# ECON 207: Problem Set 1 - KEY

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September 15, 2018

## 1 Macroeconomic Data

### 1.1 GDP and Inflation

For the following problems, use 2016 as the base year.

| | 2016 | 2017 |
|---|---|---|
| Good 1 | Price: $20<br>Quantity: 100 | Price: $25<br>Quantity: 80 |
| Good 2 | Price: $ 20<br>Quantity: 100 | Price: $20<br>120 |

1. Calculate the real and nominal GDP in this economy for each 2016 and 2017

   Nominal GDP is calculated using current prices and sales in each year.

$$Y_{2016}^{Nominal} = (\$20 \times 100) + (\$20 \times 100) = \$4000$$

$$Y_{2017}^{Nominal} = (\$25 \times 80) + (\$20 \times 120) = \$4400$$

   Real GDP is calculated using base year prices and current sales in each year

$$Y_{2016}^{Real} = (\$20 \times 100) + (\$20 \times 100) = \$4000$$

$$Y_{2017}^{Real} = (\$20 \times 80) + (\$20 \times 120) = \$4000$$

2. Calculate the GDP deflator in each year. Use this to calculate inflation from 2016 to 2017.

$$Deflator = \frac{Y^{Nominal}}{Y^{Real}}$$

$$Deflator_{2016} = \frac{\$4000}{\$4000} \times 100 = 100$$

$$Deflator_{2017} = \frac{\$4400}{\$4000} \times 100 = 110$$

$$Inflation = \frac{Deflator_{2017} - Deflator_{2016}}{Deflator_{2016}} = 0.10$$

We see 10% inflation between 2016 and 2017.

3. Suppose the household basket of goods consists of 100 units of Good 1 and 100 units of Good 2. Calculate the CPI in each year. Use this to calculate inflation from 2016 to 2017.

$$Cost\,of\,Bundle_{2016} = (\$20 \times 100) + (\$20 \times 100) = \$4000$$

$$Cost\,of\,Bundle_{2017} = (\$25 \times 100) + (\$20 \times 100) = \$4500$$

$$CPI = \frac{Cost\,of\,Bundle_{Year}}{Cost\,of\,Bundle_{Base\,Year}} \times 100$$

$$CPI_{2016} = \frac{\$4000}{\$4000} \times 100 = 100$$

$$CPI_{2017} = \frac{\$4500}{\$4000} \times 100 = 112.5$$

$$Inflation = \frac{CPI_{2017} - CPI_{2016}}{CPI_{2016}} = 0.125$$

We see 12.5% inflation between 2016 and 2017.

4. Why may the inflation rate you calculated in Question 3 be overstated? *Hint: Consider the sources of bias in the CPI.*

The inflation rate calculated using the CPI may be overstated due to substitution bias. When the price of Good 1 increases, consumers substitute from Good 1 to Good 2 and are therefore are not actually paying 12.5% more for their consumption.

### 1.2 Labor Market Variables

| Variable | Description | Number |
|---|---|---|
| E | Number of Employed Persons | 90 million |
| U | Number of Unemployed Persons | 5 million |
| P | Working Age Population | 200 million |
| h | hours per worker | 6.5 hours |

1. What is the size of the labor force in this economy?

   $E + U = 90\,million + 5\,million = 95\,million$

2. Calculate the labor force participation rate for this economy. Why are economists interested in this statistic?

   $\frac{E+U}{P} = 0.475$. The labor force participation rate is 47.5%. Economists are interested in this because it tells you how many of the people in the economy are interested in working and producing the goods that the entire economy will consume.

3. How many total hours of work does this economy provide?

   Total hours: $N = E \times h = 90\,million \times 6.5\,hours = 585\,million$

4. Describe the concept of a discouraged worker and how discouraged workers impact the unemployment rate.

   Discouraged workers are people who have been unemployed so long they have stopped looking for work. To be considered officially unemployed, a person has to be actively looking for a job. Otherwise, they are considered outside of the labor force. For this reason, many economists argue that discouraged workers cause the unemployment rate to appear too low.

## 2 Modeling Growth

### 2.1 Cobb-Douglas Production

Inputs capital ($K_t$) and labor ($N_t$) combine to produce output according to the following production function:

$$Y_t = 100K_t^{\frac{1}{2}}N_t^{\frac{1}{2}} \tag{1}$$

In this economy: $K_t = 10$ and $N_t = 100$

1. Show that the production function is increasing in capital and labor at these values of $K_t$ and $N_t$.

   The production function is increasing in capital (labor) if the first partial derivative with respect to capital (labor) is positive:

$$\begin{aligned}\frac{\partial Y_t}{\partial K_t} &= 100\left(\frac{1}{2}\right)K_t^{-\frac{1}{2}}N_t^{\frac{1}{2}}\\ &= 50(10)^{-\frac{1}{2}}(100)^{\frac{1}{2}} = \frac{500}{3.16} = 158.23 > 0\end{aligned}$$

$$\begin{aligned}\frac{\partial Y_t}{\partial N_t} &= 100\left(\frac{1}{2}\right)K_t^{\frac{1}{2}}N_t^{-\frac{1}{2}}\\ &= 50(10)^{\frac{1}{2}}(100)^{-\frac{1}{2}} = \frac{158}{10} = 15.8 > 0\end{aligned}$$

