M. Sc. Geodesy and Geoinformation Science

Module:

Analysis of Stochastic Processes WS 25/26

Exercise 2: Time Series Analysis - Part II (Cross-Correlation Functions)

GROUP 2

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*Group work completed by all group members. This assignment is therefore valid without physical signature

1. Introduction

Careful signal analysis supports the integrity and reliability of geodetic monitoring. Therefore, cross-correlation is a fundamental tool in time-series analysis used to quantify the similarity between two signals as a function of a temporal shift applied to one of them. For two discrete sequences x_t and y_t, the normalized sample cross-correlation at lag τ is defined as

$$r_{xy}(\tau) = \frac{c_{xy}(\tau)}{\sqrt{c_x(0)\, c_y(0)}},$$

where the sample cross-covariance is given by

$$c_{xy}(\tau) = \frac{1}{n-\tau} \sum_{t=1}^{n-\tau} (x_t - \bar{x})(y_{t+\tau} - \bar{y}).$$

The MATLAB® function `xcorr()` computes the raw (unnormalized) cross-correlation according to

$$\widehat{R}_{xy}(m) = \begin{cases} \displaystyle\sum_{n=0}^{N-m-1} x_{n+m}\, y_n^*, & m \geq 0, \\ \widehat{R}_{yx}^*(-m), & m < 0, \end{cases}$$

where the returned integer lag m specifies how the *second* input signal must be shifted to best match the first one. This behaviour is consistent with the MATLAB documentation: *"r = xcorr(x,y) returns the cross-correlation of two discrete-time sequences. Cross-correlation measures the similarity between a vector x and shifted (lagged) copies of a vector y as a function of the lag."* Therefore, the lag values obtained from `xcorr(x,y)` indicate how the second argument y must be shifted to achieve maximum similarity with the first argument x.

The maximum of the cross-correlation function marks the time shift at which two signals are most strongly aligned. This property makes cross-correlation essential for time-delay estimation, sensor alignment, and multi-sensor data fusion. It allows the detection of leading or lagging behaviour, supports the synchronization of heterogeneous measurements, and reveals systematic temporal offsets between instruments.

In this assignment, cross-correlation is applied to several datasets—including temperature, inclination, TLS, IBIS-S, and accelerometer recordings—to quantify temporal dependencies and to align all observations onto a common time axis. Through plotting, lag estimation, and interpretation of cross-correlation functions, the analysis demonstrates how cross-correlation facilitates time alignment, dependency evaluation, and multi-sensor integration.

2. Methods

All computations were carried out in MATLAB®, following the procedure presented in the lectures of the course *Analysis of Stochastic Processes*. The analysis consisted of the following steps:

(1) Data preparation

Each dataset was imported as a two-column time series $[t_i, y_i]$. All signals were demeaned prior to cross-correlation to remove constant offsets and to emphasise the oscillatory components relevant for lag estimation.

(2) Computation of the cross-correlation

For each sensor pair, the normalized cross-correlation

$$\rho_{xy}(k)$$

was computed using

```
[xcf, lags] = xcorr(x, y, 'normalized');
```

where the vector `lags` contains the integer shifts k expressed in samples. The corresponding physical delay was obtained via

$$\tau = k\,\Delta t,$$

with Δt the sampling interval of the reference signal.

(3) Selection of the physically meaningful lag

Two criteria were considered, depending on the expected sensor behaviour:

- maximum absolute correlation, when signals may be inverted or out of phase,
- maximum positive correlation, when an in-phase physical relationship is required.

The sensor configuration and the sign conventions determined which criterion was applied.

(4) Time-axis correction

Once the lag τ was identified, the time axis of the affected signal was corrected according to

$$t_{\text{corrected}} = t + \tau.$$

For accelerometer data, the signal was additionally inverted to match the displacement convention of TLS and IBIS-S.

