DBA5101 Analytics in Managerial Economics

Homework 2: Third-degree Price Discrimination by DID Analysis

Submitted by Wong Cheuk Wah (A0280543H)

Question 1

Find the control and treatment groups for business customers, and find the control and treatment groups for education customers separately. Describe shortly how you distinguish the control and treatment groups.

To distinguish the control and treatment groups, the prices in every quarter are extracted. Within each sector, if the customers experience a rise in price since Quarter 5, they are considered the treatment group. Otherwise, if their price remains unchanged throughout the 8 quarters, they are the control group.

Within business customers, there are 93 customers belonging to the treatment group whose price rose to $325 from Quarters 5 to 8. The remaining (72 of them) are the control group as their price is $250 at all times.

Within education customers, there are 40 customers in the treatment group as their price increased to $130 starting from Quarter 5. The remaining 45 customers belong to the control group with the price remaining unchanged throughout the 8 quarters.

The control and treatment groups for business customers and education customers are identified in the code file.

Question 2

Estimate price elasticities for the business customers and the education customers by performing necessary regression analysis. Write down your regression equation and take a screenshot of the regression summary and attach it in your answer file. Explain your findings shortly.

For each sector, differences-in-differences (DID) analysis is performed to estimate the expected percentage change of quantity demanded because of the increase in price. After that, with the expected percentage change of quantity demanded, we find the price elasticity using the corresponding formula,

$$Price\ Elasticity = \frac{\%\ change\ in\ quantity\ demand}{\%\ change\ in\ price}$$

The DID regression equation is as follows,

$$\log(quantity_{it}) = \beta_0 + \beta_1 * treatment_i * after_t + \beta_2 * \log(revenue_{it}) + \epsilon_{it} + \gamma_i + \delta_t$$

- i is the notation for customers and t is the notation for quarters
- β_1 is the coefficient of interest measuring the expected percentage change of quantity demanded due to the price rise
- $quantity_{it}$ refers to the quantity demanded by customer i in quarter t
- $treatment_i$ is a treatment group indicator
    - $treatment_i$ = 1 if customer i is a treatment
    - $treatment_i$ = 0 if customer I is a control

- $after_t$ is a dummy indicator for the price rise
  - $after_t$ = 1 indicates quarters 5-8 in which the price has increased.
  - $after_t$ = 0 indicates quarters 1-4 in which the price has not increased yet.
- $revenue_{it}$ refers to the customer i's revenue earned in quarter t
- $\gamma_i \; and \; \delta_t$ are customer fixed effect and time fixed effect respectively, which are used to control for influences constant either over time or across customers.
- ϵ_{it} is the error term

This regression equation is applied to the business and education sectors separately.

Within business customers, the result is as follows,

PanelOLS Estimation Summary

Dep. Variable:	log(quantity)	R-squared:	0.2398
Estimator:	PanelOLS	R-squared (Between):	0.9366
No. Observations:	1320	R-squared (Within):	0.2464
Date:	Thu, Oct 26 2023	R-squared (Overall):	0.8851
Time:	18:30:38	Log-likelihood	787.37
Cov. Estimator:	Unadjusted		
		F-statistic:	180.72
Entities:	165	P-value	0.0000
Avg Obs:	8.0000	Distribution:	F(2,1146)
Min Obs:	8.0000		
Max Obs:	8.0000	F-statistic (robust):	180.72
		P-value	0.0000
Time periods:	8	Distribution:	F(2,1146)
Avg Obs:	165.00		
Min Obs:	165.00		
Max Obs:	165.00		

Parameter Estimates

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	2.4293	0.2946	8.2459	0.0000	1.8513	3.0074
treatment*after	-0.1475	0.0159	-9.2725	0.0000	-0.1787	-0.1163
log(revenue)	0.7821	0.0489	16.008	0.0000	0.6862	0.8780

F-test for Poolability: 3.0885
P-value: 0.0000
Distribution: F(171,1146)

Included effects: Entity, Time

According to the dataset, the price of some business customers increases from \$250 to \$325 (i.e., by $\frac{325-250}{250} * 100\%$ = 30%) starting from Quarter 5. The regression result reveals that when there is a 30% increase in the price, the percentage decrease of quantity demanded of the treatment group is expected to be 14.75%, holding all the variables constant. It is statistically significant at 1% level with the p-value less than 0.01.

Therefore, the price elasticity of business customers = $\frac{-14.75\%}{30\%} = -0.49158$. Since the value is less than 1, it means the product is inelastic to business customers and changes in price result in a smaller change in the demand for the product.

