# Chap 4: Continuous Random Variable

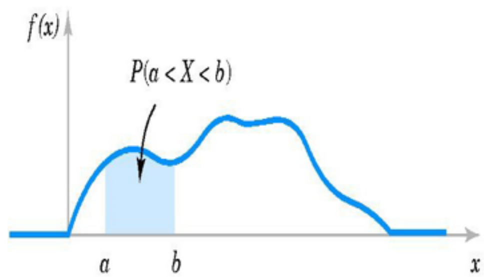
## 1. Continuous Random Variable

Continuous variables produce outcomes that come from a measurement (e.g. your annual salary, or your weight).

## 2. Probability density function f(x)

The **probability density function (pdf)** of a continuous random variable $X$ is a function such that:

1. $f(x) \geq 0, \forall x$
2. $\int_{-\infty}^{+\infty} f(x)dx = 1$
3. $P(a \leq X \leq b) = \int_a^b f(x)dx$




**Example:**

**Let X be a continuous random variable with probability density function defined by**

$$f(x) = \begin{cases} kx, 0 \leq x < 1 \\ k, 1 \leq x \leq 2 \\ 0, \text{otherwise} \end{cases}$$

**What value must k take for this to be a valid density?**

Select one:

- a. 3/2
- b. 2
- c. 2/3
- d. 3

**Suppose that X is a continuous random variable whose probability density function is given by $f(x)=C(4x-2x^2), 0<x<2$ and $f(x)=0$ for other values of $x$. What is the value of C?**

Select one:

- a. 1.520
- b. 0.125
- c. 2.500
- d. 0.375

**The age (in years) of randomly chosen T-shirts in your wardrobe from last summer is distributed according to the density function $f(x)=\frac{10}{9x^2}$ with $1 \leq x \leq 10$. Find the probability that a randomly chosen T-shirt is between 2 and 8 years old**

Select one:

- a. 0.417
- b. 0.3111
- c. 0.3311
- d. 0.1331

**The number of hours you spend looking at YouTube on a typical Saturday night is distributed according to the density function $f(x) = 2xe^{-x^2}$ with $0 \le x$. Find the probability that, on a typical Saturday night, you spend between 0.75 and 1.25 hours watching YouTube.**

Select one:

- a. 0.6315
- b. 0.3602
- c. 0.4523
- d. 0.5102

## 3. Cumulative distribution function F(x)

The **cumulative distribution function (cdf )** of a continuous random variable $X$ is

$$F(x) := \int_{-\infty}^{x} f(t)\,dt$$

for $-\infty < x < +\infty$.

**Example:**

**Suppose the cumulative distribution function of the random variable X is**

$$F(x)=\begin{cases}0 & ,x<0\\ 0.2x & ,0\leq x<4\\ 0.04x+0.64 & ,4\leq x<9\\ 1 & ,x\geq 9\end{cases}$$

**Find the value of P(X>5).**

Select one:

- a. 0.16
- b. 0.32
- c. 0.84
- d. 1.00

**Suppose the cumulative distribution of the random variable X is**

$$F(x)=\begin{cases}0, & x<-2\\ 0.25x+0.5, & -2\leq x<1\\ 0.5x+0.25, & 1\leq x<1.5\\ 1, & 1.5\leq x\end{cases}$$

**Detemine** $P(X<-1\ or\ X>1.6)$

Select one:

- a. 0.29
- b. 0.47
- c. 0.16
- d. 0.25

The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by $f(x) = \frac{10}{x^2}, x > 10$. Determine the value of $F(20)$.

Select one:

- a. 0.6
- b. 0.5
- c. 0.2
- d. 0.9

## 4. Mean, variance, standard deviation

$$Mean = \mu = E(X) = \int_{-\infty}^{+\infty} x * f(x)dx$$

$$Variance = \sigma^2 = V(X) = \int_{-\infty}^{+\infty} x^2 * f(x)dx - \mu^2$$

$$Standard\ deviation = \sigma = \sqrt{V(X)} = \sqrt{\int_{-\infty}^{+\infty} x^2 * f(x)dx - \mu^2}$$

Example:

**Let X be a continuous random variable with probability density function defined by**

$$f(x) = \begin{cases} \frac{3}{8}x^2 & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

**Find the mean of X**

Select one:

- a. 3/2
- b. 1/3
- c. 1/2
- d. 2/3

**Suppose the probability density function of the length of computer cables is $f(x) = 0.5$ from 10 to 12 millimeters. Determine the mean and standard deviation of the cable length.**

