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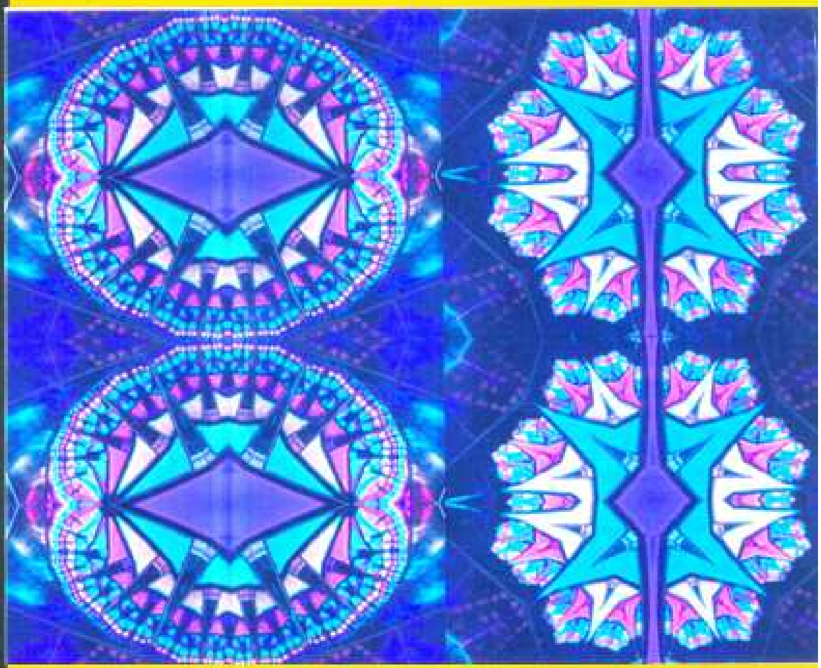

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SUNSPOTS DATA ANALYSIS USING TIME SERIES

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Abstract

The record of the sunspot number visible on the sun is regularly collected over the centuries by various observatories for studying the different factors influencing the sunspot cycle and solar activity. Sunspots appear in cycles, and last several years. These cycles follow a certain pattern which is well known. We analyzed monthly and yearly averages of sunspot data observed from year 1818 to 2002 using rescaled range analysis. The Hurst exponent calculated for monthly data sets are 0.8899, 0.8800 and 0.8597 and for yearly data set is 0.7187. Fractal dimensions[1] calculated are 1.1100, 1.1200, 1.1403 and 1.2813. From the study of Hurst exponent and fractal dimension, we conclude that time series of sunspots show persistent behavior.

The fundamental tool of signal processing is the fast Fourier transform technique (FFT). The sunspot data is also analyzed using FFT. The power spectrum of monthly and yearly

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averages of sunspot shows distinct peaks at 11 years confirming the well known 11-year cycle. The monthly sunspot data is also analyzed using FFT to filter the noise in the data.

Keywords: Sunspot; Time Series; R/S Analysis; Fractal Dimension; FFT; Power Spectrum.

1. INTRODUCTION

Sunspots[2–4] are dark which appear on the surface of the sun. They are the cooler areas in the centers of sunspots where temperatures go down to about 3700 K (compared to 5700 K for the surrounding photosphere). Sunspots typically last for several days, although very large ones may live for several weeks. Sunspots are magnetic regions on the sun where magnetic field strengths are thousands of times stronger than the Earth's magnetic field. Sunspots generally seen in pairs or a group of pairs on both sides of the solar equator come in groups with two sets of spots. One set will have a positive or north directed magnetic field, while the other set will have a negative or South directed magnetic field. The field is strongest in the darker parts of the sunspots (umbra).[5] The field is weaker and more horizontal in the lighter part of the sun (penumbra). Magnetic fields of the sunspots are very strong and keep heat away from these regions on the sun surface. Sunspots are formed when magnetic field lines are twisted and push through the solar photosphere.[6] The twisted magnetic fields above sunspots are where solar flares are observed. It has been found that solar flares have a tremendous amount of energy and are produced by sunspots.[7–9] There is a close relation between the variation of the number of sunspots and the solar activity.[10,11] In fact, the solar maximum activity corresponds to a higher number of sunspots present, while the minimum of activity is related to very few sunspots. Another effect of the solar activity is the presence of fluctuations in the interplanetary magnetic field.

