Concrete Column Biaxial Bending Calculation Report

1. Column Inputs

Column section width (x dimension);	$w = 24\ \text{in}$
Column section height (y dimension);	$h = 18\ \text{in}$
Longitudinal rebar size (Imperial);	$= \#6$
Longitudinal rebar cover;	$= 1.5\ \text{in}$
Cover is to bar center or bar edge (clear cover);	$= \text{Edge}$
Transverse reinforcement type;	$= \text{Tied}$
Number of bars on the top/bottom edges;	$= 5$
Number of bars on the left/right edges;	$= 4$
Concrete strength;	$f_c' = 8000\ \text{psi}$
Steel strength;	$\mathrm{f_y} = 60\text{ksi}$

Load Cases

Pu (kip)	Mux (kip-ft)	Muy (kip-ft)	Show Calc in Report
1400	-300	100	True

Area of one bar
$A_{\text{bar}} = 0.44\ \text{in}^2$

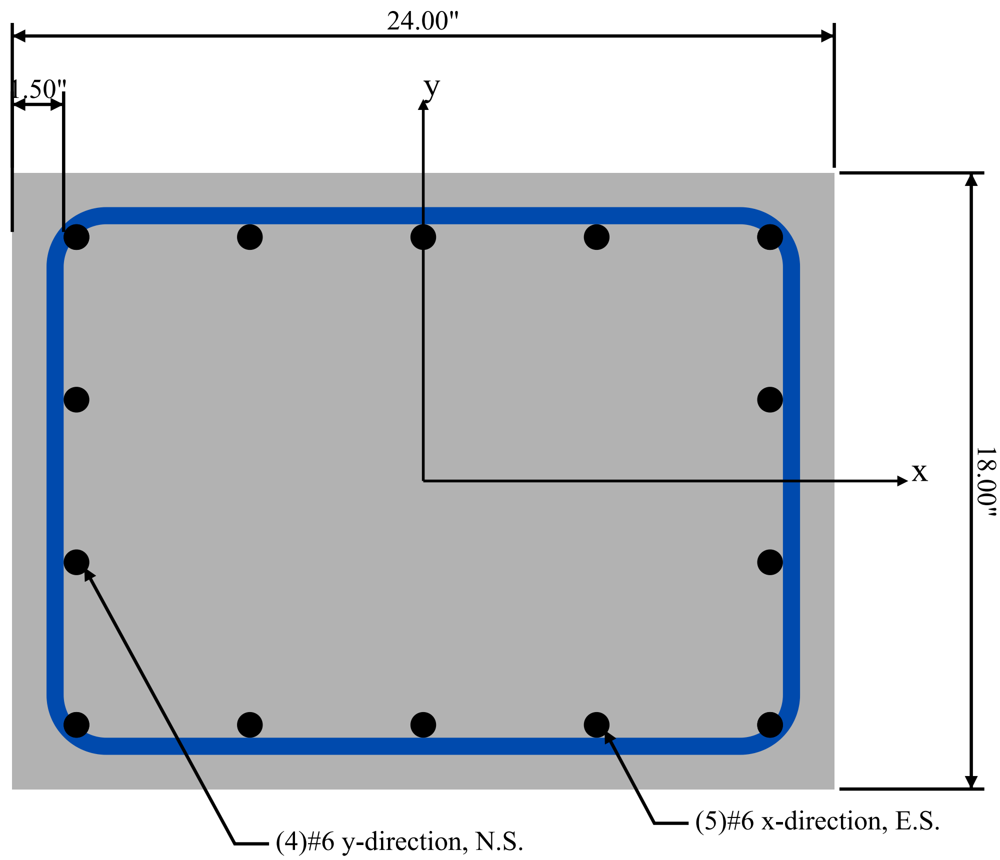
$f_y = 60\ \text{ksi}$

Steel modulus of elasticity
$E_s = 29000\ \text{ksi}$ [ACI 318-19 20.2.2.2]

Concrete strain at f'c
$\epsilon_u = 0.003$ [ACI 318-19 22.2.2.1]

2. Assumptions

[ASSUME] ACI 318-19 controls the design
[ASSUME] Reinforcement is non-prestressed
[ASSUME] Lap splices of longitudinal reinforcement are in accordance with ACI 318-19 Table 10.7.5.2.2
[ASSUME] Strain in concrete and reinforcement is proportional to distance from the neutral axis, per ACI 318-19 22.2.1.2



Section of Column

3. Axial Capacity Calculations

Total area of longitudinal reinforcement:

$$A_{st} = A_{\text{bar}} \cdot (2 \cdot \ + 2 \cdot \ - 4) = 0.44 \text{ in}^2 \cdot (2 \ \cdot 5 \ + 2 \ \cdot 4 \ - 4 \ )$$
$$\therefore A_{st} = 6.16 \text{ in}^2$$

Gross section area

$$A_g = w \cdot h = 24 \text{ in} \cdot 18 \text{ in}$$
$$\therefore A_g = 432 \text{ in}^2$$

