Periodicity and topology.

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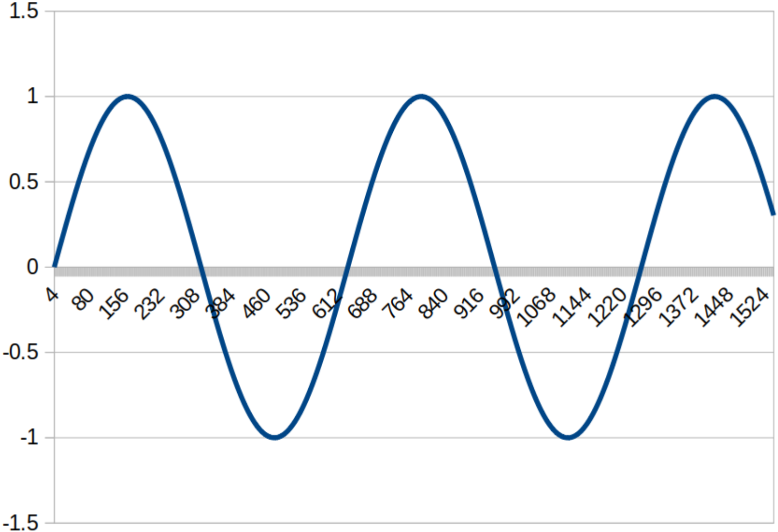
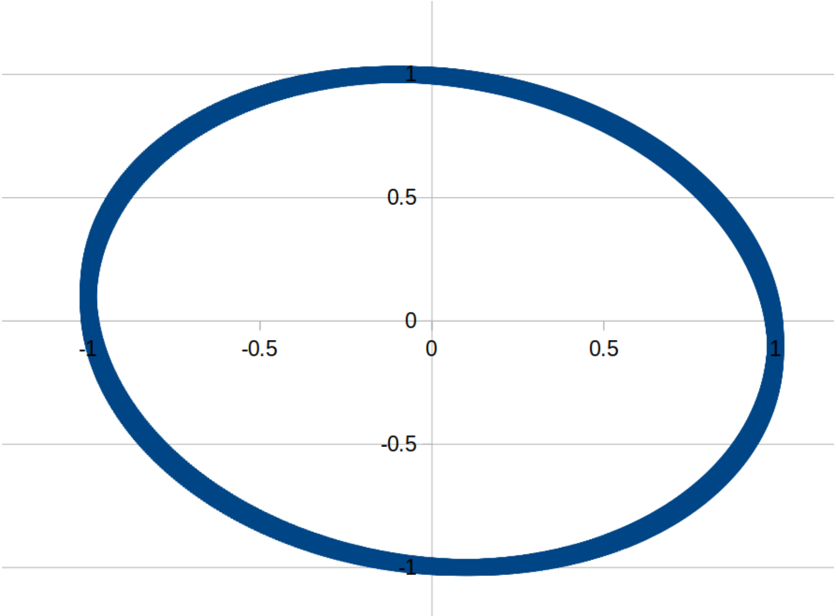
Let us start with something nice, periodic, noiseless



Moving away and coming back.

