier is displayed in the figure below.

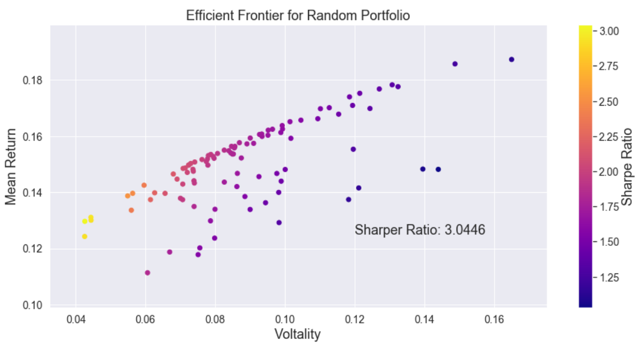

Fig 2.1: Efficient Frontier for 100 Random Portfolio

The first two securities (A,B) were considered and their efficient frontier is shown below.

m_A_B = [0.10,0.20]

cov_A_B = [[0.005,-0.010],
[-0.010,0.040]]

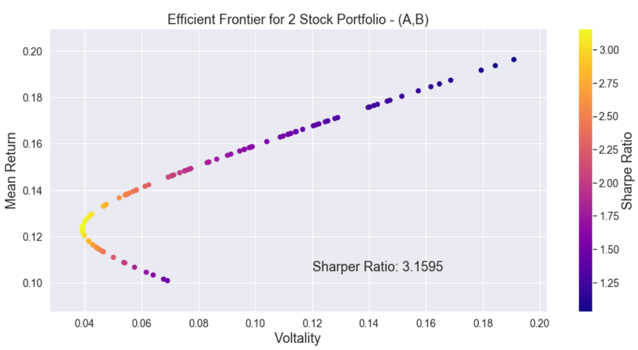

Fig 2.2: Efficient Frontier for Portfolio – (A , B)

The B and C securities were considered and their efficient frontier is shown below.

m_B_C= [0.20,0.15]

cov_B_C = [[0.040, -0.002],
[-0.002, 0.023]]

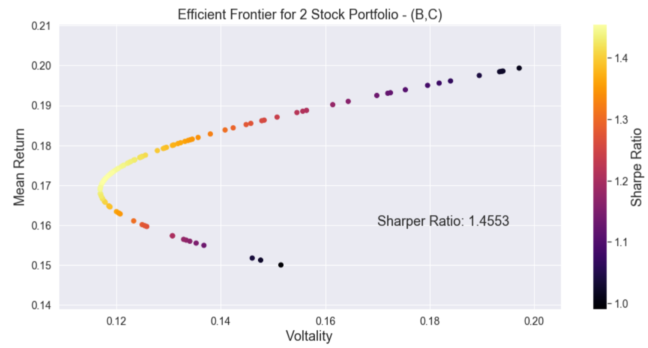

Fig 2.3: Efficient Frontier for Portfolio – (B , C)

The securities (A,C) were considered and their efficient frontier is shown below.

m_A_C= [0.10,0.15]

cov_A_C = [[-0.005,0.004],
[0.004,0.023]]

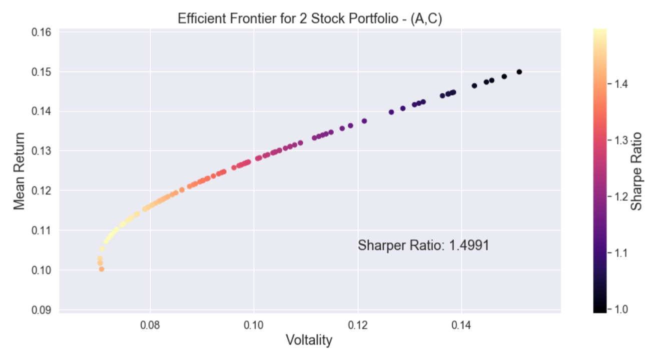

Fig 2.4: Efficient Frontier for Portfolio – (A , C)

The Efficient Frontier for all the three 2-asset combinations is shown below.