Solar activity has a well-known impact on the Earth's magnetosphere and ionosphere. There is also mounting evidence that solar activity has an influence on terrestrial climate.

2. SUNSPOT DATA

Sunspots are first seen around 1610 AD by Galileo with his telescope[12] (Bray and Loughhead, 1964). The 11-year variation in the number of sunspots was first noted by Schwabe (1844). Shortly thereafter (1848), Rudolf Wolf at the Swiss Federal Observatory in Zurich devised a technique to measure the number of sunspots which continues till today and is known as the International Sunspot Number (Kiepenheuer, 1953; Waldmeier, 1961; McKinnon, 1987).[13]

Sunspots are continuously changing with time in a random fashion and constitute a typically random time series. Figure 1 shows the monthly number of sunspots, the starting point on the x-axis, i.e. 0 corresponds to the data for the month of January of the year 1818. Figure 2 show the yearly number of sunspots.

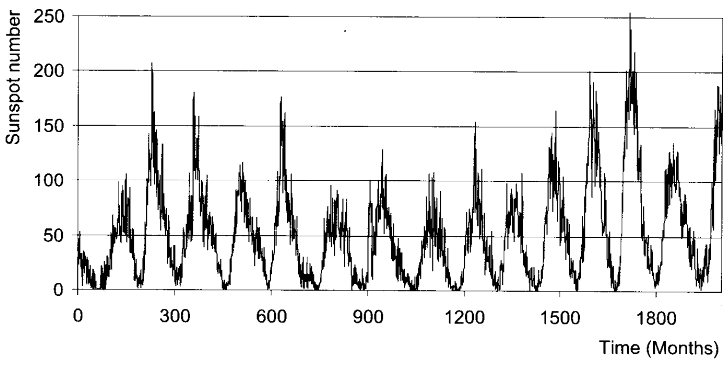

Fig. 1 Monthwise data of number of sunspots (from January 1818).

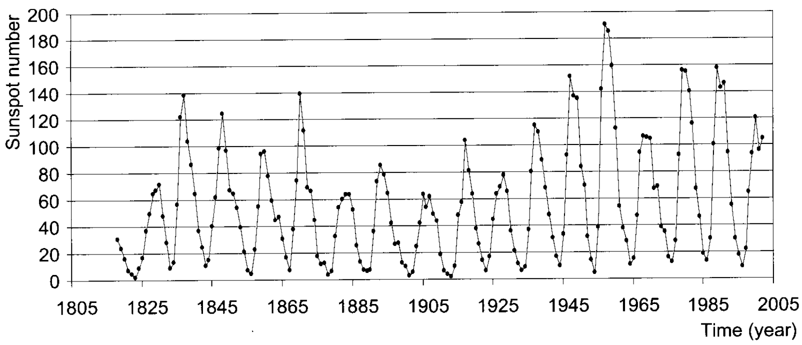

Fig. 2 Yearwise data of number of sunspots.

These plots include 2268 monthly data points and 184 yearly data points, respectively (data from January 1818 to December 2002). The approximate 11-year cycle can clearly be seen on this plot.

3. RESCALE RANGE ANALYSIS OF TIME SERIES

Rescaled range (R/S) analysis[14–17] is a statistical method to analyze the records of natural phenomena. The theory of the rescaled range analysis was first given by Hurst (1965). Mandelbrot and Wallis further elaborated the method. Feder (1988) gives an excellent review of the analysis of data using time series, history, theory and applications, and adds some more statistical experiments.

In this model Hurst exponent "H" is calculated which tells that whether time series is random or non-random. Hurst exponent is also related to the fractal dimension. There are different techniques for the estimation of Hurst exponent. R/S analysis is a method usually used for distinguishing a completely random time series from a correlated time series. Let us define an average of a given sequence of observations $Z(t)$ as

$$\langle Z\rangle_\tau = \frac{1}{\tau}\sum_{t=1}^{\tau} Z(t) \tag{1}$$

$$X(t,\tau) = \sum_{u=1}^{t} [Z(u) - \langle Z\rangle_\tau]. \tag{2}$$

The self-adjusted range $R(\tau)$ is defined as

$$R(\tau) = \max_{t=1}^{\tau} X(t,\tau) - \min_{t=1}^{\tau} X(t,\tau). \tag{3}$$

Hurst used a dimensionless ratio R/S where $S(\tau)$ is the standard deviation as a function of τ

$$S(\tau) = \left(\frac{1}{\tau}\sum_{t=1}^{\tau}(Z(t) - \langle Z\rangle_\tau)^2\right)^{\frac{1}{2}} \tag{4}$$

and found that observed rescaled range for a time series (R/S), Eqs. (4) and (5)

$$(R/S) = (\tau/2)^H. \tag{5}$$

Plotting log (R/S) against log $(\tau/2)$, the Hurst exponent H can be found which is the slope of a resulting straight line.