(5) Visualisation

For each task, the workflow included:

1. plotting the raw time series,
2. plotting the cross-correlation function and marking the selected lag,
3. applying the temporal correction,
4. plotting the aligned time series for comparison.

These visualisations allow verification of the estimated delays and demonstrate the effectiveness of the temporal alignment.

3. Task 1: Cross-Correlation Function

Figure 1 shows the two time series $y_1(t)$ and $y_2(t)$, both recorded over the same time interval with equally spaced sampling times. Equal sampling is important because the MATLAB function `xcorr()` assumes a constant sampling interval Δt. Only under this condition the lag index returned by `xcorr()` can be converted into a physical time lag via the equation:

$$\tau = \text{lags} \cdot \Delta t.$$

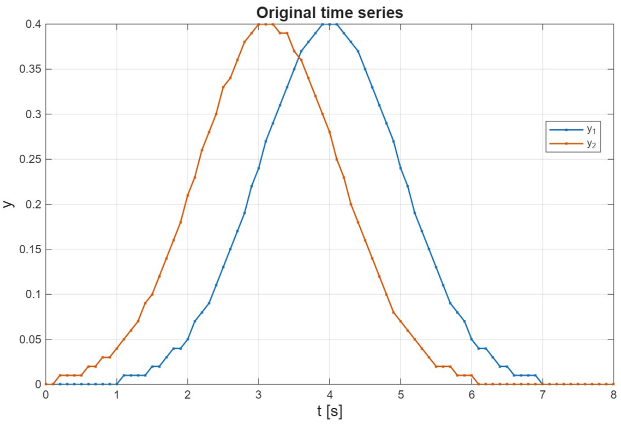

Figure 1: Time series $y_1(t)$ and $y_2(t)$.

The sample cross-correlation between two discrete signals is defined as

$$R_{12}[k] = \sum_n y_1[n]\, y_2[n-k],$$

where n is the discrete time index and k is the lag (in samples). According to the sign convention used by `xcorr()`, a positive lag k means that the second signal y_2 must be shifted to the right (backwards in time) to match y_1, whereas a negative lag shifts y_2 to the left (forwards in time). This specifies how the second input signal must be moved to achieve the highest similarity with the first.

The normalized cross-correlation used in this exercise is given by

$$\rho_{12}[k] = \frac{R_{12}[k]}{\sqrt{R_{11}[0]\, R_{22}[0]}} \in [-1, 1].$$

Here $R_{12}[k]$ is the (unnormalized) cross-correlation between y_1 and y_2, while $R_{11}[0]$ and $R_{22}[0]$ denote the autocorrelations of y_1 and y_2 at zero lag. These terms correspond to the signal energies and are used to scale the result so that $\rho_{12}[k]$ always lies between -1 and 1.

Using the raw input signals, the resulting normalized cross-correlation function is shown in Figure 2.

Interpretation

The time lag between the two signals can be determined by locating the maximum of the cross-correlation function. From Figure 2, the maximum

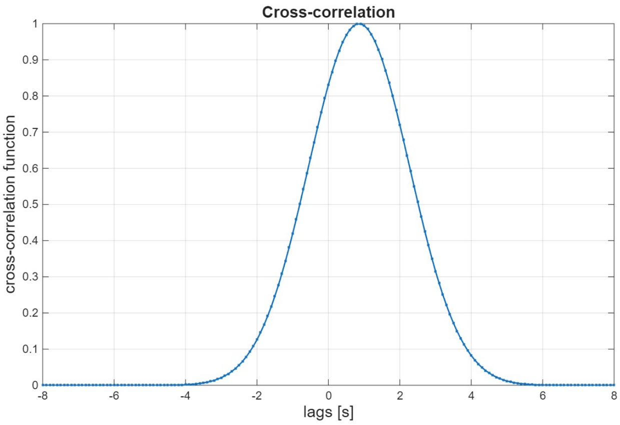

Figure 2: Normalized cross-correlation between y_1 and y_2.

occurs at

$$\tau_{\max} = 0.9\,\text{s}.$$

This means that shifting $y_2(t)$ by $0.9\,\text{s}$ produces the strongest alignment with $y_1(t)$.