2. Show that the production function is constant returns to scale.

   Constant returns to scale means that if we increase both $K_t$ and $N_t$ by the same factor, we will also increase $Y_t$ by this factor. We can show that the production function is constant returns to scale by showing that doubling both $K_t$ and $N_t$ will also double $Y_t$.

$$\begin{aligned}Y_t &= 100(K_t)^{\frac{1}{2}}(N_t)^{\frac{1}{2}}\\ &= 100(10)^{\frac{1}{2}}(100)^{\frac{1}{2}}\\ &= 3162.3\end{aligned}$$

$$\begin{aligned}Y_t &= 100(K_t)^{\frac{1}{2}}(N_t)^{\frac{1}{2}}\\ &= 100(20)^{\frac{1}{2}}(200)^{\frac{1}{2}}\\ &= 6324.6 = (2 \times 3162.3)\end{aligned}$$

   Note: Depending how you round, you may find that the production only approximately doubles when we double capital and labor.

3. How much would this economy produce if $N_t = 0$?

   The economy would not produce anything. Both capital and labor are required to make positive output with this production function.

4. Write an expression showing the total costs of a firm. *Hint: This will include payments to labor at a wage rage, $w_t$, and capital at a rental rate, $R_t$.*

   The total costs of the firm include total payments to labor and total payments to capital: $w_t N_t + R_t K_t$

5. Derive the marginal product of labor and the marginal product of capital. What is the relationship between these marginal products and the wage rate and rental rate on capital for a profit-maximizing firm?

   The marginal product of labor (capital) is the first derivative of the production function with respect to that variable labor (capital).

$$\begin{aligned}\frac{\partial Y_t}{\partial K_t} &= 100\left(\frac{1}{2}\right) K_t^{-\frac{1}{2}} N_t^{\frac{1}{2}} \\ &= 50(10)^{-\frac{1}{2}}(100)^{\frac{1}{2}} = \frac{500}{3.16} = 158.23 > 0\end{aligned}$$

$$\begin{aligned}\frac{\partial Y_t}{\partial N_t} &= 100\left(\frac{1}{2}\right) K_t^{\frac{1}{2}} N_t^{-\frac{1}{2}} \\ &= 50(10)^{\frac{1}{2}}(100)^{-\frac{1}{2}} = \frac{158}{10} = 15.8 > 0\end{aligned}$$

   For a profit-maximizing firm, tthe marginal product of labor is equal to the wage rate and the marginal product of capital is equal to the rental rate on capital.

## 2.2 Solow Model

The following equations can be used to characterize the basic solow model (without wages or rental rate on capital).

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha} \tag{2}$$

$$Y_t = C_t + I_t \tag{3}$$

$$K_{t+1} = I_t + (1-\delta)K_t \tag{4}$$

$$I_t = sY_t \tag{5}$$

1. Describe each of these equations in words.

   In order, these equations are the production function, the household budget constraint, the capital accumulation equation, and the savings function.

2. Show that: $K_{t+1} = sAK_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t$

   Sustitute Equation 5 into Equation 4:

$$K_{t+1} = sY_t + (1-\delta)K_t$$

   Then substitute Equation 2 in for $Y_t$:

$$K_{t+1} = sAK_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t$$

3. Show that $C_t = (1-s)Y_t$ Combining Equations 3 and 5:

$$Y_t = C_t + sY_t$$

   And rearranging:

$$C_t = (1-s)Y_t$$

4. Put each equation into per worker terms. For Equation (4), use the form in Part 2 of this question.

$$\frac{Y_t}{N_t} = A\left(\frac{K_t}{N_t}\right)^{\alpha} (1)^{1-\alpha}$$

$$y_t = Ak_t^{\alpha}$$

$$\frac{Y_t}{N_t} = \frac{C_t}{N_t} + \frac{I_t}{N_t}$$

$$y_t = c_t + i_t$$

$$\frac{K_{t+1}}{N_t} = sA\left(\frac{K_t}{N_t}\right)^{\alpha}(1)^{1-\alpha} + (1-\delta)\frac{K_t}{N_t} = sAk_t^{\alpha} + (1-\delta)k_t$$

Because in the simple Solow model $N_t = N_{t+1}$, we can rewrite this as:

$$k_{t+1} = sAk_t^{\alpha} + (1-\delta)k_t$$

$$\frac{I_t}{N_t} = s\frac{Y_t}{N_t}$$

$$i_t = sy_t$$

5. Allow: $A = 100$, $\alpha = \frac{1}{2}$, $s = 0.2$, $\delta = 0.1$. Solve for the steady state level of capital per worker ($k^*$), output per worker ($y^*$), consumption per worker ($c^*$), investment per worker ($i^*$).