3.1. Compressive Capacity

$$P_0 = \frac{0.85 \cdot f_c'}{1000} \cdot (A_g - A_{st}) + f_y \cdot A_{st}$$
$$= \frac{0.85 \ \cdot 8000 \text{ psi}}{1000} \cdot \left(432 \text{ in}^2 - 6.16 \text{ in}^2\right) + 60 \text{ ksi} \cdot 6.16 \text{ in}^2$$
$$\therefore P_0 = 3265 \text{ kips}$$

[ACI 318-19 22.4.2.2]

Because the transverse reinforcement is tied:

$$P_{\mathrm{n,max}} = 0.8 \cdot P_0 = 0.8 \ \cdot 3265 \text{ kips}$$
$$\therefore P_{\mathrm{n,max}} = 2612 \text{ kips}$$

[ACI 318-19 22.4.2.1(a)]

$$\phi = 0.65$$

[ACI 318-19 Table 21.2.2(b)]

$$\phi P_{\mathrm{n,max}} = \phi \cdot P_{\mathrm{n,max}} = 0.65 \ \cdot 2612 \text{ kips}$$
$$\therefore \phi P_{\mathrm{n,max}} = 1698 \text{ kips}$$

3.2. Tensile Capacity

$$P_{\mathrm{nt,max}} = f_y \cdot A_{st} = 60 \text{ ksi} \cdot 6.16 \text{ in}^2$$
$$\therefore P_{\mathrm{nt,max}} = 369.6 \text{ kips}$$

[ACI 318-19 22.4.3.1]

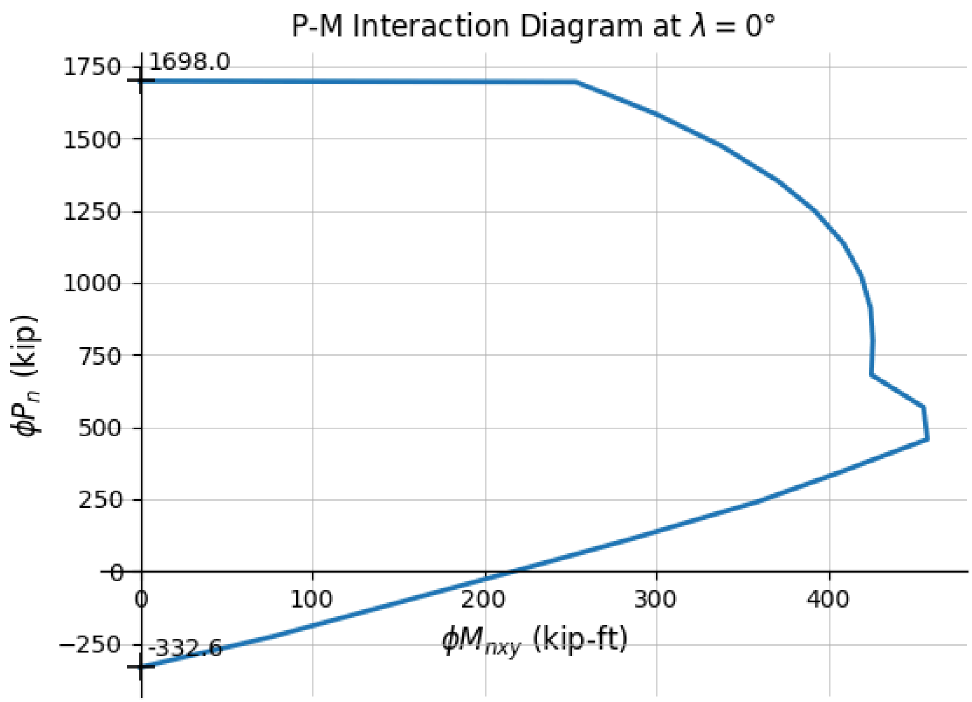
Because failure is tension-controlled:

$$\phi = 0.9$$

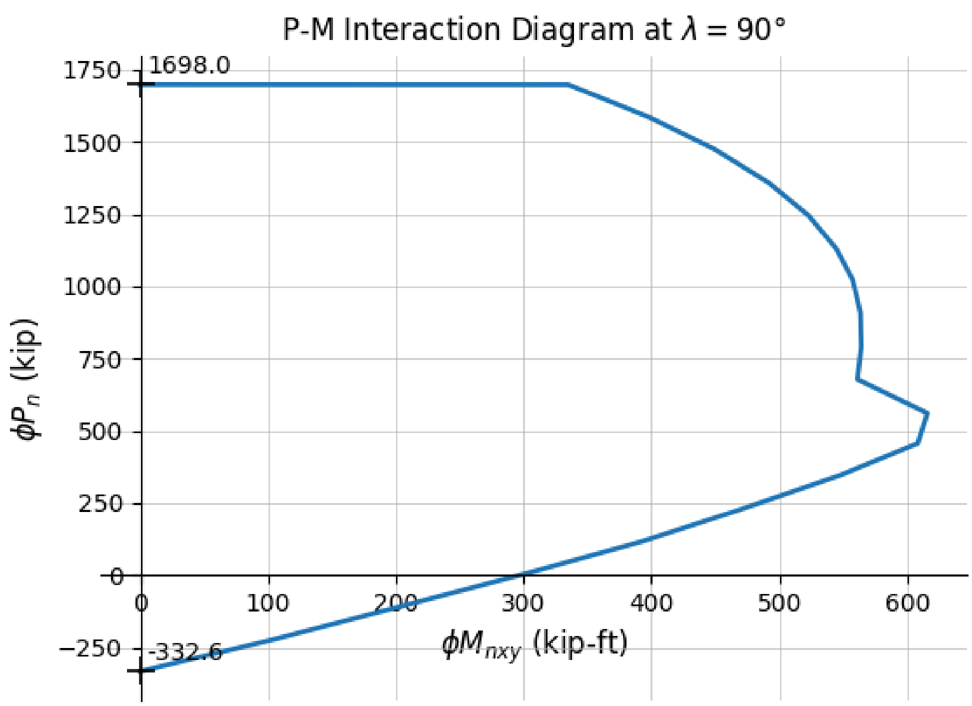
[ACI 318-19 21.2.2(e)]

$$\phi P_{\mathrm{nt,max}} = \phi \cdot P_{\mathrm{nt,max}} = 0.9 \ \cdot 369.6 \text{ kips}$$
$$\therefore \phi P_{\mathrm{nt,max}} = 332.6 \text{ kips}$$

4. PM Diagrams for Pure Mx and My

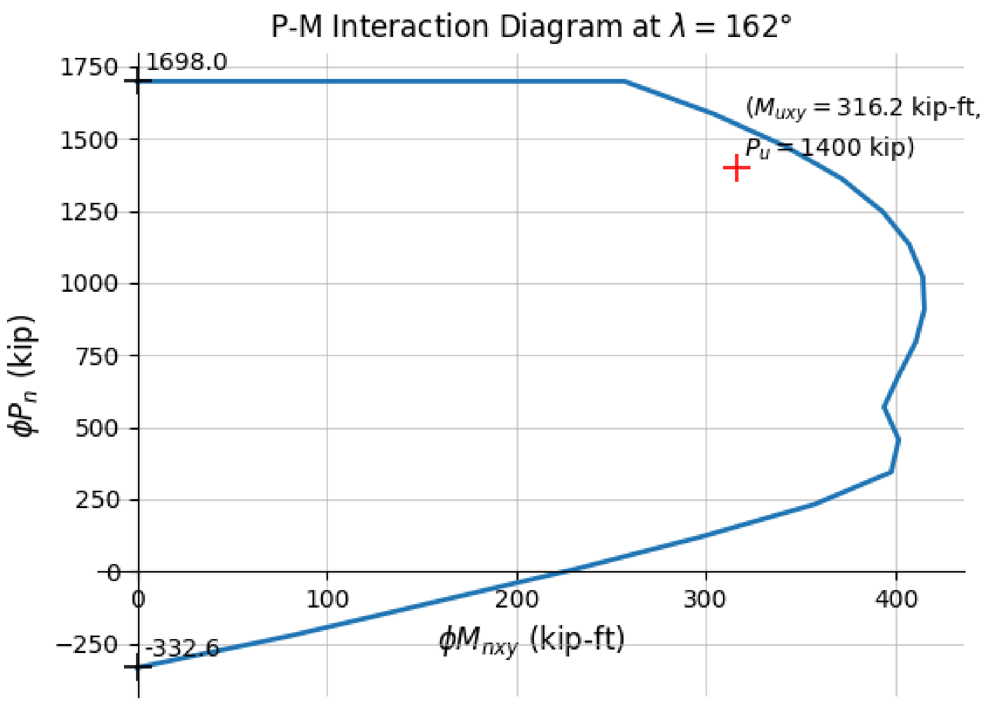


PM interaction diagram for pure Mx.



PM interaction diagram for pure My.

5. DCR Calculation for Load Case P=1400, Mx=-300, My=100



PM interaction diagram for this load case.

The neutral axis angle and depth below are chosen to produce a capacity point aligning exactly with the PMM vector of the applied load.

