Case Study 4 - Group 4

Annika Janson h11829506
Jan Beck h11814291
Franz Uchatzi h1451890

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2 Model

2.1 Model estimation

2.1.1 and 2.1.2

(See page 2 for model comparison and regression output.)

The R^2 value of model 1 and 2 is **0.348** and **0.828** respectively.

The estimates and standard errors for the non-brand explanatory variables of model 1 and 2 are identical.

The estimates for `rq`, `vo`, `wa`, `ju`/intercept, `education`, `income`, `age` and `price` are significant at the 5%-level.

2.2

Model 1: the estimate for `kr` is **-0.287950**, which means that on average the rating is changing by **-0.2887950** c.p. In other words, we shift the regression line down by 0.2887950.

Model 2: the estimate for `kr` is **20.560087**, this is the intercept for `kr`. On average, if the brand kr and all other variables were 0, the rating would be **20.560087** c.p.

2.3

We can calculate the regression parameter associated with `kr` in Model 1 by subtracting the value of `ju` in Model 2 from the value of `kr`in Model 2.

This is because `ju` was our reference group, so the intercept of Model 1 is equivalent to the intercept of `ju`, which is also shown in Model 2. Model 1 shows us the difference between choosing "kr" or any other group and Model 2 shows us each groups intercept.

Table 1: Model comparison

	Dependent variable:	
	rating	
	(1)	(2)
rq	3.884***	24.732***
	(0.312)	(0.478)
vo	3.557***	24.405***
	(0.312)	(0.478)
wa	0.596*	21.444***
	(0.312)	(0.478)
kr	−0.288	20.560***
	(0.312)	(0.478)
ju		20.848***
		(0.478)
education	−0.257	−0.257
	(0.218)	(0.218)
gender	−0.107	−0.107
	(0.200)	(0.200)
income	−0.641***	−0.641***
	(0.205)	(0.205)
age	0.012**	0.012**
	(0.006)	(0.006)
price	−0.303***	−0.303***
	(0.008)	(0.008)
Constant	20.848***	
	(0.478)	
Observations	3,195	3,195
R^2	0.348	0.828
Adjusted R^2	0.346	0.828
Residual Std. Error (df = 3185)	5.584	5.584
F Statistic	188.881*** (df = 9; 3185)	1,537.900*** (df = 10; 3185)
Note:		*p<0.1; **p<0.05; ***p<0.01

2.4

H0: $\beta_{wa} = 0$ H1: $\beta_{wa} \neq 0$

In model 1, the p-value for β_{wa} is **0.05641**. Therefore, at the $\alpha = 0.05$, we can not reject the null hypothesis. We conclude, that there is no difference in the average rating between the brands `ju` and `wa` c.p.

Bonus question:

```
## Linear hypothesis test
##
## Hypothesis:
## wa - ju = 0
##
## Model 1: restricted model
## Model 2: rating ~ 0 + rq + vo + wa + kr + ju + education + gender + income +
##     age + price
##
##   Res.Df   RSS Df Sum of Sq      F  Pr(>F)
## 1   3186 99433
## 2   3185 99320  1    113.58 3.6425 0.05641 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The linear hypothesis shows that the p-value again is **0.05641**, which is exactly the p-value we expected, as it was the one we could see in the results of `wa` in Model 1.

2.5

2.5.1

```
## Linear hypothesis test
##
## Hypothesis:
## rq = 0
## vo = 0
## wa = 0
## kr = 0
##
## Model 1: restricted model
## Model 2: rating ~ rq + vo + wa + kr + education + gender + income + age +
##     price
##
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1   3189 109650
## 2   3185  99320  4     10331 82.823 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

To check whether the brand information is helpful to determine the rating of mineral water, we perform a linear hypothesis test for Model 1 with the following H0 and H1. However, we need to exclude the variable "ju" as it works as the baseline for the brand effect in model 1.

H0: $H_0 : \beta_{rq} = \beta_{vo} = \beta_{wa} = \beta_{kr} = 0$ H1: $H_1 : H_0$ is not true.

We run a linear hypothesis test and find that the p-value is $5.5040372 \times 10^{-67}$ and the F-statistic is **82.8229128**. We find little evidence in the data that we should reject the null hypothesis that the coefficients for `rq`, `vo`, `wa` and `kr` are equal to 0 and therefore can be jointly excluded from the model, c.p.

Bonus question:

```
## Linear hypothesis test
##
## Hypothesis:
## rq = 0
## ju = 0
## vo = 0
## wa = 0
## kr = 0
##
## Model 1: restricted model
## Model 2: rating ~ 0 + rq + vo + wa + kr + ju + education + gender + income +
##     age + price
##
##   Res.Df    RSS Df Sum of Sq      F    Pr(>F)
## 1   3190 192342
## 2   3185  99320  5     93023 596.61 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
## [1] 0
```

```
## [1] 596.6138
```

For the bonus question, we take same approach as for Model 1 with the difference that now, all brands of mineral water are included in the H0. In Model 2 the intercept β_0 is excluded.

