Indexing: B+ Tree

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CS411: Database Systems

Learning Objectives

After this lecture, you should be able to:

- Describe how to search B+ Trees
- Describe how to insert new key(s) into a B+ tree
- Describe how to delete key(s) from a B+ tree

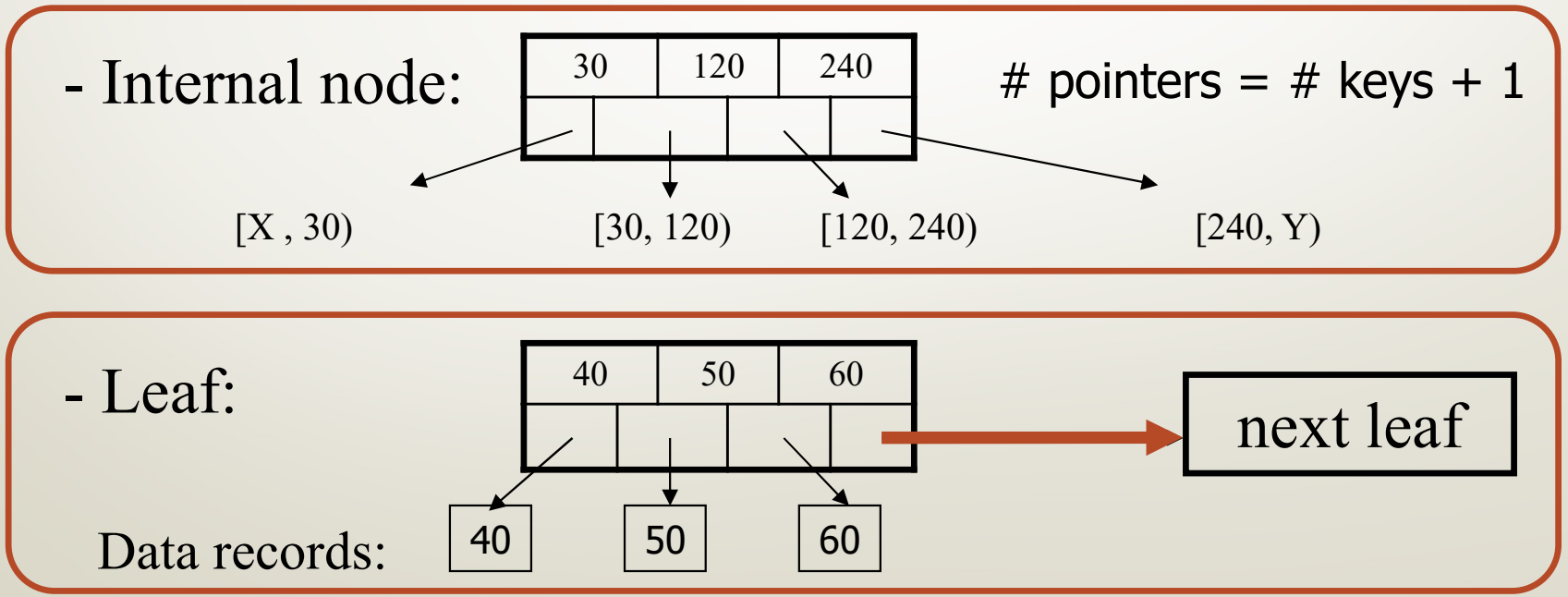
B+ Trees

- Intuition:
  - The index can be very large.
  - Index of index?
  - Index of index of index?
  - How best to create such a multi-level index?
- B+ trees:
  - Textbook refers to B+ trees (a popular variant) as B-trees (as most people do)

Focus on the dense version:
applies to clustered and unclustered settings

B+ Trees Basics

- B+ Trees are trees with nodes:
  Nodes have keys and pointers to:
  - Other nodes [if the node is an internal node]
  - Data Records [if the node is a leaf]



B+ Trees Basics

- Parameter d = the *degree* ; n = max keys
- When n is even [*this is our focus for simplicity*]
  - each node has [d, 2d] keys (except root); $n = 2d$
- At least half full at all times
  - d is the minimum amount it needs to be full.

