Supplemental Material for *Allies as Armaments Explaining the Specialization of State Military Capabilities*

Anonymized

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This appendix accompanies the paper "Allies as Armaments: Explaining the Specialization of State Military Capabilities". It provides supplemental information concerning descriptive statistics of the data used in the model and robustness checks and alternate specifications as described in the results section of the manuscript.

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1 Descriptive statistics

Table A1: Summary statistics for model variables. Year polynomials are omitted for simplicity.

	Unique (#)	Missing (%)	Mean	SD	Min	Median	Max
Year	45	0	1992.94	12.88	1970.00	1994.00	2014.00
Specialization	5728	10	0.00	0.50	-1.00	-0.05	2.34
Ally Mil Spend (log)	3952	32	11.01	3.09	0.00	11.64	13.97
Ally CINC Ratio	4866	32	0.92	0.15	0.05	0.98	1.00
GDP (log)	6909	4	24.04	2.16	18.92	23.79	30.42
CINC	7128	1	0.01	0.02	0.00	0.00	0.23

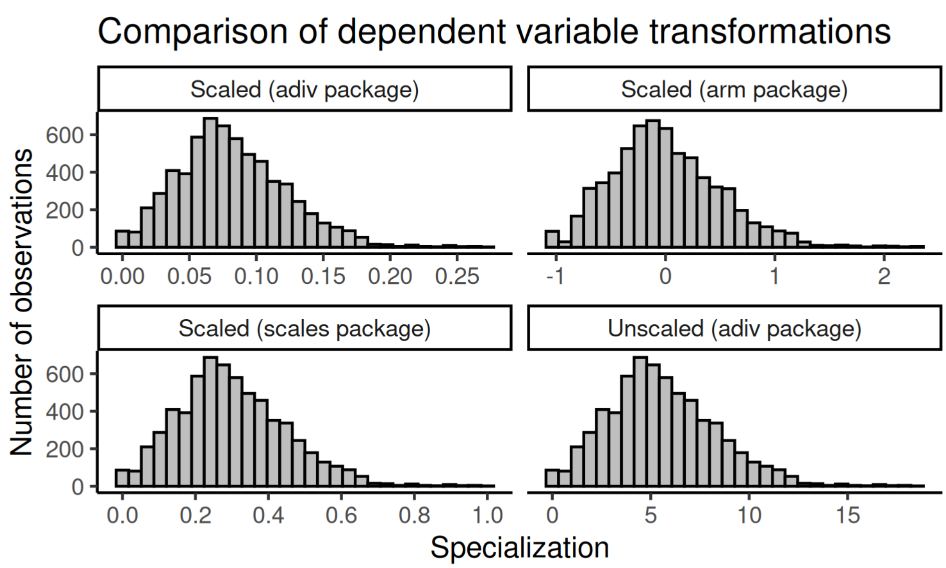

Figure A1: The distribution is consistent across transformations and since it is a linear transformation, does not change the model results other than the interpretation of the coefficients. The main text uses the scaling from the arm package and the fractional, beta, ZOIB, and ordered beta regressions use the bounded version in adiv.

2 Measurement Validation

2.1 Synthetic Observations

I create three synthetic observations where we have confident prior beliefs about how the specialization index should measure them. Using the existing data for 2010, I create new synthetic measures for countries with: 1) synthetic minimum - a country that had the smallest quantity for each military capability that at least one country possessed in that year, 2) synthetic median - a country that had the median quantity for each military capability, and 3) a super navy - a country that had the largest navy in the world across all naval platforms but possessed no ground or air assets. We would expect the synthetic minimum country to be diversified, since what matters is not its size but the fact that it is in the same percentile range across the board, we would expect the median country to be close to 0 since it should represent the specialization of a country composed entirely of averages across weapons platforms, and a super navy to be highly specialized. In running the simulation, the measurements for the synthetic observations are consistent with these expectations, suggesting the index is validating cases we would expect to have low, medium, and high values of specialization. The synthetic minimum's index value is -0.16, the synthetic median's is -0.02, and the synthetic super navy is 1.14. The synthetic super navy is among the most specialized 1% of real world observations, and states with specialization similar to the synthetic super navy are those like Australia which is described in the manuscript.