The resolution of the estimated time shift is limited by the sampling period,

$$\Delta t = t_{i+1} - t_i.$$

Since `xcorr()` returns lags in integer numbers of samples, the smallest detectable time shift is one sampling interval Δt. Fractional or sub-sample shifts (e.g. $0.5\,\Delta t$) cannot be resolved directly, because the correlation can only be evaluated at discrete sample positions.

Additional Remarks

The computation above includes the mean values of both signals. If the signals have large positive means, the term

$$\bar{y}_1\, \bar{y}_2$$

can dominate the correlation and produce a cross-correlation function that remains mostly positive for all lags. In such cases, the result mainly reflects the constant offsets rather than the relationship between the fluctuations of the two signals.

To focus only on the variations, the sample mean can be subtracted from each signal:

$$\tilde{y}_1(t) = y_1(t) - \bar{y}_1, \qquad \tilde{y}_2(t) = y_2(t) - \bar{y}_2.$$

The cross-correlation of the demeaned signals,

$$\rho_{\tilde{y}_1\tilde{y}_2}(\tau),$$

indicates now how the oscillatory components of the two signals relate, without the influence of constant offsets. This approach is useful when the mean does not carry information (e.g., zero-mean stochastic processes), when focusing on dynamic behaviour, or when comparing sensors with different baselines.

The demeaned cross-correlation is shown in Figure 3. In contrast to the non-demeaned case, it contains both positive and negative values and reveals the "underlying" oscillatory structure.

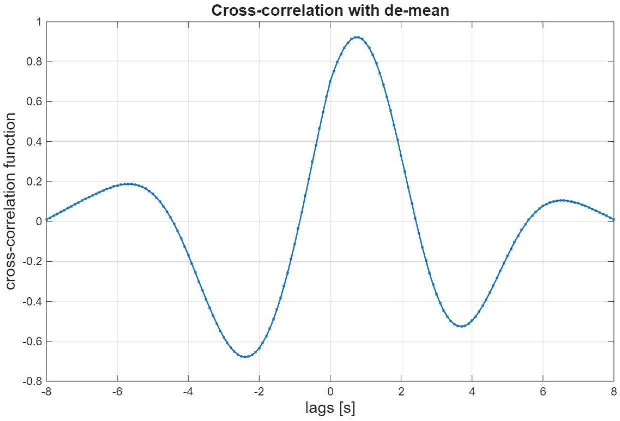

Figure 3: Normalized cross-correlation after removing the mean from both signals.

Both versions of the cross-correlation are valid, depending on the objective. The non-demeaned correlation is suitable when absolute signal levels are relevant or when offsets between sensors matter. The demeaned version is adequate for the analysis of fluctuations, periodic components, or dynamic responses.

4. Task 2: Cross Correlation Function

Since both time series are sampled at equal time intervals, the same procedure as in Task 1 can be applied. Figure 4 shows the signals $y_1(t)$ and $y_2(t)$, which show approximately harmonic behavior and remain within the interval $[-1, 1]$ throughout the sampling period.