Within education customers, the result is as follows,

PanelOLS Estimation Summary

Dep. Variable:	log(quantity)	R-squared:	0.2168
Estimator:	PanelOLS	R-squared (Between):	0.9655
No. Observations:	680	R-squared (Within):	0.2662
Date:	Thu, Oct 26 2023	R-squared (Overall):	0.8417
Time:	18:35:03	Log-likelihood	23.122
Cov. Estimator:	Unadjusted		
		F-statistic:	81.083
Entities:	85	P-value	0.0000
Avg Obs:	8.0000	Distribution:	F(2,586)
Min Obs:	8.0000		
Max Obs:	8.0000	F-statistic (robust):	81.083
		P-value	0.0000
Time periods:	8	Distribution:	F(2,586)
Avg Obs:	85.000		
Min Obs:	85.000		
Max Obs:	85.000		

Parameter Estimates

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	2.0119	0.7013	2.8686	0.0043	0.6345	3.3894
treatment*after	-0.4054	0.0387	-10.472	0.0000	-0.4815	-0.3294
log(revenue)	0.8505	0.1158	7.3466	0.0000	0.6231	1.0779

F-test for Poolability: 0.9828
P-value: 0.5277
Distribution: F(91,586)

Included effects: Entity, Time

According to the dataset, the price of a few education customers increases from $100 to $130 (i.e., by $\frac{130-100}{100}$ = 30%) since Quarter 5. Holding everything constant, this regression result shows that when there is a 30% increase in price, customers in the treatment group are expected to decrease their quantity demanded by 40.54%, which is statistically significant at 1% level because of the p-value less than 0.01.

As such, the price elasticity of education customers = $\frac{40.54\%}{30\%} = -1.35141$. Since the value is greater than 1, the product is elastic and changes in the price cause a greater than proportional change in demand. The demand is sensitive to an increase in price.

Question 3

How much more revenue would the company generate by using price discrimination compared to uniform pricing?

To find the increase in revenue, we first calculate the quantity and the revenue at each price point under each approach. After that, we compare the maximum revenues generated by price discrimination and uniform pricing.

Only the data before the price rise (i.e., Quarters 1-4) is considered as it provides the initial price point for each sector, allowing us to find the percentage change of price with different possible new prices.

Price Discrimination

Given the different observable characteristics of customers, different uniform prices are charged to different customers according to the price elasticity of the group they belong to.

For each group of customers, a potential range of prices from 1 to an arbitrary large value is set up. At each new price point if a non-negative revenue is yielded, we apply the formula of price elasticity to find the expected change in quantity demanded, thus the new quantity and the revenue of the sector. The following formulas are applied.

Assume P_0, Qd_0 are the initial price point and quantity and P_i is one of the new price points in the potential range of prices. We are finding the corresponding new quantity Qd_i and the revenue R_i.

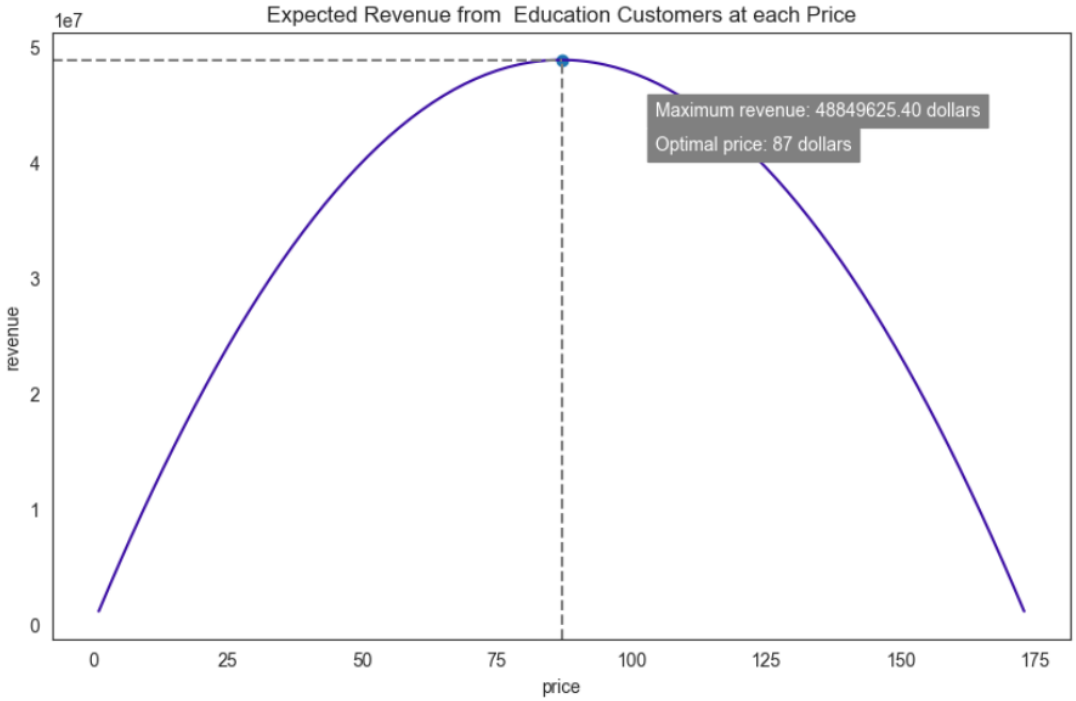
$$\%\ change\ of\ P_i = \frac{P_i - P_0}{P_0}$$

