0a. import liblaries

In [3]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from pulp import *
```

0b. load data

In [4]:

```
distances = pd.read_csv("Distances.csv")
```

In [5]:

```
distances = distances.drop("Unnamed: 0", axis=1)
distances = distances.set_index('Unnamed: 1')
distances = distances[distances.index.notnull()]
distances
```

Out[5]:

Unnamed: 1	BS16 5TR	BS8 4HY	BS4 2NZ	BS35 3RY	BS3 3NA	BS29 6AB	BS10 6DQ	BS25 1TW	BS16 4DD	BS10 5DB	BS48 3RE	BS37 6RX
BS16 5TR	0.00	8.16	7.85	8.85	8.56	29.96	6.44	22.39	1.80	5.95	13.01	6.92
BS8 4HY	8.28	0.00	4.33	10.60	2.87	20.23	4.07	15.03	5.51	3.92	5.65	15.53
BS4 2NZ	7.63	4.87	0.00	13.04	2.93	17.66	6.57	15.87	4.46	5.24	7.98	13.60
BS35 3RY	8.88	15.10	13.62	0.00	13.41	25.83	7.44	29.49	9.99	6.81	16.48	14.74
BS3 3NA	8.57	2.37	2.63	13.09	0.00	15.25	7.33	13.46	5.80	5.93	5.48	15.82
BS29 6AB	30.64	16.68	17.51	26.27	15.15	0.00	24.67	3.96	31.75	25.89	13.26	36.51
BS10 6DQ	6.30	3.99	7.12	7.42	7.71	23.97	0.00	19.87	7.41	1.78	1.05	13.57
BS25 1TW	22.77	14.89	15.71	29.94	13.36	3.96	19.85	0.00	20.00	18.44	11.46	30.02
BS16 4DD	1.53	5.48	4.74	9.92	5.87	31.04	5.70	19.71	0.00	4.37	10.33	8.42
BS10 5DB	6.03	4.07	5.78	6.82	6.47	25.55	1.88	18.63	4.60	0.00	9.25	13.30
BS48 3RE	13.31	5.42	7.98	16.56	5.28	13.25	10.38	11.45	10.54	8.98	0.00	20.56
BS37 6RX	6.92	15.58	16.19	14.87	15.98	35.98	17.59	29.81	8.45	14.49	20.43	0.00
BS13 8BW	12.15	4.26	4.48	17.71	2.15	13.78	9.22	11.98	9.38	7.82	5.11	19.40
BS5 8EJ	3.24	5.85	2.88	11.66	4.16	18.89	5.89	17.10	2.15	4.56	8.96	11.22

1. Construct a delivery route for one lorry by starting at the base (BS16 5TR) and consistenly visiting the nearest non-visited remaining address until all addresses are visited and then going back to base. What is the total distance that is travelled?

In [6]:

```
home = "BS16 5TR"
base = "BS16 5TR"
total_distance = 0;
i = 0;
distances_editable = distances.copy()
while (i < len(distances.columns)-1):
    shortest_path = distances_editable.loc[base,:][distances_editable.loc[base,:] > 0.0
].min()
    new_destination = distances_editable.loc[base,:][distances_editable.loc[base,:] >
0.0].idxmin()
    total_distance += shortest_path
    distances_editable = distances_editable.drop(base,axis=1)
    i += 1
    base = new_destination

# go back home
total_distance += distances.loc[base,home]

print("total distance: ", total_distance, "km")
print("CORRECT")
```

```
total distance:  86.27 km
CORRECT
```

2. What is the best route for a single lorry to deliver products to all of the addresses? How does it compare to the ”Nearest Neighbour” approach?

In [14]:

```
# convert column names of pandas 'dataframe' into a python list
adresses = distances.columns.tolist()
```

In [15]:

```
# Create the 'prob' variable to contain the problem data
prob = LpProblem("Shortest path - basic",LpMinimize)
```

In [16]:

```
# A dictionary called 'adress_vars' is created to contain the referenced Variables
adresses_vars = LpVariable.dicts("path",(adresses,adresses),0,1,LpInteger)
```

In [17]:

```
# The objective function
prob += lpSum([distances.loc[i,j]*adresses_vars[i][j]
               for i in adresses
               for j in adresses]), "Total distance"
```

Add constraints

In [18]:

```
# each node can have only one edge out and one edge in
for i in adresses:
    prob += lpSum([adresses_vars[i][j] for j in adresses]) == 1, ("one edge out per " +
i)
    prob += lpSum([adresses_vars[j][i] for j in adresses]) == 1, ("one edge in per nod
e" + i)

    # no node can lead to itself
    prob += adresses_vars[i][i] == 0, ("no going to itself" + i)

# eliminate sub-loops
for i in adresses:
    for j in adresses:
        prob += adresses_vars[j][i] + adresses_vars[i][j] <= 1, ("avoid 2-loops" + i +
"_" + j)

prob.solve()
```

Out[18]:

```
1
```

In [19]:

```
print("Total distance: ", value(prob.objective))
print("correct values is: 75.84")
```

```
Total distance:  75.84000000000002
correct values is: 75.84
```

Use following code to find sub-loops

In [20]:

```
distances_copy = distances.copy()
distances_copy.iloc[:,:] = 0
for v in prob.variables():
    if (v.varValue > 0.0):
        adress_array = v.name.split('_')
        from_ = adress_array[1] + " " + adress_array[2]
        to_ = adress_array[3] + " " + adress_array[4]
        distances_copy.loc[from_,to_] = 1
```

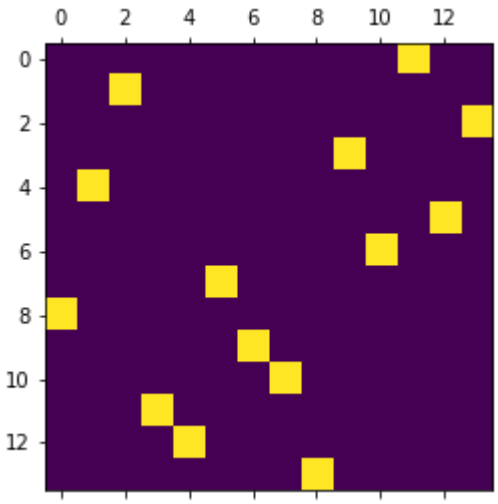
In [21]:

```
# calculate "binary matrix" of variables to the power of n (for n = [1:14])

distances_p2 = distances_copy.dot(distances_copy)
distances_p3 = distances_p2.dot(distances_copy)
distances_p4 = distances_p3.dot(distances_copy)
distances_p5 = distances_p4.dot(distances_copy)
distances_p6 = distances_p5.dot(distances_copy)
distances_p7 = distances_p6.dot(distances_copy)
distances_p8 = distances_p7.dot(distances_copy)
distances_p9 = distances_p8.dot(distances_copy)
distances_p10 = distances_p9.dot(distances_copy)
distances_p11 = distances_p10.dot(distances_copy)
distances_p12 = distances_p11.dot(distances_copy)
distances_p13 = distances_p12.dot(distances_copy)
distances_p14 = distances_p13.dot(distances_copy)
```

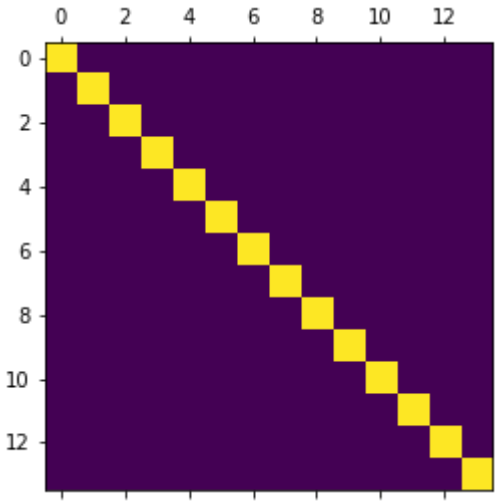
In [22]:

```
plt.matshow(distances_copy.iloc[:,:] == 1)
plt.show()
```



In [23]:

```
plt.matshow(distances_p14.iloc[:,:] == 1)
plt.show()
```



above matrix is diagonal which indicates that all nodes come back to itself after 14 steps

3. What are the best routes if you have two lorries?

In [24]:

```
# Create the 'prob' variable to contain the problem data
prob_2 = LpProblem("Shortest path - 2 lorries",LpMinimize)

# A dictionary called is created to contain the referenced Variables
adress_vars = LpVariable.dicts("path",(adresses,adresses),0,1,LpInteger)