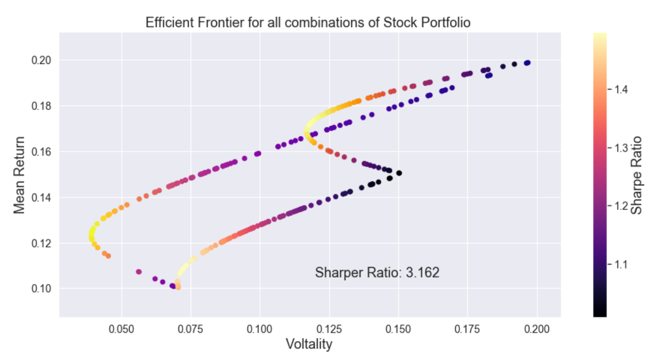

Fig 2.5: Efficient Frontier for Portfolio – (A,B), (B,C),(A,C)

2.1 Conclusion

The portfolio with stocks A and B is the best because the sharper ratio is highest for this combination.

3 CVX OPTIMIZATION

According to Markowitz' "Modern Portfolio Theory", an efficient frontier of portfolios is the one with highest expected return for a given level of risk. The portfolio at which the efficient frontier starts is called the Minimum variance portfolio. There are no constraints on the expected return for this portfolio. All the other portfolios on efficient frontier have higher volatility and higher return than the minimum variance portfolio.

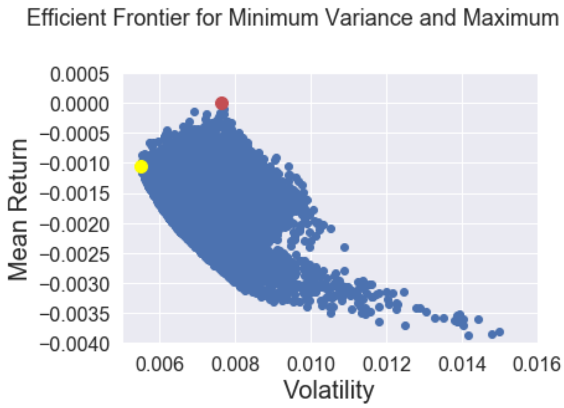

Fig 3.1 : Efficient Frontier with Miinum and Maximum Variance

In The Fig 3.1, obtained using plotly and cvxopt packages in python, the yellow dot represents the Minimum Variance Portfolio for four stocks. This portfolio has the least variance among all the portfolios. The red dot represents the Maximum Variance portfolio and it has the highest variance and also has the highest mean among all the portfolios. It is similar to the efficeint frontier obtained from Naïve MV function for the same four stocks.

4 FTSE 100

The analysis is performed for thirty of the "FTSE 100" companies stock data for the time period 1st January 2015 to 31st December 2017. The adjusted value at close for all the thirty companies were picked for analysis. Out of 30 companies, 3 companies were picked after the below analysis. The 30 companies were split into 3 groups of 10 companies each. As per Markowitz "Modern portfolio Theory", it is best to pick stocks that are uncorrelated from one another so that even if one stock is affected by market conditions, the other one will not be affected, and the loss can be minimized. The correlation plot for all the ten companies in each group were plotted and two stocks with least correlation were picked from each group. From the six companies, final three companies that are negatively correlated with each other were picked. The companies which were picked for analysis are GVC Holdings – GVC, Standard Chartered Bank – STAN, Nursing and Midwifery Council – NMC.

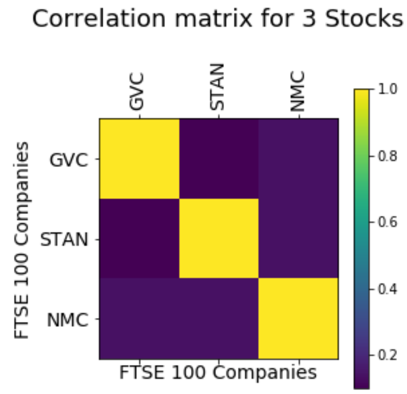

Fig 4.1: Correlation Plot of 3 stocks

The data is split into two parts. First part from 1/1/2015 to 05/31/2016 are used for analyzing the stock data using E -V Model and the second part dated from 6/1/2016 to 12/31/2017 is used for analysing the stocks using Naïve 1/N Portfolio model.