$$\%\ change\ of\ Qd_i = price\ elasticity * \%\ change\ of\ P_i$$

$$Qd_i\ at\ P_i = Qd_0 * (1 + \%\ change\ of\ Qd_i)$$

$$R_i = Qd_i * P_i$$

In the business sector, it is found that the optimal price is $379. With this price, we can yield revenue of $ 257447710.29.



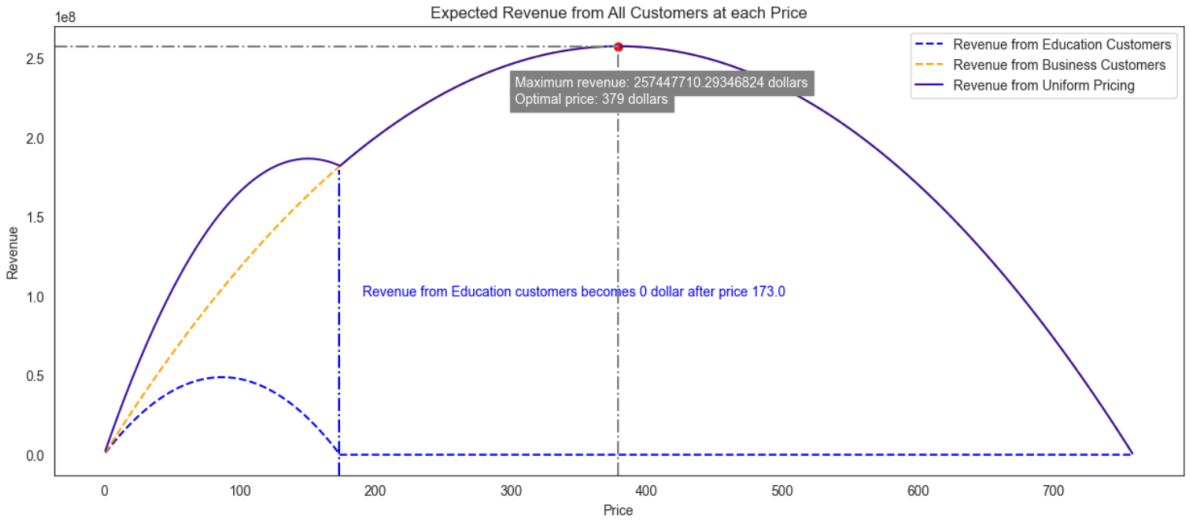
In the education sector, the optimal price is $87 at which the revenue is $ 48849625.40.

As a result, performing price discrimination, the total revenue we can collect from the two sectors = 257447710.29 + 48849625.40 = $306297335.69.

<u>Uniform Pricing</u>

Uniform pricing means charging the same price to all the customers regardless of their sectors or characteristics.

During the optimal price and maximum revenue search when implementing price discrimination, the quantity and revenue at each price point have been calculated for each sector. Therefore, to perform uniform pricing, the aggregate revenue of all customers at each price is found by merging the revenues of the business and education customers based on their available prices.



The calculation results show that a maximum revenue of $257447710.29 can be achieved with the price of $379.

Increase in Revenue

The increase in revenue = Total revenue under price discrimination – Revenue under uniform pricing

= (Maximum possible revenue from business sectors + Maximum possible revenue from education sectors) – Revenue under uniform pricing

= 257447710.29 + 48849625.40 - 257447710.29 = $48849625.40

Due to the elastic demand of education customers who are sensitive to the price rise, when the price is greater than $173, their quantity and revenue become zero. Hereby, if uniform pricing is implemented, the sales of product only cater to the business customers with the optimal price of $379. However, no education customers will buy the product.

Performing price discrimination with different prices for different sectors based on their price elasticities, both business and education customers are accommodated. In other words, the market for education customers is expanded by adopting price discrimination. $48849625.40 more of sales are generated from the education sector as a result.