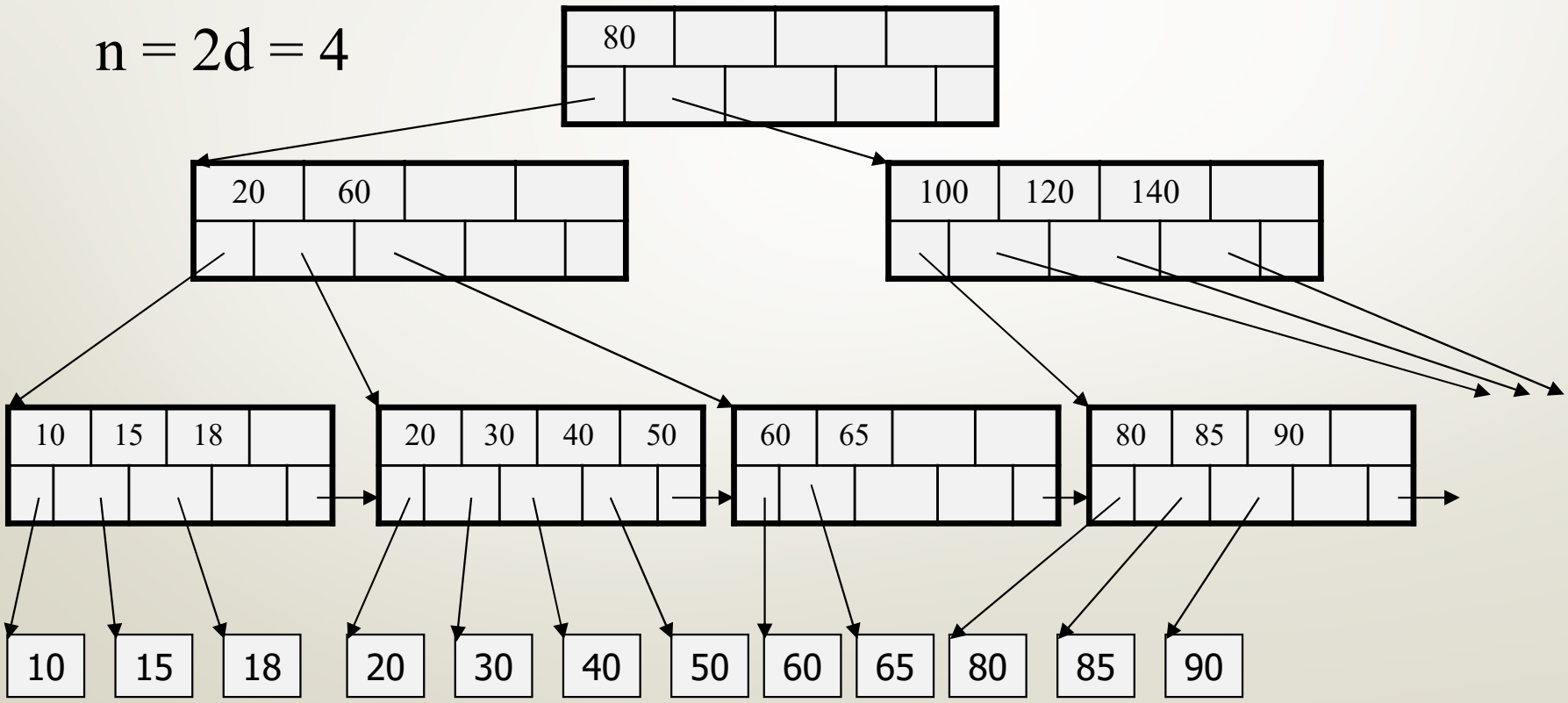
B+ Tree Example

Root can have 1 or more filled in keys

Rest have at least d

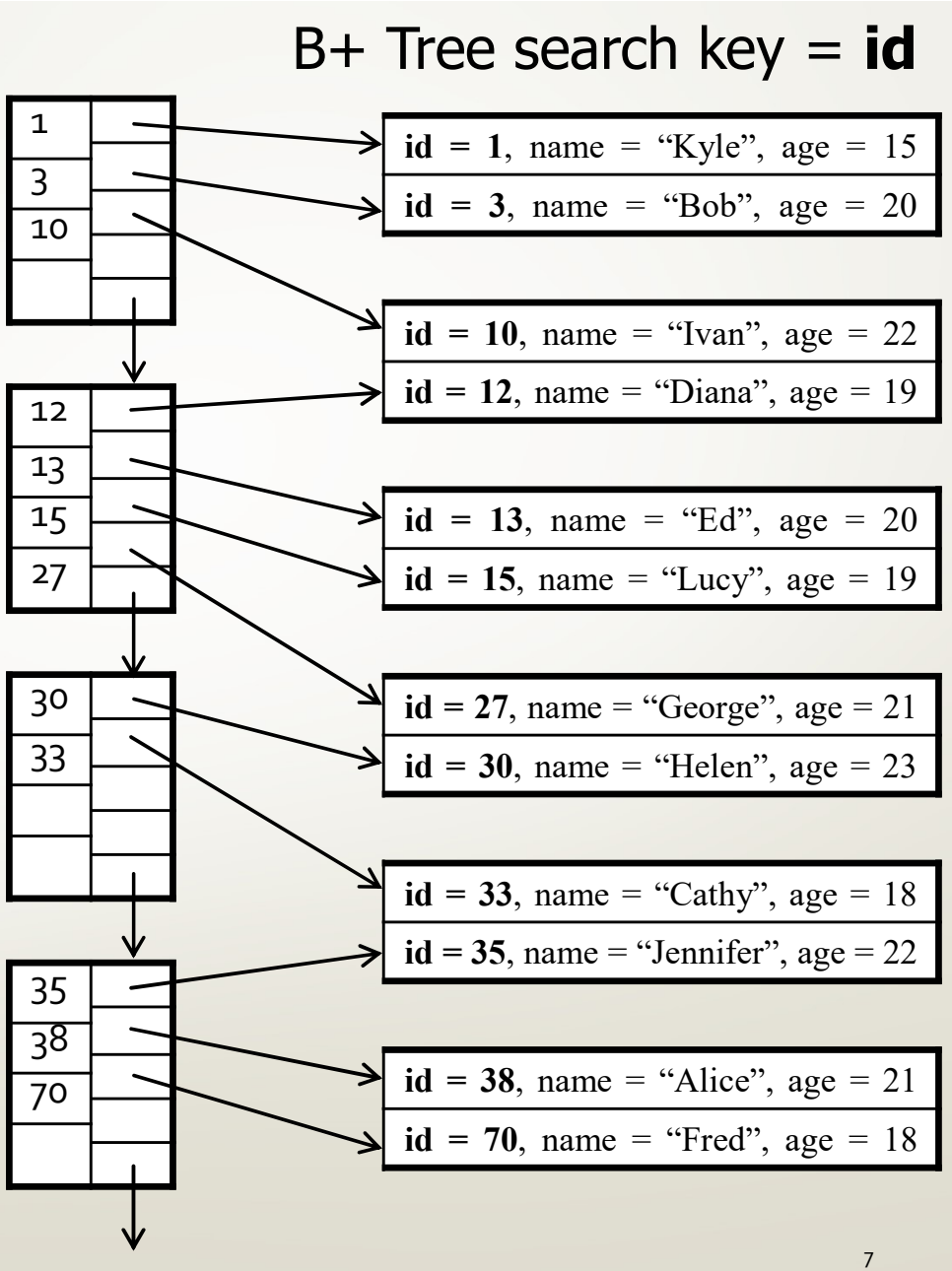
d = 2

n = 2d = 4



Clustered dense (entry for every record)

d = 2; n = 4



Clustered dense (entry for every record)

