1. Column Inputs

Column section width (x dimension);	$w = 30$ in
Column section height (y dimension);	$h = 40$ in
Longitudinal rebar size (Imperial);	You can't use 'macro parameter character #' in math mode
Longitudinal rebar cover;	$= 2.5$ in
Cover is to bar center or bar edge (clear cover);	$=$ Edge
Transverse reinforcement type;	$=$ Tied
Number of bars on the top/bottom edges;	$= 5$
Number of bars on the left/right edges;	$= 8$
Concrete strength;	$f_c' = 5000$ psi
Steel strength;	$\mathrm{f_y} = 60\mathrm{ksi}$

Load Cases

Pu (kip)	Mux (kip-ft)	Muy (kip-ft)	Show Calc in Report
1500	1000	200	True
0	0	-2200	False

Area of one bar

$A_{\mathrm{bar}} = 0.44\ \mathrm{in}^2$

Steel modulus of elasticity

$E_s = 29000$ ksi [ACI 318-19 20.2.2.2]

Concrete strain at f'c

$\epsilon_u = 0.003$ [ACI 318-19 22.2.2.1]

2. Assumptions

[ASSUME] ACI 318-19 controls the design

[ASSUME] Reinforcement is non-prestressed

[ASSUME] Lap splices of longitudinal reinforcement are in accordance with ACI 318-19 Table 10.7.5.2.2

[ASSUME] Strain in concrete and reinforcement is proportional to distance from the neutral axis, per ACI 318-19 22.2.1.2

$f_y = 60$ ksi

3. Axial Capacity Calculations

Total area of longitudinal reinforcement:

$$A_{st} = A_{\text{bar}} \cdot (2 \cdot \; + 2 \cdot \; - 4) = 0.44 \text{ in}^2 \cdot (2 \;\; \cdot 5 \;\; + 2 \;\; \cdot 8 \;\; - 4 \;)$$
$$\therefore A_{st} = 9.68 \text{ in}^2$$

Gross section area

$$A_g = w \cdot h = 30 \text{ in} \cdot 40 \text{ in}$$
$$\therefore A_g = 1200 \text{ in}^2$$

3.1. Compressive Capacity

$$P_0 = \frac{0.85 \cdot f'_c}{1000} \cdot (A_g - A_{st}) + f_y \cdot A_{st}$$
$$= \frac{0.85 \;\; \cdot 5000 \text{ psi}}{1000} \cdot \left(1200 \text{ in}^2 - 9.68 \text{ in}^2\right) + 60 \text{ ksi} \cdot 9.68 \text{ in}^2$$
[ACI 318-19 22.4.2.2]
$$\therefore P_0 = 5640 \text{ kips}$$

Because the transverse reinforcement is tied:

$$P_{\text{n,max}} = 0.8 \cdot P_0 = 0.8 \;\; \cdot 5640 \text{ kips}$$
$$\therefore P_{\text{n,max}} = 4512 \text{ kips}$$
[ACI 318-19 22.4.2.1(a)]

$$\phi = 0.65$$
[ACI 318-19 Table 21.2.2(b)]

$$\phi P_{\text{n,max}} = \phi \cdot P_{\text{n,max}} = 0.65 \;\; \cdot 4512 \text{ kips}$$
$$\therefore \phi P_{\text{n,max}} = 2933 \text{ kips}$$