4.1 E-V Portfolio

The Daily returns for E-V Model Portfolio is plotted and shown in Fig 4.2

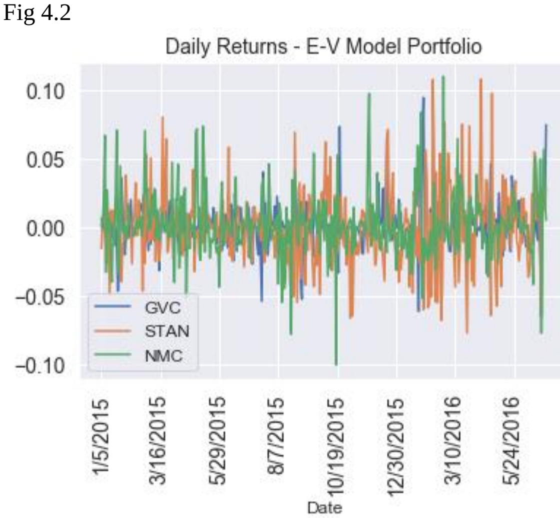

Fig 4.2: Daily Returns Plot for 1/1/2015 to 5/31/2016

The Daily Mean Return, Variance and Standard Deviation were calcualted and are show in Table 4.1.

Stock	Mean Return (Daily)	SD	Var	Skew (Mean Return)	Kutosis
GVC	0.08980	0.01700	0.00028	0.73005	5.39815
STAN	-0.02043	0.02593	0.00067	0.46233	1.86024
NMC	0.27079	0.02326	0.00054	0.28874	3.77415

Table 4.1: E-V Portfolio Calculations

In the above Table 4.1, there is one negative return. Apart from STAN, other stocks were not losing for the time period considered. Generally, most of the financial returns are leptokurtic and have kurtosis greater than 3. GVC and NMC have excess kurtosis suggesting non-normality and extreme high and low return are common and STAN has kurtosis less than 3 suggesting it is normally distributed.The Effcient Frontier for E-V model with 100 random portfolio with mean and covariance same as the three stocks are shown in Fig 4.3

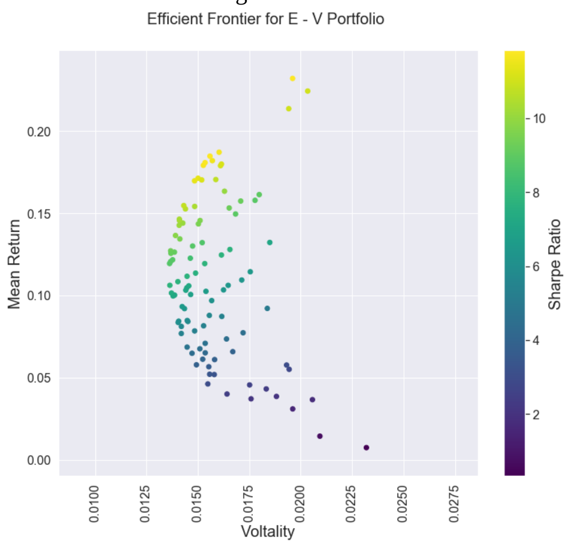

Fig 4.3: Efficinet Frontier for E-V Portfolio

The cumulative Portfolio over return for E-V Model is shown below. The highest cumulative return from E-V model is between 0.5 to 0.6 percent.

