INTRODUCTION TO PARALLEL COMPUTING IN R

prof. dr. Janez Povh dr. Tomas Martinović (Rcpp) EuroHPC Competence Center, September 25, 2024

Outline/next



Introduction to HPC

Introduction to R

• Parallel R within one node

Parallelization with Rmpi

Introduction to HPC

What is HPC

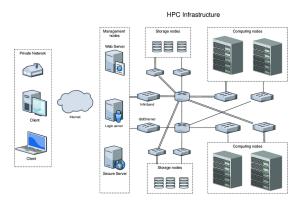




Source: https://www.vyzkumne-infrastruktury.cz/en/2022/06/lumi-supercomputer-has-been-inaugurated/

What is HPC

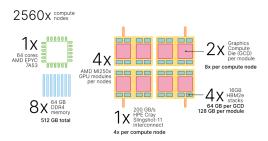




Source: Reghenzani, F. et al, IEEE Access, 8, 208566-208582.

Node structure

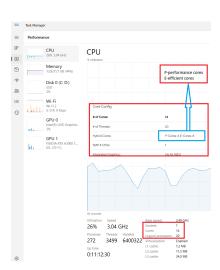




Source: https://docs.lumi-supercomputer.eu/hardware/lumig/

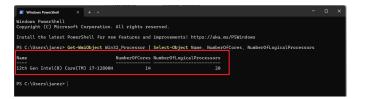
How does my computer look like





How does my computer looks like





How does Barbora cluster look like



192 standard computing nodes (2x18 cores), 8 GPU nodes, 1 fat node (6TB RAM)





Some computations are (very) extensive:

• cpu-extensive: take too much cpu time



Some computations are (very) extensive:

- cpu-extensive: take too much cpu time
- memory-extensive: Take too much memory



Some computations are (very) extensive:

- cpu-extensive: take too much cpu time
- memory-extensive: Take too much memory
- I/O-extensive: Take too much time to read/write from disk



Some computations are (very) extensive:

- cpu-extensive: take too much cpu time
- memory-extensive: Take too much memory
- I/O-extensive: Take too much time to read/write from disk
- network-extensive: Take too much time to transfer over the network.

User experience



```
PuTTY login.vega.izum.si - PuTTY
                                                                          X
 Using username "jpovh".
 Authenticating with public key "rsa-key-20220609"
 Further authentication required
 Keyboard-interactive authentication prompts from server:
 Verification code:
```

How to access HPC



HPCs are easily available.



Euro CC2



• Network of EU Competence centers for HPC.

Euro CC2



- Network of EU Competence centers for HPC.
- All members of EuroHPC JU involved.



- Network of EU Competence centers for HPC.
- All members of EuroHPC JU involved.
- Training, support for industry, talent attraction,...

Outline/next



Introduction to HPC

Introduction to R

• Parallel R within one node

Parallelization with Rmpi



What is R



- Software for Statistical Data Analysis
- Based on S
- Cores routines are written and Fortran
- Programming Environment
- Interpreted Language
- Data Storage, Analysis, Visualization
- Free and Open Source Software

How to obtain R



- R current version 4.4.1 (released June 2024).
- http://cran.r-project.org
- Binary/Windows executable code

Pros and Cons



Pros:

- Free and Open Source
- Strong User Community
- Highly extensible, flexible
- Implementation of high-end statistical methods
- Flexible graphics and intelligent defaults

Pros and Cons



Pros:

- Free and Open Source
- Strong User Community
- Highly extensible, flexible
- Implementation of high-end statistical methods
- Flexible graphics and intelligent defaults

Cons

- Steep learning curve
- Slow for large datasets

Data types



- R Supports virtually any type of data
- Numbers, characters, logicals (TRUE/ FALSE)
- Arrays of virtually unlimited sizes
- Simplest: Vectors and Matrices
- Lists: Can Contain mixed type variables
- Data Frame: Rectangular Data Set
- Datasets in tidy format.

Data structures in R



Linear

- vectors (all same type)
- lists (mixed types)

Data structures in R



Linear

- vectors (all same type)
- lists (mixed types)

Rectangular

- data frame
- matrix

Running R



• I recommend RStudio, an IDE for R.