3.2. Tensile Capacity

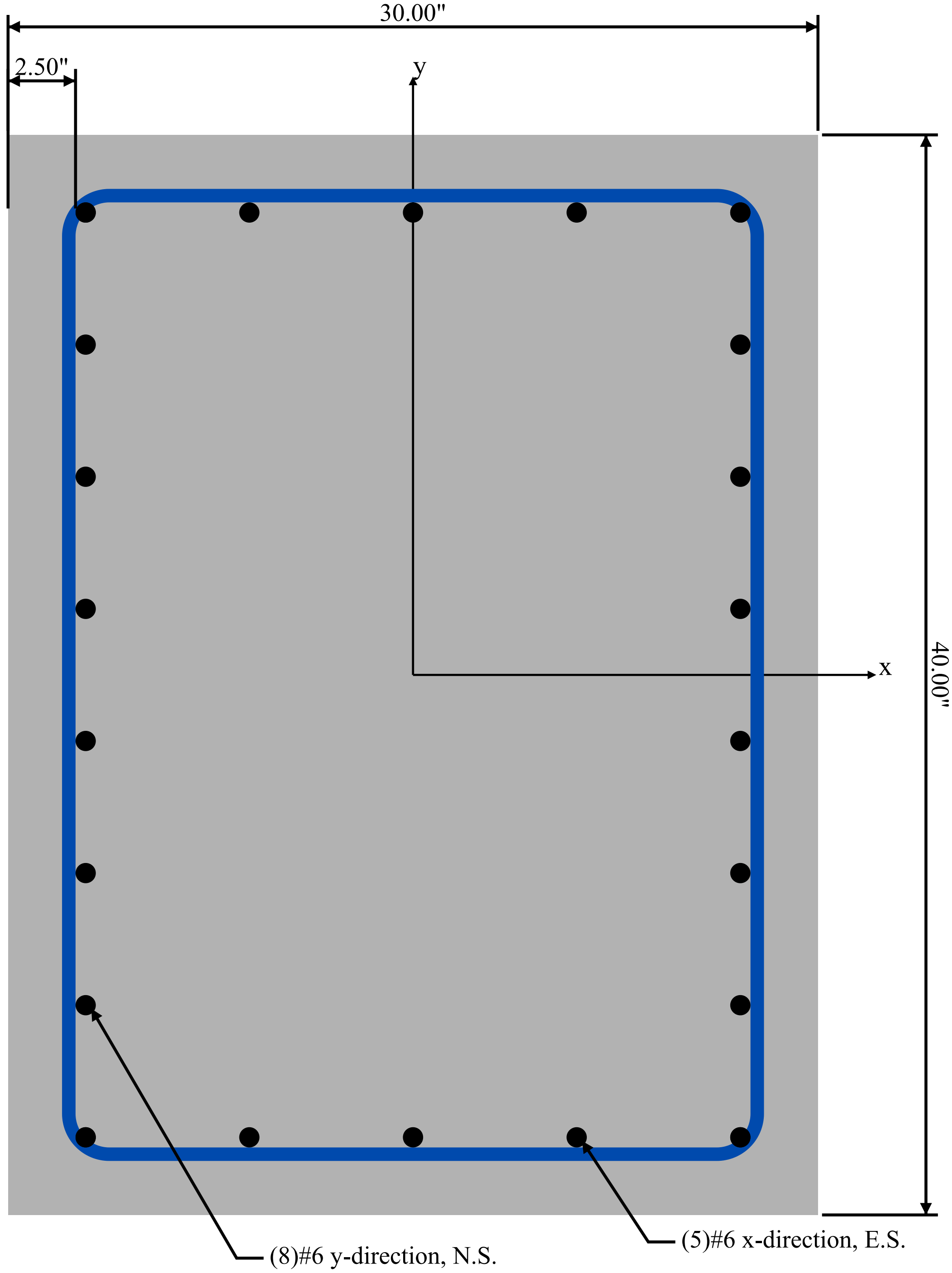
$$P_{\text{nt,max}} = f_y \cdot A_{st} = 60 \text{ ksi} \cdot 9.68 \text{ in}^2$$
$$\therefore P_{\text{nt,max}} = 580.8 \text{ kips}$$
[ACI 318-19 22.4.3.1]

Because failure is tension-controlled:

$$\phi = 0.9$$
[ACI 318-19 21.2.2(e)]

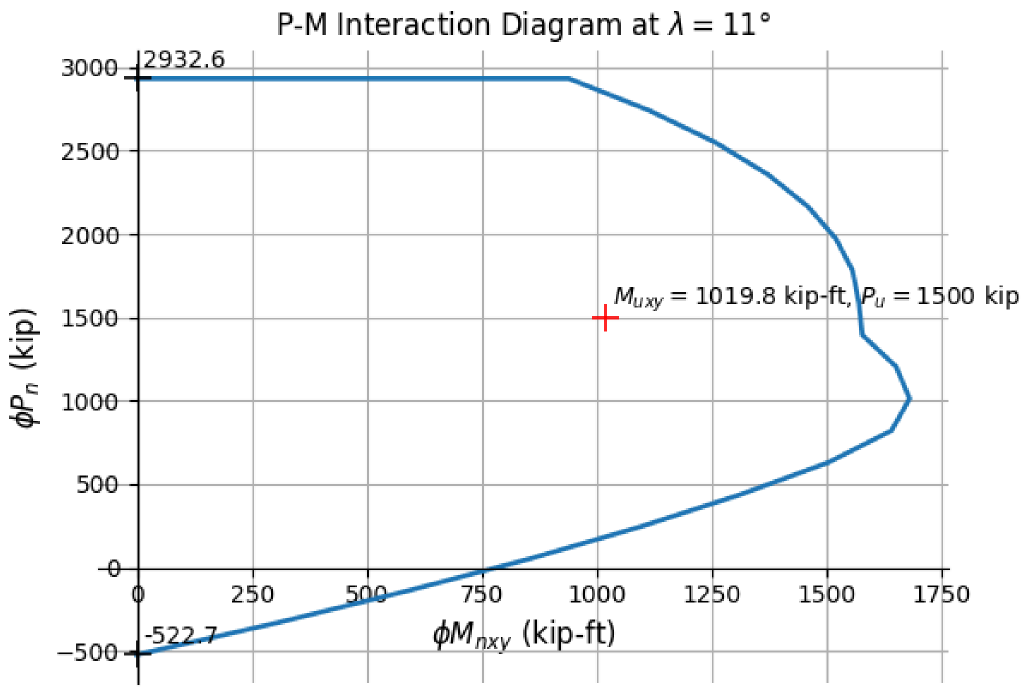
$$\phi P_{\text{nt,max}} = \phi \cdot P_{\text{nt,max}} = 0.9 \;\; \cdot 580.8 \text{ kips}$$
$$\therefore \phi P_{\text{nt,max}} = 522.7 \text{ kips}$$

Calculation Report



Section of Column

4. DCR Calculation for Load Case P=1500, Mx=1000, My=200



PM interaction diagram for this load case.