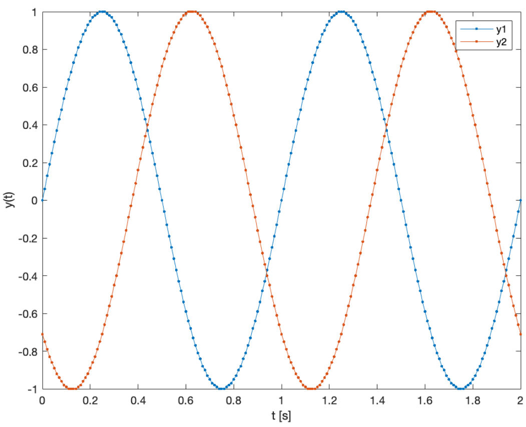

Figure 4: Harmonic time series $y_1(t)$ and $y_2(t)$

The normalized MATLAB function `xcorr()` is used to compute the cross-correlation between y_1 and y_2. As in Task 1, the lag index is converted into physical time lags using $\tau = \text{lags} \cdot \Delta t$. The resulting cross-correlation function is shown in Figure 5. Since the signals are harmonic, the cross-correlation is also oscillatory and contains several clear maxima and minima.

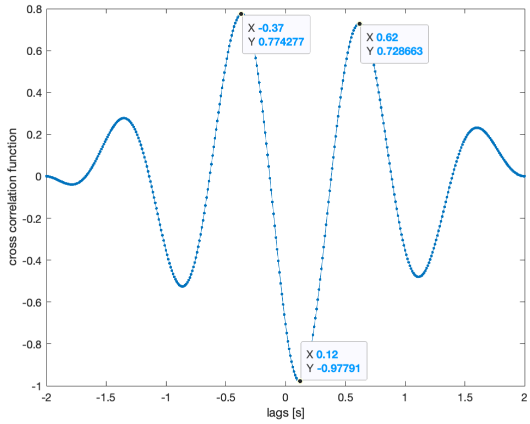

Figure 5: Normalized cross-correlation between y_1 and y_2 in Task 2.

Interpretation

The cross-correlation function shows three clear extrema close to zero lag. The largest *positive* correlation is obtained at

$$\tau_{\text{pos}} \approx -0.37 \text{ s}, \qquad \rho(\tau_{\text{pos}}) \approx 0.77,$$

which indicates that shifting $y_1(t)$ by approximately $0.37\,\text{s}$ (or $y_2(t)$ by $-0.37\,\text{s}$) aligns the two signals so that they move mostly in phase.

The largest *negative* correlation occurs at

$$\tau_{\text{neg}} \approx 0.12 \text{ s}, \qquad \rho(\tau_{\text{neg}}) \approx -0.98.$$

At this lag, the signals are almost perfectly out of phase and can be approximated by

$$y_1(t) \approx -\,y_2(t + 0.12 \text{ s}).$$

Thus, a shift of $0.12\,\text{s}$ aligns y_1 and y_2 as nearly inverted versions of each other.

If the goal is to identify the time delay that gives the strongest correlation in magnitude, the relevant quantity is the maximum of $|\rho(\tau)|$, which in this example appears at $\tau_{\text{neg}} \approx 0.12\,\text{s}$. If a positive (in-phase) relationship is required, the relevant lag is $\tau_{\text{pos}} \approx -0.37\,\text{s}$.

Because the task does not specify any physical context, both lags can be considered valid: τ_{pos} corresponds to the largest positive similarity, while τ_{neg} corresponds to the strongest similarity up to a sign (inverted response).

5. Task 3: Deformation monitoring

In this task we have given two time-series, one depicting the inclination of a bridge and the second showing temperature measured at the bridge.
The question is, whether the deformation - or inclination - could be correlated to the change in temperature, and if yes, the observed time lag between temperate change and observed deformation.
Below the observations and the respective correlation

function are plotted, having the temperature measurement as the reference time-series. Therefore we would expect a **positive** lag between temperature and a following deformation.