Running R

- I recommend RStudio, an IDE for R.
- It is available as RStudio Desktop and RStudio Server, which runs on a remote server and allows accessing RStudio using a web browser.



Figure 1: https://rstudio.com/products/rstudio/download/

RStudio on IT41



Run RStudio on VM.

RStudio on IT41

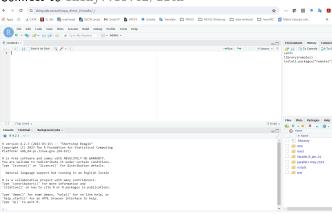


- Run RStudio on VM.
- Connect to shiny.vsb.cz/auth

RStudio on IT41



- Run RStudio on VM.
- Connect to shiny.vsb.cz/auth



Clone the data



Clone project from GITHUB

https://github.com/janezpovh/parallel-R-sep-24



Creating the first script file



Create and save simple data file

```
N = 1000;
Data = data.frame(group = character(N), ints = numeric(N), reals = numeric(N))
Data$group = sample (c("a", "b", "c"), 1000, replace = TRUE);
Data $ ints = rbinom (N, 10, 0.5);
Data$reals=rnorm(N):
head (Data)
Data
write.table(Data, file='data/Data_Ex_1.txt', append = FALSE, dec = ".",col.names =
 TRUE)
1s()
rm(list = ls())
```

Load and analyse the data



Load data

```
Data_read<-read.table(file='data/Data_Ex_1.txt', header = TRUE)</pre>
# first few rows
head (Data read)
#10 th row
Data_read[10,]
# column group
Data_read$group
Data_read[,1]
```

Analyse the data

Simple analysis

```
# compute means and counts by groups
group count_ints mean_ints
          5 014837
          5.032544
c | 325 | 4.990769
# primitive solution
Group_lev=levels(Data_read$group)
Tab_summary = data.frame(group = character(3),count_ints = integer(3),mean_ints = numeric(3))
Tab_summary $group <- Group_lev
for (i in c(1:3)){
  sub_data = subset(Data_read,group == Group_lev[i])
  Tab_summary $count_ints[i] <- nrow (sub_data)
  Tab_summary $mean_ints[i] <- mean (sub_data$ints)
```

Analyze the data by split, aggregate



split, aggregate

```
s <- split (Data_read, Data_read$group)
Tab_summary1 <- cbind (aggregate (ints~group, data = Data_read, FUN=length), aggregate (ints~
 group,data = Data_read,FUN=mean))
Tab_summary1 <- Tab_summary1[,-3]
```

Outline/next



Introduction to HPC

Introduction to R

• Parallel R within one node

Parallelization with Rmpi

Parallel R within one node

What system do I have



How many cores

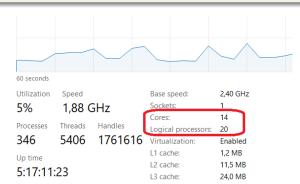
library(parallel)
detectCores()
> detectCores()
[1] 20

What system do I have



How many cores

library(parallel)
detectCores()
> detectCores()
[1] 20



Speed up without paralelization - vectorization

Vectorization:

- a programming technique used to avoid explicit loops in order to improve the performance and readability of code.
- a function can be called on a vector and operate on each element, rather than requiring a loop across elements
- Example Use sum() or mean() instead of loop variant.

Speed up without paralelization - vectorization

Vectorization:

- vectorization is faster: R is an interpreted language and not a compiled one:
 - R needs to repeatedly interpret what your code means for each iteration of the loop;
 - each iteration of the for loop requires indexing into x using the subset function.
- Vectorised functions loop over and repeatedly indexes x, but this is done in the compiled language C and has been optimized to take advantage of the fact that the elements of a vector are contiguous in memory.