The neutral axis angle and depth below are chosen to produce a capacity point aligning exactly with the PMM vector of the applied load.

Neutral axis angle; $\theta = (-0.4298)\ \text{rad}$

Neutral axis depth; $c = 35.4\ \text{in}$

4.1. Forces in the Concrete

$\rightarrow 4000 \ < \ f'_c \ < \ 8000$

$$\beta_1 = 0.85 - \frac{0.05 \cdot (f'_c - 4000)}{1000}$$

$$= 0.85 \ - \frac{0.05 \ \cdot (5000\ \text{psi} - 4000\ )}{1000}$$ [ACI 318-19 Table 22.2.2.4.3(b)]

$$\therefore \beta_1 = 0.8$$

Depth of equivalent compression zone:

$$a = \beta_1 \cdot c = 0.8 \ \cdot 35.4\ \text{in}$$

$$\therefore a = 28.32\ \text{in}$$ [ACI 318-19 22.2.2.4.1]

Concrete strength converted to ksi:

$$f_c' = \frac{f_c'}{1000} = \frac{5000\ \text{psi}}{1000}$$

$$\therefore f_c' = 5\ \text{ksi}$$

y coordinate of equivalent compression zone intersection with left edge:

$$y_{\text{left}} = \frac{h}{2} - \frac{a}{\cos(\theta)} - w \cdot \tan(\theta)$$

$$= \frac{40\ \text{in}}{2} - \frac{28.32\ \text{in}}{\cos((-0.4298)\ \ \text{rad})} - 30\ \text{in} \cdot \tan((-0.4298)\ \ \text{rad})$$

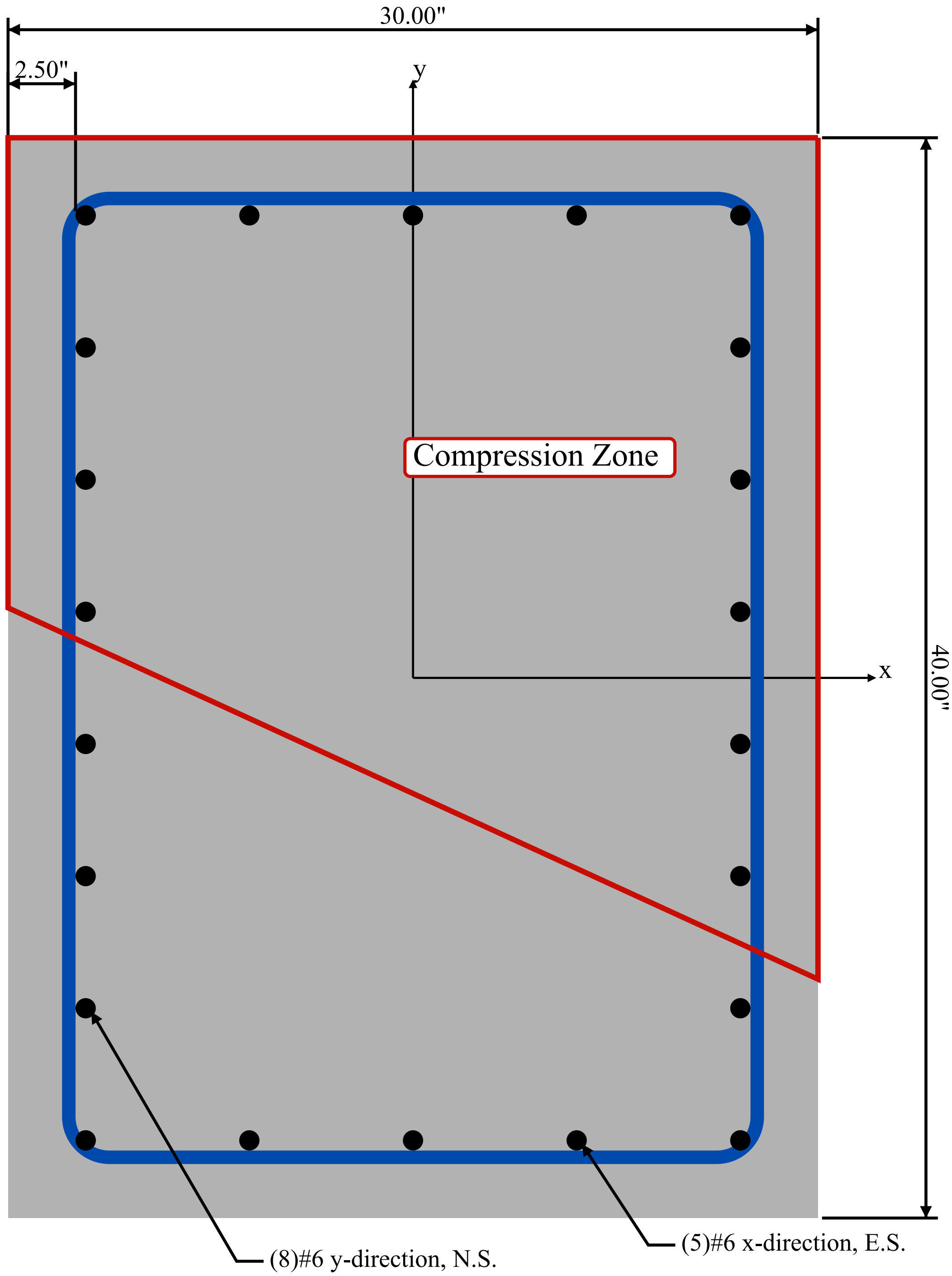
$$\therefore y_{\text{left}} = 2.594\ \text{in}$$