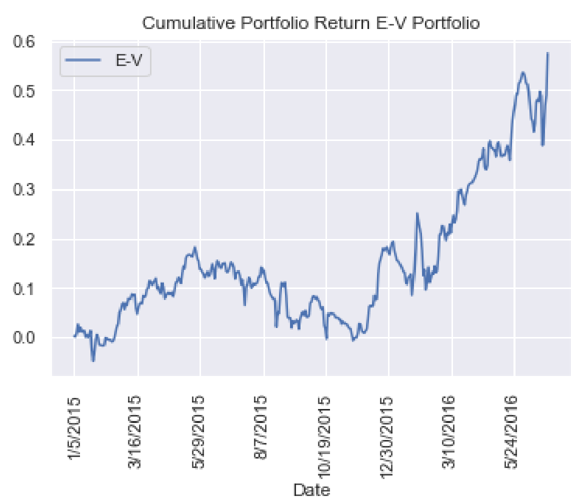

Fig 4.4: Cumulative Return for E-V Portfolio

4.2 Naïve 1/N Portfolio

The Daily returns for 1/N Portfolio is plotted and shown in Fig 4.2

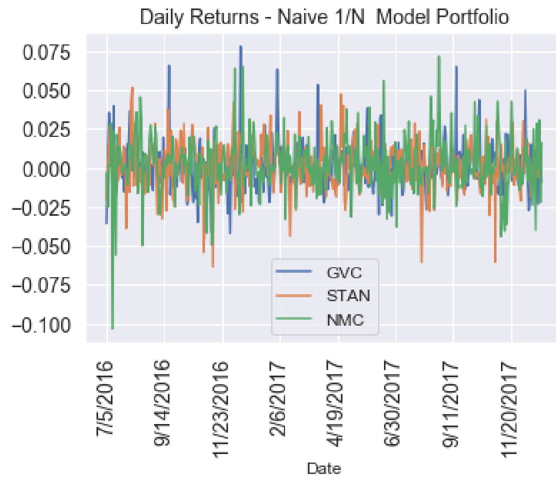

Fig 4.5: Efficinet Frontier for E-V Portfolio

The Daily Mean Return, Variance and Standard Deviation were calcualted and are show in Table 4.2.

Stock	Mean Return (Daily)%	SD	Var	Skew	Kutosis
GVC	0.15731	0.01408	0.00019	1.10651	3.31313
STAN	0.06626	0.01430	0.00020	-0.26181	2.65906
NMC	0.26323	0.01702	0.00029	0.36613	1.15632

Table 4.2: 1/N Portfolio Calculations

In the above Table 4.1, there is no negative return. None of the stocks were losing for the time period considered. Kurtosis is below 3 for STAN and NMC and little above 3 for GVC suggesting all 3 stocks are normally distributed.The Effcient Frontier for 1/N model with 100 random portfolio with mean and covariance same as the three stocks are shown in Fig 4.6

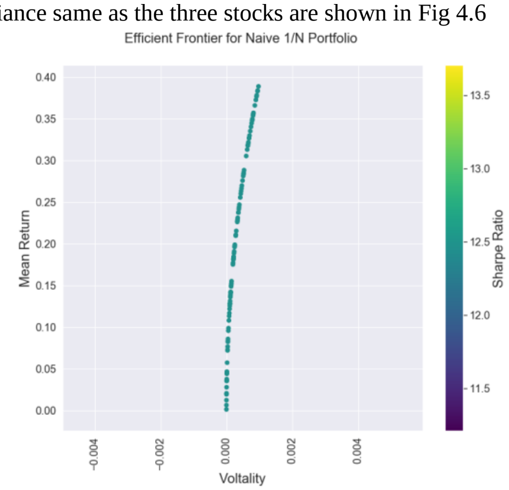

Fig 4.6: Efficinet Frontier for 1/N Portfolio

The cumulative Portfolio over return for Naïve 1/N Model is shown below. The highest cumulative return from E-V model is between 0.7 to 0.8 percent.

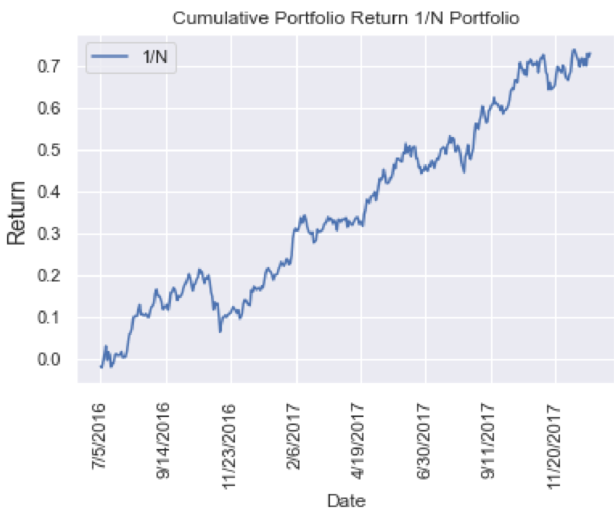

Fig 4.7: Cumulative Return for 1/N Portfolio

4.3 Conclusion

The Hence, E-V Modle Portfolio fails to perform better than Naïve 1/N portfolio model. Moreover, we can see in Fig 4.8, the efficeint frontier for 1/N portfolio(Magma color) has less risk and better mean than E-V Portfolio.