Benchmarking



Let us benchmark a loop and vectorised function:

Benchmarking

```
# Library for timing comparison
library(microbenchmark)

x <- 1:100000
microbenchmark(a = sum(x),
b = {
    s0 <- 0
    for (i in seq_along(x)) {
        s0 <- s0 + x[i]
    }
})</pre>
```

apply, lapply, sapply



These are not typical vectorised functions - they are "loop-hiding", since they they contain implicit loops. apply actually has a loop in its R code. The others do drop down to C for their loops, however, at each step in the C loop, they evaluate the R function passed in. This is what makes them not vectorized, as a true vectorized function performs its loop in C and uses C compiled functions inside that loop.

apply, lapply, sapply

```
apply (X, MARGIN, FUN)
-x: an array or matrix
-MARGIN=1: the manipulation is performed on rows
-MARGIN=2: the manipulation is performed on columns
-MARGIN=c(1,2): the manipulation is performed on rows and columns
-FUN: tells which function to apply. Built functions like mean, median, sum, min, max and
user-defined functions can be applied
```

apply

For data constructed above (Data_read) compute row and columns means using apply



For data constructed above (Data_read) compute row and columns means using apply

```
Data_read<-read.table(file='data/Data_Ex_1.txt',header = TRUE)

Data_col_means_1 <- colMeans(Data_read[,-1])
Data_col_means_2 <- apply(Data_read[,-1],2,FUN =mean)

Data_row_means_1 <- rowMeans(Data_read[,-1])
Data_row_means_2 <- apply(Data_read[,-1],1,FUN =mean)

Data_both_squares <- apply(Data_read[,-1],c(1,2),FUN = function(x) return(x^2))</pre>
```

lapply



- lapply function takes list, vector or data frame as input and returns only list as output
- sapply function takes list, vector or data frame as input. It is similar to lapply function but returns only vector as output.

For data constructed above (Data_read) compute row and columns sums using lapply



- lapply function takes list, vector or data frame as input and returns only list as output
- sapply function takes list, vector or data frame as input. It is similar to lapply function but returns only vector as output.

For data constructed above (Data_read) compute row and columns sums using lapply

lapply

```
Data_col_sums_1 <- apply(Data_read[,-1],2,FUN =sum)
Data_col_sums_2 <- lapply(Data_read[,-1],FUN =sum)
typeof (Data_col_sums_1)
typeof (Data col sums 2)
Data_abs <- lapply(Data_read[,-1],FUN =abs)
Data_sq <- lapply(Data_read[,-1],FUN = function(x) {x^2})
typeof (Data_abs)
length (Data abs)
```

sapply



For data constructed above (Data_read) compute row and columns sums using sapply

sapply



For data constructed above (Data_read) compute row and columns sums using sapply

```
Data_col_sums_1 <- apply(Data_read[,-1],2,FUN = sum)
Data_col_sums_2 <- lapply(Data_read[,-1],FUN = sum)
Data_col_sums_3 <- sapply(Data_read[,-1],FUN = sum)

typeof(Data_col_sums_1)
typeof(Data_col_sums_2)
typeof(Data_col_sums_3)

Data_col_sums_4 <- lapply(list(Data_read$ints,Data_read$reals),FUN = sum)
Data_col_sums_5 <- sapply(list(Data_read$ints,Data_read$reals),FUN = sum)
Data_col_len_1 <- lapply(list(Data_read$ints,Data_read$reals),FUN = length)
Data_col_len_2 <- sapply(list(Data_read$ints,Data_read$reals),FUN = length)</pre>
```

for loop



Let us compute sums of all elements of K random matrices of order $N \times N$

```
for
```

```
N=1000
K=60
set.seed(2021)
sum_rand=rep(0,K-1);
tic()
time_for_sys=system.time({
    for (i in c(1:K)){
        A=rand(N,N)
        sum_rand[i]=sum(A)
    }
})
time_for=toc()
```

foreach do loop



Let us compute sums of all elements of K random matrices of order $N \times N$

```
for

set.seed(2021)
sum_rand=rep(0,K-1);
tic()
time_foreach_sys=system.time({
    foreach (i = c(1:K)) %do% {
        A=rand(N,N)
        sum_rand[i]=sum(A)
    }
})
time_foreach=toc()
```

Libraries parallel, doParallel



- parallel package comes in the base R installation
- parallel works great for any task that you pass to the apply family (e.g., lapply becomes parLapply).
- doParallel package works great when you want to use parallel variant of for-loops (foreach -do), and might be a little easier to use.

