Stationarity and Unit Root Tests

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Agenda

- Motivation
- Theoretical Background
- Simulation
- Case Study
- Conclusions

Motivation

ARMA and ARIMA models assume constant variance, mean and autocovariance in order to make reliable predictions (weak stationarity).

When a series is nonstationary, the models parameters become unstable and its predictions unreliable.

This makes it essential to diagnose and address non-stationarity before attempting modeling and forecasting in order to produce reliable estimates.

Theoretical Background

Definition (Hyndman 2018)

Strict Stationarity: the joint distributions of a time series are time-invariant.

Weak stationarity: constant mean and variance, and autocovariance that is only dependent on lags, not on time.

Granger & Newbold (1974)

Landmark paper demonstrating spurious correlations between non-stationary time series that shared stochastic trends.

Called for methods such as first-differencing before modeling and forecasting.

Dickey & Fuller (1981)

One of the first tests to determine the presence of a unit root.

Sensitive to heteroskedasticity in the errors and structural breaks.

Null:
unit root exists

Alternative:
no unit root

Phillips-Perron (1988)

Allows for heteroskedasticity and serial correlation in the errors.

Depends on kernel choices to estimate long-run variance of the errors.

Null:
unit root exists

Alternative:
no unit root

KPSS (1991)

Complementary null hypothesis to ADF and PP tests.

Sensitive to structural breaks and small, finite samples.

Null: series is level or trend-stationary

Alternative: unit root exists

Simulation

Granger and Newbold (1974) demonstrated that shared stochastic trends lead to spurious correlations, advocating for transformations like first-differencing before modeling and forecasting.

A Monte Carlo simulation of regressing two independent random walks produced a type 1 error rate of 76.3%.

First differencing to achieve stationarity prior to modeling led to insignificant results and a low R squared value.

Monte Carlo Simulation

Simulations	Error_Rate	Average_R_Squared
1000	0.763	0.242012

Spurious Correlation between X and Y

	Dependent variable:
	Y
X	-0.677***
	(0.113)
Constant	-0.836
	(0.510)
Observations	100
R2	0.268
Adjusted R2	0.261
Residual Std. Error	2.451 (df = 98)
F Statistic	35.940*** (df = 1; 98)
Note:	*p<0.1; **p<0.05; ***p<0.01

Differenced Regression of X and Y

	Dependent variable:
	diff(Y)
diff(X)	0.023
	(0.093)
Constant	-0.106
	(0.091)
Observations	99
R2	0.001
Adjusted R2	-0.010
Residual Std. Error	0.907 (df = 97)
F Statistic	0.058 (df = 1; 97)
Note:	*p<0.1; **p<0.05; ***p<0.01

Case Study

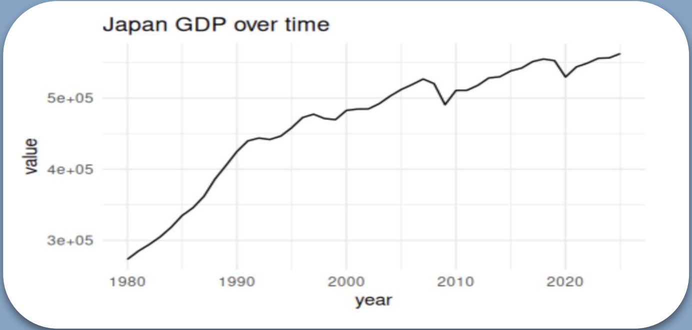
Use GDP data* from the USA, Japan and Brazil to assess non-stationarity and appropriate techniques to achieve stationarity prior to modeling and forecasting.

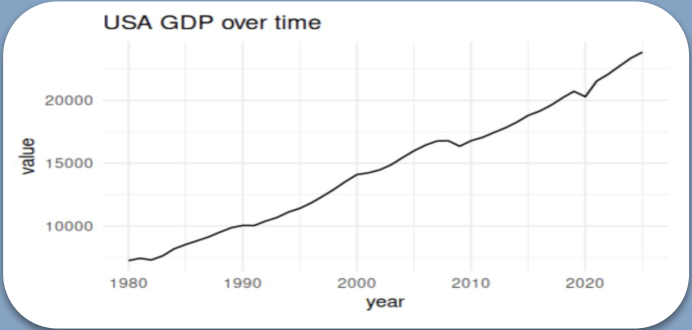
Japan
- Rapid growth into 1990
- Relative stagnation afterwards

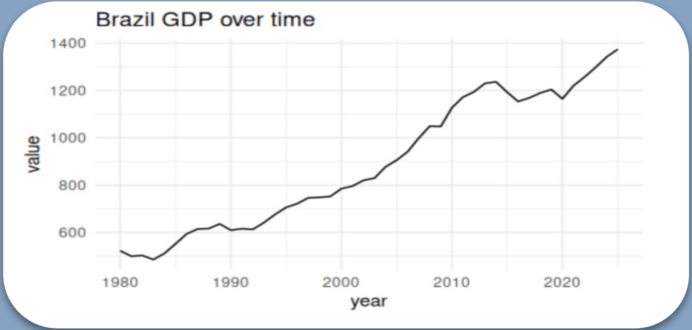
USA
- Persistent upward trend
- Small structural breaks in 2008 (Financial Crisis) and 2020 (Covid-19)