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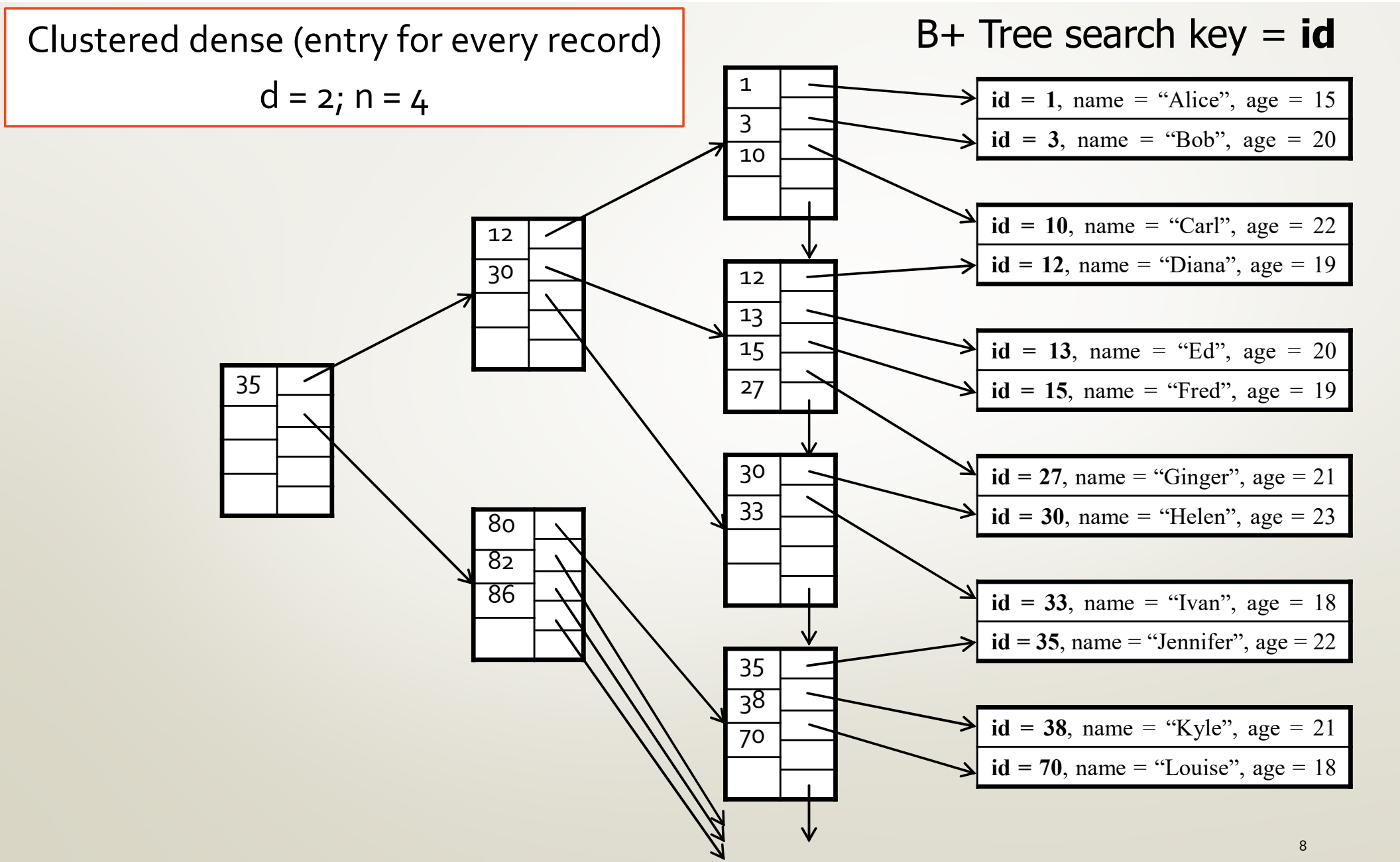
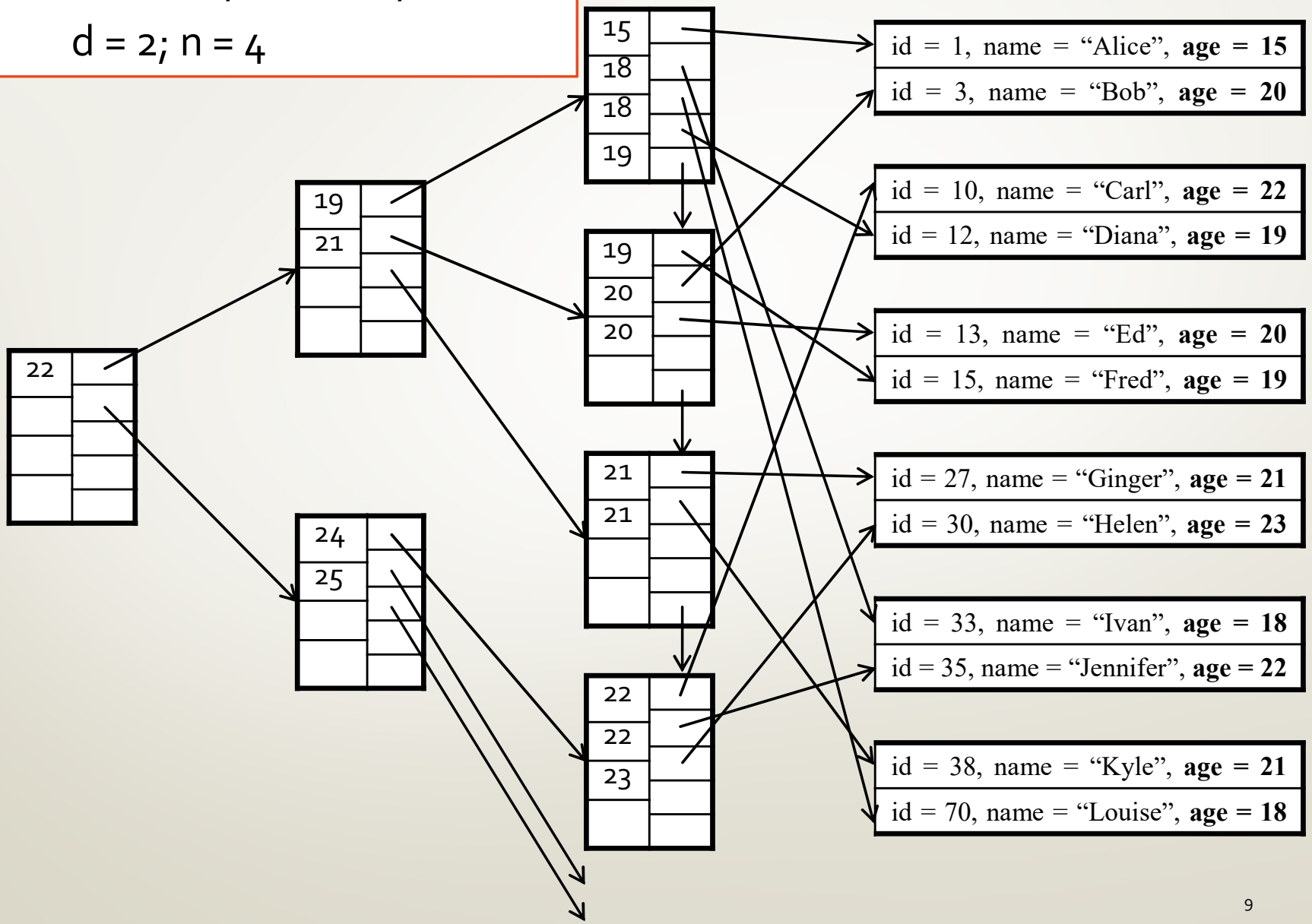


Unclustered dense (entry for every record)

d = 2; n = 4

B+ Tree search key = **age**



B+ Tree Design

- How large should *d* be?
- Example:
  - Key size = 4 bytes
  - Pointer size = 8 bytes
  - Block size = 4096 byes
- 2d x 4 + (2d+1) x 8 <= 4096
- d = 170; 2d = 340

So up to 340 records in leaf blocks

B+ Trees in Practice

- Typical d: 100. Typical fill-factor: 66.5%.
  - average "fanout" = 66.5 * 2 [66.5/100 * 200] =133
- Typical capacities:
  - Height 4: 133^4 = 312,900,700 records
  - Height 3: 133^3 = 2,352,637 records
- Can often hold top levels in main memory:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 Mbytes

When Do B+ Trees Help?