doParallel



 This library is meant for use with foreach, which lets you use a particular type of for-loop, that looks like:

```
foreach(i=list_of_elements) %do% {thing with i}.
```

doParallel



 This library is meant for use with foreach, which lets you use a particular type of for-loop, that looks like:

```
foreach(i=list_of_elements) %do% {thing with i}.
```

• Foreach allows this to be parallelized, using dopar:

```
foreach(i=listOfThings) %dopar% {thing with i}.
```



doParallel

 This library is meant for use with foreach, which lets you use a particular type of for-loop, that looks like:

```
foreach(i=list_of_elements) %do% {thing with i}.
```

Foreach allows this to be parallelized, using dopar:

```
foreach(i=listOfThings) %dopar% {thing with i}.
```

- Note that: parallelization with dopar depends on which backend you use.
 - doParallel is one such backend it tells foreach to use parallel.
 - There are others: doFuture, doMPI (another parallel backend, using message passing interface), doSnow (another backend, using the snow package for creating parallel processes),...
 - By default, doParallel uses multicore functionality on Unix-like systems and snow functionality on Windows.

Parallel foreach-dopar loop



Let us compute sums of all elements of K random matrices of order $N \times N$ using foreach ...dopar from foreach and doParallel

Parallel foreach-dopar loop



Let us compute sums of all elements of K random matrices of order $N \times N$ using foreach ...dopar from foreach and doParallel

Do you observe any difference?

Creating cluster with doParallel



• Option 1: (use doParallel)

Create cluster

```
clust <- makeCluster(n_cores-1)
registerDoParallel(clust)
getDoParName()
.
.
stopCluster(clust)
#registerDoSEQ()  # alternative - register sequential mode</pre>
```

Creating cluster with doParallel



• Option 2: use parallel

Create cluster

```
registerDoParallel(cores=n_cores-1)
getDoParName()
.
.
registerDoSEQ() # alternative - register sequential mode
```

- In linux:
 - the first option uses SNOW library and and utilizes parallel::parLapply()
 - the second creates FORK cluster and uses MULTICORE library (it effectively utilizes parallel::mclapply()).
- In windows: both use SNOW. They create PSOCK clusters and basically utilize parallel::parLapply()



Let us compute sums of all elements of K random matrices of order $N \times N$ using foreach ...dopar from foreach, doParallel. Create cluster!

Option 1

```
set.seed (2021)
clust <- makeCluster (n_cores-1)
registerDoParallel(clust) # use multicore, set to the number of our cores - needed for
     foerach dopar
getDoParName()
sum_rand=rep(0,K-1);
tic()
time_foreachdopar_1_sys = system.time({
  print ("for each-dopar (cluster allocated)")
  for each (i = c(1:K)) %dopar% {
    library (pracma)
    A = rand(N)
    sum rand[i] = sum(A)
}}
time foreach dopar 1=toc()
stopCluster(clust)
```

Parallel foreach dopar loop - option 2



Let us compute sums of all elements of K random matrices of order $N \times N$ using foreach ...dopar from foreach, doParallel. Create cluster!

```
Option 2
```

```
set.seed (2021)
registerDoParallel(n_cores-1) # use multicore, set to the number of our cores - needed
     for foerach dopar
getDoParName()
sum_rand = rep(0, K-1);
tic()
time foreachdopar 2 sys=system.time({
  print("for each-dopar (cluster allocated)")
  for each (i = c(1:K)) %dopar% {
    library (pracma)
    A=rand(N)
    sum_rand[i] = sum(A)
}}
time_foreach_dopar_1=toc()
registerDoSEQ() #this registers sequential mode - equivalent
```



Timings

Timings

	user.self	sys.self	elapsed
time_for_sys	4.16	0.50	4.93
time_for_each_sys	4.21	0.37	4.92
time_for_each_dopar_sys	4.11	0.55	4.98
time_for_each_dopar_1_sys	0.13	0.01	1.90
time for each dopar 2 sys	0.11	0.00	1.80