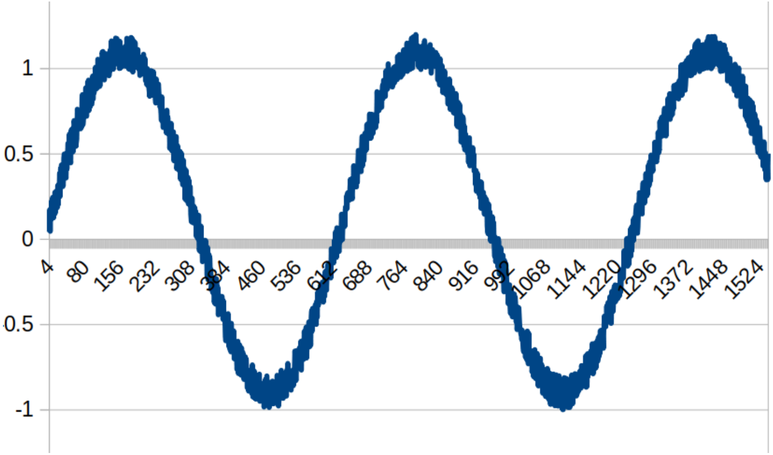
Take its 2-dimensional embedding



Periodicity implies cyclicity

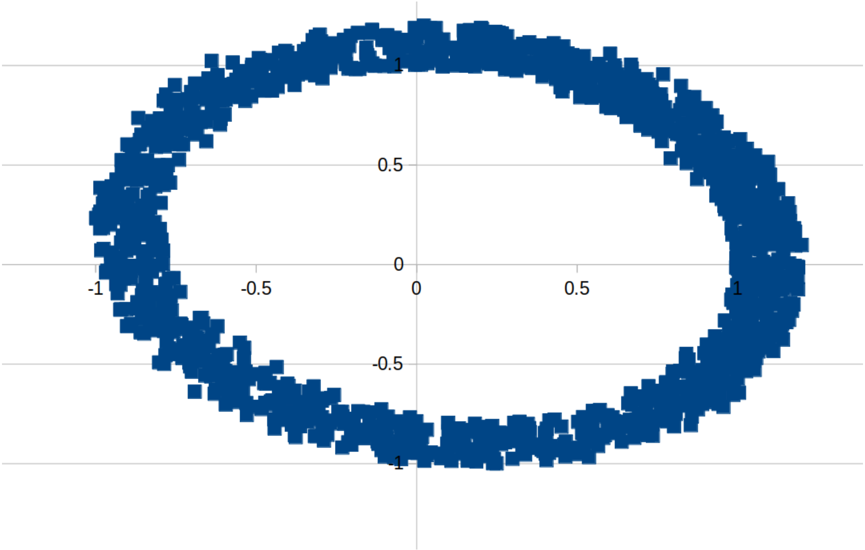
But not vice versa

Let us take something less ideal



Moving away and coming back.

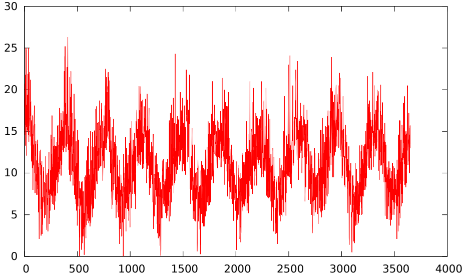
Still cyclic, but noisy



Why it is nice to look at embeddings?

1. The technique generalizes to higher dimensions (image sequence, Jan's example), and even more,
2. Topology gives score on how periodic or aperiodic things are,
3. We can compare how motion of one joint (of half of body) against the motion of another joint (half of the body),
4. We get numerical scale telling us how similar, or different, things are.

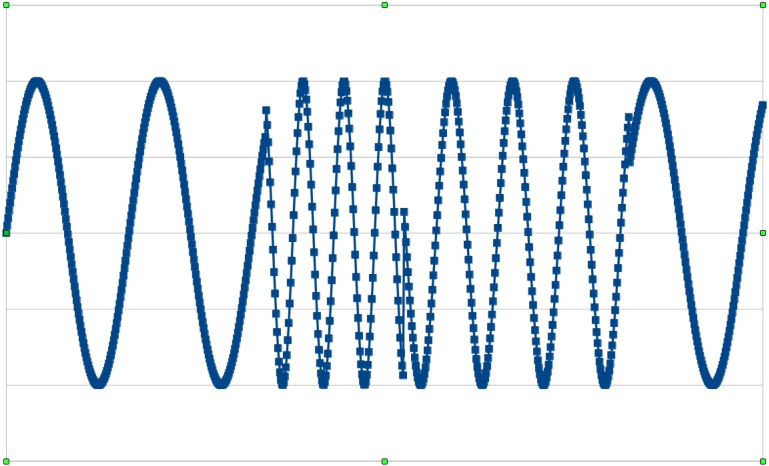
Why not to use spectral methods?