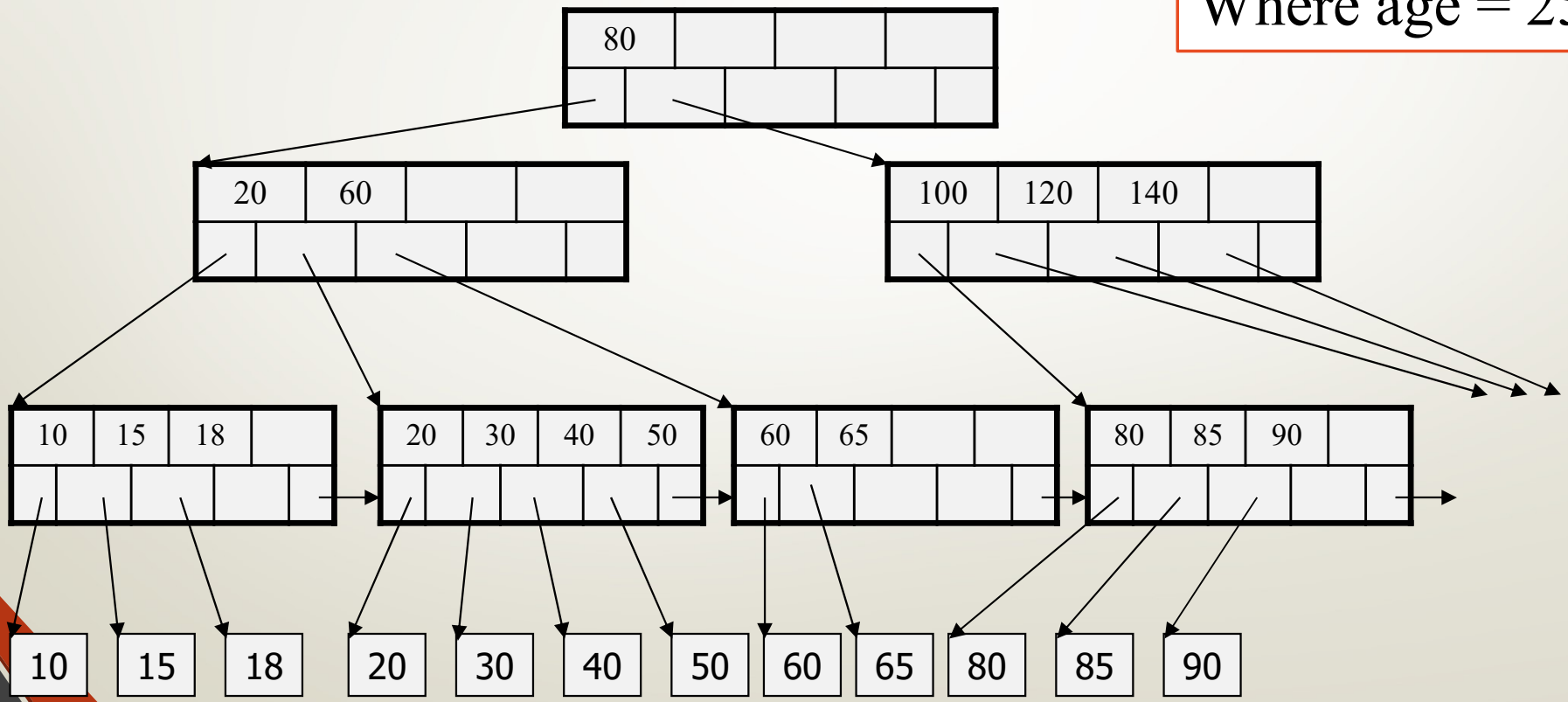
- Do B+ Trees always help?
  - No. e.g., an array of sorted integers.
- Types of queries to answer with a B+ Tree:
  - *Exact key value*, e.g., SELECT name FROM people WHERE age=20
  - *Range queries*, e.g., SELECT name FROM people WHERE age>=20 and age<=70

Searching a B+ Tree

Exact key values:

- Start at the root;
- Proceed down to the leaf

Select name
From people
Where age = 25

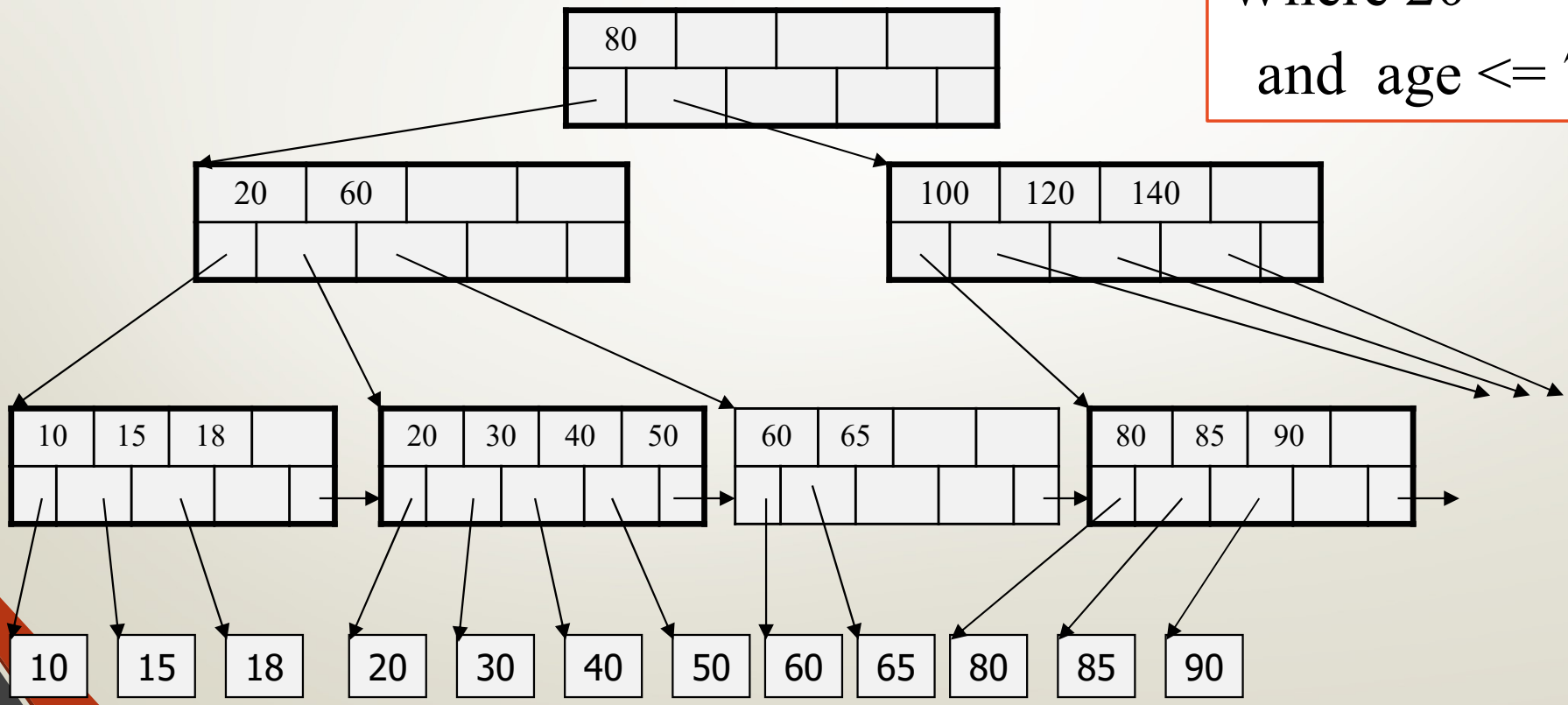


Searching a B+ Tree

Range queries:

- As above
- Then sequential traversal using "next leaf" pointers

```
Select name
From people
Where 20 <= age
  and  age <= 70
```

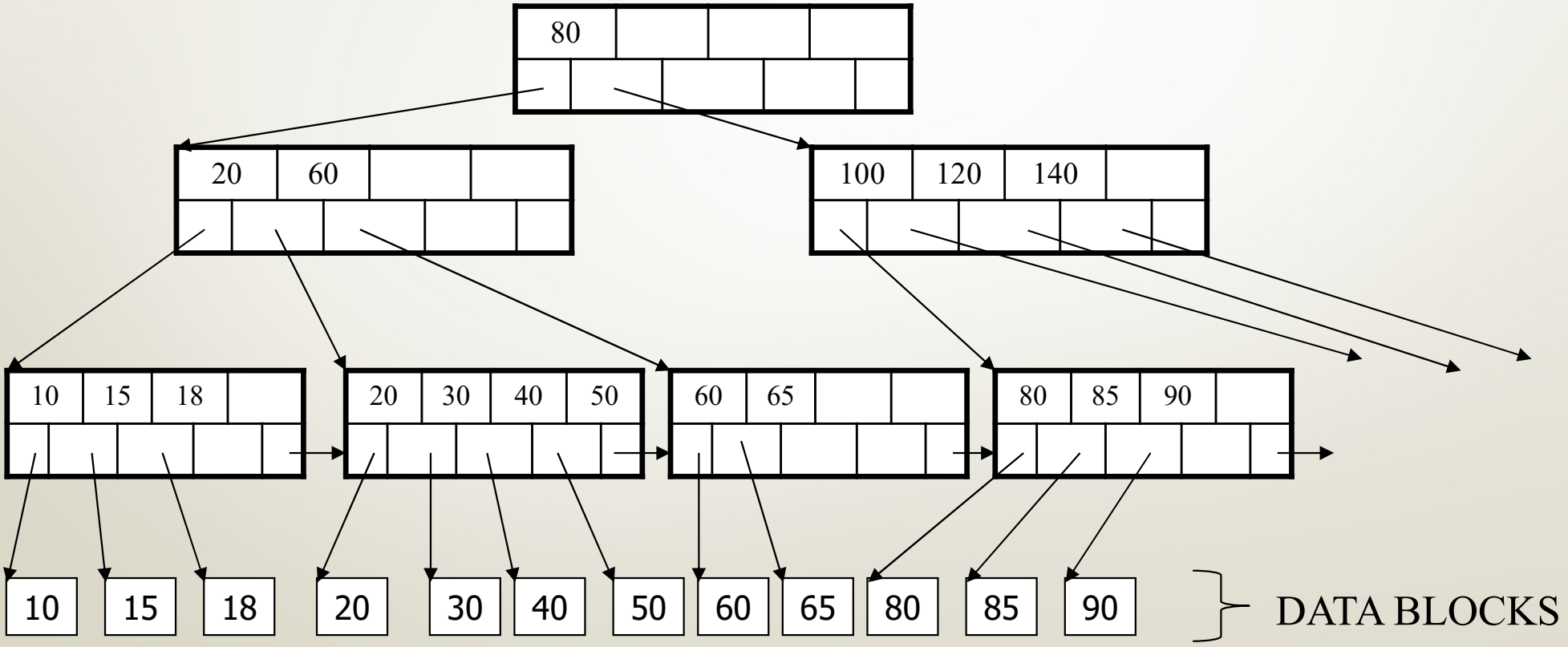


Handling data changes in B+ Trees

Insertion in a B+ Tree

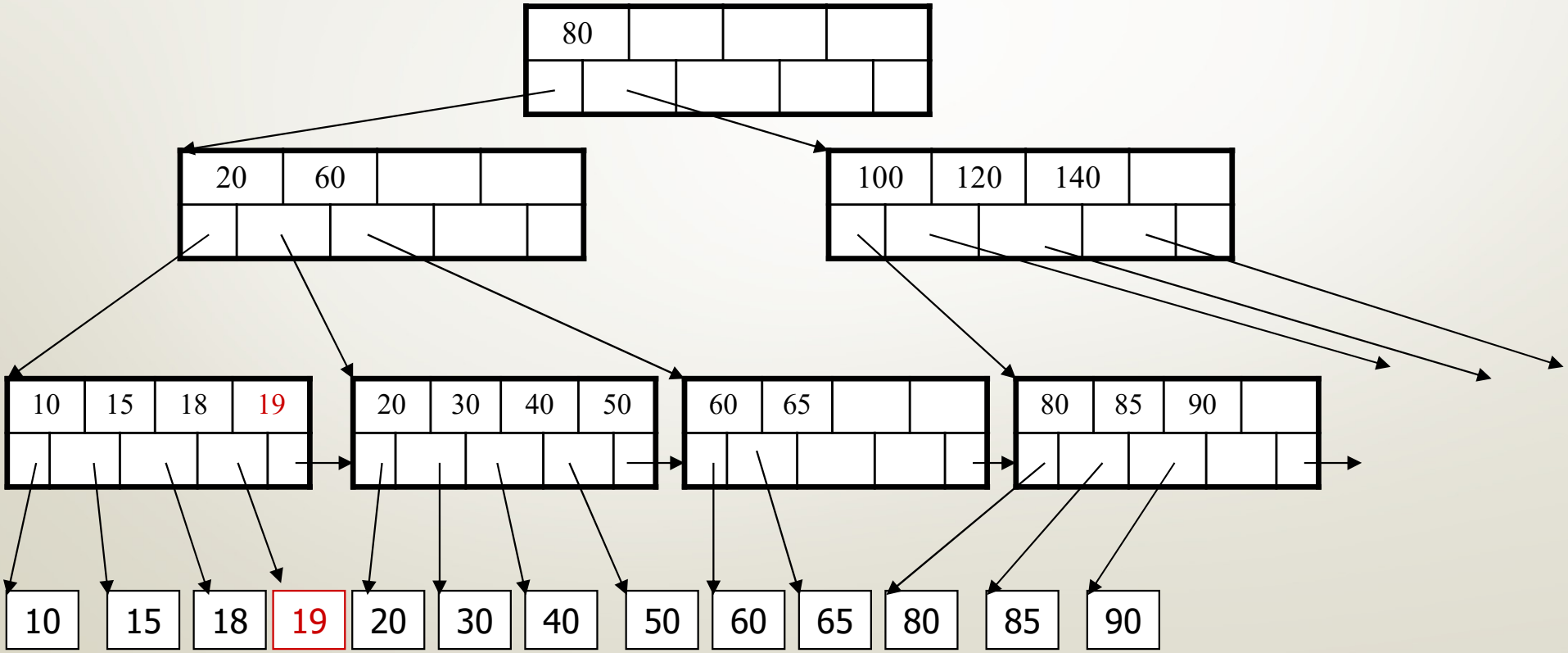
Assume d=2.