3 Alternate Model Specifications

We run a set of alternate model specifications as robustness checks.[1] We choose the OLS model specification for the primary results shown in the manuscript given it is appropriate for a continuous and normally distributed dependent variable, the most easily interpretable, and consistent with existing research. Explanations for additional model specifications as well as their results are shown below. These alternate model specifications have been chosen based on existing research using similar dependent variables in both ecology and political science (Chiu 2022; Kubinec 2022).

The dependent variable, military specialization, is an entropy-based measures of deviations from a country's count of each military capability from a baseline determined by the composition of their military as well as that of other countries. In its original calculation, it is bounded between [0, 1) with 0 representing no specialization (a state's composite military perfectly matches prior expectations) and 1 representing the theoretical entropic maximum. In the observed data (n = 7,203), there are 85 rows where the dependent variable is 0 and none where the dependent variable is 1.

I avoid modeling with two-way fixed effects given recent research identifying the shortfalls and biases of such modeling (Callaway and Sant'Anna 2021; Goodman-Bacon 2021; Sun and Abraham 2021; Borusyak, Jaravel, and Spiess 2022). The new estimators that resolve these problems cannot be easily applied here because the treatment variable is continuous and there are more than two time periods (Callaway, Goodman-Bacon, and Sant'Anna 2021).

[1] Much of the modeling done here follows the protocol described by Heiss (2021). The author is thankful for their open-source code and strongly suggests readers refer to the original material from which much of this code originates.

3.1 Un-adjusted standard errors

Table A2: Non-country clustered SE

	Military Specialization			
	(1)	(2)	(3)	(4)
Allies' Mil Spend. (log)	0.018***	0.018***		
	(0.002)	(0.002)		
Allies' CINC Ratio			0.303***	0.302***
			(0.044)	(0.044)
Democracy	-0.026+	-0.026+	-0.002	-0.002
	(0.014)	(0.014)	(0.013)	(0.013)
Interstate War (5yr lag)	0.014	0.014	0.029	0.031
	(0.038)	(0.039)	(0.038)	(0.039)
GDP (log)	0.151***	0.151***	0.164***	0.164***
	(0.004)	(0.004)	(0.003)	(0.003)
CINC	2.239***	2.244***	2.489***	2.490***
	(0.317)	(0.319)	(0.344)	(0.346)
Num.Obs.	3900	3900	3900	3900
Time trend	Cubic poly	Year FE	Cubic poly	Year FE
Robust SE	No	No	No	No
AIC	2691.6	2678.1	2709.8	2696.6
BIC	2754.3	2715.7	2772.5	2734.2

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

3.2 Standardized coefficients

Table A3: Standardized coefficient estimates for original model results.

	Military Specialization			
	(1)	(2)	(3)	(4)
Allies' Mil Spend. (log)	0.018*	0.018***		
	(0.009)	(0.001)		
Allies' CINC Ratio			0.303*	0.302***
			(0.143)	(0.052)
Democracy	-0.026	-0.026**	-0.002	-0.002
	(0.041)	(0.010)	(0.042)	(0.011)
Interstate War (5yr lag)	0.014	0.014	0.029	0.031
	(0.073)	(0.056)	(0.070)	(0.055)
GDP (log)	0.151***	0.151***	0.164***	0.164***
	(0.013)	(0.005)	(0.013)	(0.004)
CINC	2.239	2.244***	2.489	2.490***
	(2.130)	(0.307)	(2.294)	(0.467)
Num.Obs.	3900	3900	3900	3900
Time trend	Cubic poly	Year FE	Cubic poly	Year FE
Robust SE	Yes	Yes	Yes	Yes
AIC	10453.6	10376.1	10471.8	10394.6
BIC	34845.2	34542.1	34863.5	34560.5