Minimum temperature in Melbourne, 1981-1990.

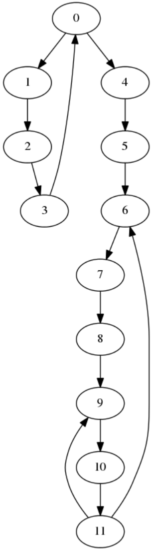
- State of the art spectral methods do not allow us to recover that year has 365 days.
- Topological methods, give an estimate between 300 and 370 days.
- Topology generalize to higher dimensions.

Change of period.

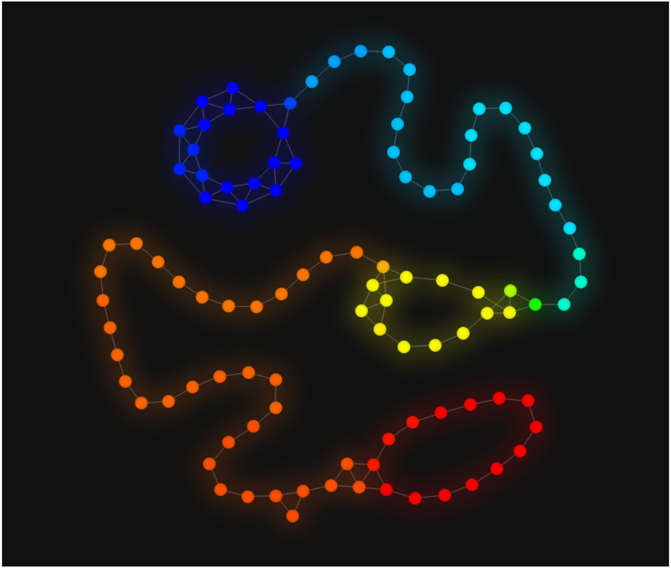


This is where Fourier analysis will fail, but topology still provides important insights.

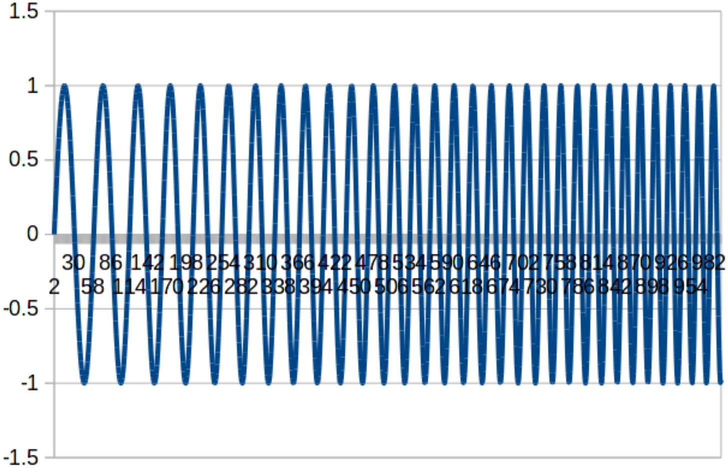
Dynamic graph (50 dimensional embedding, r=5).



Discontinuous change of period (100 dimensional embedding, r=5).

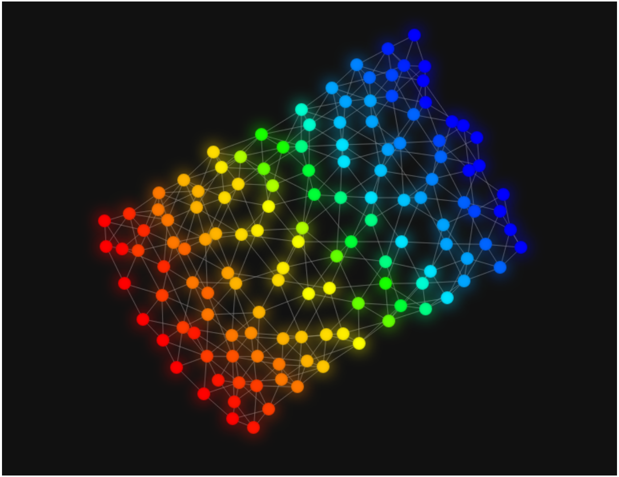


Continuous change of period.