H0: $H_0 : \beta_{ju} = \beta_{rq} = \beta_{vo} = \beta_{wa} = \beta_{kr} = 0$ H1: $H_1 : H_0$ is not true.

We run a linear hypothesis test and find that the p-value is **0** and the F-statistic is **596.613813**. We find little evidence in the data that we should reject the null hypothesis that the coefficients for `ju`, `rq`, `vo`, `wa` and `kr` are equal to 0 and therefore can be jointly excluded from the model, c.p.

2.5.2

```
##         R-squared Adj. R-squared      AIC      BIC
## Model 1 0.3479941      0.3461517 20069.45 20136.21
## Model 3 0.2801749      0.3461517 20377.61 20420.10
```

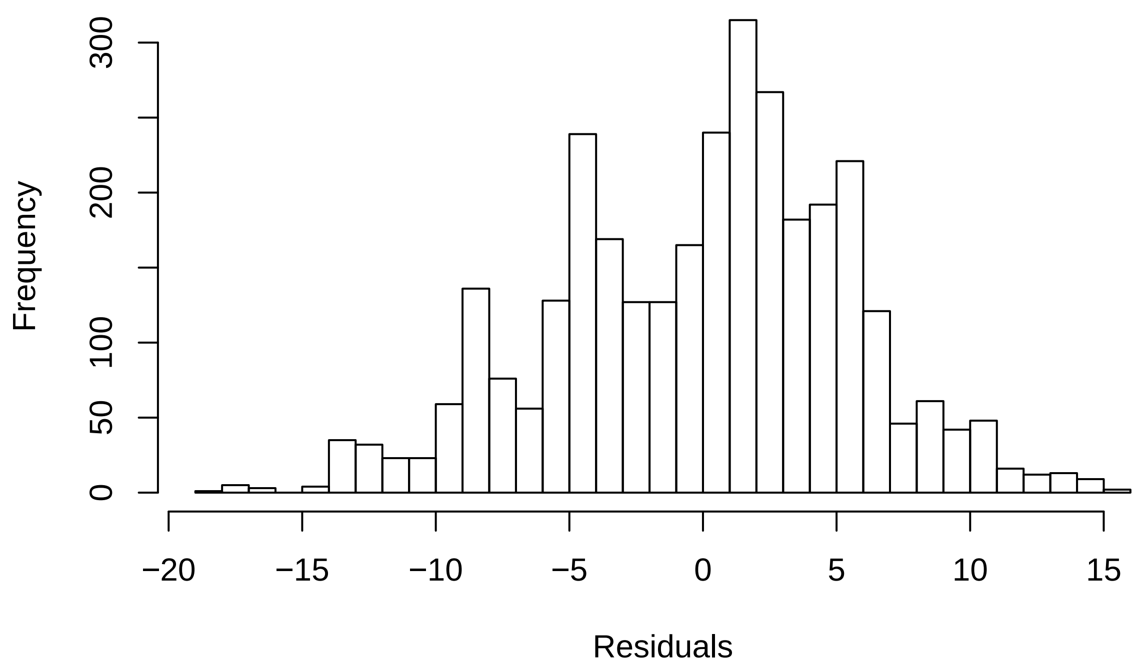
The matrix shows different values for model selection criteria for model 1 and model 3, which is a reduced version of model 1 in terms of explanatory variables of the mineral water brands. We can see that R-squared of model 1 is **0.0678192** higher than for model 3, suggesting that model one explains **6.7819** percent more variation in rating can be explained with variation of the independent variables, altough, the percentage increase for each variable is comparatively low. Furthermore, model 1 consists of five more explanatory variables than model 3 and R-squared has the property to increase for each additionally explanatory variable added to the model.

Therefore, we take a look at the adjusted R-squared, which is the same value for model 1 and model 3 respectively with **0.3461517** . This result shows that the percentage increase of R-squared in model 1 is expected to be by chance and that adding the variables of mineral water brands does not actually increase the model fit.

If we compare the AIC and BIC values for each model we see that for model 1 the AIC is **308.1600653** and the BIC is **283.8826959** units smaller than for model 3. The smaller AIC and BIC values of model 1 indicate a better fit of the model in comparison to model 3. As a result, we find that model 1 explains a change in rating better than model 3.