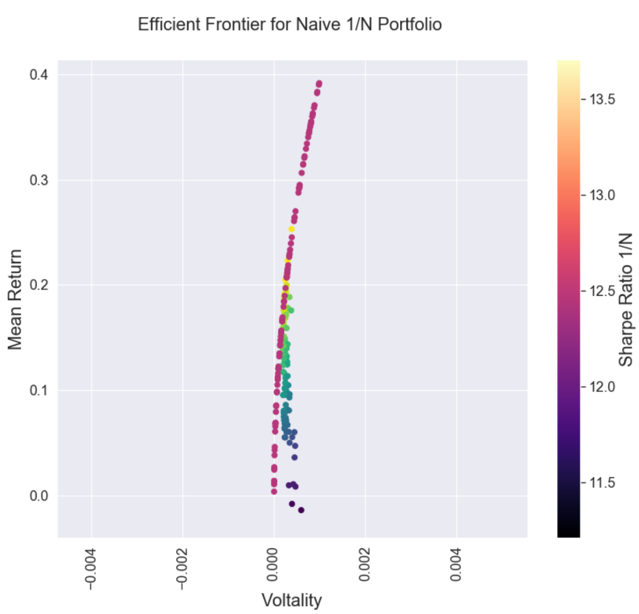

Fig 4.8: Efficinet Frontier for E-V & 1/N Portfolio

5 SHORTSALE CONSTRAINT

Short sale constraint refers to the restriction of short sale in a portfolio. This constrained is imposed by putting a restriction that the weight assigned to each stock should not be negative. Here the shortsale constraint is analyzed on a minimum variance portfolio. The analysis is performed on three stocks NMC, GVC and STAN selected by Question 4 for the same time period. The following libraries are used to calculate the Efficient Frontier for a portfolio with short sale constraint in R: quadprog, IntroCompFinR, PerformanceAnalytics, zoo.

The Mean Return and Covariance matrix were calculated.

mean M= [0.11 0.01 0.26] and

covariance matrix C = [[0.005, -0.010, 0.004],

[-0.010, 0.040, -0.002],

[0.004, -0.002, 0.023]]

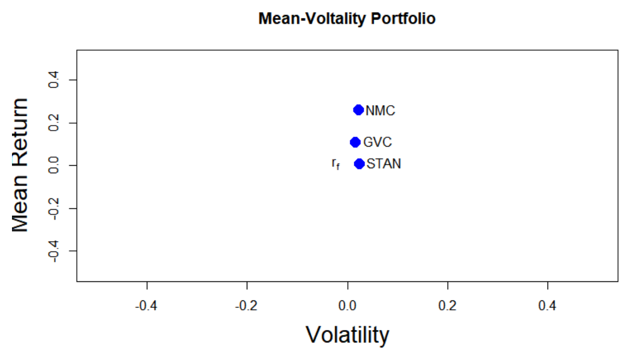

Fig 5.1: Mean-Volatility for 3 stocks

The above fig 5.1 shows the position of stocks based on their mean and volatility.

Efficient Frontier is plotted for 10 different portfolios with 3 stocks with shortsale(blue) and without shortsale(red) constraint using "**efficient.frontier**" function from **"IntroCompFinR"** library based on guidance provided in [6]. We can see the no-short sales efficient frontier lies "inside" and "to the right" of the short sales frontier [6].

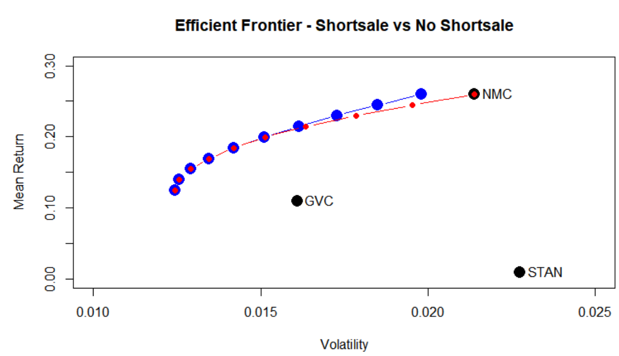

Fig 5.2: Efficient Frontier with and without shortsale

We can see in Fig 5.2, the Efficient Frontier with shortsale constraint is deviating from the efficient frontier with short sale constraint.

