Continuous-time Derivatives Pricing: Take-Home Assignment

Jan Besler, Student ID: 5629079

Discussion partner: Jannis Hollwedel

I – II: Product selection and characteristics

This assignment aims at analysing the the express certificate (ISIN: DE000UBS1KS0) on the stock of Morphosys (ISIN: DE0006632003). The certificate is issued by UBS Group AG. It was emitted on the 25 August 2021, while its maturity date is the 31 August 2026 with four observation dates in between. The early redemption level of the Certificate is lowered by 10 % of the initial stock price of 48.68 EUR on each observation date until it hits the barrier of 29.208 EUR at maturity. The payoff structure is depicted in figure 1.

At each of the obersavtion dates there are two possible scenarios. It is evaluated whether the underlying has touched the early redemption level, if this is the case an early payoff is triggered consisting of the nominal amount and the coupon. The certificate is best suited for investors who expect the underlying to slightly rise above the corresponding redemption levels. As can be seen in figure 1, the certificate outperforms the underlying at a range of 32 EUR to 150 EUR, depending on the redemption date. Beyond this point the underlying outperforms the certificate. Hence the certificate is not recommended to investors who anticipate a sharply falling or rising price movement in the underlying.

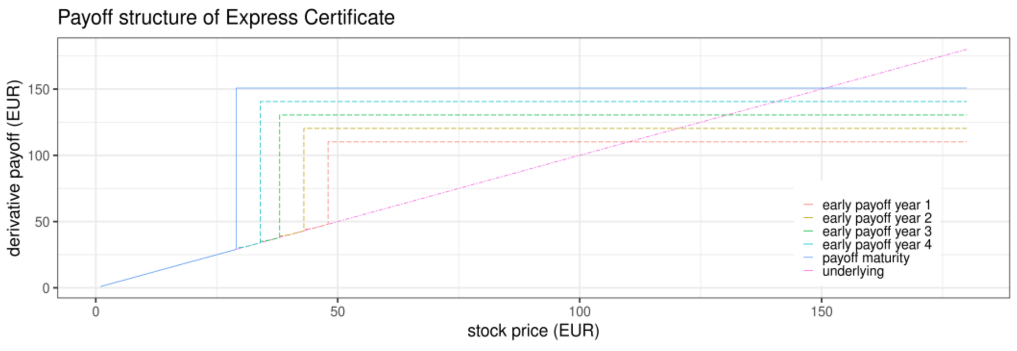

Figure 1: Payoff structure of express certificate

III – IV: Parameter selection and Valuation

The parameters used to evaluate the certificate can be viewed in table 1. The number of traiding days has been chosen as 240 to reflect 20 days per month.

Parameter	Type	Source
Price of the Derivative	Daily Closing Price	Onvista
Stock Price RWE	Daily Closing Price	Yahoo finance
Risk-free interest rate	Daily Interest Rate	Deutsche Bundesbank – Svensson method
Traiding days per year	Traiding Days (240)	-
Stock price volatility	Daily Moving Average	Standard deviation of stock returns

Table 1: Financial parameters

Due to the path-dependent nature of an express certificate a general Black Scholes formula can not be used, hence the method used to evaluate the express certificate is a geometric brownian motion, based on the before mentioned parameters. For each of the 1'000 paths the stock price has been simulated from the emission date until maturity. The price of the certificate is computed as the discounted average payoff of the single paths (figure 1). Parameters like the risk-free rate computed with the svensson method as well as the time to maturity and the volatility are daily values to better reflect the valuation settings of the express certificate.

For valuation the volatility is used as the standard deviation of the daily log returns of the underlying. This has been computed for the 1-year volatility as well as the 2-year volatility.

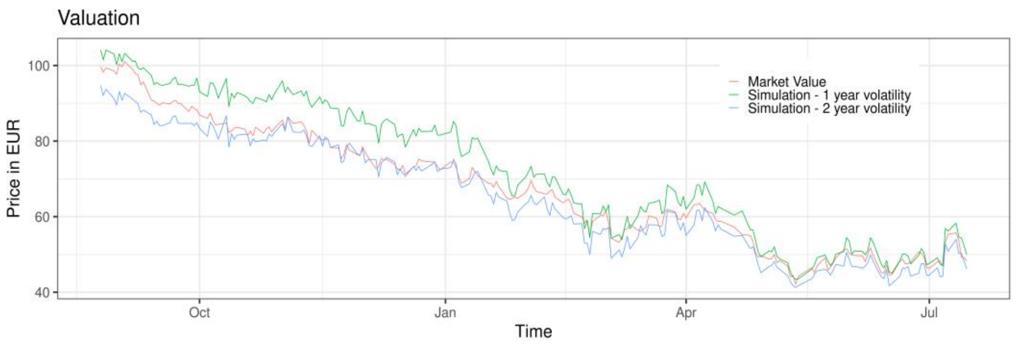

Figure 2: Valuation different volatilities

As can be seen in figure 2, the valuation based on the 2-year volatility is very close to that of the acutal market value. Especially in the time horizons of October to January and March to July the Simulation is capturing the price fluctuations of the certificate well. Where as the 1-year volatility measure is overestimating the certificate values. These discrepencies may be reduced by using more paths or a different measure such as implied volatility.