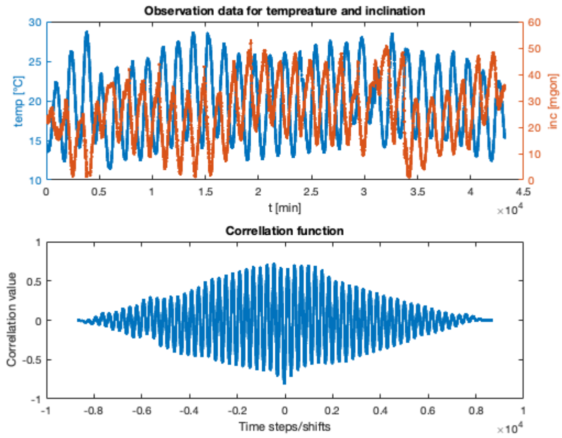

Figure 6: Above: raw observations, Below: Correlation function

Interpretation

A harmonic-looking correlation function is obtained, with its maximum **absolute** correlation value of

$$|xcf_{\max}| = 0.804,$$

at a delay of +29 steps.

However, this corresponds to a **negative** correlation. In this case it is necessary to consider the expected physical relationship between temperature and inclination. An increase in the inclination time series is expected after a period of solar heating (temperature rising). A time shift of

$$29 \text{ steps} \times 5 \text{ min} = 145 \text{ min},$$

produces the alignment shown in Fig. 7.

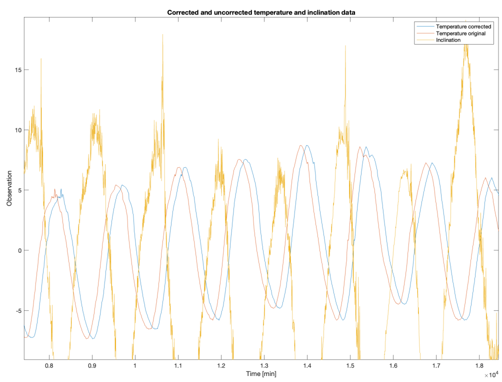

Figure 7: Shift-Corrected Temperature observation, using max(abs(xcf))

This may be the numerically largest correlation value; however, the expected relationship between temperature and inclination is **positive**. For this reason it is necessary to specify how the relevant maximum is chosen. In this case, the appropriate choice is the closest positive peak in the correlation function.

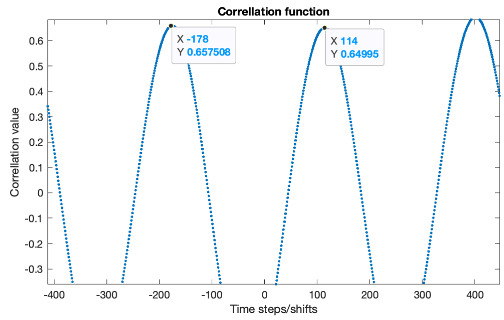

Figure 8: Correlation function

The closest positive peak occurs at 114 shifts, giving

$$r_{\text{corr}} = 0.64995.$$

Converting this to a time delay yields

$$114 \times 5\,\text{min} = 570\,\text{min}.$$

As the `xcorr` function was not applied directly to the shifted signals, the 570 min delay can be subtracted from the inclination time axis to obtain the alignment shown in Fig. 9. Based on this interpretation, and using the expected positive relationship between the two quantities, the deformation of the bridge follows the temperature signal with a delay of approximately **+9.5 h**, with a correlation of about **0.65**.

This delay is derived from the global correlation function, while only a single peak of interest is considered. Consequently, the correlation value is not very high. A more refined estimate could be obtained by applying the correlation analysis to shorter subsets of the data, which would likely increase the peak correlation. For the purpose of this task, however, the global approach provides a sufficient estimate of the delay between temperature changes and deformation.

For illustration, Fig. 9 shows the temperature series shifted by 570 min. The arithmetic mean has been subtracted from both series for better visualization.

6. Task 4: Deformation monitoring with TLS, IBIS-S and accelerometer

During a deformation monitoring experiment, the vertical oscillation of a point on a bridge was measured simultaneously with a terrestrial laser scanner (TLS) and the ground–based radar interferometer IBIS-S. Both instruments were configured such that their positive measurement axis pointed towards the zenith. The corresponding displacement time series are shown in Figure 10.

