Asymmetric Agents and their Willingness to Fight Customer Loyalty and its Impact on Competition

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1 Model Set Up

Consider the market for smartphones. Two firms compete over customers with heterogeneous preferences. Devices are sold at an exogenously given price such that firm 1 derives a profit of γ_i from each customer

Customer k exhibits a preference for one of the two products given by θ_k. $\theta_k \gg 0$ implies that customers k has a strong preference for the device sold by firm 2, $\theta_k \ll 0$ that she has a strong preference for product 1 and $\theta_k \approxeq 0$ that she is indifferent between the firms. Preferences of the customers are exogenously determined and distributed according to the CDF $G(\cdot)$ and the PDF $g(\cdot)$. $G(\cdot)$ is assumed twice continuously differentiable such that $g(\cdot)$ is continuously differentiable and the distribution lacks atoms.

Preferences are taken as exogenously given. They are either derived from a non-marketable unique characteristic of the products (some people dislike the sharp edges of the device, others love it), or they are derived from previous interactions with the firm. This can be used as the starting point for a dynamic model, but more about this later.

Firms compete in R&D efforts to increase the quality of their own products. A higher quality level helps to gain customers from the competitors. It can also help to increase the revenue per customer. People are willing to pay a certain amount for a smartphone, and the quality might have a strong impact on their decision which device to buy, while there willingness to pay could remain unchanged ($\frac{\partial \gamma_i}{\partial x_i} = 0$) it will often increase ($\frac{\partial \gamma_i}{\partial x_i} > 0$).

An alternative way is to let firms compete in marketing efforts. Marketing can either illustrate the positive characteristics of their device such that the customers' willingness to pay also increases with efforts, or it can illustrates drawbacks of the competing product such that the customers' willingness to pay remains unchanged.

Customer k compares the utility derived from both choices and purchases product 2 if and only if the difference in quality overcompensates for her own loyalty. The distribution has no atoms, such that ties occur with measure 0. Thus customer k purchases 2 if and only if: $\theta_k \geq x_1 - x_2$.

If firm 1 provides a product superior to its competitor's, only customers highly loyal to firm 2 will continue to purchase its devices. Thus, the demand for firm 1 is given by:

$$G(x_1 - x_2)$$

The costs of providing efforts are convex and the profits of both firms are given by:

$$G(x_1 - x_2)\gamma_1 - c_1 x_1^2/2 \tag{1}$$

$$\big(1 - G(x_1 - x_2)\big)\gamma_2 - c_2 x_2^2/2 \tag{2}$$

2 Optimal Firm Behavior

The optimal effort level for both firms is:

$$x_1^C = g(x_1^C - x_2^C)\frac{\gamma_1}{c_1} + G(x_1 - x_2)\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1} \tag{3}$$
$$x_2^C = g(x_1^C - x_2^C)\frac{\gamma_2}{c_2} + \big(1 - G(x_1 - x_2)\big)\frac{\partial \gamma_2}{\partial x_2}\frac{1}{c_2} \tag{4}$$

As long as $g(\cdot) > 0$, efforts are non-zero. If the return per customer is independent of the efforts, we can normalize the effort costs to 1 without loss of generality. Depending on the shape of $G(\cdot)$ the game might have multiple equilibria. To avoid having to assume uniqueness the following discussion focuses only on local changes.

Efforts either increase the return per customers or leave it unchanged such that $\gamma_1'(\cdot) \geq 0$. Additionally, $\gamma_1''(\cdot) - c_1 < 0$. This implies that efforts don't decrease the valuation of the product and that without of competition the optimal level of efforts is finite.

2.1 Special Case: Per Customer Revenue independent of Efforts

The reaction function of firm i to a change in the efforts of firm j is:

$$\frac{\mathrm{d}x_1^C}{\mathrm{d}x_2} = g'(x_1^C - x_2^C)\frac{\gamma_1}{c_1}\left(\frac{\mathrm{d}x_1^C}{\mathrm{d}x_2} - 1\right) \tag{5}$$
$$\frac{\mathrm{d}x_1^C}{\mathrm{d}x_2} = \frac{-g'(x_1^C - x_2^C)\frac{\gamma_1}{c_1}}{1 - g'(x_1^C - x_2^C)\frac{\gamma_1}{c_1}} \tag{6}$$

In this case the costs can be normalized to 1 by setting the revenue per customer to $\frac{\gamma_1}{c_1}$. If $g'(x_1^C - x_2^C)\gamma_1 > 1$ the market cannot be in equilibrium as the second order condition for the optimal efforts of firm 1 implies that the optimal efforts are not a maximum.

If $-\frac{1}{\gamma_1} < g'(x_1^C - x_2^C) < 0$ this expression is positive, such that an increase in x_2 leads to an increase in x_1 and thus an overall escalation. The increase in x_2 increases the number of indifferent customers, such that competition becomes more intense. If $g'(x_1^C - x_2^C) > 0$ this expression is negative and an increase in x_2 leads to crowding out of x_1.