y coordinate of equivalent compression zone intersection with right edge:

$$y_{\text{right}} = \frac{h}{2} - \frac{a}{\cos(\theta)}$$

$$= \frac{40\ \text{in}}{2} - \frac{28.32\ \text{in}}{\cos((-0.4298)\ \ \text{rad})}$$

$$\therefore y_{\text{right}} = (-11.16)\ \ \text{in}$$



Equivalent compression zone outlined in red.

The equivalent stress block is now broken down into triangular areas and the forces are calculated for each.

4.1.1. Forces in Concrete Area 1

Below are the coordinates of the three points A, B, and C which define compression area number 1.

$$A_x = \frac{(-w)}{2} = \frac{(-30\text{ in})}{2}$$
$$\therefore A_x = (-15)$$

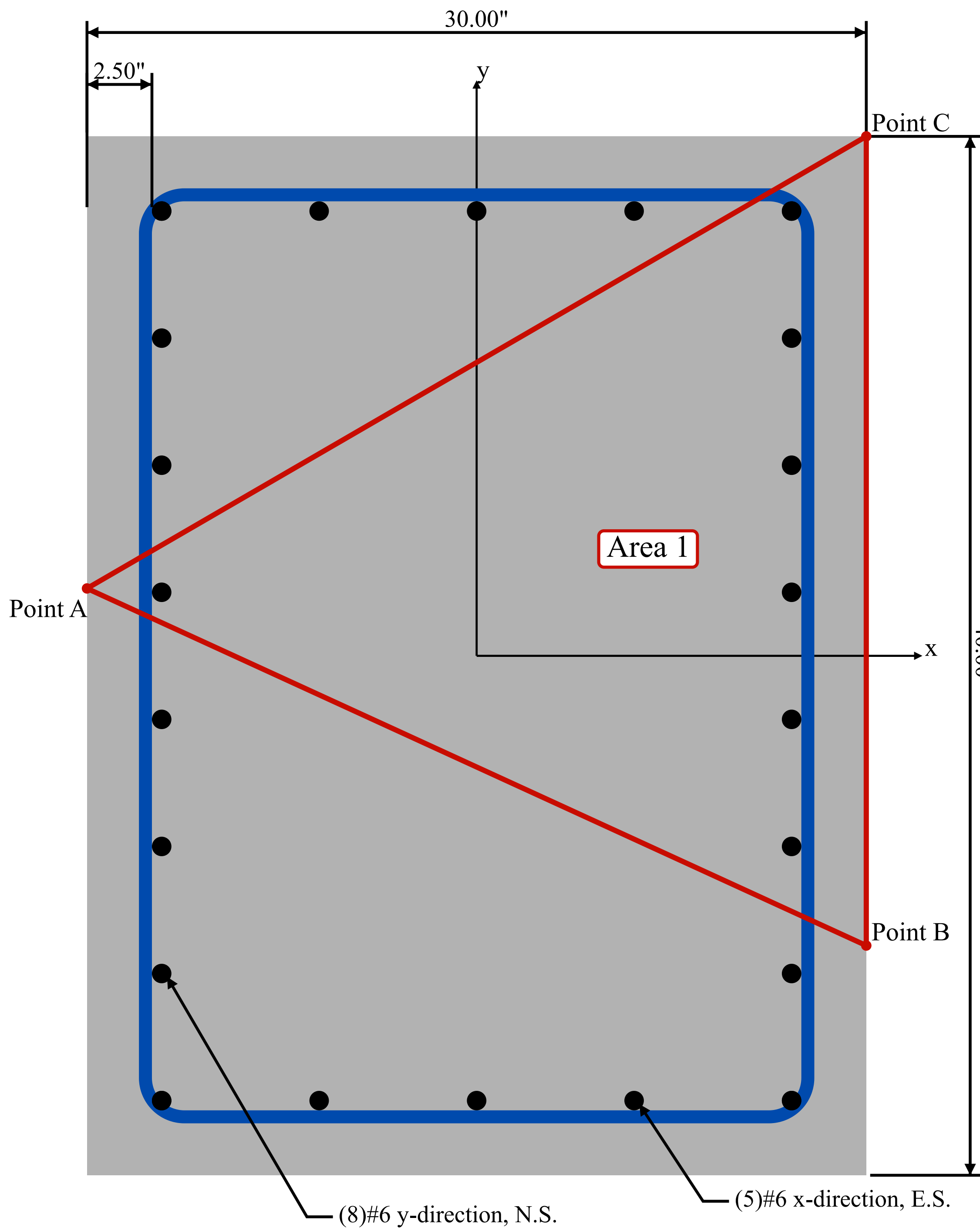
$$A_y = \frac{h}{2} - \frac{a}{\cos(\theta)} - w \cdot \tan(\theta)$$
$$= \frac{40\text{ in}}{2} - \frac{28.32\text{ in}}{\cos((-0.4298)\ \text{ rad})} - 30\text{ in} \cdot \tan((-0.4298)\ \text{ rad})$$
$$\therefore A_y = 2.594$$