Neutral axis angle; $\theta = (-0.2207)$ rad

Neutral axis depth; $c = 23.07$ in

5.1. Forces in the Concrete

$\rightarrow f'_c \geq 8000$

$\beta_1 = 0.65$ [ACI 318-19 Table 22.2.2.4.3(c)]

Depth of equivalent compression zone:

$a = \beta_1 \cdot c = 0.65 \cdot 23.07$ in

$\therefore a = 14.99$ in [ACI 318-19 22.2.2.4.1]

Concrete strength converted to ksi:

$$f'_c = \frac{f'_c}{1000} = \frac{8000 \text{ psi}}{1000}$$

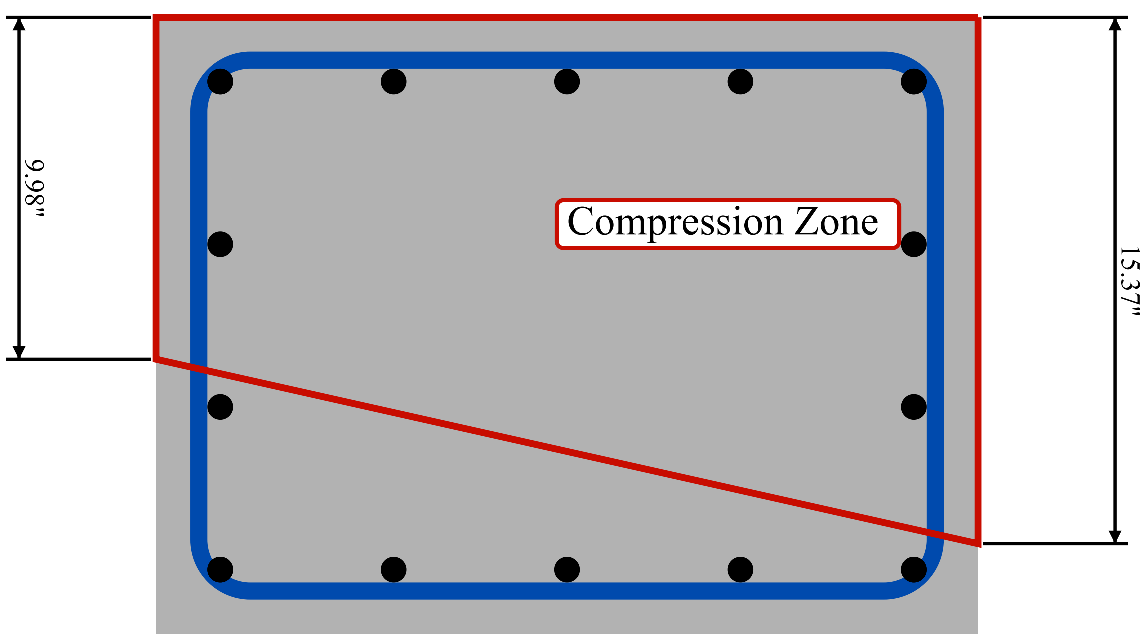
$\therefore f'_c = 8$ ksi

y coordinate of equivalent compression zone intersection with left edge:

$$y_{\text{left}} = \frac{h}{2} - \frac{a}{\cos(\theta)} - w \cdot \tan(\theta)$$
$$= \frac{18 \text{ in}}{2} - \frac{14.99 \text{ in}}{\cos((-0.2207) \text{ rad})} - 24 \text{ in} \cdot \tan((-0.2207) \text{ rad})$$
$$\therefore y_{\text{left}} = (-0.9824) \text{ in}$$

y coordinate of equivalent compression zone intersection with right edge:

$$y_{\text{right}} = \frac{h}{2} - \frac{a}{\cos(\theta)}$$
$$= \frac{18 \text{ in}}{2} - \frac{14.99 \text{ in}}{\cos((-0.2207) \text{ rad})}$$
$$\therefore y_{\text{right}} = (-6.368) \text{ in}$$



Equivalent compression zone outlined in red.

The equivalent stress block is now broken down into triangular areas and the forces are calculated for each.

5.1.1. Forces in Concrete Area 1

Below are the coordinates of the three points A, B, and C which define compression area number 1.