Though as table 2 shows the 2-year volatility already performs best.

	MAE	RMSE	0.25 Quantile	0.5 Quantile	0.75 Quantile
1-year volatility	4.7611	5.8073	1.8165	4.0690	7.8373
2-year volatility	2.6993	3.3245	1.1990	2.2327	3.8612

Table 2: Valuation error metrics

The 2-year voaltility outperfoms the 1-year voaltility in every category and would thus be be the better estimator to evaluate the certificate price. The differences between the the two volatility measures could be explained by the effects of the COVID-19 pandemic, as the 2-year voaltility measure is more robust against market movements.

V: Sensitivity Analysis

It is common to consider the *Greeks* (*Delta* Δ, *Gamma* Γ and *Vega* V and *Theta* θ) as risk measures to develop a more detailed understanding of the sensitivity of the certificate.

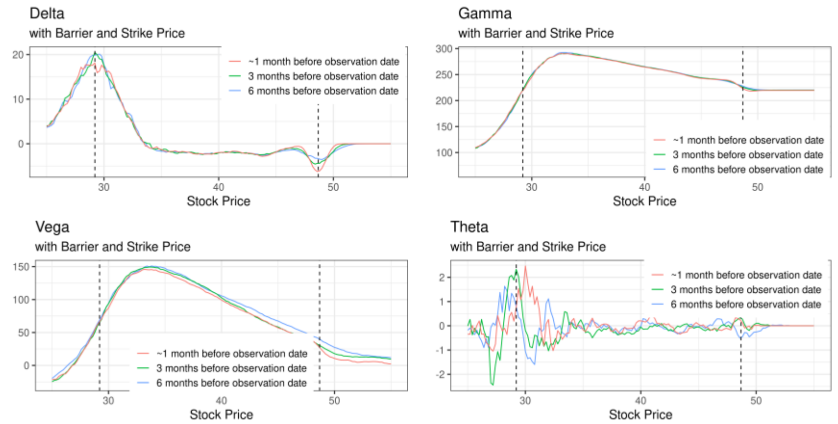

Figure 3: Greeks- Δ - Γ - V - θ

Δ measures the certificate price w.r.t the price of the stock. The plot shows that an increase in the underlying around the barrier drives the certificate price up significantly. This can be explained by being even slightly above the barrier increases the payoff. Similar but with lower intensity and inverse, being closer to the strike price decreases the price since the next higher payoff with a lower redemption level can't be reached if an early redemption level has been triggered. In between the two thresholds and beyond the strike price, the investor is not participating in the gains of the underlying. Further the closer to maturity the higher Δ, since there is less room for price movements but they have a higher impact. Γ is the derivative of Δ w.r.t the underlying. The plot shows that the most movements in Δ happen close to the barrier and strike price for the exact reasons described previously. V measure the sensitivity of the certificate towards volatility. Close to the barrier higher volatility results in a decrease in the price, while being close or above the barrier volatility significantly increases the price. This can be explained through higher losses below the barrier and higher gains above the barrier caused by increased volatility. Thsi effect increases further with less time to maturity as the volatility has an higher impact. If the price is above the strike price the participation in the development of the underlying is again approaching zero. Θ is the derivative w.r.t to time to maturity. The plot shows that only the barrier is heavily influenced by θ. This indicates that a movement across the threshold is less likely.

VI – VII: Performance Analysis

To determine the possible return of the fictive investor, the same method as in Part 1 has been chosen, using the geometric brownian motion 1'000 paths have been created. The parameters for the simulation are taken from the 15th July 2022 with a stock price of 20.28 EUR.

In the first part the investment is not hedged and thus without risk management. Only in the second part the investment in hedged with different put options.

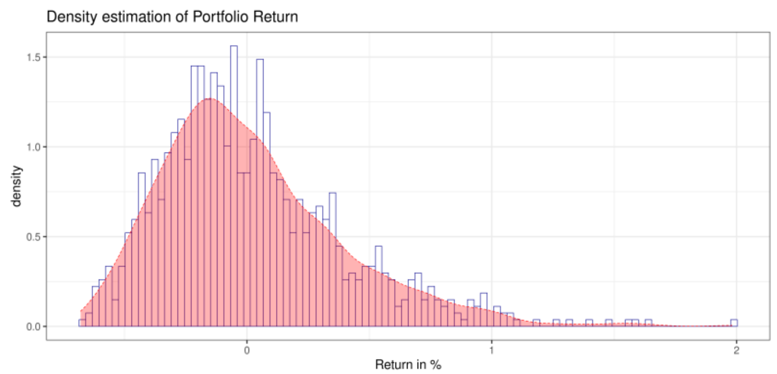

Figure 4: Portfolio return

The figure 4 shows a slight skewness towards the left and some outliers of high returns, but the majority of the returns are centred around 0 %.