$$B_x = \frac{w}{2} = \frac{30\text{ in}}{2}$$
$$\therefore B_x = 15$$

$$B_y = \frac{h}{2} - \frac{a}{\cos(\theta)}$$
$$= \frac{40\text{ in}}{2} - \frac{28.32\text{ in}}{\cos((-0.4298)\ \text{ rad})}$$
$$\therefore B_y = (-11.16)$$

$$C_x = \frac{w}{2} = \frac{30\text{ in}}{2}$$
$$\therefore C_x = 15$$

$$C_y = \frac{h}{2} = \frac{40\text{ in}}{2}$$
$$\therefore C_y = 20$$



Compression Area Outline

Calculate the area of this triangular compression zone:

$$
\begin{aligned}
A_{\text{triangle}} &= 0.5 \cdot |A_x \cdot B_y + B_x \cdot C_y + C_x \cdot A_y - A_x \cdot C_y - B_x \cdot A_y - C_x \cdot B_y| \\
&= 0.5 \cdot |(-15) \cdot (-11.16) + 15 \cdot 20 + 15 \cdot 2.594 - (-15) \cdot 20 - 15 \cdot 2.594 - 15 \cdot (-11.16)| \\
\therefore A_{\text{triangle}} &= 467.3 \text{ in}^2
\end{aligned}
$$

$$P_{\text{n, Area 1}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} = 0.85 \ \cdot 5 \text{ ksi} \cdot 467.3 \text{ in}^2$$
$$\therefore P_{\text{n, Area 1}} = 1986 \text{ kips}$$

x coordinate of the centroid of this zone:

$$x_{\text{centroid}} = \frac{A_x + B_x + C_x}{3} = \frac{(-15) \ + 15 \ + 15}{3}$$
$$\therefore x_{\text{centroid}} = 5 \text{ in}$$

y coordinate of the centroid of this zone:

$$y_{\text{centroid}} = \frac{A_y + B_y + C_y}{3} = \frac{2.594 \ + (-11.16) \ + 20}{3}$$
$$\therefore y_{\text{centroid}} = 3.813 \text{ in}$$

$$M_{\text{nx, Area 1}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} \cdot y_{\text{centroid}} = 0.85 \ \cdot 5 \text{ ksi} \cdot 467.3 \text{ in}^2 \cdot 3.813 \text{ in}$$
$$\therefore M_{\text{nx, Area 1}} = 7573 \text{ kip} - \text{in}$$

$$M_{\text{ny, Area 1}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} \cdot x_{\text{centroid}} = 0.85 \ \cdot 5 \text{ ksi} \cdot 467.3 \text{ in}^2 \cdot 5 \text{ in}$$
$$\therefore M_{\text{ny, Area 1}} = 9931 \text{ kip} - \text{in}$$

4.1.2. Forces in Concrete Area 2

Below are the coordinates of the three points A, B, and C which define compression area number 2.