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

3.3 Fractional logit and beta regression

Since the original dependent variable is continuous, but bounded between [0, 1] we can use a quasi-binomial fractional logistic regression (Papke and Wooldridge 1996). The coefficients cannot be compared to the original model because they are provided on a log odds scale and can instead be interpreted as percentage change in the dependent variable predicted by a one unit increase in the independent variable value. We can also use a beta regression treating the dependent variable as a proportion bounded between (0, 1) non-inclusive (Grün, Kosmidis, and Zeileis 2012). Coefficients for the beta regression are similarly on a logit scale. Since the actual dependent variable does include values of 0, we transform it using the modification suggested by Smithson and Verkuilen (2006). I do not report these as the primary results supporting the theory because of concerns about bias introduced by the lack of a normalizing constant in the fractional logit Kubinec (2023). The results from these models (with and without robust standard errors for the fractional logit) are provided in Table A4 and are consistent with the original results provided in the manuscript.[2] Both independent variables retain statistical significance in the expected direction and of comparable magnitudes.

[2]As the beta regression is a distributional, rather than mean-focused regression, it provides a precision parameter that is omitted here (Kneib, Silbersdorff, and Säfken 2021).

Table A4: Fractional logit and beta regression results

	(1)	(2)	(3)	(4)	(5)	(6)
Allies' Mil Spend. (log)	0.019***	0.019***	0.020***			
	(0.002)	(0.002)	(0.003)			
Allies' CINC Ratio				0.243***	0.243***	0.189***
				(0.045)	(0.042)	(0.053)
Democracy	-0.051***	-0.051***	-0.067***	-0.023+	-0.023+	-0.029+
	(0.014)	(0.013)	(0.017)	(0.013)	(0.013)	(0.016)
Interstate War (5yr lag)	0.014	0.014	0.001	0.026	0.026	0.015
	(0.035)	(0.041)	(0.042)	(0.035)	(0.041)	(0.043)
GDP (log)	0.157***	0.157***	0.182***	0.169***	0.169***	0.196***
	(0.004)	(0.003)	(0.004)	(0.003)	(0.003)	(0.004)
CINC	0.479+	0.479+	-0.264	0.509+	0.509+	-0.549
	(0.268)	(0.263)	(0.327)	(0.299)	(0.303)	(0.363)
Robust SE	No	Yes	No	No	Yes	No
Num.Obs.	3900	3900	3900	3900	3900	3900

Note: ^^ + $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

3.4 ZOIB and ordered beta regression

Rather than transforming the zeros for the beta regression, we can also implemented a zero-inflated or ordered beta which models a separate data generating process for the country-years with no military specialization at all (Tang et al. 2022; Kubinec 2022). As both models are Bayesian, the table specifies only confidence intervals since there are no other analogues for frequentist statistics.

Table A5: Zero-inflated (1, 3) and ordered beta (2, 4) regression results

	(1)	(2)	(3)	(4)
Intercept	-6.133	-6.320	-6.454	-6.606
	[-6.306, -5.966]	[-6.481, -6.161]	[-6.642, -6.262]	[-6.783, -6.429]
Allies' Mil Spend. (log)	0.017	0.014		
	[0.013, 0.021]	[0.009, 0.019]		
Allies' CINC Ratio			0.201	0.222
			[0.118, 0.286]	[0.133, 0.312]
Democracy	-0.031	-0.054	-0.020	-0.033
	[-0.059, -0.004]	[-0.082, -0.025]	[-0.047, 0.007]	[-0.059, -0.007]
Interstate War (5yr lag)	-0.004	-0.019	0.005	-0.008
	[-0.088, 0.083]	[-0.091, 0.051]	[-0.080, 0.089]	[-0.082, 0.065]
GDP (log)	0.147	0.156	0.160	0.165
	[0.140, 0.155]	[0.149, 0.164]	[0.153, 0.167]	[0.159, 0.172]
CINC	0.250	0.373	0.463	0.527
	[-0.304, 0.800]	[-0.186, 0.916]	[-0.173, 1.113]	[-0.080, 1.145]

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