# The objective function
prob_2 += lpSum([distances.loc[i,j]*adress_vars[i][j]
               for i in adresses
               for j in adresses]), "Total distance"
```

In [25]:

```
# ADD CONSTRAINTS

# node (1,1) (our starting location) can have 2 edges out and 2 edges in (as there are
 two lorries living it)
prob_2 += lpSum([adress_vars[adresses[0]][j] for j in adresses]) == 2, ("two edges out
 from the starting location")
prob_2 += lpSum([adress_vars[i][adresses[0]] for i in adresses]) == 2, ("two edges in t
o the starting location")

# each remaining node can have only one edge out and one edge in
for i in adresses[1:]:
    prob_2 += lpSum([adress_vars[i][j] for j in adresses]) == 1, ("one edge out per " +
i)
    prob_2 += lpSum([adress_vars[j][i] for j in adresses]) == 1, ("one edge in per nod
e" + i)

# no node can lead to itself
for i in adresses:
    prob_2 += adress_vars[i][i] == 0, ("no going to itself" + i)

# eliminate sub-loops
for i in adresses[1:]:
    for j in adresses[1:]:
        prob_2 += adress_vars[j][i] + adress_vars[i][j] <= 1, ("avoid 2-loops" + i +
"_" + j)

prob_2.solve()
```

Out[25]:

```
1
```

In [26]:

```
# The status of the solution is printed to the screen
print("Status:", LpStatus[prob.status])

# Each of the variables is printed with it's resolved optimum value
for v in prob.variables():
#     print(v.name, "=", v.varValue)
    if (v.varValue > 0.0):
        print(v.name, "=", v.varValue)
```

```
Status: Optimal
path_BS10_5DB_BS10_6DQ = 1.0
path_BS10_6DQ_BS48_3RE = 1.0
path_BS13_8BW_BS3_3NA = 1.0
path_BS16_4DD_BS16_5TR = 1.0
path_BS16_5TR_BS37_6RX = 1.0
path_BS25_1TW_BS29_6AB = 1.0
path_BS29_6AB_BS13_8BW = 1.0
path_BS35_3RY_BS10_5DB = 1.0
path_BS37_6RX_BS35_3RY = 1.0
path_BS3_3NA_BS8_4HY = 1.0
path_BS48_3RE_BS25_1TW = 1.0
path_BS4_2NZ_BS5_8EJ = 1.0
path_BS5_8EJ_BS16_4DD = 1.0
path_BS8_4HY_BS4_2NZ = 1.0
```

In [27]:

```
print("Total distance: ", value(prob_2.objective))
print("correct values is:  76.74")
```

```
Total distance:  76.74000000000002
correct values is:  76.74
```

4. What are the best routes if you have two lorries, each with a capacity of 10?

In this solution we introduce a new dimenison: 'lorry_no' (lorry number) which can take either value '1' or '2'. This variable will indicate which lorry has visited which adress.

Following are some guidlines needed for constructing constrains:

- First adress is visited by both lorries
- only one lorry can visit all other adresses
- for a given location the lorry that entered a location has to be the one that left it.
- single lorry can visit at most 10 locations

In [51]:

```
lorry_no = ['1','2']

# Create the 'prob' variable to contain the problem data
prob_3 = LpProblem("Shortest path - 2 lorries",LpMinimize)

# A dictionary called is created to contain the referenced Variables
adresses_vars_3 = LpVariable.dicts("path",(adresses,adresses,lorry_no),0,1,LpInteger)

# The objective function
prob_3 += lpSum([distances.loc[i,j]*adresses_vars_3[i][j][k]
                 for i in adresses
                 for j in adresses
                  for k in lorry_no]), "Total distance"
```

In [52]:

```
# ADD CONSTRAINTS

# node (1,1) (our starting location) can have 2 edges out and 2 edges in (as there are
 two lorries living it)
# however each edge in or out has to be associated with different lorry
for k in lorry_no:
    prob_3 += lpSum([adresses_vars_3[adresses[0]][j][k] for j in adresses]) == 1, ("two
edges out from the starting location" + k)
    prob_3 += lpSum([adresses_vars_3[i][adresses[0]][k] for i in adresses]) == 1, ("two
edges in to the starting location" + k)