   Solving for the steady state using the per worker capital accumulations function: $k^* = \left(\frac{As}{\delta}\right)^{\frac{1}{1-\alpha}} = 40000$, $y^* = A(k^*)^{\alpha} = 20000$, $i^* = sy^* = 4000$, $c^* = y^* - i^* = 16000$.

### 2.3 Augmented Solow Model

The capital accumulation equation in the augmented Solow model is:

$$K_{t+1} = sAF(K_t, Z_tN_t) + (1-\delta)K_t \tag{6}$$

1. Labor grows at rate $n$. Labor-augmenting productivity grows at rate z. Write an expression for each $N_t$ and $Z_t$ in terms of $N_{t-1}$ and $Z_{t-1}$.

$$N_t = (1+n)N_{t-1}$$

$$Z_t = (1+n)Z_{t-1}$$

2. Show that $\frac{K_{t+1}}{Z_t N_t} = (1+n+z)\frac{K_{t+1}}{Z_{t+1}N_{t+1}}$.. *Assume $n \times z \approx 0$ for this question and all remaining.*

   Multiply $\frac{K_{t+1}}{Z_t N_t}$ by $\frac{Z_{t+1}N_{t+1}}{Z_{t+1}N_{t+1}}$. Because $Z_{t+1}N_{t+1} = (1+z)Z_t(1+n)N_t$,

$$\frac{K_{t+1}}{Z_t N_t}\frac{Z_{t+1}N_{t+1}}{Z_{t+1}N_{t+1}} = \frac{K_{t+1}}{Z_{t+1}N_{t+1}} \times (1+z+n)$$

3. Rewrite the capital accumulation equation in terms of capital per effective units of labor ($\hat{k}_t$, $\hat{k}_{t+1}$).

   Divide both sides of the capital accumulation equation by $Z_t N_t$:

$$\frac{K_{t+1}}{Z_t N_t} = sAF\left(\frac{K_t}{Z_t N_t}, 1\right) + (1-\delta)K_t Z_t N_t$$

   From the Part 3 of this question, we can substitute:

$$\frac{K_{t+1}}{Z_{t+1}N_{t+1}} \times (1+z+n) = sAF\left(\frac{K_t}{Z_t N_t}, 1\right) + (1-\delta)K_t Z_t N_t$$

   Which can be rewritten:

$$\hat{k}_{t+1} \times (1+z+n) = sAf(\hat{k}_t) + (1-\delta)\hat{k}_t$$

4. What is the amount of break even investment in this economy? How is this different than break even investment in the basic Solow model?

   In the steady state:

$$\hat{k}^* \times (1+z+n) = sAf(\hat{k}^*) + (1-\delta)\hat{k}^*$$

$$\hat{k}^* \times (z+n+\delta) = sAf(\hat{k}^*)$$

   The right hand side of this equation is total investment, so break even investment is $\hat{k}^* \times (z+n+\delta)$. This is different than break even investment in the basic Solow model: $k^*\delta$. This is because in the basic Solow model, savings only has to cover the depreciation of capital per worker (rather than capital per units of effective labor) because labor does not grow and there is no labor-augmenting technology (or growth in labor-augmenting technology).

5. Solve for the steady state value of $\hat{k}$, $\hat{k}^*$, in terms of $s$, $A$, $n$, $z$, $\delta$.

$$\hat{k}^* = \frac{1}{z+n+\delta} sAf(\hat{k}^*)$$

$$\hat{k}^* = \frac{1}{z+n+\delta} sA(\hat{k}^*)^\alpha$$

$$(\hat{k}^*)^{1-\alpha} = \frac{1}{z+n+\delta} sA$$

$$(\hat{k}^*) = \left[\frac{sA}{z+n+\delta}\right]^{\frac{1}{1-\alpha}}$$

6. How does $\hat{c}^*$, the steady state value of consumption per unit of effective labor, change when we increase the savings rate, $s$?

   The effect of an increase in the savings rate on the consumption per unit of effective labor is ambiguous. Saving more will increase $\hat{k}^*$ which will increase $\hat{y}^*$. The increase in output has the effect of increasing consumption. Increasing the savings rate also increases $\hat{\imath}^*$, which has the effect of decreasing consumption. Because we do not know which of these effects is larger, we don't know what the total effect on consumption will be.