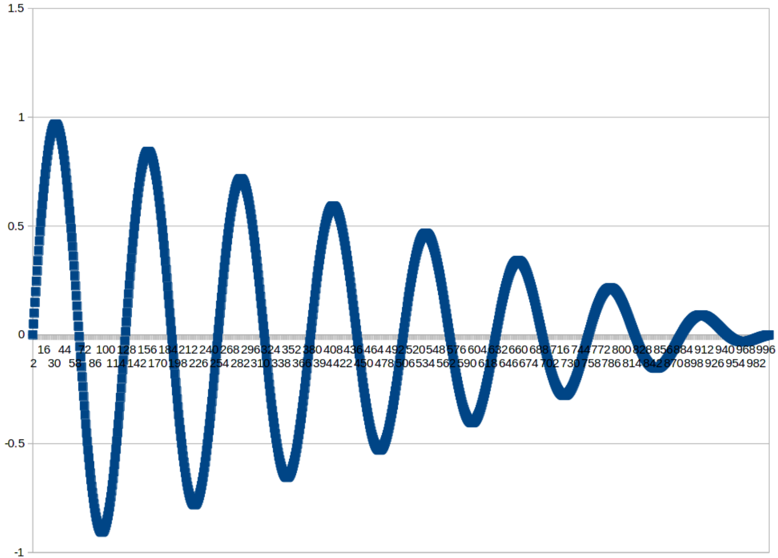
$sin(f(x))$.

Continuous change of period.

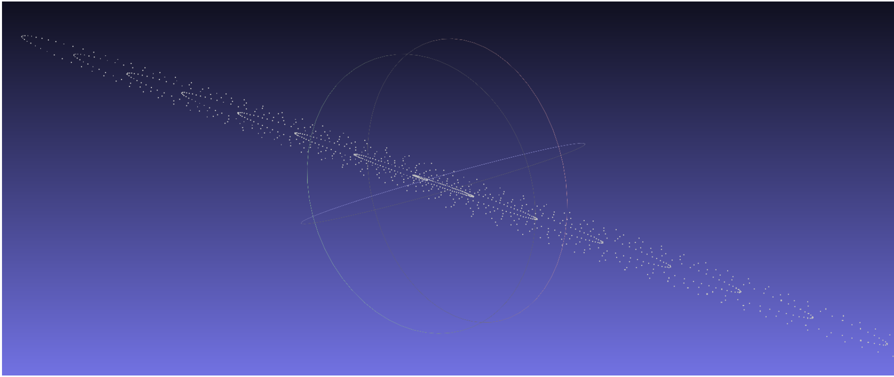


SWE in $\mathbb{R}^{100}$.
(See the animation!!)

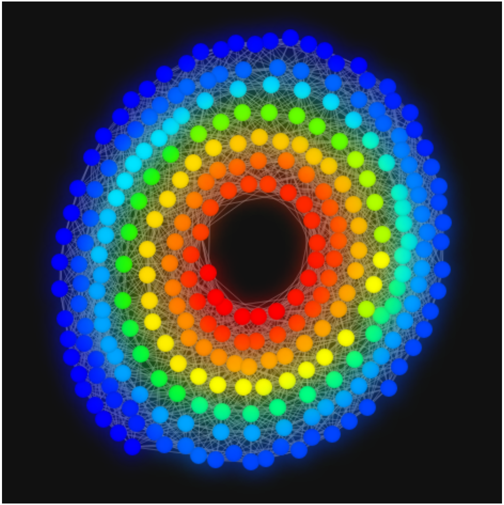
Damping and sliding window embedding.



And Sliding Window Embedding



And Sliding Window Embedding



Why this is relevant for medical images (2/3 d)?

1. Agnostic to dimension,
2. Robust...
3. Tool that quantify how far away we are from perfectly periodic behavior,
4. Or, how two periodic behaviors are far away from each other.

Let us see Jan's example!