Delaunay Triangulation of Imprecise Points

Preprocess and actually get a fast query time

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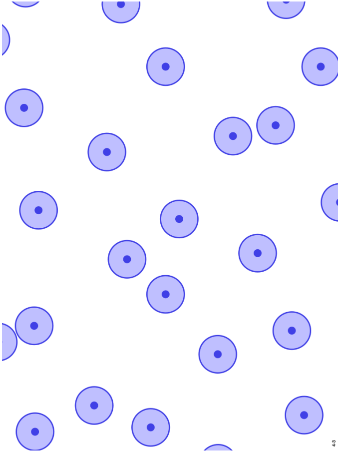
Problem

Given: a set of regions (imprecise points)

Is there an advantage we can take and find Delaunay triangulation effectively?



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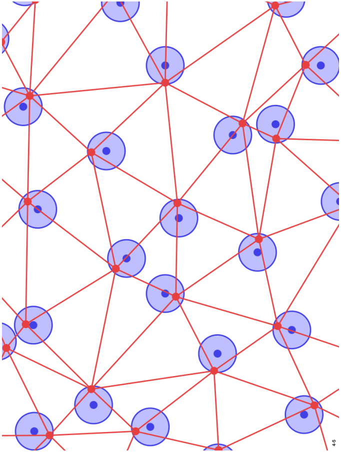
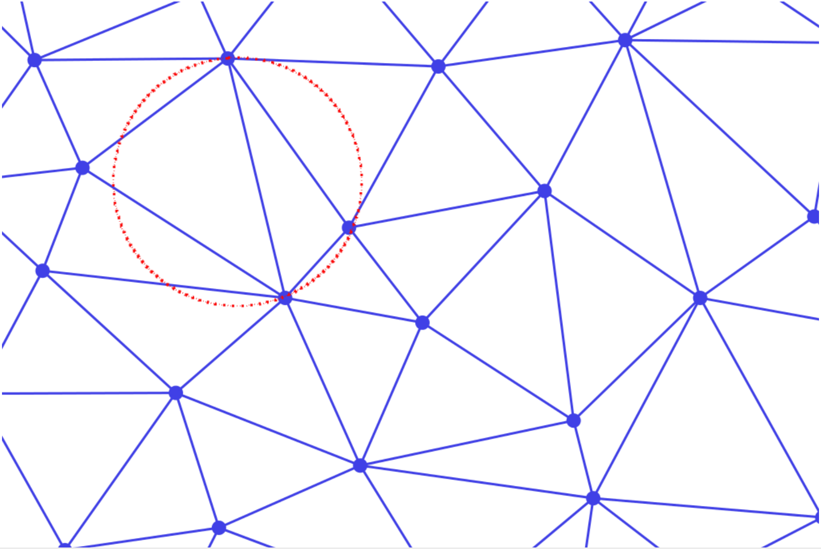


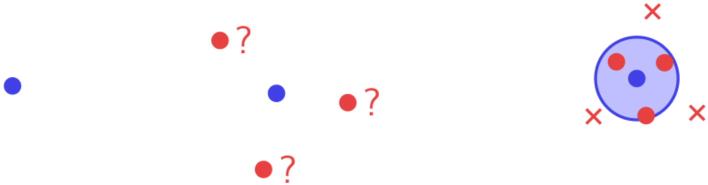
Outline

Delaunay Triangulation



Imprecise point

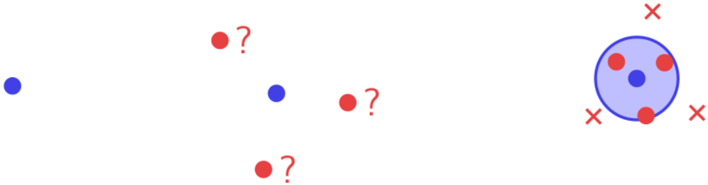
Imprecise point: extending a point to some region



Supposing we have precise locations within the regions (point instances)

Imprecise point

Imprecise point: extending a point to some region



Supposing we have precise locations within the regions (point instances)

Imprecise point

- $|W| \rightarrow$ a set W
- $|xy| \rightarrow$ distance between points x and y.
- $DT_P \rightarrow$ Delaunay triangulation of set of points P
- $NNP(v) \rightarrow$ nearest neighbour graph of $v \in P$ in $P \setminus \{v\}$
- $d^\circ G(v) \rightarrow$ degree of point v in graph G
- $\dot{p} \rightarrow$ the center of imprecise point p
- $\hat{p} \rightarrow$ an instance of imprecise point p.
- $S, \dot{S}, \hat{S}$ analogously being the sets of points
- $D(p) \rightarrow$ the disk with center p and radius $||\dot{p}NN\dot{S}(\dot{p})|| + 1$, in case of disjoint unit disks S
- $W(q) = \{p \in S \setminus q; \hat{q} \in D(p)\}$

