Exercise 5: Time Series Analysis – Part V

- Fourier Transformation and filtering in the frequency domain -

Group:	Surname, Given Name:	Matriculation number:	Signature*:

*** With my signature I declare that I was involved in the elaboration of this exercise.**

Deadline: 28.01.2026

Objective

This exercise deals with time series analysis in the frequency domain.

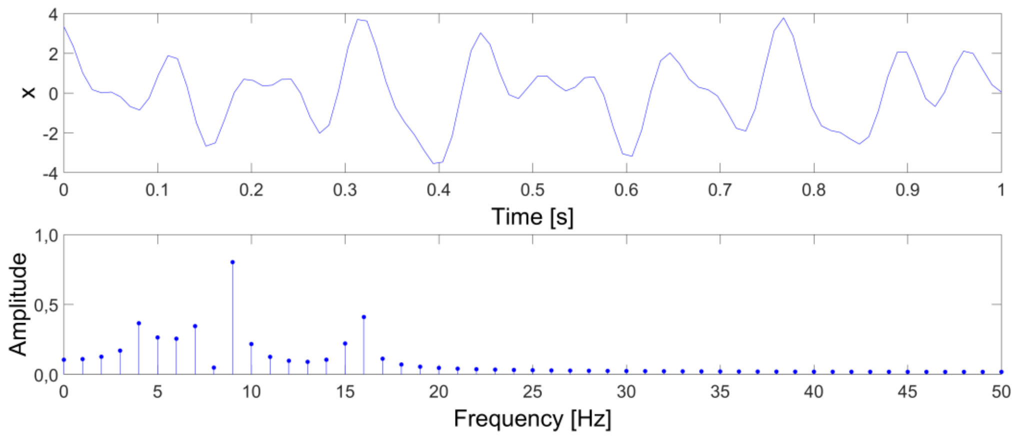

Figure 1: Time series (top) and related Fourier spectrum (bottom)

in time domain the base function iare the base vectors/base functions of “nomials”

more general the base functions is a function, defined in its different spaces/use cases

in fourier we can use the fourier basis, a set of harmonic basae functiobs with different coefficients

Task 1: Amplitude and phase spectra

1) Create a time series of $n = 100$ observations using the following functional model and chose appropriate values for amplitude A, frequency f and phase shift φ

$$y(t) = A \cdot \sin(2\pi f t + \varphi)$$

2) Calculate the Fast Fourier Transform of the time series using the implemented Matlab function *fft*
3) Plot the time series as well as the Fast Fourier Spectrum
4) Repeat steps 1) to 3) while changing the amplitude A and interpret the results
5) Repeat steps 1) to 3) while changing the phase shift φ and interpret the results

Task 2: Fourier analysis

- Calculate the Fast Fourier Transform of the given time series
  - "*TimeSeries.txt*"
  - "*TimeSeries_Offset.txt*"
  - "*TimeSeries_Trend.txt*"
  - "*TimeSeries_Trend_Offset.txt*"
  - "*TimeSeries_Data_Gap.txt*"
- Plot each Fast Fourier spectrum
- Compare and interpret the results

Task 3: Filtering

1) Use the file "E*xercise5-1.txt*" from Exercise 5 – Task 1
2) Implement a moving average filter in the frequency domain for different filter length $\tau = [5\ 15\ 25]$
3) Compare the results with those from the filtering in the time domain in Exercise 5 – Task 1

Task 4: Frequency response of different periodic kernels

Consider a discrete, periodic signal with $N = 127$ samples and the discrete index $n = 0, \dots, N-1$. Furthermore, the signal is assumed to be periodic with the period N, with the centred frequency index $k = -\frac{N-1}{2}, \dots 0, \dots, \frac{N-1}{2}$. For a periodic kernel $h[n]$, the frequency response (eigenvalues) is defined as:

$$\lambda_k = \mathrm{FFT}\{h[n]\}_k$$

Construct the following kernels. In each case, build a length-N kernel vector h by placing the nonzero coefficients centred around n=0 (circularly), and setting all other entries to zero.

- Moving average, $\tau = 11$
$$h_1[n] = \frac{1}{11}[1, 1, \dots, 1]$$
- Moving average, $\tau = 21$
$$h_2[n] = \frac{1}{21}[1, 1, \dots, 1]$$
- Triangular kernel, $\tau = 21$
Construct $h_3[n]$ by convolving the length-11 moving average kernel $h_1[n]$ with itself, and normalize such that $\sum_n h_3[n] = 1$.
- Random symmetric compact kernel, $\tau = 21$
Create random coefficients on the support length 21, enforce symmetry $h_4[-n] = h_4[n]$, and normalize such that $\sum_n h_4[n] = 1$.

1) Compute the frequency response $\lambda_k^{(i)}$ for each kernel $h_i[n]$ and plot the magnitude spectrum $\left|\lambda_k^{(i)}\right|$ versus k.
2) For each kernel, briefly discuss:
    a. Is it mainly a low-pass filter?
    b. How quickly does $\left|\lambda_k^{(i)}\right|$ decay with increasing $|k|$.
    c. Do you observe side lobes? What do they indicate?
    d. Compare the rectangular and triangular kernels: which one has the smoother frequency response and why?

Task 5: Ideal low pass filtering and the corresponding kernel

Consider a discrete, periodic signal with $N = 127$ samples and the discrete index $n = 0, \dots, N - 1$. Furthermore, the signal is assumed to be periodic with the period N. A filter is defined in the frequency domain by the following frequency response:

$$\lambda_k = \begin{cases} 1, & |k| \le k_0, \\ 0, & \text{otherwise}, \end{cases} \qquad k = -\frac{N-1}{2}, \dots 0, \dots, \frac{N-1}{2}$$

The parameter k_0 is defined via the filter length

$$k_0 = \frac{\tau - 1}{2}, \qquad \tau = 11.$$

Here, λ_k denotes the eigenvalues of the circular convolution operator associated with the filter.

1) Interpretation in the frequency domain
   a. Identify the type of filter defined by λ_k.
   b. Plot the frequency response λ_k.
   c. Explain why this filter is referred to as an ideal low-pass filter.
2) Determination of the kernel
   a. Determine the corresponding filter kernel $h[n]$ by applying the inverse discrete Fourier transform to λ_k (e.g. using *ifft*). Ensure that the kernel is properly centred with respect to the discrete index n.
   b. Plot the kernel $h[n]$ over all $N = 127$ samples.
   c. Describe the characteristic shape of the kernel.
3) Analysis of the Kernel properties
   a. Explain how the global, oscillatory shape of the kernel influences the filtered signal in terms of locality, ringing artifacts, and phase behaviour.