Select one:

- a. mean = 11 and standard deviation = 0.33
- b. mean = 22 and standard deviation = 1.16
- c. mean = 22 and standard deviation = 0.66
- d. mean = 11 and standard deviation = 0.58

## 5. Uniform distribution on [a, b]

$$Mean = Expected\ Value = \mu = E(X) = \frac{b+a}{2}$$

$$Variance = \sigma^2 = V(X) = \frac{(b-a)^2}{12}$$

$$Standard\ deviation = \sigma = \sqrt{V(X)} = \sqrt{\frac{(b-a)^2}{12}}$$

Example:

**The diameters of ball bearings produced in a manufacturing process can be described using a uniform distribution over the interval 2.55 to 4.75 millimeters. What is the mean diameter of ball bearings produced in this manufacturing process?**

Select one:

- a. 3.5 millimeters
- b. 3.65 millimeters
- c. 4.25 millimeters
- d. 4.55 millimeters

**Suppose a uniform random variable can be used to describe the outcome of an experiment with outcomes ranging from 50 to 60. What is the mean outcome of this experiment?**

Select one:

- a. 60
- b. 70
- c. 65
- d. 55

**Suppose *X* is a uniform random variable over the interval [40, 70]. Find the standard deviation of *X*.**

Select one:

- a. 3.03
- b. 1.58
- c. 8.66
- d. 31.75

**A machine is set to pump cleanser into a process at the rate of 10 gallons per minute. Upon inspection, it is learned that the machine actually pumps cleanser at a rate described by the uniform distribution over the interval 9 to 13.5 gallons per minute. Find the variance of the distribution.**

Select one:

- a. 1.3345
- b. 1.6875
- c. 8.3321
- d. 44.0802

$$P(X > x) = \frac{b - x}{b - a}$$

$$P(X < x) = \frac{x - a}{b - a}$$

Example:

**Suppose *X* is a uniform random variable over [10, 70]. Find the probability that a randomly selected observation is between 13 and 65.**

Select one:

- a. 0.87
- b. 0.5
- c. 0.8
- d. 0.13

**Suppose X is a uniform random variable over the interval [20, 90]. Find the probability that a randomly selected observation is between 23 and 85.**

Select one:

- a. 0.8
- b. 0.89
- c. 0.11
- d. 0.5

**Let *X* be a uniform random variable over the interval [0.1, 5] . What is the probability that the random variable *X* has a value less than 2.1?**

Select one:

a. 0.0125

b. 0.3875

c. 0.1375

d. 0.408

**The diameters of ball bearings produced in a manufacturing process can be described using a uniform distribution over the interval 4.5 to 7.5 millimeters. Any ball bearing with a diameter of over 6.25 millimeters or under 4.55 millimeters is considered defective. What is the probability that a randomly selected ball bearing is defective?**

Select one:

a. 0.505

b. 0.433

c. 0

d. 0.250

**The diameters of ball bearings produced in a manufacturing process can be described using a uniform distribution over the interval 8.5 to 10.5 millimeters. What is the probability that a randomly selected ball bearing has a diameter greater than 9.8 millimeters?**

Select one:

- a. 0.876
- b. 0.350
- c. 0.650
- d. 0.484

## 6. Normal distribution $(\mu, \sigma)$

$P(X > x) \rightarrow CASIO$

$P(X < x) \rightarrow CASIO$

$P(x_1 < X < x_2) \rightarrow CASIO$

Example:

**A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 350 and a standard deviation of 50. If an applicant is randomly selected, find the probability of a rating that is between 310 and 295.**

Select one:

- a. 0.0668
- b. 0.0762
- c. 0.9032
- d. 0.4332

**A new phone system was installed last year to help reduce the expense of personal calls that were being made by employees. Before the new system was installed, the amount being spent on personal calls follows a normal distribution with an average of $705 per month and a standard deviation of $48 per month. Refer to such expenses as PCE's (personal call expenses). Find the probability that a randomly selected month had a PCE that falls below $650.**

Select one:

- a. 0.2143
- b. 0.1259
- c. 0.0013
- d. 0.6857

**A machine pours beer into 16 oz. bottles. Experience has shown that the number of ounces poured is normally distributed with a standard deviation of 1.5 ounces. Find the probabilities that the amount of beer the machine will pour into the next bottle will be more than 16.5 ounces.**

Select one:

- a. 0.471
- b. 0.251
- c. 0.369
- d. 0.452

**The amount of soda a dispensing machine pours into a 12 ounce can of soda follows a normal distribution with a mean of 12.27 ounces and a standard deviation of 0.18 ounce. The cans only hold 12.51 ounces of soda. Every can that has more than 12.51 ounces of soda poured into it causes a spill and the can needs to go through a special cleaning process before it can be sold. What is the probability a randomly selected can will need to go through this process?**

Select one:

- a. 0.8432
- b. 0.1587
- c. 0.0912
- d. 0.3413

**The lifetimes of light bulbs of a particular type are normally distributed with a mean of 420 hours and a standard deviation of 15 hours. What percentage of the bulbs have lifetimes that lie within 2 standard deviations of the mean?**

Select one:

- a. 99.74%
- b. 34%
- c. 95%
- d. 97.72%

**The lengths of human pregnancies are normally distributed with a mean of 269 days and a standard deviation of 16 days. What is the probability that a pregnancy lasts at least 302 days?**

Select one:

- a. 0.0332
- b. 0.0196
- c. 0.9834
- d. 0.0166
- e. 0.0179

**Assume that the weights of quarters are normally distributed with a mean of 5.73 g and a standard deviation 0.071 g. A vending machine will only accept coins weighing between 5.48 g and 5.82 g. What percentage of legal quarters will be rejected?**

Select one:

- a. 10.27%
- b. 62.54%
- c. 1.96%
- d. 2.48%

**For a standard normal distribution, find the percentage of data that are more than 1 standard deviation away from the mean.**

Select one:

- a. 68.26%
- b. 15.87%
- c. 34.13%
- d. 31.74%

**If Z is a standard normal variable, find the the probability that Z is less than 1.13.**

Select one:

- a. 0.8485
- b. 0.8907
- c. 0.8708
- d. 0.1292

**When considering area under the standard normal curve, decide whether the area between z = -0.2 and z = 0.2 is bigger than, smaller than, or equal to the area between z = -0.3 and z = 0.3.**

Select one:

- a. equal to
- b. smaller than
- c. bigger than

**Let *Z* is a standard normal variable, find the the probability that Z lies between 0 and 3.01.**

Select one:

a. 0.1217

b. 0.5013

c. 0.4987

d. 0.9987

**The area to the right of *z* = 1.0 is equal to**

Select one:

a. 0.6816.

b. 0.1587.

c. 0.8413.

d. 0.3413.

$P(X < x) = 0.9 \rightarrow Find\ x \rightarrow CASIO$

Example:

**Assume that *X* is normally distributed with a mean of 23 and a standard deviation of 5. Find the value of c if *P(X > c)* = 0.0592.**

Select one:

a. 23.72

b. 4.23

c. 30.81

d. 15.19

**Assume that z scores are normally distributed with a mean of 0 and a standard deviation of 1. If $P(-a < Z < a) = 0.4314$, find $a$.**

Select one:

- a. -0.18
- b. 0.57
- c. 0.3328
- d. 1.49

**Assume that z scores are normally distributed with a mean of 0 and a standard deviation of 1. If $P(0.2 < Z < a) = 0.2314$, find $a$.**

Select one:

- a. -0.1821
- b. 0.3542
- c. 1.4906
- d. 0.8805

**The number of ounces of soda that a vending machine dispenses per cup is normally distributed with a mean of 12.4 ounces and a standard deviation of 4.3 ounces. Find the number of ounces above which 86% of the dispensed sodas will fall.**

Select one:

- a. 9.1
- b. 12.4
- c. 8.6
- d. 7.8

**A supermarket manager has determined that the amount of time customers spend in the supermarket is approximately normally distributed with a mean of 51 minutes and a standard deviation of 6.5 minutes. Find the number of minutes, m, for which the probability that a customer spends less than m minutes in the supermarket is 0.20.**

Select one:

- a. 37.3
- b. 47.3
- c. 45.5
- d. 27.3

**The tread life of a particular brand of tire is a random variable best described by a normal distribution with a mean of 65,000 miles and a standard deviation of 1500 miles. What warranty should the company use if they want 95% of the tires to outlast the warranty?**

Select one:

- a. 65, 550 miles
- b. 62,533 miles
- c. 57,900 miles
- d. 67,467 miles

**The owner of a fish market determined that the weights of catfish are normally distributed with the average weight for a catfish is 3.2 pounds with a standard deviation of 0.6 pound. A citation catfish should be one of the top 5% in weight. At what weight (in pounds) should the citation designation be established?**