Insert K=19



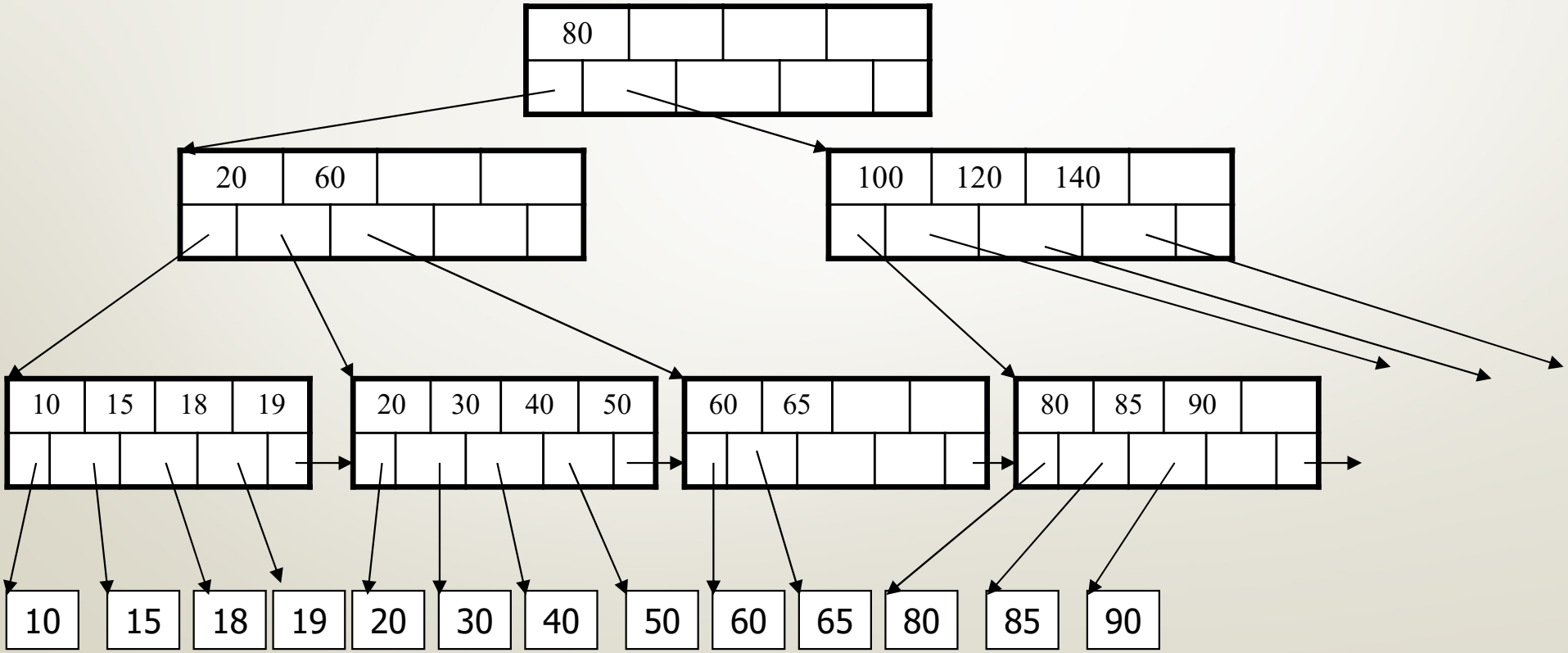
Insertion in a B+ Tree

After insertion



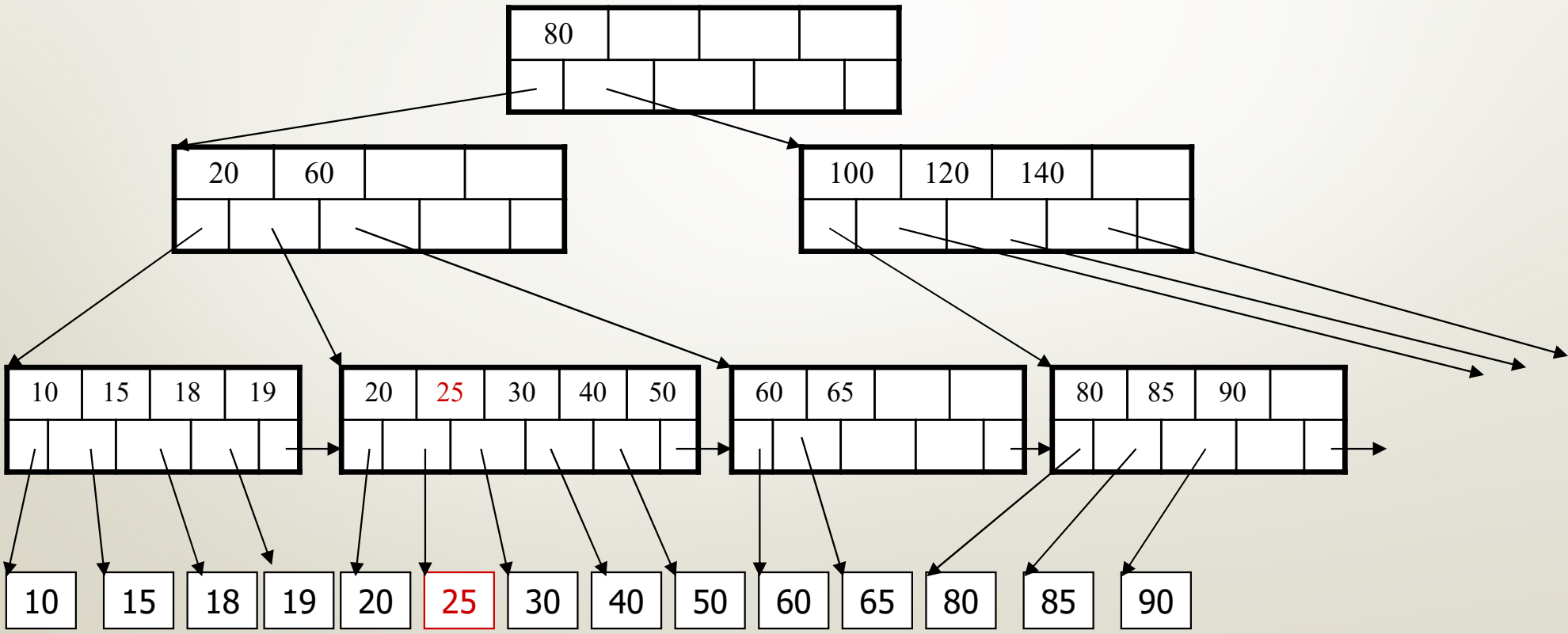
Insertion in a B+ Tree

Now insert 25