Library parallel



- encapsulates existing libraries multicore, snow
- multicore functionality supports multiple workers only on those operating systems that support the **fork** system call - this excludes Windows.
- two ways of parallelization:
 - The socket approach: launches a new version of R on each core via networking (e.g. the same as if you connected to a remote server), but the connection is happening all on your own computer.
 - pros: (i) Works on any system (including Windows); (ii) Each process on each node is unique so it can't cross-contaminate.
 - cons: (i) Each process is unique so it will be slower (ii) Things such as package loading need to be done in each process separately. Variables defined on your main version of R don't exist on each core unless explicitly placed there. (iii) More complicated to implement.
 - use parLapply, parSapply

Library parallel



- The forking approach copies the entire current version of R and moves it to a new core.
 - (i) Faster than sockets. (ii) Because it copies the existing version of R, your entire workspace exists in each process. (iii) Easy to implement.
 - Cons (i) Only works on POSIX systems (Mac, Linux, Unix, BSD) and not Windows. (ii) it can cause issues specifically with random number generation or when running in a GUI (such as RStudio). This doesn't come up often.
- use mclapply

Parallel versions of lapply



By using library parallel and parSapply, mclapply compute sums of all elements of K random matrices of order $N \times N$. Create cluster!

parallel versions of apply

```
mat_sum<-function(x){
   library(pracma)
   A=rand(x)
   return(sum(A))
}

time_lapply<-system.time({
   set.seed(2021)
   sum_rand_lapply=lapply(rep(N,K),FUN=mat_sum)
})

time_sapply<-system.time({
   set.seed(2021)
   sum_rand_sapply=sapply(rep(N,K),FUN=mat_sum)
})</pre>
```

Parallel versions of lapply



parallel versions of apply

```
#forking
time_mcLapply <- system.time ({
  set . seed (2021)
  sum_rand_mcLapply = mclapply (X = rep(N,K), FUN = mat_sum, mc.cores = n_cores)
3)
# socketing
clust <- makeCluster(n_cores, type="PSOCK")
time_parLapply <- system.time({
  set.seed(2021)
  sum_rand_parLapply = parLapply (clust , rep(N,K), fun = mat_sum)
stopCluster(clust)
clust <- makeCluster(n cores, type="PSOCK")
time_parSapply <- system.time({
  set . seed (2021)
  sum_rand_parSapply = parSapply (clust , rep(N , K) , FUN = mat_sum)
3)
stopCluster(clust)
```

Parallel versions of lapply



parallel versions of apply

```
times_apply <-rbind(time_lapply, time_sapply, time_parLapply, time_parSapply, time_
     mcLapply)
    > times_apply[,1:3]
                    user.self
                               sys.self
                                         elapsed
time_lapply
                   1.741
                               0.011
                                         1.751
time_sapply
                   1.726
                               0.007
                                        1.731
time_parLapply
                    0.007
                                      1.940
                               0.004
                               0.005
                                      1.842
time_parSapply
                    0.005
time_mcLapply
                    0.004
                               0.238
                                         1.679
```

Libraries for shared memory parallelization in R

- Parallel for-loop (foreach...dopar). Cluster created by registerDoParallel(N) and registerDoSEQ(). Library foreach, doParalel needed.
- Parallel apply: parLapply, parSapply, mcLapply need library parallel.

Very paralleizable task



Perfectly paralelizable computing task

```
# simple very parallel
library(parallel)
library(tictoc)

f <- function(...) {
    Sys.sleep(1)
    "DONE"
}

tic()
res <- lapply(1:25, f)
ti=toc()
#> 5.025 sec elapsed

tic()
res <- mclapply(1:25, f, mc.cores = 25)
t2=toc()
#> 1.019 sec elapsed
```

Cross-validation of models



- Suppose we want to cretae N GLM models, where training data sets is random sample of size 70%, and test data sets is the remaing set.
- If N=10 and we use 10% training set, we have the usual 10-fold cross-validation.
- We do it on dataset K_data_clean.txt.
- Use R script parallel_cross-validation.R
- Timings with 10 cores

user.s	elf sys.s	elf elaps	sed
time_ser	139.71	0.11	141.89
time_par	0.76	0.13	30.82