Both records show a damped harmonic oscillation with very similar frequency content and amplitude

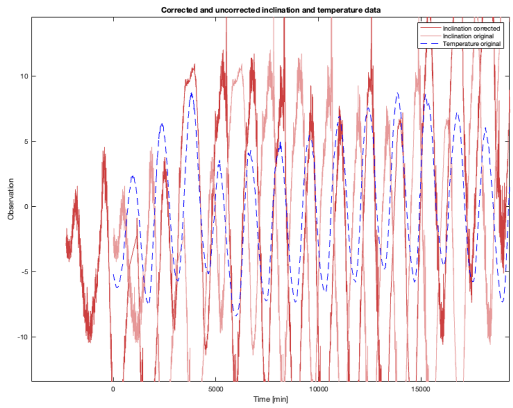

Figure 9: Temperature time series shifted by 570 min.

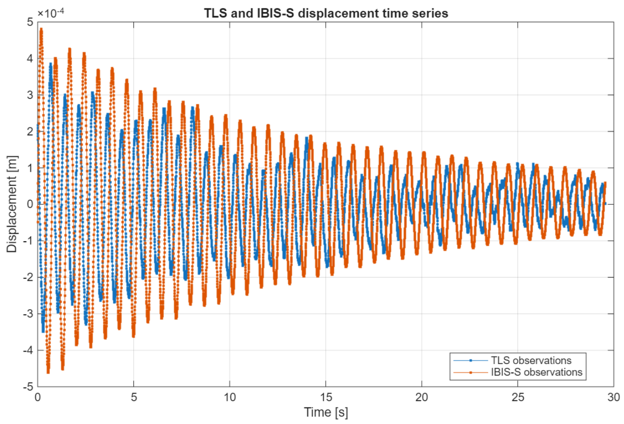

Figure 10: Displacement time series recorded by TLS and IBIS-S.

range. The sampling intervals are approximately $\Delta t_{\text{TLS}} \approx \Delta t_{\text{IBIS}} \approx 0.005\,\text{s}$ (about 200 Hz).

TLS–IBIS time shift from cross-correlation

To estimate the relative time shift between the two series, the normalized sample cross-correlation

$$\rho_{y_{\text{TLS}}y_{\text{IBIS}}}[k]$$

is evaluated for demeaned signals, using the MATLAB function `xcorr`. The resulting cross-correlation function is shown in Figure 11.

The largest correlation in absolute value is obtained at

$$\max_k |\rho_{y_{\text{TLS}}y_{\text{IBIS}}}[k]| \approx 0.96 \quad \text{for} \quad k = 23.$$

With a TLS sampling interval of $\Delta t_{\text{TLS}} \approx 0.005\,\text{s}$, this corresponds to a time lag of

$$\tau_{\text{TLS–IBIS}} = k\,\Delta t_{\text{TLS}} \approx 23 \cdot 0.005\,\text{s} \approx 0.115\,\text{s}.$$

The corresponding correlation value at this lag is negative ($\rho \approx -0.962$), indicating that the two displacement records are approximately out of phase (inverted) at the point of maximum similarity. For the purpose of estimating the delay, only the position of

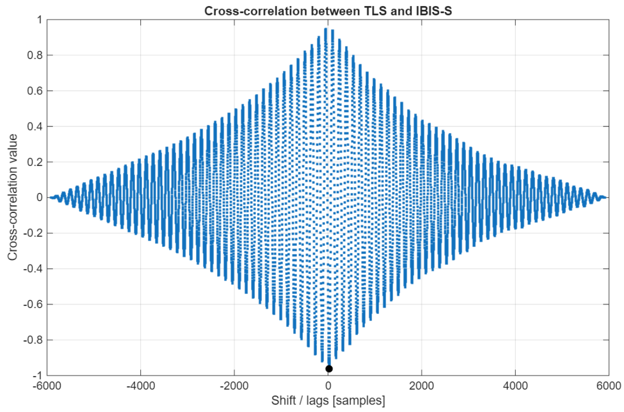

Figure 11: Normalized cross-correlation between TLS and IBIS-S displacement time series.

the peak in $|\rho|$ is used, not its sign.

