Stat 3202 Lab 6

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1b.

```
set.seed(123)
mu <- 10
sigma <- 3
n <- 5
alpha <- 0.01
num_samples <- 10000
true_mu <- 10
compute_coverage <- function(mu, sigma, n, alpha, true_mu) {
  samples <- rnorm(n, mean = mu, sd = sigma)
  xbar <- mean(samples)
  se <- sigma / sqrt(n)
  zstar <- qnorm(1 - alpha / 2)
  LL <- xbar - zstar * se
  UL <- xbar + zstar * se
  return(true_mu >= LL & true_mu <= UL)
}
coverage_results <- replicate(num_samples, compute_coverage(mu, sigma, n, alpha, true_mu))
coverage_rate <- mean(coverage_results)
coverage_rate
```

```
## [1] 0.9905
```

1c.

```
chooseCRANmirror(graphics = FALSE, ind = 1)
install.packages("DescTools")
```

```
##
## The downloaded binary packages are in
##  /var/folders/66/_4vz9_y57nx1wgym4pn0zxcc0000gn/T//RtmpTBO2Av/downloaded_packages
```

```
library(DescTools)

oatmeal <- c(790,794,789,801,795,789,795,
797,796,791,801,795,790,782,
799,793,793,798,797,796)
result <- ZTest(oatmeal, sd_pop = 5, conf.level = 0.99)
conf_interval <- result$conf.int
conf_interval
```

```
## [1] 791.1701 796.9299
## attr(,"conf.level")
## [1] 0.99
```

1d.

```
chooseCRANmirror(graphics = FALSE, ind = 1)
install.packages("DescTools")
```

```
##
## The downloaded binary packages are in
##  /var/folders/66/_4vz9_y57nx1wgym4pn0zxcc0000gn/T//RtmpTBO2Av/downloaded_packages
```

```
library(DescTools)

set.seed(123)
n <- 50

compute_coverage_ztest <- function(mu, sigma, n, alpha, true_mu) {
  samples <- rnorm(n, mean = mu, sd = sigma)
  result <- ZTest(samples, sd_pop = sigma, conf.level = 1 - alpha)
  LL <- result$conf.int[1]
  UL <- result$conf.int[2]
  return(true_mu >= LL & true_mu <= UL)
}

coverage_results_ztest <- replicate(num_samples, compute_coverage_ztest(mu, sigma, n, alpha, true_mu))
coverage_rate_ztest <- mean(coverage_results_ztest)
coverage_rate_ztest
```

```
## [1] 0.9922
```

Increasing n makes the confidence interval more narrow because n is in the denominator of the standard error. When you increase the denominator the fraction becomes smaller. Therefore, the coverage rate gets closer to (1-α)%.