Returning to the example of the smartphone sector, we consider a distribution that satisfies $-\frac{1}{\gamma_1} < g'(\theta) < \frac{1}{\gamma_1}$ for all θ. Additionally, most customers are centered in the middle with only few highly loyal customers. This can be described by $g'(0) = 0$ and $g'(\theta) < 0, \theta > 0; g'(\theta) < 0, \theta < 0$. Most customer are willing to switch to the superior product, while a small number has some inherent preference for one.

I need to address symmetry earlier in the text.

2.2 General Case

The reaction function of firm 1 to a change in the efforts of firm 2 is given by:

$$\frac{\mathrm{d}x_1}{\mathrm{d}x_2} = g'(x_1^C - x_2^C)\left(\frac{\mathrm{d}x_1}{\mathrm{d}x_2} - 1\right)\frac{\gamma_1}{c_1} + g(x_1 - x_2)\left(\frac{\mathrm{d}x_1}{\mathrm{d}x_2} - 1\right)\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1} + G(x_1 - x_2)\frac{\partial^2 \gamma_1}{\partial^2 x_1}\frac{\mathrm{d}x_1}{\mathrm{d}x_2}\frac{1}{c_1} \tag{7}$$

$$\frac{\mathrm{d}x_1}{\mathrm{d}x_2} = A_i^{-1}\left(-g'(x_1^C - x_2^C)\frac{\gamma_1}{c_1} - g(x_1 - x_2)\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1}\right) \tag{8}$$
$$A_i = 1 - g'(x_1^C - x_2^C)\frac{\gamma_1}{c_1} - g(x_1 - x_2)\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1} - G(x_1 - x_2)\frac{\partial^2 \gamma_1}{\partial^2 x_1}\frac{1}{c_1} \tag{9}$$

Assuming that $A_i > 0$ and that $\frac{\partial \gamma_1}{\partial x_1} > 0$, we find that the sign of the reaction function depends on $g'(\cdot)$. The reaction is positive if $g'(\cdot)$ is sufficiently negative. In this case also $A_i > 0$. Then the competition intensity described by $g(\theta^*)$ increases sufficiently to raise the incentives to compete more intensively. If $g'(\cdot)$ is close to zero, competition intensity does still increase, but is offset by the lower market share. A lower market share makes extracting profits from the customers less valuable, thus making $\frac{\partial \gamma_i}{\partial x_i}$ less important. Thus, for small negative and large positive $g'(\cdot)$ and increase in x_2 leads to lower competition and a lower effort by 1.

3 Third Party

Now consider a third party, that is maximizing the efforts x_1 and x_2, weighted with some function. The utility derived from the R&D expenditures is given by $U(x_1, x_2)$ with $\frac{\partial U(x_1,x_2)}{\partial x_1}, \frac{\partial U(x_1,x_2)}{\partial x_2} > 0$. The third party is able to affect the incentives of the firms. To simplify the problem we assume that the third party intervenes in γ_2 and that firm 2 is the disadvantaged firm ($\gamma_1 \geq \gamma_2$). The optimal γ_2 is given by:

$$\frac{\mathrm{d}U(x_1,x_2)}{\mathrm{d}\gamma_2} = \frac{\partial U(x_1,x_2)}{\partial x_1}\frac{\mathrm{d}x_1}{\mathrm{d}\gamma_2} + \frac{\partial U(x_1,x_2)}{\partial x_2}\frac{\mathrm{d}x_2}{\mathrm{d}\gamma_2} \tag{10}$$

$$\frac{\mathrm{d}U(x_1,x_2)}{\mathrm{d}\gamma_2} = \left(\frac{\partial U(x_1,x_2)}{\partial x_1}\frac{\mathrm{d}x_1}{\mathrm{d}x_2} + \frac{\partial U(x_1,x_2)}{\partial x_2}\right)\frac{\mathrm{d}x_2}{\mathrm{d}\gamma_2} \tag{11}$$

The second line follows from the fact that γ_2 does not directly impact x_1 but only through x_2. An increase in the incentives of firm 2 leads to an increase of its efforts such that $\frac{\mathrm{d}x_2}{\mathrm{d}\gamma_2} > 0$.

Thus, if the reaction function is positive or only slightly negative an increase in the incentives of the weaker firm (2) leads to an increase in the total efforts of both firms. Firm 2 will exert more efforts which in turn will encourage firm 1 to also increases hers, or only slightly reduces its efforts.