Port	Shortsale Allowed			Shortsale not allowed		
	GVC	STAN	NMC	GVC	STAN	NMC
1	0.374	-0.224	0.851	0.520	0.231	0.250
2	0.390	-0.174	0.784	0.503	0.180	0.317
3	0.406	-0.123	0.717	0.487	0.129	0.383
4	0.422	-0.073	0.650	0.471	0.079	0.450
5	0.439	-0.022	0.584	0.455	0.028	0.517
6	0.455	0.028	0.517	0.402	0.000	0.598
7	0.471	0.079	0.450	0.302	0.000	0.698
8	0.487	0.129	0.383	0.201	0.000	0.799
9	0.503	0.180	0.317	0.101	0.000	0.899
10	0.520	0.231	0.250	0.000	0.000	1.000

Table 5.1: Weights for Shortsale and no shortsale portfolio

We can see that negative weights are being assigned to stock “STAN” for no shortsale constraint portfolio and there are no negative weights assigned to any stock where there is a shortsale constraint.

6 INDEX TRACKING

There are two types of index tracking: Active and Passive. Active investments assume that the market is not perfectly efficient and try to beat the market, whereas passive investments assume that the market cannot be beaten in the long run [1].

6.1 Greedy Forward Selection

The data was split into training set and test set. The time period considered for analysis is from 1/1/2015 to 12/31/2017 and it was split into two parts 1/1/2015 to 5/31/2016 for training and 6/1/2016 to 12/31/2017 for testing.

6.1.1 **Selection Process**

First data was sorted based on best return and by doing cross validation of size 10. The following features were selected – 0,2,1,12,21. The first stock 0 was picked and again feature selection was performed and stock 2 was picked. Now based on stocks 0 and 2, stock 1 was picked. Then based on stocks 0,2,1 stock 12 was picked and then finally based on stocks 0,2,1,12 stock 21 was picked.

In the figure 6.1, we can see the error is decreasing as the number of stocks selected is increasing.

The stocks chosen by Greedy Forward Selection algorithm are PRU, IAG, RIO, SPY, CPG. The Mean Square Error is 0.239383.

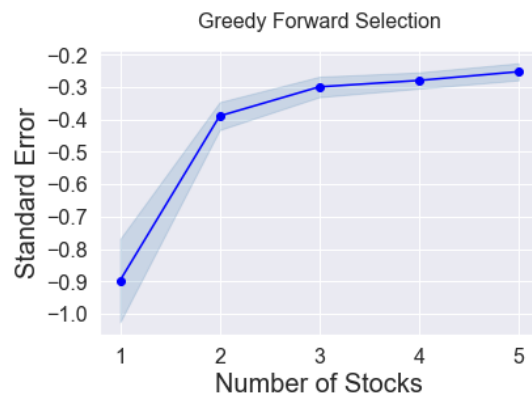

Fig 6.1: Number of Stocks selected by Greedy Forward Selection

6.2 Sparse Index Tracking

Sparse Index Tracking is performed through library(sparseIndexTracking) in R. 30 companies form FTSE 100 were picked for the time period “1/1/2015 to 12/31/2017” and FTSE price is also selected for same time range. The number of stocks selected is determined by a regularization parameter called lambda. Tracking measure selected for sparse index tracking is Empirical Tracking Error method. Plots are generated using library(plotly). The sparse index tracking was done using “**spIndexTrack**” function for various values of parameter “**lambda**” till 5 stocks were selected [8].

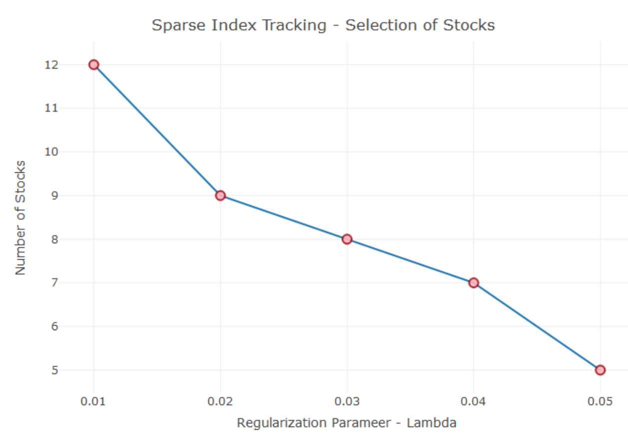

Fig 6.2: Number of Stocks selected Sparse Index Tracking

The stocks chosen by Sparse Index Tracking are MSFT, TSCO, IAG, EVR, DBO. The Mean Square Error is 0.279783

6.3 Conclusion

This Greedy Forward Selection performs better than Sparse Index Tracking based on the above analysis for index pricing of FTSE 100 companies.