$$A_x = \frac{(-w)}{2} = \frac{(-24\ \text{in})}{2}$$
$$\therefore A_x = (-12)$$

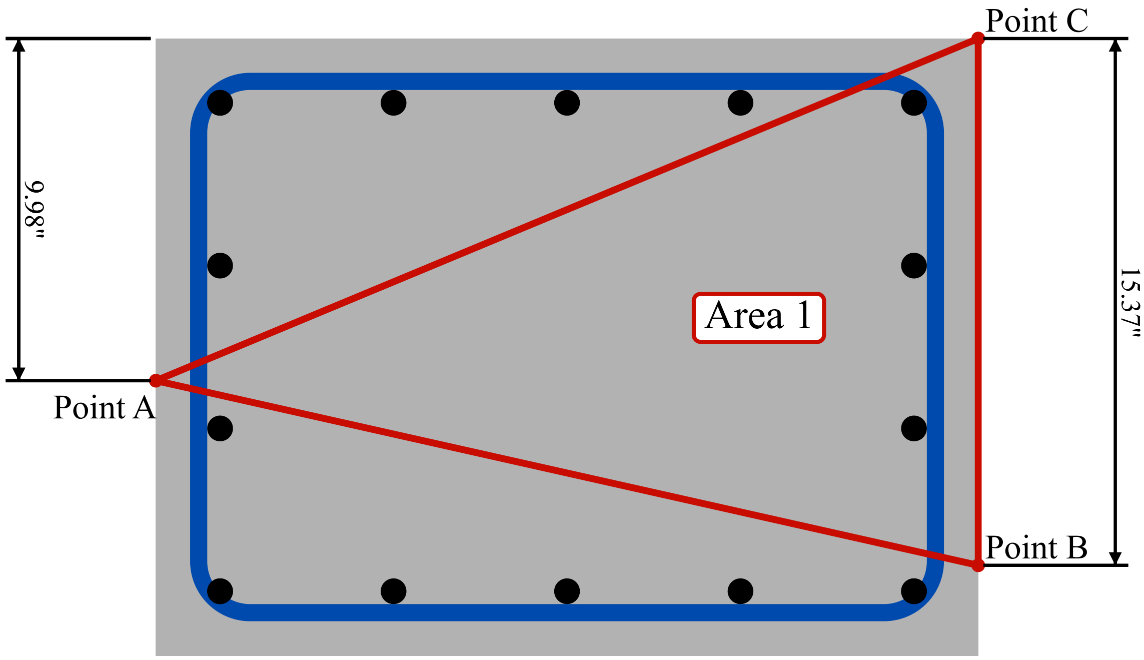
$$A_y = \frac{h}{2} - \frac{a}{\cos(\theta)} - w \cdot \tan(\theta)$$
$$= \frac{18\ \text{in}}{2} - \frac{14.99\ \text{in}}{\cos((-0.2207)\ \text{rad})} - 24\ \text{in} \cdot \tan((-0.2207)\ \text{rad})$$
$$\therefore A_y = (-0.9824)$$

$$B_x = \frac{w}{2} = \frac{24\ \text{in}}{2}$$
$$\therefore B_x = 12$$

$$B_y = \frac{h}{2} - \frac{a}{\cos(\theta)}$$
$$= \frac{18\ \text{in}}{2} - \frac{14.99\ \text{in}}{\cos((-0.2207)\ \text{rad})}$$
$$\therefore B_y = (-6.368)$$

$$C_x = \frac{w}{2} = \frac{24\ \text{in}}{2}$$
$$\therefore C_x = 12$$

$$C_y = \frac{h}{2} = \frac{18\ \text{in}}{2}$$
$$\therefore C_y = 9$$



Compression Area Outline

Calculate the area of this triangular compression zone:

$$A_{\text{triangle}} = 0.5 \cdot |A_x \cdot B_y + B_x \cdot C_y + C_x \cdot A_y - A_x \cdot C_y - B_x \cdot A_y - C_x \cdot B_y|$$
$$= 0.5 \cdot |(-12) \cdot (-6.368) + 12 \cdot 9 + 12 \cdot (-0.9824) - (-12) \cdot 9 - 12 \cdot (-0.9824) - 12 \cdot (-6.368)|$$
$$\therefore A_{\text{triangle}} = 184.4 \text{ in}^2$$

$$P_{\text{n, Area 1}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} = 0.85 \cdot 8 \text{ ksi} \cdot 184.4 \text{ in}^2$$
$$\therefore P_{\text{n, Area 1}} = 1254 \text{ kips}$$

x coordinate of the centroid of this zone:

$$x_{\text{centroid}} = \frac{A_x + B_x + C_x}{3} = \frac{(-12) + 12 + 12}{3}$$
$$\therefore x_{\text{centroid}} = 4 \text{ in}$$

y coordinate of the centroid of this zone:

$$y_{\text{centroid}} = \frac{A_y + B_y + C_y}{3} = \frac{(-0.9824) \ + (-6.368) \ + 9}{3}$$
$$\therefore y_{\text{centroid}} = 0.55 \text{ in}$$

$$M_{\text{nx, Area 1}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} \cdot y_{\text{centroid}} = 0.85 \ \cdot 8 \text{ ksi} \cdot 184.4 \text{ in}^2 \cdot 0.55 \text{ in}$$
$$\therefore M_{\text{nx, Area 1}} = 689.6 \text{ kip} - \text{in}$$

$$M_{\text{ny, Area 1}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} \cdot x_{\text{centroid}} = 0.85 \ \cdot 8 \text{ ksi} \cdot 184.4 \text{ in}^2 \cdot 4 \text{ in}$$
$$\therefore M_{\text{ny, Area 1}} = 5016 \text{ kip} - \text{in}$$

5.1.2. Forces in Concrete Area 2

Below are the coordinates of the three points A, B, and C which define compression area number 2.