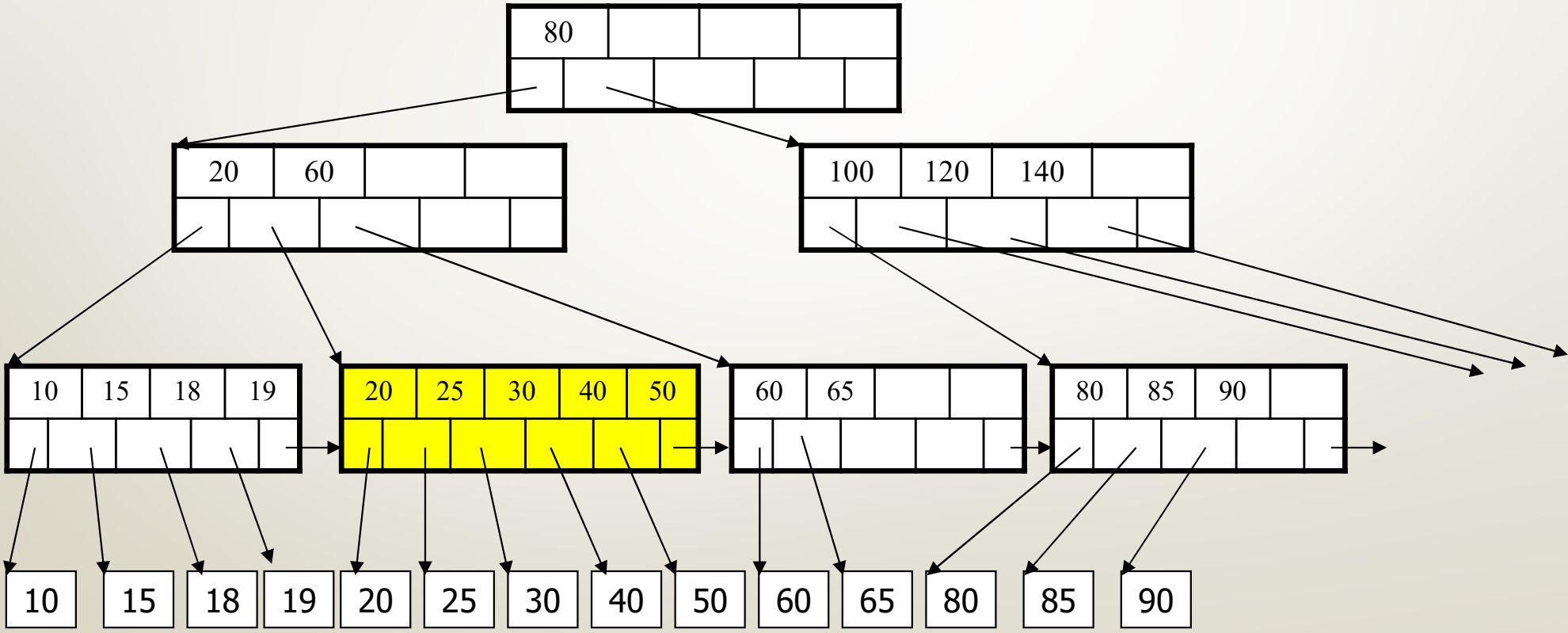
Insertion in a B+ Tree

After insertion



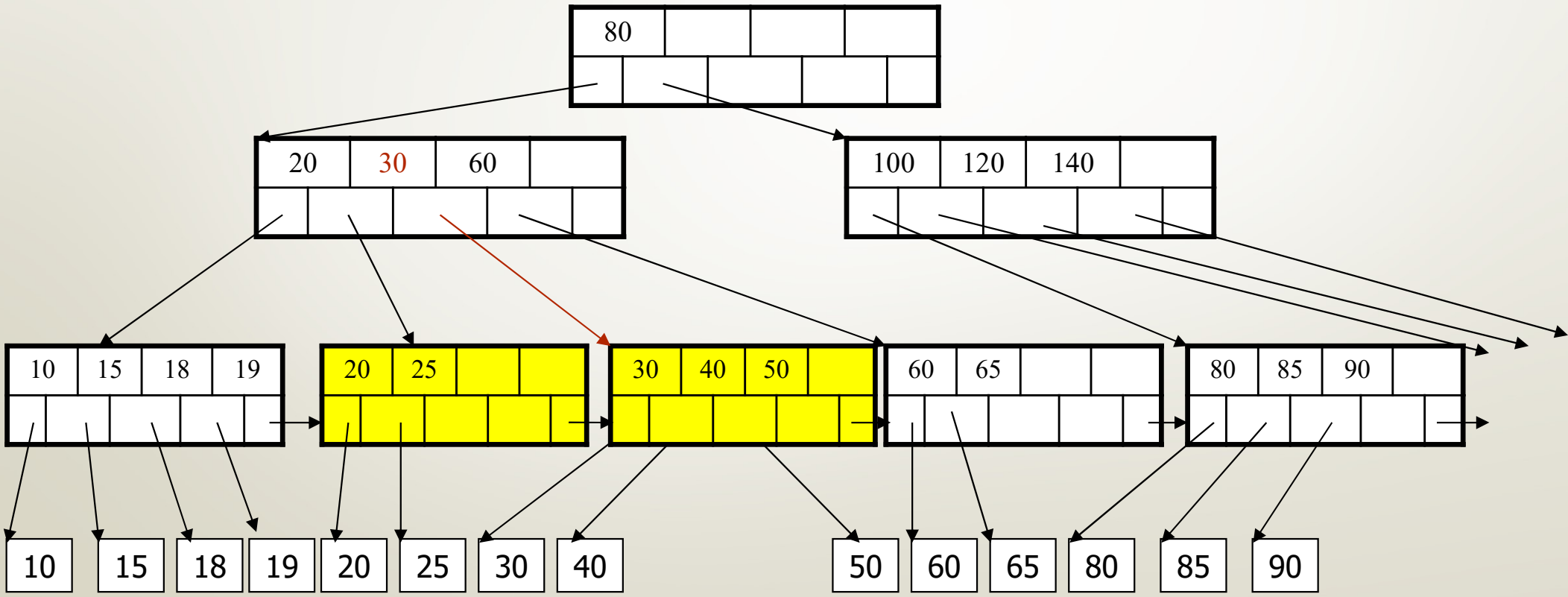
Insertion in a B+ Tree

But now have to split !