To align the TLS record with the IBIS-S reference, the TLS time axis is shifted by $+0.115\,\text{s}$. Figure 12 shows the original and corrected TLS data together with the IBIS-S displacement.

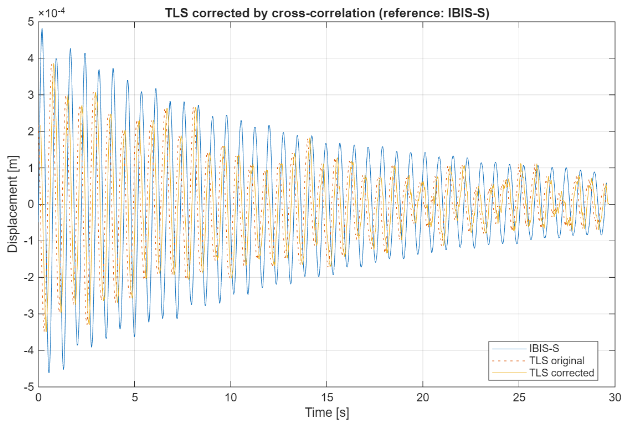

Figure 12: TLS displacement before and after time-shift correction, using IBIS-S as reference.

After applying this shift, the oscillation cycles of both instruments coincide well over the entire observation interval, confirming the estimated delay.

Inclusion of accelerometer measurements

In a second step, acceleration of the same bridge point was recorded using an accelerometer, whose positive axis points towards the ground. To make the accelerometer record comparable with the displacement measurements (positive upwards), the sign of the acceleration is inverted. Figure 13 displays the IBIS-S displacement, the time–corrected TLS displacement and the raw accelerometer record.

The accelerometer and IBIS-S signals do not share exactly the same time grid. Therefore, for the delay estimation the IBIS-S displacement is linearly interpolated onto the accelerometer time stamps over the common time interval. The normalized cross-correlation

$$\rho_{a\,y_{\text{IBIS}}}[k]$$

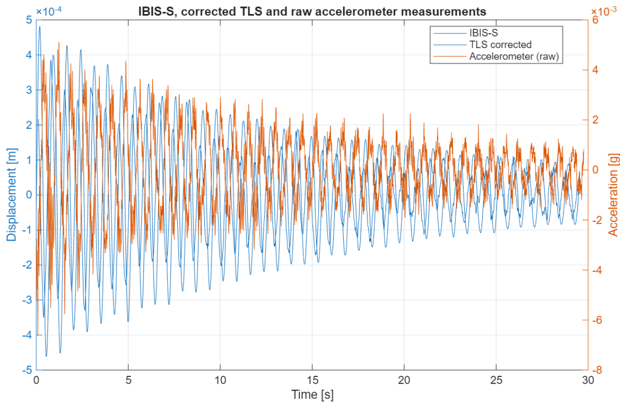

Figure 13: IBIS-S displacement, corrected TLS displacement and raw accelerometer time series.

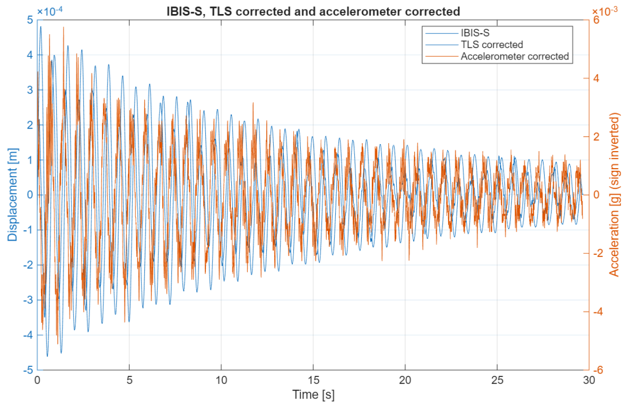

Figure 15: IBIS-S, time–corrected TLS and time–corrected accelerometer signals.

is then computed between the demeaned, sign–inverted acceleration and the resampled IBIS-S displacement. The resulting cross-correlation function is plotted in Figure 14.