$$A_x = \frac{(-w)}{2} = \frac{(-24 \text{ in})}{2}$$
$$\therefore A_x = (-12)$$

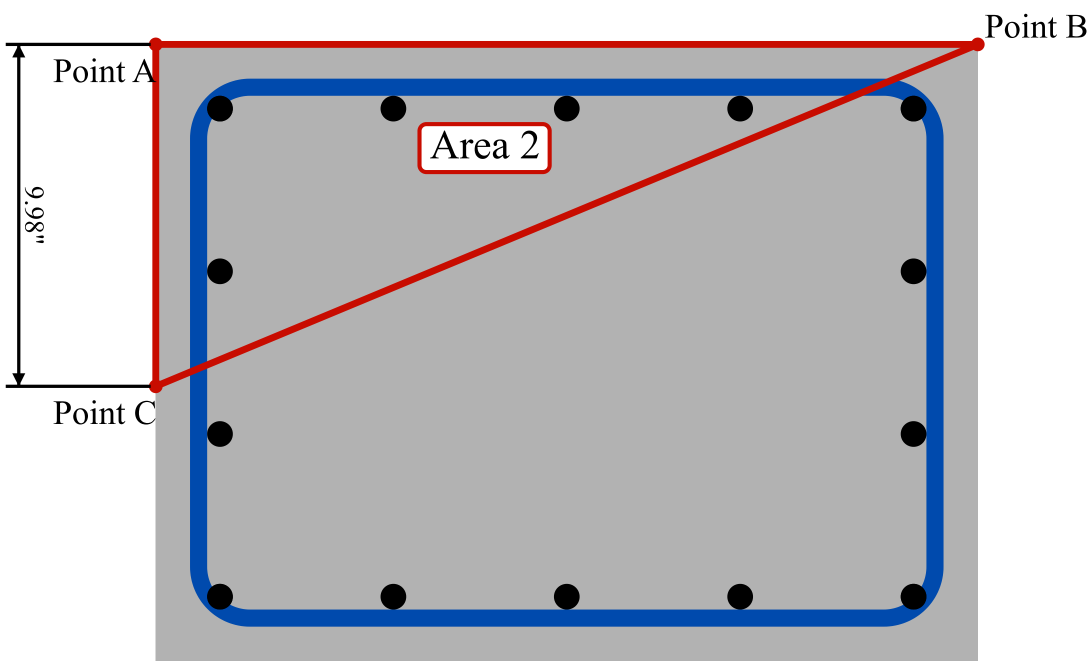
$$A_y = \frac{h}{2} = \frac{18 \text{ in}}{2}$$
$$\therefore A_y = 9$$

$$B_x = \frac{w}{2} = \frac{24 \text{ in}}{2}$$
$$\therefore B_x = 12$$

$$B_y = \frac{h}{2} = \frac{18 \text{ in}}{2}$$
$$\therefore B_y = 9$$

$$C_x = \frac{(-w)}{2} = \frac{(-24 \text{ in})}{2}$$
$$\therefore C_x = (-12)$$

$$C_y = \frac{h}{2} - \frac{a}{\cos(\theta)} - w \cdot \tan(\theta)$$
$$= \frac{18 \text{ in}}{2} - \frac{14.99 \text{ in}}{\cos((-0.2207) \text{ rad})} - 24 \text{ in} \cdot \tan((-0.2207) \text{ rad})$$
$$\therefore C_y = (-0.9824)$$



Compression Area Outline

Calculate the area of this triangular compression zone:

$$A_{\text{triangle}} = 0.5 \cdot |A_x \cdot B_y + B_x \cdot C_y + C_x \cdot A_y - A_x \cdot C_y - B_x \cdot A_y - C_x \cdot B_y|$$
$$= 0.5 \cdot |(-12) \cdot 9 + 12 \cdot (-0.9824) + (-12) \cdot 9 - (-12) \cdot (-0.9824) - 12 \cdot 9 - (-12) \cdot 9|$$
$$\therefore A_{\text{triangle}} = 119.8 \text{ in}^2$$

$$P_{\text{n, Area 2}} = 0.85 \cdot f'_c \cdot A_{\text{triangle}} = 0.85 \cdot 8 \text{ ksi} \cdot 119.8 \text{ in}^2$$
$$\therefore P_{\text{n, Area 2}} = 814.6 \text{ kips}$$

x coordinate of the centroid of this zone:

$$x_{\text{centroid}} = \frac{A_x + B_x + C_x}{3} = \frac{(-12) \ \ + 12 \ + (-12)}{3}$$
$$\therefore x_{\text{centroid}} = (-4) \ \text{in}$$

y coordinate of the centroid of this zone:

$$y_{\text{centroid}} = \frac{A_y + B_y + C_y}{3} = \frac{9 \ \ + 9 \ \ + (-0.9824)}{3}$$
$$\therefore y_{\text{centroid}} = 5.673 \ \text{in}$$

$$M_{\text{nx, Area 2}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} \cdot y_{\text{centroid}} = 0.85 \ \ \cdot 8 \ \text{ksi} \cdot 119.8 \ \text{in}^2 \cdot 5.673 \ \text{in}$$
$$\therefore M_{\text{nx, Area 2}} = 4621 \ \text{kip} - \text{in}$$

$$M_{\text{ny, Area 2}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} \cdot x_{\text{centroid}} = 0.85 \ \ \cdot 8 \ \text{ksi} \cdot 119.8 \ \text{in}^2 \cdot (-4) \ \text{in}$$
$$\therefore M_{\text{ny, Area 2}} = (-3258) \ \text{kip} - \text{in}$$