In contrast, if $\frac{\mathrm{d}x_1}{\mathrm{d}x_2} \ll 0$ and/or if $\frac{\partial U(x_1,x_2)}{\partial x_1} \gg \frac{\partial U(x_1,x_2)}{\partial x_2}$ the third party should reduce the incentives of firm 2 such that it reduces its efforts which encourages firm 1 to increase its efforts. While the total amount of effects does necessarily decrease, firm 1's efforts are seen as more important than firm 2, creating a net benefit. If the effort do not change the firm's profit extraction, the latter case requires that $\frac{1}{\gamma_1} < g'(x_1^C - x_2^C) < 0$.

In the smartphone sector we observe: a small level of loyalty and a medium amount of asymmetry. Thus, supporting the weak firm will raise overall competition leading to a welfare increase. Similar, damaging the profit margin of the leader might lead to an overall increase in the efforts if the change in the number of indifferent customers is sufficiently big. Here moving closer to symmetry increases competition intensity.

In contrast, the market for cola-flavored soft-drinks features a large amount of loyalty, with a moderate amount of asymmetry. Thus, $g'(\theta) > 0$ and increasing the incentives of the weaker firm or reducing the incentives of the leader causes a decrease in competition and an overall decrease in total efforts.

3.1 Value-Augmenting Efforts

If the efforts of the firm also augment its profit extraction from customers, the reaction function becomes:

$$\frac{\mathrm{d}x_1}{\mathrm{d}x_2} = A_i^{-1}\left(-g'(x_1^C - x_2^C)\frac{\gamma_1}{c_1} - g(x_1 - x_2)\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1}\right) \tag{12}$$

$$A_i = 1 - g'(x_1^C - x_2^C)\frac{\gamma_1}{c_1} - g(x_1 - x_2)\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1} - G(x_1 - x_2)\frac{\partial^2 \gamma_1}{\partial^2 x_1}\frac{1}{c_1} \tag{13}$$

A sufficiently positive $g'(\theta)$ leads to an decrease in the effort of firm 1 in response to an increase in the efforts of firm 2. However, the reaction is now also determined by the changes to the market share. An increase in the efforts of firm 2 leads to a decrease in the market shares of firm 1. This lowers it's incentives to use high efforts to extract monetary value from the firms. By encouraging the laggards in the smartphone sector the third party also reduces the market share of the leading firms, who will then invest less in R&D.

4 Examples

4.1 Basic Case - Uniform Distribution

First, consider the basic case of a uniform distribution of customers. Thus $g(\theta) \equiv \tilde{g}$ and $g'(\cdot) = 0$. The solutions to the firms problem becomes:

$$\begin{aligned} x_1^C &= \tilde{g}\frac{\gamma_1}{c_1} + G(x_1 - x_2)\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1} \\ x_2^C &= \tilde{g}\frac{\gamma_2}{c_2} + \big(1 - G(x_1 - x_2)\big)\frac{\partial \gamma_2}{\partial x_2}\frac{1}{c_2} \end{aligned} \tag{14}$$

The reaction function is given by:

$$\begin{aligned} \frac{\mathrm{d}x_1}{\mathrm{d}x_2} &= A_i^{-1}\left(-\tilde{g}\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1}\right) \\ A_i &= 1 - \tilde{g}\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1} - G(x_1 - x_2)\frac{\partial^2 \gamma_1}{\partial^2 x_1}\frac{1}{c_1} \end{aligned}$$

If the efforts do not affect the value extraction of the firms, the optimal efforts of the firms are given by $\tilde{g}\frac{\gamma_i}{c_i}$ and thus proportional to the value and the cost ratio. Most importantly, the efforts are independent of the competitors efforts. Firms compete for market shares, but their optimal decision are independent of the other firms decision.

If the firm also derive value from the efforts, the reaction is always negative. A higher effort by firm 2 leads to a lower market share of firm 1, which reduces its incentives to exert efforts. Market share is the dominant motivator to perform product improvement and a smaller one leads to less incentives to innovate.

Can the increase of firm 2's efforts in this case lead to an increase of total efforts? This would require that $\frac{\mathrm{d}x_1}{\mathrm{d}x_2} > -1$.

$$\begin{aligned} -1 &< \frac{\mathrm{d}x_1}{\mathrm{d}x_2} \leq 0 \\ 0 \leq \tilde{g}\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1} &< 1 - \tilde{g}\frac{\partial \gamma_1}{\partial x_1}\frac{1}{c_1} - G(x_1 - x_2)\frac{\partial^2 \gamma_1}{\partial^2 x_1}\frac{1}{c_1} \end{aligned}$$

Thus, total efforts only increase if the affect of x_1 on the value extraction is very small. If $\frac{\partial \gamma_1}{\partial x_1} = 0$ it holds as the efforts are independent of the market share. However, if the efforts are very important in extracting value from the customers an increase in the efforts of any party leads to a decrease in the total efforts in the market.

5 Other aspects

- Market shares determine investments. Are returns convex or concave?
- Customers profit from investment. The large firm has more customers. If it invests more, it helps more people.
- What is the social optimum?
- Switching costs?