Problem Set 2

Due: January 7, 2026

I expect you to prepare well-written answers to the problem set—not just computer outputs or numbers. Please also upload your code that replicates your results through Waseda Moodle. For quantitative questions, you may use any statistical software as long as you can submit all replication materials.

You may use generative AI tools (such as ChatGPT, Claude, or Gemini) to support your learning—for example, to check R syntax or clarify statistical concepts. However, you may not upload or quote this entire problem set to any AI system, nor may you submit AI-generated work as your own. All submitted materials must reflect your own reasoning, understanding, and interpretation.

If your output appears inconsistent with your demonstrated understanding, I reserve the right to conduct an individual oral interview to confirm authorship and comprehension of your code, analysis, and written explanations. Failure to demonstrate genuine understanding may be treated as an academic integrity violation.

1. Check whether the following VAR models satisfy the stationary condition or not. (Hint: You can express any VAR(p) as VAR(1).)

   (a)

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 \\ 1.2 & 0.2 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{y,t} \\ \epsilon_{z,t} \end{bmatrix} \tag{1}$$

   (b)

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.2 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} y_{t-2} \\ z_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_{y,t} \\ \epsilon_{z,t} \end{bmatrix} \tag{2}$$

2. Download the following time-series data from FRED (https://fred.stlouisfed.org).[1]

   - Real GDP (`GDPC1`)
   - GDP Deflator (`GDPDEF`)
   - Producer Price Index of Commodities (`PPIACO`)
   - Federal Funds Rate (`FEDFUNDS`)
   - M2 Money Stock (`M2SL`)

   FRED series ID are in perentheses. Except for the Fed funds rate, use them in the log level.

   (a) It is always a good idea to visually check obtained data series. Plot the raw data.

[1] You may find the R package `fredr` useful for downloading the data.

(b) Using this data set, estimate a 5-variable VAR(p) (no need to report the reduced-form coefficients). What would be your choice of p?

(c) Estimate dynamic responses of these variables to the contractionary monetary policy shock, which is defined as a positive innovation to the Fed funds rate in the structural form. Use the Cholesky decomposition together with the following identifying assumption:

- Real GDP, GDP deflator, and the producer price index of commodities do not react to changes in the Fed funds rate simultaneously.
- M2 is the only variable that responds to changes in the Fed funds rate contemporaneously.

Present impulse response functions for 12 quarters together with the 95% confidence bands.

(d) Explain the estimated responses. Do they make sense?

3. Download unemployment rate (`UNRATE`) from FRED as well. Estimate a bivariate VAR model, consisting of real GDP growth rate and unemployment rate. By imposing the long-run restriction in the spirit of Blanchard and Quah (1989), estimate the responses of GDP growth rate and unemployment rate to the technology shock.

4. Considere a class of GARCH models.

(a) Pick a variant of GARCH models (other than the ordinary GARCH model, such as, AGARCH, BGARCH, CGARCH, etc...).[2] Briefly describe details of your pick.

(b) Download daily S&P 500 index (`SP500`) data for the last 5 years from FRED (including the pandemic period). Compute the log return as

$$r_t = \log(P_t/P_{t-1}) \times 100, \tag{3}$$

where P_t is the S&P 500 index at time t. Estimate your choice of GARCH-type model to account for volatility of the daily log returns. Compare performance of your model with GARCH(1,1).

5. Dowload dairy NASDAQ index (`NASDAQCOM`) from FRED for the last 5 years.

(a) Peform the Engle-Granger cointegration test for S&P 500 and NASDAQ index in the log level. What do you find?

(b) Repeat this for the observations in the last 1 year. Does your conclusion change? What do you conclude?

[2] Bollerslev (2009) provides a non-exhaustive list of various types of GARCH models. The working paper version is available at http://public.econ.duke.edu/~boller/Papers/glossary_arch.pdf. Teräsvirta (2009) surveys univariate GARCH models, which is available at https://link.springer.com/book/10.1007/978-3-540-71297-8

References

[1] Bollerslev, T. (2009) "Glossary to ARCH (GARCH)." In *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle* (eds. Tim Bollerslev, Jeffrey R. Russell and Mark W. Watson), Chapter 8, pp.137-163. Oxford University Press.

[2] Teräsvirta T. (2009) "An Introduction to Univariate GARCH Models." In *Handbook of Financial Time Series* (eds. T.G. Andersen, R.A. Davis, J.-P. Kreiß, T. Mikosch), pp.17-42. Springer.