5.1.3. Total Forces in Concrete

$$P_{\text{n,conc.}} = 2069 \ \text{kips}$$

$$M_{\text{nx,conc.}} = 5310 \ \text{kip} - \text{in}$$

$$M_{\text{ny,conc.}} = 1758 \ \text{kip} - \text{in}$$

5.2. Equations for Rebar Axial and Moment Calculations

For a bar located at coordinates (x,y) relative to the column centroid (as an example, it could be located at (5,5):

$$x_{\text{bar}} = 5$$

$$y_{\text{bar}} = 5$$

Effective depth:

$$d_{\text{bar}} = \left(\frac{w}{2} - x_{\text{bar}}\right) \cdot \cos\left(\theta + \frac{\pi}{2}\right) + \left(\frac{h}{2} - y_{\text{bar}}\right) \cdot \sin\left(\theta + \frac{\pi}{2}\right)$$

Strain:

$$\epsilon_{\text{bar}} = \frac{\epsilon_u \cdot d_{\text{bar}} - c}{c}$$

Stress:

$$\sigma_{\text{bar}} = \min\left(f_y, \max\left(\left(-f_y\right), \epsilon_u \cdot \epsilon_{\text{bar}}\right)\right)$$

If the bar is in the equivalent compression zone, add the concrete stress to avoid double-counting:

$$\sigma_{\text{bar}} = \sigma_{\text{bar}} + 0.85 \cdot f_c'$$

Note that the sign of the following expressions is reversed because positive stress in the rebar is defined as tension while positive axial moment in the column is defined as compression.

Contribution to moment about the x axis:

$$P_{\text{bar}} = \left(-A_{\text{bar}}\right) \cdot \sigma_{\text{bar}} \cdot y_{\text{bar}}$$

Contribution to moment about the x axis:

$$M_{\text{x,bar}} = \left(-A_{\text{bar}}\right) \cdot \sigma_{\text{bar}} \cdot y_{\text{bar}}$$

Contribution to moment about the y axis:

$$M_{\text{y,bar}} = \left(-A_{\text{bar}}\right) \cdot \sigma_{\text{bar}} \cdot x_{\text{bar}}$$

5.3. Tabulated Rebar Results

Bar Number	**X Coord. (in)**	**Y Coord. (in)**	**Effective Depth d (in)**	**Strain (unitless)**	**Stress (ksi)**	**Stress Correction for Displaced Concrete (ksi)**	**Axial Force (kips)**	**Contribution to Mx (kip-in)**	**Contribution to My (kip-in)**
1	-10.12	-7.12	20.58	-0.0003	-9.39	0	4.1	-29.5	-41.9
2	10.12	-7.12	16.14	-0.0009	-26.12	0	11.5	-81.9	116.3
3	-10.12	-2.38	15.94	-0.0009	-26.87	0	11.8	-28.1	-119.7
4	10.12	-2.38	11.51	-0.0015	-43.59	6.8	16.2	-38.4	163.9
5	-10.12	2.38	11.31	-0.0015	-44.35	6.8	16.5	39.2	-167.3
6	10.12	2.38	6.87	-0.0021	-60.0	6.8	23.4	55.6	237.0
7	-10.12	7.12	6.67	-0.0021	-60.0	6.8	23.4	166.8	-237.0
8	10.12	7.12	2.24	-0.0027	-60.0	6.8	23.4	166.8	237.0
9	-5.06	-7.12	19.47	-0.0005	-13.57	0	6.0	-42.6	-30.2

Bar Number	X Coord. (in)	Y Coord. (in)	Effective Depth d (in)	Strain (unitless)	Stress (ksi)	Stress Correction for Displaced Concrete (ksi)	Axial Force (kips)	Contribution to Mx (kip-in)	Contribution to My (kip-in)
10	-5.06	7.12	5.57	-0.0023	-60.0	6.8	23.4	166.8	-118.5
11	0.0	-7.12	18.36	-0.0006	-17.75	0	7.8	-55.7	0.0
12	0.0	7.12	4.46	-0.0024	-60.0	6.8	23.4	166.8	0.0
13	5.06	-7.12	17.25	-0.0008	-21.94	0	9.7	-68.8	48.9
14	5.06	7.12	3.35	-0.0026	-60.0	6.8	23.4	166.8	118.5

