Estimate regression parameters

random variable $x_t \in R^N, y_t \in R^1$ and t =1,2,..., then

$$y_t = X_t^T \theta + \xi_t \tag{1}$$

$\theta \in R^N$ is unknown parameters and ξ is a random noise.

$$\begin{aligned} \theta_t &= \theta_{t-1} + \gamma_t \varphi(y_t - \theta_{t-1}^T x_t) x_t \\ \bar{\theta}_t &= \frac{1}{t} \sum_{i=0}^{t} \theta_i \end{aligned} \tag{2}$$

By compare SGD's gradient with MLS, we can find out that

$$\varphi(x) = x \tag{3}$$

Now,let N=5, and $x_t \sim N(5,1) \; \forall \; t$

$$B = E x_1 x_1^t = \begin{pmatrix} 26 & 25 & 25 & 25 & 25 \\ 25 & 26 & 25 & 25 & 25 \\ 25 & 25 & 26 & 25 & 25 \\ 25 & 25 & 25 & 26 & 25 \\ 25 & 25 & 25 & 25 & 26 \end{pmatrix} \tag{4}$$

$$\psi(x) = E_{\psi}(x + \xi_1) \quad = \quad x \tag{5}$$

$$\chi(x) = E_{\psi^2}(x + \xi_1) \quad = \quad x^2 + 1 \tag{6}$$

$\bar{\theta}_t \to \theta$ almost surely and $(\bar{\theta}_t - \theta)\sqrt{t} \xrightarrow{D} N(0, V)$ where

$$V = B^{-1} \frac{\chi(0)}{\psi'^2(0)} = B^{-1}$$