For a hedged strategy one can observe that with a higher investment in put options the return in our case increases, while showing lower risk measures. These risk measures include volatility, skewness and the value-at-risk (VaR) for the 95 % and 99 % quantile. Hence the investor would be advised to hedge the investment to protect himself against downside risks. Regarding the different strike prices (K) for the put options, The mean return increases with higher strike prices and the VaR measures decrease. The volatility stays about the same. As a result the investor should pick put options with higher strike prices to minimise the VaR and maximise the return. Some examples are presented in table 3

	Mean return	volatility	skewness	VaR 95%	VaR 99 %
0% put options	1.55 %	37.44 %	1.08	- 47.39 %	- 59.30 %
6% put options	3.17 %	28.22 %	1.51	- 19.59 %	- 21.81 %
12% put options	4.78 %	22.62 %	2.28	- 16.10 %	- 16.81 %
K = 18.0	0.71 %	29.41 %	1.84	- 23.30 %	- 25.52 %
K = 19.5	4.10 %	27.61 %	2.06	- 17.73 %	- 19.96 %
K = 22.0	9.47 %	24.48 %	2.48	- 8.46 %	- 10.68 %

Table 3: Risk measures

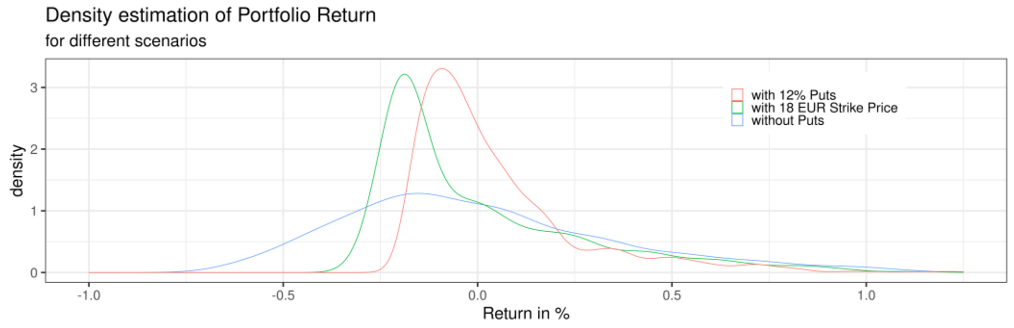

Figure 5: Portfolio hedging

The density estimations in figure 5 demonstrate how effective the downside protection of the put strategy can be. The scenario without risk management has the highest probability of generating deeply negativ returns with only a similar probability of generating positive returns compared to the other two scenarios. These are better protected against loss which can be seen in their skewness. Hence the VaR for the scenarios with risk management is lower than the scenario without hedging.

VIII: Stress Scenario

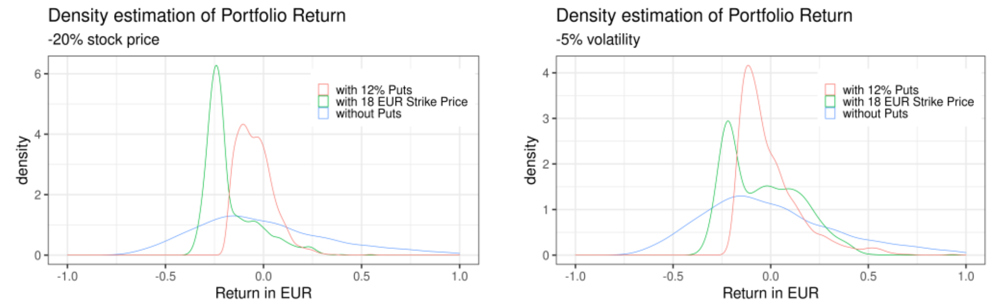

Figure 6: portfolio return for different scenarios

For the stress scenarios the simulated paths have been manipulated with half a year left until maturity. In one instance the stock price dropped 20 % and in the other case the volatility decreased by 5 %. This has been achieved by running new simulations with each of the newly changed price of a path as the new input for the simulation.

Table 4 shows the effects of the different scenarios with the example of the 12 % put hedging strategy.

	Mean return	volatility	skewness	VaR 95%	VaR 99 %
Base case	4.78 %	22.62 %	2.28	- 16.10 %	- 16.80 %
-5% volatility	- 0.42%	15.67 %	1.71	- 16.14 %	- 16.94 %
-20% stock price	- 4.15 %	9.22 %	1.06	- 16.36 %	- 16.82 %
-20% stock price & +5% volatility	- 4.13 %	9.20 %	1.04	- 16.40 %	- 16.79 %

Table 4: Portfolio scenarios

It can be seen that especially the drop in the stock price has a significant effect on the mean return, but also decreases the volatility. This can be explained by the smaller range the simulation can move in. Also the VaR stays about the same, this is due to the way it is calculated, since it only uses values at the beginning and end of the runtime the lowest prices stay roughly at the same position as before due to the lowered volatility.