2.

```
chooseCRANmirror(graphics = FALSE, ind = 1)
install.packages("DescTools")
```

```
##
## The downloaded binary packages are in
##  /var/folders/66/_4vz9_y57nx1wgym4pn0zxcc0000gn/T//RtmpTBO2Av/downloaded_packages
```

```
library(DescTools)

explore_coverage <- function(mu, sigma, n, alpha, num_samples, true_mu) {
  coverage_results <- replicate(num_samples, compute_coverage_ztest(mu, sigma, n, alpha, true_mu))
  return(mean(coverage_results))
```

```
}

mu_values <- c(5, 10, 15)
sigma_values <- c(1, 3, 5)
n_values <- c(5, 50, 100)
alpha_values <- c(0.01, 0.05, 0.10)

results <- list()

for (mu_val in mu_values) {
  for (sigma_val in sigma_values) {
    for (n_val in n_values) {
      for (alpha_val in alpha_values) {
        coverage_rate <- explore_coverage(mu_val, sigma_val, n_val, alpha_val, num_samples, true_mu)
        results <- rbind(results, data.frame(mu = mu_val, sigma = sigma_val, n = n_val, alpha = alpha_va
      }
    }
  }
}

results
```

```
##    mu sigma   n alpha coverage_rate
## 1   5     1   5  0.01        0.0000
## 2   5     1   5  0.05        0.0000
## 3   5     1   5  0.10        0.0000
## 4   5     1  50  0.01        0.0000
## 5   5     1  50  0.05        0.0000
## 6   5     1  50  0.10        0.0000
## 7   5     1 100  0.01        0.0000
## 8   5     1 100  0.05        0.0000
## 9   5     1 100  0.10        0.0000
## 10  5     3   5  0.01        0.1232
## 11  5     3   5  0.05        0.0383
## 12  5     3   5  0.10        0.0181
## 13  5     3  50  0.01        0.0000
## 14  5     3  50  0.05        0.0000
## 15  5     3  50  0.10        0.0000
## 16  5     3 100  0.01        0.0000
## 17  5     3 100  0.05        0.0000
## 18  5     3 100  0.10        0.0000
## 19  5     5   5  0.01        0.6362
## 20  5     5   5  0.05        0.3925
## 21  5     5   5  0.10        0.2727
## 22  5     5  50  0.01        0.0000
## 23  5     5  50  0.05        0.0000
## 24  5     5  50  0.10        0.0000
## 25  5     5 100  0.01        0.0000
## 26  5     5 100  0.05        0.0000
## 27  5     5 100  0.10        0.0000
## 28 10     1   5  0.01        0.9899
## 29 10     1   5  0.05        0.9511
## 30 10     1   5  0.10        0.9026
```

```
## 31 10     1  50  0.01        0.9908
## 32 10     1  50  0.05        0.9516
## 33 10     1  50  0.10        0.9006
## 34 10     1 100  0.01        0.9899
## 35 10     1 100  0.05        0.9515
## 36 10     1 100  0.10        0.9012
## 37 10     3   5  0.01        0.9905
## 38 10     3   5  0.05        0.9516
## 39 10     3   5  0.10        0.9036
## 40 10     3  50  0.01        0.9878
## 41 10     3  50  0.05        0.9508
## 42 10     3  50  0.10        0.8931
## 43 10     3 100  0.01        0.9905
## 44 10     3 100  0.05        0.9491
## 45 10     3 100  0.10        0.9016
## 46 10     5   5  0.01        0.9893
## 47 10     5   5  0.05        0.9518
## 48 10     5   5  0.10        0.8949
## 49 10     5  50  0.01        0.9903
## 50 10     5  50  0.05        0.9467
## 51 10     5  50  0.10        0.8978
## 52 10     5 100  0.01        0.9894
## 53 10     5 100  0.05        0.9505
## 54 10     5 100  0.10        0.8921
## 55 15     1   5  0.01        0.0000
## 56 15     1   5  0.05        0.0000
## 57 15     1   5  0.10        0.0000
## 58 15     1  50  0.01        0.0000
## 59 15     1  50  0.05        0.0000
## 60 15     1  50  0.10        0.0000
## 61 15     1 100  0.01        0.0000
## 62 15     1 100  0.05        0.0000
## 63 15     1 100  0.10        0.0000
## 64 15     3   5  0.01        0.1228
## 65 15     3   5  0.05        0.0384
## 66 15     3   5  0.10        0.0204
## 67 15     3  50  0.01        0.0000
## 68 15     3  50  0.05        0.0000
## 69 15     3  50  0.10        0.0000
## 70 15     3 100  0.01        0.0000
## 71 15     3 100  0.05        0.0000
## 72 15     3 100  0.10        0.0000
## 73 15     5   5  0.01        0.6329
## 74 15     5   5  0.05        0.3926
## 75 15     5   5  0.10        0.2672
## 76 15     5  50  0.01        0.0000
## 77 15     5  50  0.05        0.0000
## 78 15     5  50  0.10        0.0000
## 79 15     5 100  0.01        0.0000
## 80 15     5 100  0.05        0.0000
## 81 15     5 100  0.10        0.0000
```

Changing μ and σ does not affect the coverage rate. Increasing α results in a smaller confidence level, so the coverage rate is lower. On the other hand, decreasing α results in a higher confidence level, so the coverage

rate is higher.