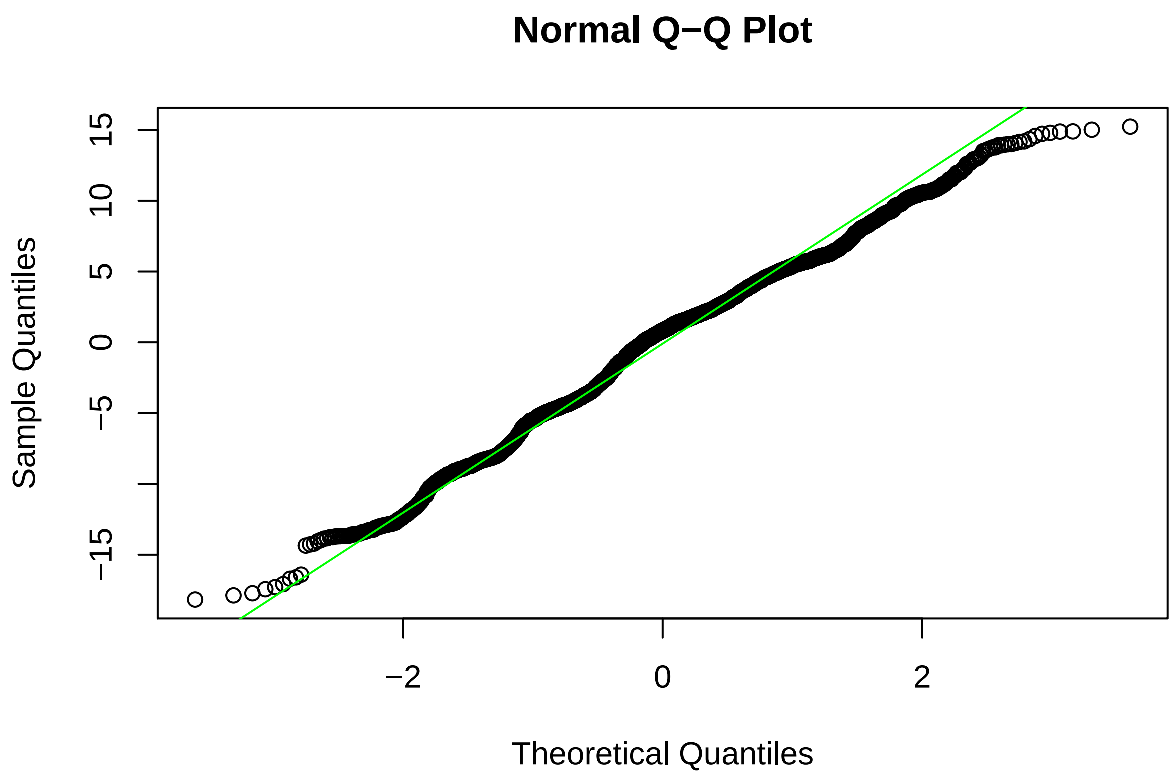
2.6

```
## [1] TRUE
```




```
##
## Call:
## lm(formula = rating ~ rq + vo + wa + kr + education + gender +
##     income + age + price, data = marketing)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -18.167  -4.118   0.827   3.931  15.232
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 20.848037   0.477726  43.640  < 2e-16 ***
## rq           3.884194   0.312412  12.433  < 2e-16 ***
```

```
## vo            3.557121   0.312412  11.386  < 2e-16 ***
## wa            0.596244   0.312412   1.909  0.05641 .
## kr           -0.287950   0.312412  -0.922  0.35675
## education    -0.256875   0.218121  -1.178  0.23902
## gender       -0.106798   0.199892  -0.534  0.59319
## income       -0.641044   0.204691  -3.132  0.00175 **
## age           0.012078   0.006017   2.007  0.04483 *
## price        -0.302541   0.008232 -36.750  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.584 on 3185 degrees of freedom
## Multiple R-squared:  0.348,  Adjusted R-squared:  0.3462
## F-statistic: 188.9 on 9 and 3185 DF,  p-value: < 2.2e-16
```




```
##
##  Jarque Bera Test
##
## data:  resids
## X-squared = 36.524, df = 2, p-value = 1.172e-08
```

H0: Residuals are normally distributed H1: Residuals are not normally distributed

Histogramm: Looking at the Histogramm, it does not look like a symmetric distribution around 0. it seems that the residuals are not normally distributed, as they are located around 1.5 and not around 0. And they have outliners on both sides which they do ot have not the other side.

QQ-Plot: Till 1.5 it seems the reisduals follow a normal distribution. But for values higher than 1.5, they seem to differ from normal distribution.

Jarque- Bera- Test: The Jarque-Bera Test confirms our observations from the Histogramm and the QQ-Plot. With X-squared = **36.525** it is bigger than **6**, which is the limit. Additional the p-value is **1.172e-08**, so very small. At a 95% confidence level, the p-value of "J" is smaller than 0.05 and we reject the H0.

Summarising our observations, our error term is not normaly distributed, we have a problem with our model.

