

Linear Regression

Newton's three sisters

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- 1 Least Squares Method
- 2 Multiple Regression
- 3 Distribution $\hat{\beta}$
- 4 Distribution of RSS Values
- 5 Hypothesis Testing for $\hat{\beta}_j \neq 0$
- 6 Coefficient of Determination and the Detection of Collinearity
- 7 Confidence and Prediction Intervals

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- Hat matrix defined by $\hat{y} = Hy$

$$\hat{y} = X\hat{\beta} = X(X^T X)^{-1} X^T y = Hy$$

$$H \triangleq X(X^T X)^{-1} X^T$$

- some properties

$$H^2 = X(X^T X)^{-1} X^T \cdot X(X^T X)^{-1} X^T = X(X^T X)^{-1} X^T = H$$

$$(I - H)^2 = I - 2H + H^2 = I - H$$

$$HX = X(X^T X)^{-1} X^T \cdot X = X$$

- RSS defined

$$RSS \triangleq ||y - \hat{y}||^2$$

- using hat matrix

$$\begin{aligned} y - \hat{y} &= y - Hy = (I - H)y = (I - H)(X\beta + \varepsilon) \\ &= (X - HX)\beta + (I - H)\varepsilon = (I - H)\varepsilon \end{aligned}$$

$$RSS \triangleq ||y - \hat{y}||^2 = \{(I - H)\varepsilon\}^T (I - H)\varepsilon = \varepsilon^T (I - H)^2 \varepsilon = \varepsilon^T (I - H)\varepsilon$$

- They are only zeros and ones
- Dimensions of the eigenspaces of H and $I - H$ are both $p + 1$

Proof using $\text{rank}(X) = p + 1$

$$\text{rank}(H) \leq \min\{\text{rank}(X(X^T X)^{-1}), \text{rank}(X)\} \leq \text{rank}(X) = p + 1$$

$$\text{rank}(H) \geq \text{rank}(HX) = \text{rank}(X) = p + 1$$

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Q & A

Thank you :)