# each node can have only one edge out and one in
for i in adresses[1:]:
    prob_3 += lpSum([adresses_vars_3[i][j][k] for j in adresses for k in lorry_no]) ==
1, ("one edge out per " + i)
    prob_3 += lpSum([adresses_vars_3[j][i][k] for j in adresses for k in lorry_no]) ==
1, ("one edge in per node" + i)

# each lorry can visit only 10 adresses at maximum + 1 path to get back home = 11
for k in lorry_no:
    prob_3 += lpSum([adresses_vars_3[j][i][k] for i in adresses for j in adresses]) <=
11, ("each lorry has capacity max 10" + k)


# the lorry that left an adress has to be the same one the came in
for i in adresses[1:]:
    for k in lorry_no:
        prob_3 += lpSum([adresses_vars_3[i][j][k] for j in adresses]) == lpSum([adresse
s_vars_3[j][i][k] for j in adresses]), ("ensure same lorry drives in and out: " + i +
"_" + k)

# no node can lead to itself
for k in lorry_no:
    for i in adresses:
        prob_3 += adresses_vars_3[i][i][k] == 0, ("no going to itself" + i +"_" + k)

#  eliminate 2-node sub-loops
for k in lorry_no:
    for i in adresses:
        for j in adresses:
            prob_3 += adresses_vars_3[j][i][k] + adresses_vars_3[i][j][k] <= 1, ("avoid
2-loops" + i + "_" + j + "_" + k)


# eliminate some manually detected 3-node sub-loops
adresses_sub = ['BS13 8BW', 'BS25 1TW', 'BS29 6AB']
for k in lorry_no:
    prob_3 += lpSum([adresses_vars_3[j][i][k] for i in adresses_sub for j in adresses_s
ub]) <= 2, ("avoid 3-loop - a" + k)

adresses_sub_2 = ['BS16 5TR', 'BS37 6RX', 'BS16 4DD']
for k in lorry_no:
    prob_3 += lpSum([adresses_vars_3[j][i][k] for i in adresses_sub_2 for j in adresses
_sub_2]) <= 2, ("avoid 3-loop - b" + k)

adresses_sub_3 = ['BS16 5TR', 'BS35 3RY', 'BS37 6RX']
for k in lorry_no:
    prob_3 += lpSum([adresses_vars_3[j][i][k] for i in adresses_sub_3 for j in adresses
_sub_3]) <= 2, ("avoid 3-loop - c" + k)
```

```
adresses_sub_4 = ['BS16 5TR', 'BS5 8EJ', 'BS16 4DD']
for k in lorry_no:
    prob_3 += lpSum([adresses_vars_3[j][i][k] for i in adresses_sub_4 for j in adresses
_sub_4]) <= 3, ("avoid 3-loop - d" + k)


prob_3.solve()
```

Out[52]:

```
1
```

In [53]:

```
print("Total distance: ", value(prob_3.objective))
print("correct values is:  84.92")
```

```
Total distance:  84.92
correct values is:  84.92
```

In [54]:

```
# The status of the solution is printed to the screen
print("Status:", LpStatus[prob_3.status])

# Each of the variables is printed with it's resolved optimum value
for v in prob_3.variables():
    if (v.varValue > 0.0):
        print(v.name, "=", v.varValue)
```

```
Status: Optimal
path_BS10_5DB_BS10_6DQ_2 = 1.0
path_BS10_6DQ_BS48_3RE_2 = 1.0
path_BS13_8BW_BS3_3NA_2 = 1.0
path_BS16_4DD_BS5_8EJ_1 = 1.0
path_BS16_5TR_BS35_3RY_2 = 1.0
path_BS16_5TR_BS37_6RX_1 = 1.0
path_BS25_1TW_BS29_6AB_2 = 1.0
path_BS29_6AB_BS13_8BW_2 = 1.0
path_BS35_3RY_BS10_5DB_2 = 1.0
path_BS37_6RX_BS16_4DD_1 = 1.0
path_BS3_3NA_BS8_4HY_2 = 1.0
path_BS48_3RE_BS25_1TW_2 = 1.0
path_BS4_2NZ_BS16_5TR_2 = 1.0
path_BS5_8EJ_BS16_5TR_1 = 1.0
path_BS8_4HY_BS4_2NZ_2 = 1.0
```

there are no inner loops in the solution above each node is visited once, but the first one. (checked manually)

In [ ]:

```

```