2.7

We add interactions between dummy variables and continuous explanatory variables in three steps. First, the interaction between `kr` and `age`. Second, between the variables `vo` and `income`. The last interaction added is between the variables `wa` and `price`. The results betwwen each step are shown in 2.8.

2.8

```
##                    R-squared Adj. R-squared      AIC      BIC
## Model 1            0.3479941      0.3461517 20069.45 20136.21
## Step1 (kr:age)     0.3480051      0.3459574 20071.40 20144.23
## Step2 (vo:income) 0.3483455      0.3460935 20071.73 20150.63
## Step3 (wa:price)  0.3484292      0.3459719 20073.32 20158.29
```

The matrix shows the incorporation of each interaction between a pair of selected variables and the effect on R-squared, adjusted R-squared, AIC and BIC. As a reference we compare each change in the parameters with the respective parameters of model 1.

2.8.1

```
##
## Call:
## lm(formula = rating ~ rq + vo + wa + kr + education + gender +
##     income + age + price + kr:age + vo:income + wa:price, data = marketing)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -18.3439  -4.1330   0.7936   3.9543  15.0952
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 20.716642   0.513885  40.314   <2e-16 ***
## rq           3.884194   0.312455  12.431   <2e-16 ***
## vo           3.856810   0.389425   9.904   <2e-16 ***
## wa           1.064896   0.797066   1.336   0.1816
## kr          -0.398865   0.629402  -0.634   0.5263
## education   -0.256875   0.218151  -1.178   0.2391
## gender      -0.106798   0.199920  -0.534   0.5932
## income      -0.513376   0.227407  -2.258   0.0240 *
## age          0.011491   0.006676   1.721   0.0853 .
## price       -0.299911   0.009204 -32.584   <2e-16 ***
## kr:age       0.002934   0.014451   0.203   0.8391
## vo:income   -0.638339   0.495079  -1.289   0.1974
## wa:price    -0.013161   0.020591  -0.639   0.5228
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.585 on 3182 degrees of freedom
## Multiple R-squared:  0.3484, Adjusted R-squared:  0.346
## F-statistic: 141.8 on 12 and 3182 DF,  p-value: < 2.2e-16
```

2.8.2

2.9

3 Theorie

3.1

That is true. R^2 is always increasing with each additional variable, no matter how good the new variable is. In general SSR are always smaller than TSS, and R^2 is close to 1 the smaller SSR is. If SSR = 0, then $R^2 = 1$. In this case we don't make any errors and were able to explain the variance of our model completely. In a model with a fixed number of observations N, R^2 will be always 1 if we add N-1 explanatory variables, no matter how useful they are.

For example:

```
##
## Call:
## lm(formula = log(consum) ~ log(income) + log(pchick) + log(pbeef) +
##     log(ppork), data = chick1)
##
## Residuals:
## ALL 5 residuals are 0: no residual degrees of freedom!
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  14.6347         NA      NA       NA
## log(income)  -1.0030         NA      NA       NA
## log(pchick)  -0.7657         NA      NA       NA
## log(pbeef)   -2.9596         NA      NA       NA
## log(ppork)    2.6654         NA      NA       NA
##
## Residual standard error: NaN on 0 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:    NaN
## F-statistic:   NaN on 4 and 0 DF,  p-value: NA
```

The adjusted R^2 in comparison, is taking in to account how good the new variable is. So the $ R^2adj $ is only increasing, if the change in R^^2 is large.

The formula: $R^2_{adj} = 1 - \frac{N-1}{N-K-1} * (1 - R^2)$ So with increasing "K", the term 1 $\frac{N-1}{N-K-1}$ gets bigger and $R^2 adj$ smaller, but with the term $(1 - R^2)$ it is still increasing if the change is large.

3.2

We consider the model $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + u$

The null hypothesis for a statistical test that the point where the effect of a marginal increase in X on the conditional expectation E(Y |X) changes its sign is 1:

H0: $\beta_2 = 0$ H1: $\beta_2 \neq 0$ We can use a t-test, to test if we should include quadratic part of the function or not. If $\beta_0 \neq 0$ non-linearity is given in our model and we should not exclude the quaratic term.

We are looking for the point where the marginal increase in X on the conditional expectation $E(Y|X = 1) = 0. H0 : 1 = -\beta_1/(2\beta_2)$

We can calculate the point where the signs change with $X_0 = -\beta_1/(2\beta_2)$. If β_1 *and* β_2 have different signs, the vertex can be positive. So only for different signs of β_1 *and* β_2 the vertex can be 1. In our case this is true if we set $ X_0= 1$, so $-\beta_1/(2\beta_2) = 1$ So If X<1, there is a positive effect on increasing X, if, X>1, there is a negative effect on increasing X.

??? Welcher Test

3.3