If H is between 0.5 and 1, the trend is persistent which indicates long memory effects. This also means that the increasing trend in the past implies increasing trend in the future also or decreasing trend in the past implies decreasing trend in the future also. In contrast, if H is between 0 and 0.5 then an increasing trend in the past implies a decreasing trend in the future and a decreasing trend in the past implies an increasing trend in the future. It is important to note that persistent stochastic processes have little noise whereas anti-persistent processes show presence of high frequency noise.

The relationship between fractal dimensions D_f and Hurst exponent H can be expressed as[18]

$$D_f = 2 - H. \tag{6}$$

From the Hurst exponent H of a time series, the fractal dimension of the time series can be found. When $D_f = 1.5$, there is normal scaling. When D_f is between 1.5 and 2, time series is anti-persistent and when D_f is between 1 and 1.5 the time series is persistent. For $D_f = 1$, time series is a smooth curve and purely deterministic in nature and for $D_f = 1.5$ time series is purely random. Scientist have analyzed fluctuations in share markets using the Hurst exponents calculated from the studies of time series applied to stock.

4. DATA ANALYSIS

From Eqs. (1) to (4), we have a value of (R_N/S_N) for the time series, $Z(t)$. $t = 1, 2, 3, \ldots, N$. Since we are interested in how (R/S) varies with successive sub-intervals τ of N, we substitute τ for N in Eqs. (1) to (4). The Hurst exponent is obtained from the following equation

$$(R_\tau/S_\tau)_{\text{Ave}} = (\tau/2)^H. \tag{7}$$

For example, if 64 values of $X(t)$ are available for time series, the R_N and S_N for $N = 64$ are obtained, then data are broken into two parts, each with $\tau = 32\,(1, 2, \ldots, 32$ and $33, 34, \ldots, 64)$. The value for R_{32} and S_{32} are obtained for the two parts. The two values of R_{32}/S_{32} are then averaged to give $(R_{32}/S_{32})_{\text{ave}}$. The data set is then broken into four parts, each with $\tau = 16\,(1, 2, \ldots, 16;\ 17, 18, \ldots, 32;\ 33, 34, \ldots, 48;$ and $49, 50, \ldots, 64)$. The value of (R_{16}/S_{16}) are obtained for the four parts and are averaged to give $(R_{16}/S_{16})_{\text{ave}}$, this process is continued for $\tau = 8$ and 4 to give $(R_8/S_8)_{\text{ave}}$ and $(R_4/S_4)_{\text{ave}}$ for $\tau = 2$, the value for $R_2 = S_2$ so that $(R_2/S_2) = 1$. The values of $\log(R_T/S_T)_{\text{ave}}$ are plotted against $\log(\tau/2)$ and the best fit straight line gives H from Eq. (7).

5. RESCALE RANGE ANALYSIS OF SUNSPOT TIME SERIES

We used sunspot data from January 1818 to December 2002. The plot of monthly and yearly data shown in Figs. 1 and 2 gives random time series. To study the R/S analysis, 2048 monthly data points are taken. R/S analysis was implemented to the 2048 data points divided into two groups, i.e. 1–1024, 1025–2049 to see if there is a change in behavior in the first and second half of the data set of 2048 points; also R/S analyses of the 2048 data points at a time were studied. The results of R/S analysis are presented in Tables 1 to 3, respectively. From these tables the graphs of $\log(R/S)_{\text{ave}}$ versus $\log(\tau/2)$ are plotted and are shown in Figs 3 to 5. The slopes of plots give Hurst's exponents and fractal dimensions of monthly sunspot time series are determined which are presented in Table 5.

Table 1 Mean R/S and τ for Monthly Data from January 1818 to April 1900.