$$A_x = \frac{(-w)}{2} = \frac{(-30 \text{ in})}{2}$$
$$\therefore A_x = (-15)$$

$$A_y = \frac{h}{2} = \frac{40 \text{ in}}{2}$$
$$\therefore A_y = 20$$

$$B_x = \frac{w}{2} = \frac{30 \text{ in}}{2}$$
$$\therefore B_x = 15$$

$$B_y = \frac{h}{2} = \frac{40\ \text{in}}{2}$$

$$\therefore B_y = 20$$

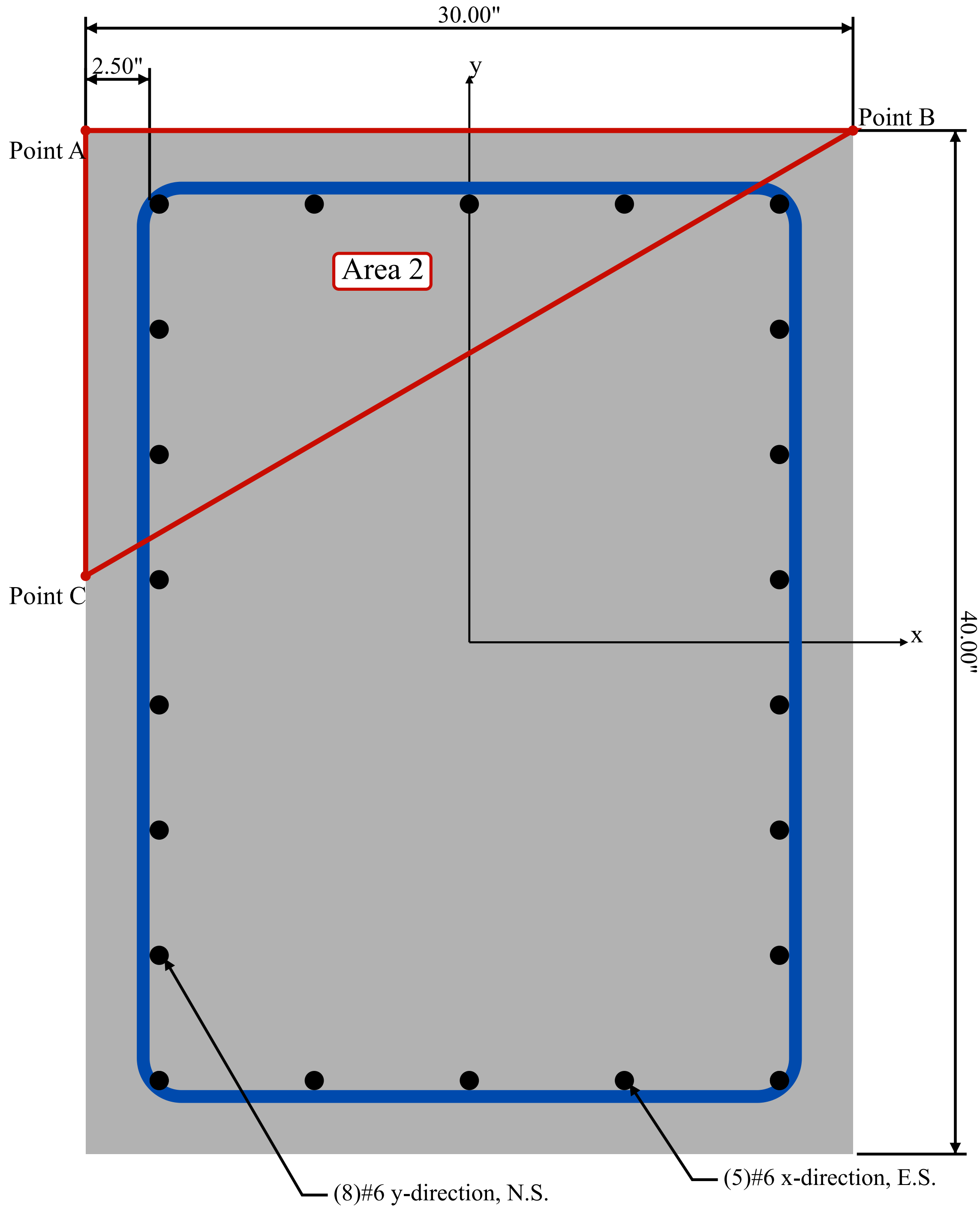
$$C_x = \frac{(-w)}{2} = \frac{(-30\ \text{in})}{2}$$

$$\therefore C_x = (-15)$$

$$C_y = \frac{h}{2} - \frac{a}{\cos(\theta)} - w \cdot \tan(\theta)$$

$$= \frac{40\ \text{in}}{2} - \frac{28.32\ \text{in}}{\cos((-0.4298)\ \text{rad})} - 30\ \text{in} \cdot \tan((-0.4298)\ \text{rad})$$

$$\therefore C_y = 2.594$$



Compression Area Outline

Calculate the area of this triangular compression zone:

$$A_{\text{triangle}} = 0.5 \cdot |A_x \cdot B_y + B_x \cdot C_y + C_x \cdot A_y - A_x \cdot C_y - B_x \cdot A_y - C_x \cdot B_y|$$

$$= 0.5 \cdot |(-15) \cdot 20 + 15 \cdot 2.594 + (-15) \cdot 20 - (-15) \cdot 2.594 - 15 \cdot 20 - (-15) \cdot 20|$$

$$\therefore A_{\text{triangle}} = 261.1 \text{ in}^2$$

$$P_{\text{n, Area 2}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} = 0.85 \ \ \cdot 5 \text{ ksi} \cdot 261.1 \text{ in}^2$$