7 TRANSACTION COST

7.1 Transaction Cost

An efficient portfolio is the one which gives maximum return for minimum risk. Risk can be reduced by constructing a diversified portfolio. A few permutations and combinations need to be performed before arriving at an optimal portfolio by buying and selling few stocks. Whenever a stock is bought or sold, a certain amount needs to be paid to an investment broker which is called “Transaction Cost”. Transaction cost depends on various factors. The factors are size of the trade, the type of asset being traded and its current market value. There are three types of Transaction cost:

1) Commission paid to the broker
2) Bid/Ask Spread
3) Market Impact

There are various transaction cost constraints. The constraints are included to limit exposure to risk and bounds on the amount held in each asset. [2]

7.2 Diversification Constraint

This constraint is introduced to limit the amount invested in each stock in the portfolio to a certain amount or percentage of the total amount to be invested for the portfolio. There are different types of diversification constraints. The predominant ones are the quality constraints and the cardinality constraints.

The quantity constraints impose an upper/lower bound on the amount that can be invested in each stock. The cardinality constraint imposes a limit on the maximum number of stocks that can be held in a portfolio [4]. There is also a constraint that prevents from investing more only on certain stocks which would results in an overly concentrated portfolio with higher risk. Diversification constraints are introduced to reduce misallocations due to estimation noise [5].

7.3 Shortselling Constraint

The short selling of stocks is not allowed. The total weight associated with each stock should be greater than 0 and cannot be negative. Short selling constraint can be imposed using minimum variance portfolio as well which is same as shrinking the extreme elements of covariance-matrix.

“DeMiguel et al. (2009b) note that imposing a short sale constraint on the sample-based mean - variance problem is equivalent to shrinking the expected return toward the average. the minimum-variance strategy with shortsale constraints (min-c) and minimum-variance with generalized constraints (g-min-c), have significantly higher Sharpe ratios than the 1/N strategy” [7].

7.4 Variance

A constraint is set on the maximum limit of variance for the portfolio. The variance of the portfolio at the end of time period considered should be less than the maximum standard deviation which is given by the convex quadratic inequality below as [2]

$$(w+x)^T \Sigma (w+x) \leq \sigma_{\max}^2.$$

Also, can be expressed in below format [2]:

$$\| \Sigma^{1/2}(w+x) \| \leq \sigma_{\max}.$$

$$\| \Sigma^{1/2}(w+x) \| \leq \sigma_{R,\max} \mathbf{1}^T w.$$

If variance is higher then risk is higher and vice versa. In risk free asset, variance is zero.

7.5 Shortfall risk constraints

Shortfalll refers to the probability that the return of the portfolio will fall below a certain level. The goal is to minimize the short fall and maximum the expected return. Formally, the asset allocation problem is given by [3] :

$$\max_{\omega \in \Re^n} \sum_{i=1}^{n} \omega_i E(R_i)$$

Subject to

$$P\left(\sum_{i=1}^{n} \omega_i R_i < R^{\text{low}} \right) \leq \theta$$

$$\sum_{i=1}^{n} \omega_i = 1, \quad \omega_i \geq 0,$$

The total sum of weights associated with each stock in a portfolio should be equal to 1 and it cannot be negative.

7.6 Convex Portfolio Optimization

When convex constraints are involved, the resulting problem also would be convex in nature. Those portfolios can be optimized using convex portfolio optimization with constraints. The constraints discussed here are linear transaction cost, shortselling constraint per asset and two short fall risk constraints. They are called second order cone constraints [2].

In Section 5 shortselling constraint was already implemented, to that function a transaction cost and shortfall risk constraint discussed in section 7.3 would be added as parameters.

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