S. No.	τ	$\tau/2$	Mean R/S	Log $(\tau/2)$	Log (R/S)
1	4	2	1.0058	0.301	0.0025
2	8	4	1.7311	0.6021	0.2383
3	16	8	2.9404	0.9031	0.4684
4	32	16	5.3857	1.2041	0.7312
5	64	32	11.1913	1.5051	1.0489
6	128	64	23.3393	1.8062	1.3681
7	256	128	44.122	2.1072	1.6447
8	512	256	72.0855	2.4082	1.8578
9	1024	512	114.5266	2.7093	2.0589

Table 2 Mean R/S and τ for Monthly Data Points from May 1900 to August 1984.

S. No.	τ	$\tau/2$	Mean R/S	Log $(\tau/2)$	Log (R/S)
1	4	2	1.0061	0.301	0.0026
2	8	4	1.7613	0.6021	0.2458
3	16	8	3.0949	0.9031	0.4906
4	32	16	5.629	1.2041	0.7504
5	64	32	11.5304	1.5051	1.0618
6	128	64	26.7637	1.8062	1.4275
7	256	128	44.9588	2.1072	1.6528
8	512	256	63.9926	2.4082	1.8061
9	1024	512	113.6337	2.7093	2.0555

Table 3 Mean R/S and τ for Monthly Data from January 1818 to August 1984.

S. No.	τ	$\tau/2$	Mean R/S	Log $(\tau/2)$	Log (R/S)
1	4	2	1.0059	0.301	0.0026
2	8	4	1.7462	0.6021	0.2421
3	16	8	3.0176	0.9031	0.4797
4	32	16	5.5073	1.2041	0.7409
5	64	32	11.3609	1.5051	1.0554
6	128	64	25.0516	1.8062	1.3988
7	256	128	44.5404	2.1072	1.6488
8	512	256	68.0391	2.4082	1.8328
9	1024	512	114.0801	2.7093	2.0572
10	2048	102400	175.9203	3.013	2.2453

We used 128 data points of yearly sunspot data from the year 1818 to 1945. R/S analysis of this data is given in Table 4. The corresponding graphs of $\log(R/S)_{\text{ave}}$ versus $\log(\tau/2)$ are shown in Fig. 6 and Hurst's exponent is given by their slopes. Fractal dimensions calculated are shown in Table 5.

The Hurst exponent H, has a value of about 0.72 for naturally occurring phenomena.[19] In our R/S analysis, it is observed that the Hurst's exponent for yearly data sets are 0.72, whereas for monthly sunspot time series data they are 0.8899, 0.8800

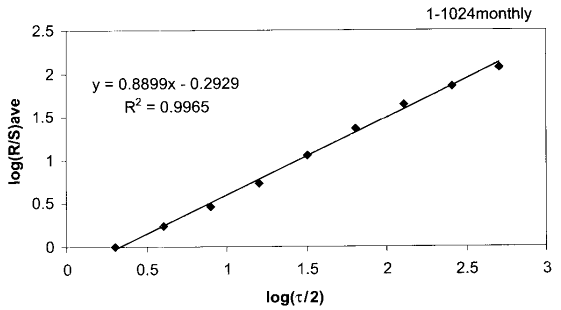

Fig. 3 A graph of $\log(R/S)_{\rm ave}$ against $\log(\tau/2)$ for monthly data from January 1818 to April 1900.

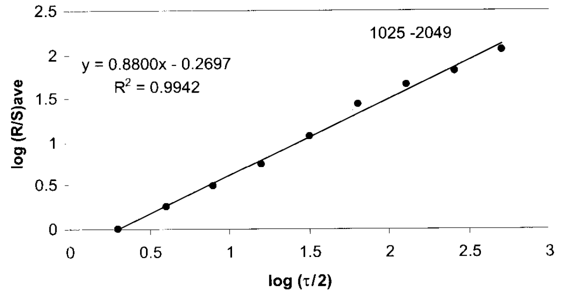

Fig. 4 A graph of $\log(R/S)_{\rm ave}$ against $\log(\tau/2)$ for monthly data from May 1900 to August 1984.