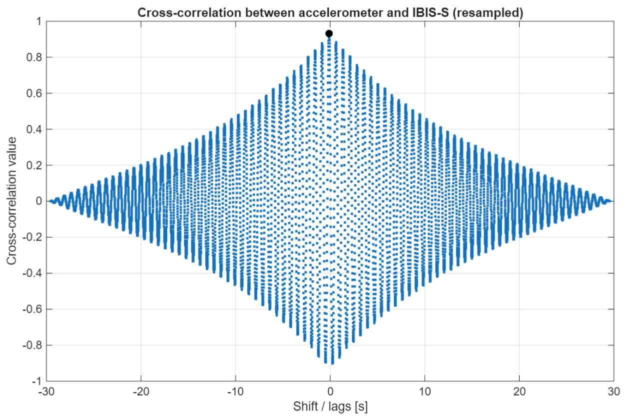

Figure 14: Normalized cross-correlation between accelerometer (sign inverted) and IBIS-S displacement (resampled to accelerometer grid).

The maximum absolute correlation is

$$\max_k |\rho_{a\, y_{\mathrm{IBIS}}}[k]| \approx 0.93 \quad \text{at} \quad k = -18,$$

on the accelerometer time grid. With a sampling interval Δt_{acc}, this corresponds to a time lag of

$$\tau_{\mathrm{acc–IBIS}} = k\, \Delta t_{\mathrm{acc}} \approx -0.118\,\mathrm{s}.$$

The negative sign indicates that the accelerometer signal is delayed with respect to IBIS-S by about 0.12 s. The accelerometer time axis is therefore shifted by $\tau_{\mathrm{acc–IBIS}}$ to align it with the IBIS-S reference.

Final comparison of all three sensors

After applying the time-shift corrections derived above, all three time series can be compared on a common time axis. Figure 15 shows the IBIS-S displacement, the corrected TLS displacement and the corrected, sign–inverted accelerometer record.

The dominant oscillation cycles of the three sensors are now largely aligned in time. The accelerometer trace is noisier and measures acceleration rather than displacement, so the agreement is not perfect. Nevertheless, the phase relationship between the peaks and troughs matches well, and the damping behaviour is consistent with the displacement records. This indicates that the estimated delays of approximately 0.115 s for TLS and 0.118 s for the accelerometer are physically plausible and that the three instruments are observing the same structural response with small, instrument–dependent timing offsets.

7. Conclusion

Overall, the results confirm that cross-correlation provides a robust and interpretable approach for estimating systematic time shifts between independent measurement systems. When applied carefully (sampling rates, overlapping intervals, and sensor-specific characteristics) cross-correlation enables reliable multi-sensor synchronization and supports a consistent physical interpretation of structural deformation processes.

Beyond the scope of this exercise, time alignment plays a central role in modern geodesy. Techniques such as VLBI, GNSS, InSAR, and ground-based ones all depend on precise synchronization to relate observations taken by different sensors, often operating at different scales and under different physical conditions. Even small temporal offsets can propagate into errors in displacement, velocity, or phase interpretation. In this sense, cross-correlation is not only a computational tool, but part of the broader geodetic principle that independent measurements must be made comparable before they can be meaningfully combined.

The workflow applied here reflects, in a simplified form, the procedures used in operational geodetic systems. Whether aligning telescopes in VLBI, synchronizing radar echoes, or integrating accelerometer data with optical sensors, these methods reinforce a core idea: precise timing is essential for understanding how the Earth and engineered structures behave.