5.4. Force Totals

$$P_{\text{n,steel}} = 224 \text{ kips}$$

$$M_{\text{nx,steel}} = 583.9 \text{ kip} - \text{in}$$

$$M_{\text{ny,steel}} = 207 \text{ kip} - \text{in}$$

$$P_{\text{n,tot}} = P_{\text{n,conc.}} + P_{\text{n,steel}} = 2069 \text{ kips} + 224 \text{ kips}$$
$$\therefore P_{\text{n,tot}} = 2293 \text{ kips}$$

$$M_{\text{nx,tot}} = M_{\text{nx,conc.}} + M_{\text{nx,steel}} = 5310 \text{ kip} - \text{in} + 583.9 \text{ kip} - \text{in}$$
$$\therefore M_{\text{nx,tot}} = 5894 \text{ kip} - \text{in}$$

$$M_{\text{ny,tot}} = M_{\text{ny,conc.}} + M_{\text{ny,steel}} = 1758 \text{ kip} - \text{in} + 207 \text{ kip} - \text{in}$$
$$\therefore M_{\text{ny,tot}} = 1965 \text{ kip} - \text{in}$$

5.5. Capacity Calculation

The extreme tension reinforcement is centered at these coordinates:

$$x_{\text{bar}} = (-10.12) \ \text{in}$$

$$y_{\text{bar}} = (-7.125) \ \text{in}$$

$$d_t = \left(\frac{w}{2} - x_{\text{bar}}\right) \cdot \cos\left(\theta + \frac{\pi}{2}\right) + \left(\frac{h}{2} - y_{\text{bar}}\right) \cdot \sin\left(\theta + \frac{\pi}{2}\right)$$
$$= \left(\frac{24 \text{ in}}{2} - (-10.12) \ \text{in}\right) \cdot \cos\left((-0.2207) \ \text{rad} + \frac{3.142}{2}\right) + \left(\frac{18 \text{ in}}{2} - (-7.125) \ \text{in}\right) \cdot \sin\left((-0.2207) \ \text{rad} + \frac{3.142}{2}\right)$$
$$\therefore d_t = 20.58 \text{ in}$$

$$\epsilon_y = \frac{\epsilon_u \cdot (d_t - c)}{c}$$
$$= \frac{0.003 \ \cdot (20.58 \text{ in} - 23.07 \text{ in})}{23.07 \text{ in}}$$
$$\therefore \epsilon_y = (-0.000324)$$

$$\epsilon_{ty} = \frac{f_y}{E_s} = \frac{60 \text{ ksi}}{29000 \text{ ksi}}$$
$$\therefore \epsilon_{ty} = 0.002069$$

$$\rightarrow \epsilon_y \ \leq \ \epsilon_{ty}$$

Failure is compression-controlled and transverse reinforcement is tied:

$\phi = 0.65$ [ACI 318-19 Table 21.2.2(b)]

Factored axial and moment capacities:

$$\phi P_n = \min\left(\phi \cdot P_{\text{n,tot}}, 1698\right) = \min\left(0.65 \ \cdot 2293 \text{ kips}, 1698 \ \right)$$
$$\therefore \phi P_n = 1490 \text{ kips}$$

$$\phi M_{nx} = \frac{\phi \cdot M_{\text{nx,tot}}}{12} = \frac{0.65 \ \cdot 5894 \text{ kip} - \text{in}}{12}$$
$$\therefore \phi M_{nx} = 319.3 \text{ kip} - \text{ft}$$

$$\phi M_{ny} = \frac{\phi \cdot M_{\text{ny,tot}}}{12} = \frac{0.65 \ \cdot 1965 \text{ kip} - \text{in}}{12}$$
$$\therefore \phi M_{ny} = 106.4 \text{ kip} - \text{ft}$$

5.6. DCR Calculation

Compare the ratios of demand to capacity for Mx, My, and P to show that the calculated capacity point is on the same PMM vector as the demand point. Note that the absolute value for the moment DCRs is because the column has equal moment capacity in opposite directions by symmetry.

$$DCR_{Mx} = \left|\frac{-300}{\phi M_{nx}}\right| = \left|\frac{(-300)}{319.3 \text{ kip} - \text{ft}}\right|$$
$$\therefore DCR_{Mx} = 0.9396$$

$$DCR_{My} = \left|\frac{100}{\phi M_{ny}}\right| = \left|\frac{100}{106.4 \text{ kip} - \text{ft}}\right|$$

$$\therefore DCR_{My} = 0.9396$$

$$DCR_P = \frac{1400}{\phi P_n} = \frac{1400}{1490 \text{ kips}}$$

$$\therefore DCR_P = 0.9395$$

The final DCR is:

$DCR = 0.9396$

$$Check\ DCR\ <\ 1.0$$

$$0.9396\ <\ 1.0$$

$$\therefore O.K.$$

6. Summary of Results

DCRs For All Load Cases

Pu (kip)	Mux (kip-ft)	Muy (kip-ft)	PM Vector DCR	Passing?
1400	-300	100	0.94	O.K.

Max DCR check:

$$Check\ 0.94\ <\ 1.0$$

$$\therefore O.K.$$