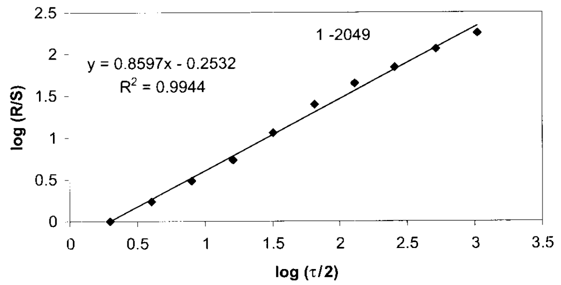

Fig. 5 A graph of $\log(R/S)_{\rm ave}$ against $\log(\tau/2)$ for monthly data from January 1818 to August 1984.

and 0.8597, respectively. Fractal dimensions for yearly data set 1.2813 and for monthly sunspot data fractal dimensions are 1.1100, 1.1200 and 1.1403, respectively.

It is seen that in all these cases the Hurst exponent lies between 0.5 and 1 indicating that there is persistency in the sunspot cycle. The changes in second and onward decimal figures in Hurst exponent H and hence the fractal dimension D_f explains complexity and variations in solar activity. This

Table 4 **Mean R/S and τ for 1 to 128 Yearly Data from 1818–1945.**

S. No.	τ	$\tau/2$	Mean R/S	Log $(\tau/2)$	Log (R/S)
1	4	2	1.0197	0.301	0.0085
2	8	4	1.9277	0.6021	0.285
3	16	8	3.5722	0.9031	0.5529
4	32	16	5.1321	1.2041	0.7103
5	64	32	7.6979	1.5051	0.8864
6	128	64	13.5109	1.8062	1.1307

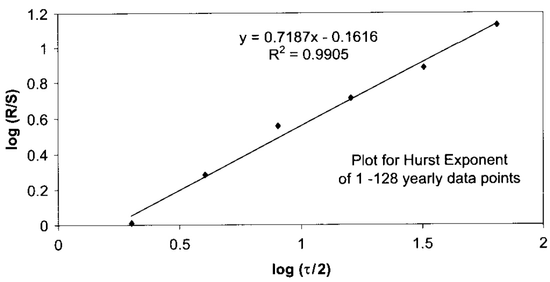

Fig. 6 Graph of $\log(R/S)_{\rm ave}$ against $\log(\tau/2)$ for yearly data 1818–1945.

Table 5 **Fractal Dimension and Hurst Exponents.**

Fig. No.	Data Type	Hurst Exponent	Fractal Dimension
Fig. 3	Monthly sunspot	0.8899	1.1100
Fig. 4	Monthly sunspot	0.8800	1.1200
Fig. 5	Monthly sunspot	0.8597	1.1403
Fig. 6	Yearly sunspot	0.7187	1.2813

is expected in a random time series of naturally occurring phenomena, many data sets of various parameters of natural phenomena show persistent behavior (as seen in Refs. 20 to 24). Benoit Mandelbrot developed the model of fractional Brownian motion to describe such phenomena.

6. FFT OF SUNSPOT DATA

The sunspot data, both yearly and monthly, were analyzed using the fast Fourier transform technique (FFT). The yearly and monthly data on mean sunspot number are shown in Figs. 1 and 2, respectively. On FFT analysis the power spectrum of the yearly data showed distinct peaks at 11 years confirming the 11-year cycle, the plot of power spectrum for the data from year 1818 to 1945 is shown in Fig. 7. The yearly data (1875–2002) on FFT analysis shows peak at 12 years indicating a 12-year cycle, this could be due to the small sample (128) years covered, however it is close to the expected 11-year cycle. The plot of power spectrum for the data from year 1875 to 2002 is shown in Fig. 8.

The monthly mean sunspot data over 1024 months (from January 1818 to April 1900) also exhibited a similar trend confirming the 11-year cycle. The plot of the power spectrum for mean monthly sunspot data is shown in Fig. 9. The monthly mean sunspot data over 1024 months (from May 1900 to August 1984). Distinct peaks are seen at 8, 16, 24 and 32 months. These correspond to 10.67 years. This is also in agreement with the 11-year sunspot number cycle. The plot of the power spectrum for mean monthly sunspot data is shown in Fig. 10. The monthly sunspot data was filtered using the FFT technique to remove the noise and fluctuations in the number of sunspots. In the resulting plot of the data of Fig. 11, the signal (in fact the random variations in the sunspot number) is filtered and the gross shape is revealed as shown in Fig. 12.