Brazil
- Relative stagnation until mid-1990s
- Upward trending growth until 2010
- Stagnation between 2010-2020 before trending upward again

*Data comes from the IMF's World Economic Outlook database (2025)







Case Study

Table 4: USA Unit Root and Stationarity Tests

	Test	Statistic	P_Value	H0
Dickey-Fuller	ADF	-1.4613697	0.7881234	Unit root (nonstationary)
Dickey-Fuller Z(alpha)	Phillips–Perron	-8.3447886	0.6110551	Unit root (nonstationary)
KPSS Level	KPSS (Level)	1.2461842	0.0100000	Stationary (Level)
KPSS Trend	KPSS (Trend)	0.1127233	0.1000000	Stationary (Trend)

Table 5: Japan Unit Root and Stationarity Tests

	Test	Statistic	P_Value	H0
Dickey-Fuller	ADF	-2.586601	0.3407153	Unit root (nonstationary)
Dickey-Fuller Z(alpha)	Phillips–Perron	-3.704918	0.8996941	Unit root (nonstationary)
KPSS Level	KPSS (Level)	1.115804	0.0100000	Stationary (Level)
KPSS Trend	KPSS (Trend)	0.272580	0.0100000	Stationary (Trend)

Table 6: Brazil Unit Root and Stationarity Tests

	Test	Statistic	P_Value	H0
Dickey-Fuller	ADF	-2.324430	0.4449583	Unit root (nonstationary)
Dickey-Fuller Z(alpha)	Phillips–Perron	-9.800383	0.5205049	Unit root (nonstationary)
KPSS Level	KPSS (Level)	1.230161	0.0100000	Stationary (Level)
KPSS Trend	KPSS (Trend)	0.123278	0.0920777	Stationary (Trend)

USA

- ADF and PP tests, rejects the null hypothesis of weak stationarity.
- KPSS trend-stationary test fails to reject the null hypothesis of trend-stationarity, suggesting that the non-stationarity in the USA data is due to a deterministic trend.
- De-trending could be more efficient than first-differencing.

Japan

- ADF and PP tests fail to reject the null hypothesis of a unit root.
- KPSS level test also reject the null hypothesis of weak stationarity. KPSS trend test rejects the null hypothesis of trend-stationarity. Japan’s GDP is not stationary around a deterministic trend, shocks to the series will be persistent throughout time.
- Japan’s GDP is a near-unit-root series, rather than a clearly trend-stationary or difference-stationary series.

Brazil

- ADF and PP tests fail to reject the null hypothesis of a unit root.
- KPSS level test rejects the null hypothesis of stationarity. KPSS trend test produces a p-value > 0.05 but less than 0.10.
- Tests struggle to deal with structural breaks in the data.

Conclusions

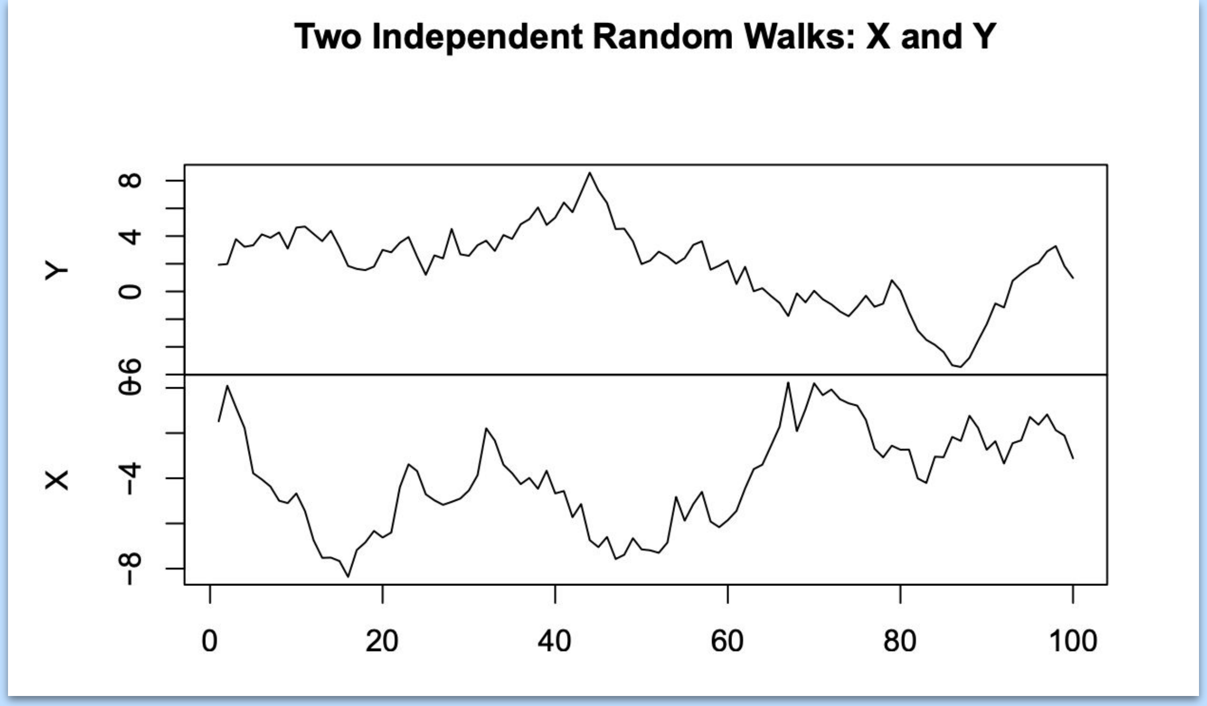
1 Detecting non-stationarity and unit roots are necessary steps for producing valid models and forecasting with accuracy.