$$\therefore P_{\text{n, Area 2}} = 1110 \text{ kips}$$

x coordinate of the centroid of this zone:

$$x_{\text{centroid}} = \frac{A_x + B_x + C_x}{3} = \frac{(-15) \ \ +15 \ \ +(-15)}{3}$$

$$\therefore x_{\text{centroid}} = (-5) \ \text{ in}$$

y coordinate of the centroid of this zone:

$$y_{\text{centroid}} = \frac{A_y + B_y + C_y}{3} = \frac{20 \ \ +20 \ \ +2.594}{3}$$

$$\therefore y_{\text{centroid}} = 14.2 \text{ in}$$

$$M_{\text{nx, Area 2}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} \cdot y_{\text{centroid}} = 0.85 \ \ \cdot 5 \text{ ksi} \cdot 261.1 \text{ in}^2 \cdot 14.2 \text{ in}$$

$$\therefore M_{\text{nx, Area 2}} = 15754 \text{ kip} - \text{in}$$

$$M_{\text{ny, Area 2}} = 0.85 \cdot f_c' \cdot A_{\text{triangle}} \cdot x_{\text{centroid}} = 0.85 \ \ \cdot 5 \text{ ksi} \cdot 261.1 \text{ in}^2 \cdot (-5) \ \text{ in}$$

$$\therefore M_{\text{ny, Area 2}} = (-5548) \ \text{ kip} - \text{in}$$

4.1.3. Total Forces in Concrete

$$P_{\text{n,conc.}} = 3096 \text{ kips}$$

$$M_{\text{nx,conc.}} = 23328 \text{ kip} - \text{in}$$

$$M_{\text{ny,conc.}} = 4383 \text{ kip} - \text{in}$$

4.2. Forces in the Rebar

Rebar Calculations

Bar Number	X Coord. (in)	Y Coord. (in)	Effective Depth d (in)	Strain (unitless)	Stress (ksi)	Stress Correction for Displaced Concrete (ksi)	Axial Force (kips)	Contribution to Mx (kip-in)	Contribution to My (kip-in)
1	-12.12	-17.12	45.05	0.0008	23.71	0	-10.4	178.6	126.5
2	12.12	-17.12	34.95	-0.0	-1.12	0	0.5	-8.4	6.0
3	-12.12	-12.23	40.6	0.0004	12.78	0	-5.6	68.8	68.2

Bar Number	X Coord. (in)	Y Coord. (in)	Effective Depth d (in)	Strain (unitless)	Stress (ksi)	Stress Correction for Displaced Concrete (ksi)	Axial Force (kips)	Contribution to Mx (kip-in)	Contribution to My (kip-in)
4	12.12	-12.23	30.5	-0.0004	-12.05	0	5.3	-64.9	64.3
5	-12.12	-7.34	36.15	0.0001	1.85	0	-0.8	6.0	9.9
6	12.12	-7.34	26.05	-0.0008	-22.98	4.25	8.2	-60.5	99.9
7	-12.12	-2.45	31.71	-0.0003	-9.08	0	4.0	-9.8	-48.5
8	12.12	-2.45	21.6	-0.0012	-33.91	4.25	13.1	-31.9	158.2
9	-12.12	2.45	27.26	-0.0007	-20.01	4.25	6.9	17.0	-84.1
10	12.12	2.45	17.16	-0.0015	-44.84	4.25	17.9	43.7	216.6
11	-12.12	7.34	22.81	-0.0011	-30.94	4.25	11.7	86.2	-142.4
12	12.12	7.34	12.71	-0.0019	-55.77	4.25	22.7	166.4	274.9
13	-12.12	12.23	18.36	-0.0014	-41.87	4.25	16.6	202.5	-200.7
14	12.12	12.23	8.26	-0.0023	-60.0	4.25	24.5	300.1	297.4
15	-12.12	17.12	13.92	-0.0018	-52.8	4.25	21.4	365.9	-259.0
16	12.12	17.12	3.81	-0.0027	-60.0	4.25	24.5	420.1	297.4
17	-6.06	-17.12	42.52	0.0006	17.5	0	-7.7	131.9	46.7
18	-6.06	17.12	11.39	-0.002	-59.01	4.25	24.1	412.6	-146.1
19	0.0	-17.12	40.0	0.0004	11.29	0	-5.0	85.1	0.0
20	0.0	17.12	8.86	-0.0022	-60.0	4.25	24.5	420.1	0.0
21	6.06	-17.12	37.47	0.0002	5.09	0	-2.2	38.3	-13.6
22	6.06	17.12	6.34	-0.0025	-60.0	4.25	24.5	420.1	148.7