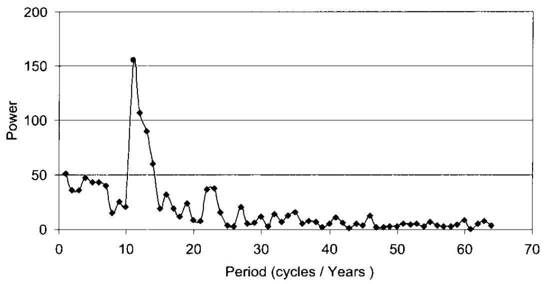

Fig. 7 The plot of power spectrum for the yearly data from year 1818 to 1945.

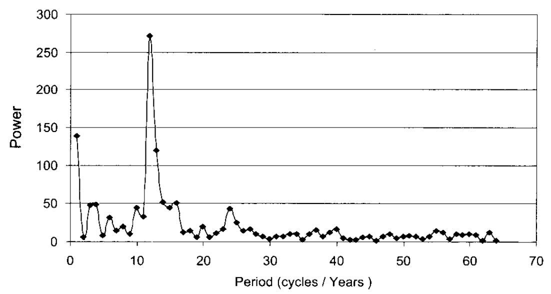

Fig. 8 The plot of power spectrum for the yearly data from year 1875 to 2002.

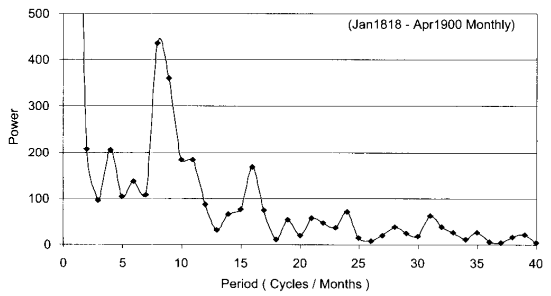

Fig. 9 The plot of power spectrum for the monthly mean sunspot data from January 1818 to April 1900.

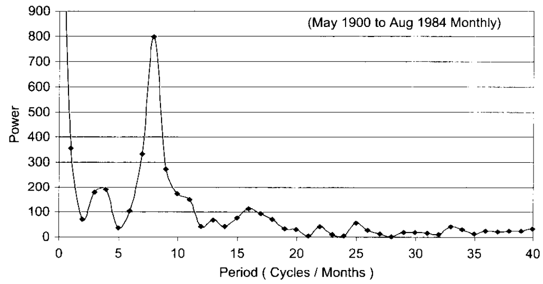

Fig. 10 The plot of power spectrum for the monthly mean sunspot data from May 1900 to August 1984.

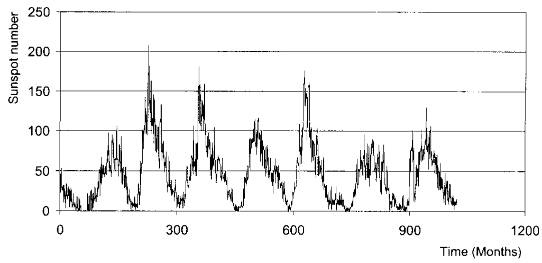

Fig. 11 Monthwise data of number of sunspots (1024 data points) (from January 1818 to April 1900).

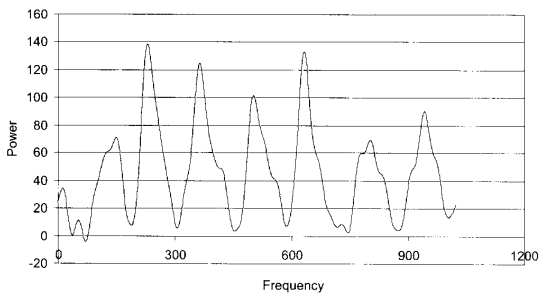

Fig. 12 Filtered signal of data in Fig. 11.

7. CONCLUSION

We analyzed time series of sunspot data by R/S analysis and estimated the Hurst exponent H and the fractal dimension D_f of the sunspot time series. We also observed that the trends shown by the yearly as well as monthly sunspot time series are persistent as is indicated by the values of the Hurst exponent lying between 0.5 and 1 and the fractal dimension 1 and 1.5.

We also analyzed the sunspot time series using FFT. We applied FFT to two different sets of yearly and three sets of monthly data. From the power spectrum for mean monthly and yearly sunspot data, it is observed that distinct peaks are found at 11-years confirming the well known 11-year cycle.

Table 5 clearly indicates a very slight increase in the fractal dimension corresponding to a slight decrease in the Hurst exponent with time.

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