2 Non-stationarity produces spurious regression relationships, inflates goodness-of-fit measures, and undermines statistical validity when left unaddressed.

3 Unit root tests come with strengths and weaknesses, and a complete analysis requires inference from multiple tests to determine the best methods for producing a stationary series.

Thank you!

Appendix



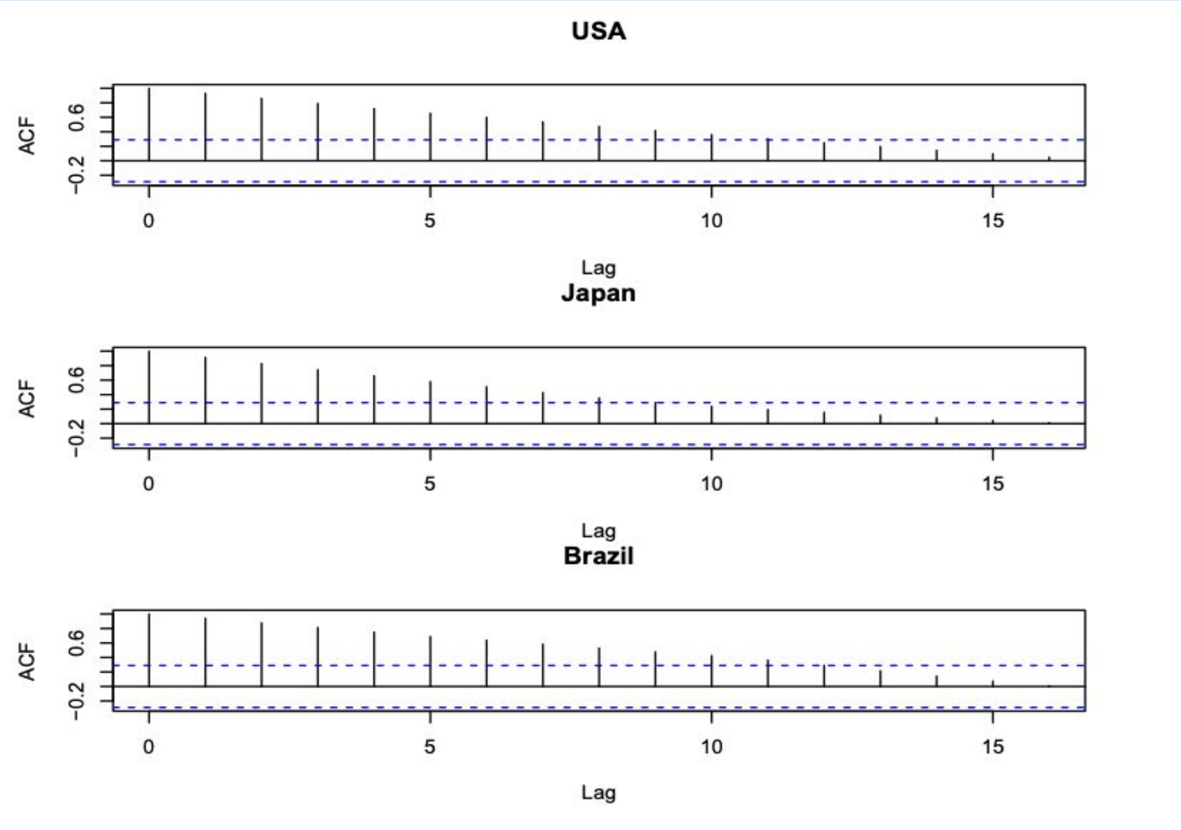
Two simulated independent random walks: X and Y

Series	ADF_Statistic	ADF_P_Value	Method
X	-1.6428	0.7246	Augmented Dickey-Fuller Test
Y	-1.5093	0.7799	Augmented Dickey-Fuller Test

ADF Test results for X and Y simulated random walk series

Series	KPSS_Statistic	KPSS_P_Value	Method
X	0.4519	0.0548	KPSS Test for Level Stationarity
Y	0.4327	0.0631	KPSS Test for Level Stationarity
X	0.2931	0.0100	KPSS Test for Trend Stationarity
Y	0.3506	0.0100	KPSS Test for Trend Stationarity

KPSS Test results for X and Y simulated random walk series



ACF Plots for GDP time series