Outline/next



Introduction to HPC

Introduction to R

• Parallel R within one node

Parallelization with Rmpi

Parallelization with Rmpi

What is Rmpi



- Rmpi library: Interface for MPI (Message Passing Interface) in R.
- Enables parallel and distributed computing in the R programming language.
- Facilitates communication and coordination between R processes across multiple nodes.
- Particularly useful for parallelizing computationally intensive tasks like simulations or data processing.
- Users can harness the power of parallel computing for improved performance in certain applications.
- Latest version from Dec 2023, see
 https://cran.r-project.org/web/packages/Rmpi/Rmpi.pdf

Few basic command



- Rmpi::mpi.comm.size(0): returns the number of active processes in current computing task/job
- Rmpi::mpi.comm.rank(0): returns the ID of current process (number from {0, 1, 2, ..., size - 1}
- Rmpi::mpi.get.processor.name() returns the name of compute node where the process runs.

Hello word example



Compute smallest eigenvalue of $n \times n$ random symmetric matrices

```
library (Rmpi)
n=30
size <- Rmpi::mpi.comm.size(0)
rank <- Rmpi::mpi.comm.rank(0)
host <- Rmpi::mpi.get.processor.name()
if (rank == 0){
    cat("size ","rank ","host ","max_eigen_value\n")
    cat(size,rank,host,"NaN\n")
} else {
    where=getwd()
    A=matrix(rnorm(n^2),nrow=n)
    A=A+t(A)
    a = max(eigen(A)$values)
    cat(size,rank,host,a,"\n")
}</pre>
```





- Save the scripts from previous slide into separate file, called e.g. Rmpi_master_slave.R
- Create separate .batch file, where the parallelization is defined, e.g.,
 Job_Rmpi_master_slave.sbatch

How to distribute this task across cluster



Compute smallest eigenvalue of n symmetric matrices of size $N \times N$

```
#!/bin/bash
#SBATCH --export=ALL,LD_PRELOAD=
#SBATCH --job-name Rmpi
#SBATCH --partition=rome --mem=24GB --time=02:00
#SBATCH --nodes=8
#SBATCH --ntasks-per-node 48 ## maximum is 48
#SBATCH --output=logs/%x_%j.out

module load OpenMPI/4.1.4-GCC-11.3.0
module load R/4.2.1-foss-2022a
srun Rscript Rmpi_master_slave.R
```

Go to Barbora



- Create directory mkdir /home/rstudio/mnt/
- copy to it files
 Job_Rmpi_master_slave.sbatch, Rmpi_master_slave.R
- mount this directory sshfs -o IdentityFile=/home/rstudio/.ssh/id_ed25519 it4i-jpovh@barbora.it4i.cz:. /home/rstudio/mnt/
- connect to barbora with ssh ssh -i /home/rstudio/.ssh/id_ed25519 it4i-jpovh@barbora.it4i.cz

Connect to Barbora



rstudio@fcfdfeXDF.4s shift -0 Identipylie.homo/rstudio,shift ed25319 itd:jpon@meron.itdi.ci: /home/rstudio/mmt/ rstudio@fcfdfeXDF.4s shi i /home/rstudio/shift ed5538 itd:jpon@meron.itdi.ci cliemt_jbdal_homtwoy.prudic_comfirm: server gave bad signature for ED2519 key 1: incorrect signature Last login: Tue May 28 0811319 204 for 00 551.131-051



...running on Red Hat Enterprise Linux 8.4

Public Service Announcement: Apptainer on the Karolina cluster
Posted: (2024-05-10 10:23:

Apptainer is now a part of the operating system, you do not need to load the module.

\$ apptainer --version apptainer version 1.3.1-1.el8

[it4i-jpovh@login2.barbora ~]\$ ▮

Run

sbatch Job_Rmpi_master_slave.sbatch

Results in log file



```
[1] "size rank host max_eigen_value"
```

- [1] "384 0 cn48 NaN"
- [1] "384 110 cn50 8.25199803297607"
- [1] "384 173 cn52 8.01187455128492"
- [1] "384 68 cn49 8.05800653948316"
- [1] "384 200 cn53 8.81600769867893"
- [1] "384 258 cn54 8.12244071842822"
- [1] "384 332 cn55 7.61927646789373"
- [1] "384 338 cn56 4.9472190383247"

How parallelise without slurm?