Select one:

- a. 7.85
- b. 4.19
- c. 2.21
- d. 4.84

### 7. Approximately Normal to Binomial

$X \sim Binomial\ (n, p)$

$Binomial: P(X \geq a) \rightarrow\ X \sim Normal(\mu = n * p, \sigma = \sqrt{n * p * (1-p)} \rightarrow P(X \geq\ a - 0.5)$

$Binomial: P(X \leq a) \rightarrow\ X \sim Normal(\mu = n * p, \sigma = \sqrt{n * p * (1-p)} \rightarrow P(X \leq\ a + 0.5)$

Example:

### 8. Approximately Normal to Poisson

$X \sim Poisson\ (\lambda)$

$Poisson: P(X \geq a) \rightarrow\ X \sim Normal\left(\mu = \lambda, \sigma = \sqrt{\lambda}\right) \rightarrow P(X \geq\ a - 0.5)$

$Poisson: P(X \leq a) \rightarrow\ X \sim Normal(\mu = \lambda, \sigma = \sqrt{\lambda}) \rightarrow P(X \leq\ a + 0.5)$

### 9. Exponential Distribution (λ)

$$Mean = \mu = E(X) = \frac{1}{\lambda}$$

$$Variance = \sigma^2 = V(X) = \frac{1}{\lambda^2}$$

$$Standard\ deviation = \sigma = \sqrt{V(X)} = \frac{1}{\lambda}$$

Example:

**Suppose that the random variable *X* has an exponential distribution with λ = 1.5. Find the mean and standard deviation of *X*.**

Select one:

a. Mean = 0.67; Standard deviation = 0.67

b. Mean = 0; Standard deviation = 1.5

c. Mean = 0; Standard deviation = 1

d. Mean = 1.5; Standard deviation = 1

$P(X > x) = e^{-\lambda * x}$

$P(X < x) = 1 - e^{-\lambda * x}$

Example:

**Let *X* represent the amount of time it takes a student to park in the library parking lot at the university. If we know that the distribution of parking times can be modeled using an exponential distribution with a mean of 4.8 minutes, find the probability that it will take a randomly selected student more than 9 minutes to park in the library lot.**

Select one:

a. 0.917915

b. 0.153355

c. 0.329680

d. 0.660321

**A catalog company that receives the majority of its orders by telephone conducted a study to determine how long customers were willing to wait on hold before ordering a product. The length of time was found to be a random variable best approximated by an exponential distribution with a mean equal to 3.3 minutes. What proportion of callers is put on hold longer than 2.8 minutes?**

Select one:

a. 0.60810

b. 0.632522

c. 0.42806

d. 0.367879

**The time (in years) until the first critical-part failure for a certain car is exponentially distributed with a mean of 3.2 years. Find the probability that the time until the first critical-part failure is less than 1 year.**

Select one:

a. 0.268384

b. 0.966627

c. 0.747184

d. 0.254811

**A catalog company that receives the majority of its orders by telephone conducted a study to determine how long customers were willing to wait on hold before ordering a product. The length of time was found to be a random variable best approximated by an exponential distribution with a mean equal to 6.5 minutes. What is the probability that a randomly selected caller is placed on hold fewer than 7.5 minutes?**

Select one:

a. 0.917915

b. 0.9990881

c. 0.082085

d. 0.684579

**The time between customer arrivals at a furniture store has an approximate exponential distribution with mean of 9.5 minutes. If a customer just arrived, find the probability that the next customer will not arrive for at least 21 minutes.**

Select one:

a. 0.109643

b. 0.504912

c. 0.653770

d. 0.095089

**A catalog company that receives the majority of its orders by telephone conducted a study to determine how long customers were willing to wait on hold before ordering a product. The length of time was found to be a random variable best approximated by an exponential distribution with a mean equal to 3 minutes. Find the waiting time at which only 10% of the customers will continue to hold.**

Select one:

- a. 2.3 minutes
- b. 13.8 minutes
- c. 6.9 minutes
- d. 3.3 minutes

**Let *X* represent the amount of time it takes a student to park in the library parking lot at the university. If we know that the distribution of parking times can be modeled using an exponential distribution with a mean of 4 minutes, find the probability that it will take a randomly selected student between 2.5 and 10 minutes to park in the library lot.**

Select one:

- a. 0.606531
- b. 0.556744
- c. 0.453176
- d. 0.656318