Preprocessing

Instance processing

Complexity – unit disjoint regions

Expected cost of building $D_{\hat{S}}$ is linear.
Proof: by Backward analysis, cost of adding last point $\hat{p}$:
3 steps:

- visit triangles incident to $\hat{x} \in DT_{\hat{S}\setminus\{\hat{p}\}}$
- visit triangles crossed by line segment $\hat{x}\hat{p}$
- update triangulation to $DT_{\hat{S}}$

Together:

$$d^{\circ}{}_{DT_{\hat{S}}}(\hat{x}) + 3 \times d^{\circ}{}_{DT_{\hat{S}}}(\hat{p}) + \sum_{\hat{q} \in D(p)} d^{\circ}{}_{DT_{\hat{S}}}(\hat{q})$$

Complexity – unit disjoint regions

- The expected degree of $\hat{p}$ in $DT_{\hat{S}}$ is less than or equal to 6
- The expected degree of $\hat{x}$ in $DT_{\hat{S}}$ is less than or equal to 36
- The expected value of $\sum_{\hat{q} \in D(P)} d^{\circ}{}_{DT_{\hat{S}}}(\hat{q})$ is less than or equal to 132.
    - proof using lemma 1

Therefore complexity of adding last point is constant. ∎

Experiment

- Imprecise points vs. classical Delaunay triangulation
- Point sets: Random discs, Brownian motion, Random Balls, 3D noisy data
- running time and # triangles visited

Running time

2D random imprecise points	running time (μs) per point					
n	10^3	10^4	10^5	10^6	10^7	10^8
spatial sort	1.1	0.85	0.83	0.90	1.0	1.13
Delaunay hierarchy	1.8	1.6	2.8	5.78	9.0	13
Skewchuch	0.96	1.12	1.05	1.61	2.4	
hint random order	0.9	0.88	1.2	2.9	3.8	5.4
hint spatial sort	1.0	0.79	0.59	0.61	0.61	0.62

Running time

2D Brownian motion	running time (μs) per point					
n	10^3	10^4	10^5	10^6	10^7	10^8
spatial sort	0.78	0.78	0.88	0.96	1.12	1.20
hint spatial sort	0.73	0.69	0.79	0.81	0.83	0.82

Running time

3D random imprecise points	running time (μs) per point				
n	10^3	10^4	10^5	10^6	10^7
spatial sort	9.0	7.6	8.0	8.2	8.4
Delaunay hierarchy	11	9.7	18	25	33
hint random order	9.2	7.8	14.2	19	23
hint spatial sort	9.5	7.5	7.8	7.9	8.0

Running time

3D noisy sample of scanned models	running time (μs) per point			
n	10^3	10^4	10^5	full size ($2 \cdot 10^6$ points)
spatial sort	7	8.2	8.6	8.9
hint spatial sort	7	7.5	7.7	7.5

Visited triangles

2D random imprecise points	number of visited triangles per point					
n	10^3	10^4	10^5	10^6	10^7	10^8
spatial sort	3.74	3.64	3.71	3.67	3.55	3.71
Delaunay hierarchy	24	28	29	38	45	47
hint random order	2.83	2.8	2.77	2.75	2.75	2.74
hint spatial sort	2.82	2.80	2.77	2.76	2.75	2.75

Visited triangles

2D Brownian motion	number of visited triangles per time step					
n	10^3	10^4	10^5	10^6	10^7	10^8
spatial sort	3.81	3.68	3.77	3.72	3.62	3.78
hint spatial sort	2.77	2.77	2.77	2.77	2.77	2.77

Visited triangles

3D random imprecise points	number of visited triangles per point				
n	10^3	10^4	10^5	10^6	10^7
hint random order	5.2	5.3	5.3	5.2	5.2
spatial sort	6.3	6.6	6.6	6.6	6.6
Delaunay hierarchy	21	29	34	42	50
hint spatial sort	4.4	4.6	4.5	4.5	4.4

Visited triangles

3D noisy sample of scanned models	number of visited triangles per point			
n	10^3	10^4	10^5	full size ($2 \cdot 10^6$ points)
spatial sort	7.0	8.0	8.6	9.5
hint spatial sort	5.7	6.0	6.1	6.4

Conclution

Do you have any questions?

Thank you for your attention.

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Bibliography

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