Compute smallest eigenvalue of n symmetric matrices of size $N \times N$

```
rm(list=ls())
                         # R code: parallel version
         library (snow)
         library (Rmpi)
         nclus = 6
         cl <-snow::makeMPIcluster(nclus)
        n = 3.0
         N_per_proc = 100
         #x=matrix(runif(n),n,1)
         #x = cbind(1.x)
         min_eig_values=function(n,N){
           a = c()
           for (ind in 1:N){
             A = matrix (rnorm (n^2), nrow = n)
             A = A + t (A)
             a[ind] = max(eigen(A)$values)
           return(a)
         ptim=proc.time()[3]
         b=clusterCall(cl,min_eig_values,n=n,N=N_per_proc)
         b=unlist(b)
         hist(b)
         tim=proc.time()[3]-ptim
         #Rmpi::mpi.quit()
         snow::stopCluster(cl)
Introduction to parallel computing in R
```



- It is a package originally develop by Dirk Eddelbuettel and Romain François
- It aims to ease the extension of R with C++ code.
- It allows to load C++ code in an interactive session.
- It has framework to help when creating package with Rcpp
- Credit: this content was prepared based on materials from dr.
 Tomas Martinovic from Technical university Ostrava.

Create C++ function within R



- Create C++ function within R by using cppFunction()
- Rcpp does all the nasty work (compiling, linking)

EURO

Mandelbrot set

- Two-dimensional set: simple definition, great complexity,
- Definition: The Mandelbrot set is set of all points c in complex plane for which the sequence

$$z_{n+1} = z_n^2 + c$$

does not diverge to infinity when iterated starting at $z_1 = c$.

• Theorem: Complex point c is in the Mandelbrot set if and only if $|z_n| \leq 2$, for all $n \in \mathbb{N}$.

Mandelbrot set



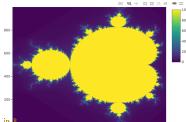
- Two-dimensional set: simple definition, great complexity,
- Definition: The Mandelbrot set is set of all points c in complex plane for which the sequence

$$z_{n+1} = z_n^2 + c$$

does not diverge to infinity when iterated starting at $z_1 = c$.

• Theorem: Complex point c is in the Mandelbrot set if and only if $|z_n| \le 2$, for all $n \in \mathbb{N}$.

Visualization: (maximum number of iterations is 100)



Inner algorithm in R



For any point $c=a+ib\in\mathbb{C}$ we check, it it is in the Mandelbrot set by the following procedure:

```
mandelbrot <- function(c, max_iter = 100) {
   z <- c
   for (i in 1:max_iter-1) {
      z <- z ^ 2 + c
      if (abs(z) > 2) {
        return(i)
      }
   }
   return(max_iter)
}
```

Inner algorithm in C++



C++ code for the inner algorithm using Rcpp:

```
Rcpp::cppFunction(
int Mandel(double real, double im,
int max_iter = 100)
  std::complex<double> c(real, im);
  std::complex<double> z = c;
  for (int i=0; i < max_iter; i++){
    z = z * z + c;
    if (std::abs(z) > 2) {
      return i;
  return max iter:
```

Outer (meta) algorithm



- 1. Input: A discrete rectangular grid of complex points between $c_{\min} = -1.5 i$ and $c_{\max} = 0.5 + i$ with resolution 1000 in each dimension.
- 2. For every point $c \in A$ check, if it is in the Mandelbrot set by the inner algorithm.
- 3. Visualise the results.





- 1. The outer and the inner algorithm written in R
- 2. The outer in R and the inner algorithm in C++
- 3. The outer and the inner algorithm written in C++

Three implementations



- 1. The outer and the inner algorithm written in R
- 2. The outer in R and the inner algorithm in C++
- 3. The outer and the inner algorithm written in C++





- 1. The outer and the inner algorithm written in R
- 2. The outer in R and the inner algorithm in C++
- 3. The outer and the inner algorithm written in C++

```
Unit: milliseconds
expr min lq mean median uq max neval
for_loop 19064.2023 19064.2023 19640.1962 19640.1962 20216.1902 2
for_c 3773.9743 3773.9743 3898.8985 3898.8985 4007.8227 4607.8227 2
c 442.7557 442.7557 463.9417 463.9417 485.1277 485.1277 2
```