4.3. Force Totals

$$P_{\text{n,steel}} = 218.7 \text{ kips}$$

$$M_{\text{nx,steel}} = 3188 \text{ kip} - \text{in}$$

$$M_{\text{ny,steel}} = 920.3 \text{ kip} - \text{in}$$

$$P_{\text{n,tot}} = P_{\text{n,conc.}} + P_{\text{n,steel}} = 3096 \text{ kips} + 218.7 \text{ kips}$$

$$\therefore P_{\text{n,tot}} = 3314 \text{ kips}$$

$$M_{\text{nx,tot}} = M_{\text{nx,conc.}} + M_{\text{nx,steel}} = 23328 \text{ kip} - \text{in} + 3188 \text{ kip} - \text{in}$$
$$\therefore M_{\text{nx,tot}} = 26515 \text{ kip} - \text{in}$$

$$M_{\text{ny,tot}} = M_{\text{ny,conc.}} + M_{\text{ny,steel}} = 4383 \text{ kip} - \text{in} + 920.3 \text{ kip} - \text{in}$$
$$\therefore M_{\text{ny,tot}} = 5303 \text{ kip} - \text{in}$$

4.4. Capacity Calculation

The extreme tension reinforcement is centered at these coordinates:

$$x_{\text{bar}} = (-12.12) \text{ in}$$
$$y_{\text{bar}} = (-17.12) \text{ in}$$

$$d_t = \left(\frac{w}{2} - x_{\text{bar}}\right) \cdot \cos\left(\theta + \frac{\pi}{2}\right) + \left(\frac{h}{2} - y_{\text{bar}}\right) \cdot \sin\left(\theta + \frac{\pi}{2}\right)$$
$$= \left(\frac{30 \text{ in}}{2} - (-12.12) \text{ in}\right) \cdot \cos\left((-0.4298) \text{ rad} + \frac{3.142}{2}\right) + \left(\frac{40 \text{ in}}{2} - (-17.12) \text{ in}\right) \cdot \sin\left((-0.4298) \text{ rad} + \frac{3.142}{2}\right)$$
$$\therefore d_t = 45.05 \text{ in}$$

$$\epsilon_y = \frac{\epsilon_u \cdot (d_t - c)}{c} = \frac{0.003 \cdot (45.05 \text{ in} - 35.4 \text{ in})}{35.4 \text{ in}}$$
$$\therefore \epsilon_y = 0.0008175$$

$$\epsilon_{ty} = \frac{f_y}{E_s} = \frac{60 \text{ ksi}}{29000 \text{ ksi}}$$
$$\therefore \epsilon_{ty} = 0.002069$$

$$\rightarrow \epsilon_y \leq \epsilon_{ty}$$

Failure is compression-controlled and transverse reinforcement is tied:

$\phi = 0.65$ [ACI 318-19 Table 21.2.2(b)]

Factored axial and moment capacities:

$$\phi P_n = \min\left(\phi \cdot P_{\text{n,tot}}, 2933\right) = \min\left(0.65 \cdot 3314 \text{ kips}, 2933\right)$$
$$\therefore \phi P_n = 2154 \text{ kips}$$

$$\phi M_{nx} = \frac{\phi \cdot M_{\text{nx,tot}}}{12} = \frac{0.65 \ \cdot 26515 \text{ kip} - \text{in}}{12}$$
$$\therefore \phi M_{nx} = 1436 \text{ kip} - \text{ft}$$

$$\phi M_{ny} = \frac{\phi \cdot M_{\text{ny,tot}}}{12} = \frac{0.65 \ \cdot 5303 \text{ kip} - \text{in}}{12}$$
$$\therefore \phi M_{ny} = 287.3 \text{ kip} - \text{ft}$$

4.5. DCR Calculation

Compare the ratios of demand to capacity for Mx, My, and P to show that the calculated capacity point is on the same PMM vector as the demand point. Note that the absolute value for the moment DCRs is because the column has equal moment capacity in opposite directions by symmetry.

$$DCR_{Mx} = \left|\frac{1000}{\phi M_{nx}}\right| = \left|\frac{1000}{1436 \text{ kip} - \text{ft}}\right|$$
$$\therefore DCR_{Mx} = 0.6963$$

$$DCR_{My} = \left|\frac{200}{\phi M_{ny}}\right| = \left|\frac{200}{287.3 \text{ kip} - \text{ft}}\right|$$
$$\therefore DCR_{My} = 0.6963$$

$$DCR_{P} = \frac{1500}{\phi P_n} = \frac{1500}{2154 \text{ kips}}$$
$$\therefore DCR_{P} = 0.6963$$

The final DCR is:

$DCR = 0.6963$

$$Check\ DCR < 1.0$$
$$0.6963 < 1.0$$
$$\therefore OK$$

5. Summary of Results

DCRs For All Load Cases

Pu (kip)	Mux (kip-ft)	Muy (kip-ft)	PM Vector DCR	Passing?
1500	1000	200	0.7	O.K.

Pu (kip)	Mux (kip-ft)	Muy (kip-ft)	PM Vector DCR	Passing?
0	0	-2200	4.21	N.G.