Insertion in a B+ Tree

After the split

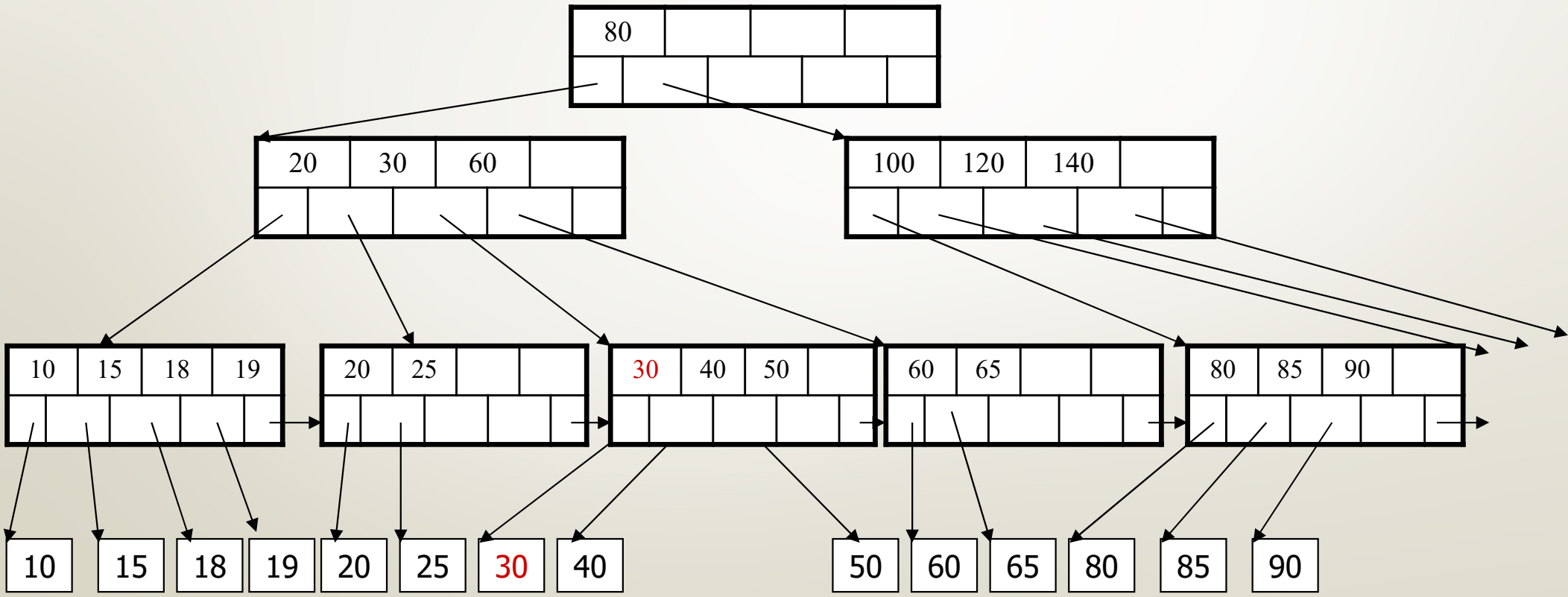


Outline

- B+ Trees
  - ✓ Inserting
  - Deletion

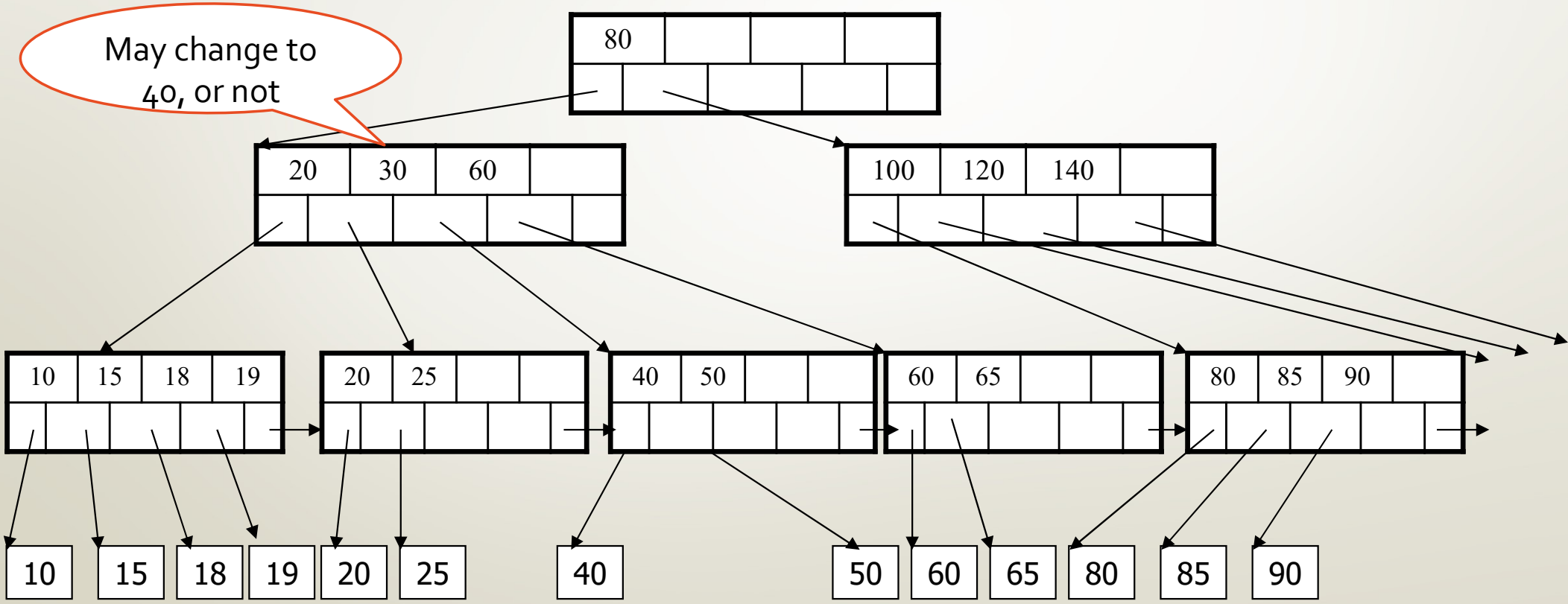
Deletion from a B+ Tree

Delete 30



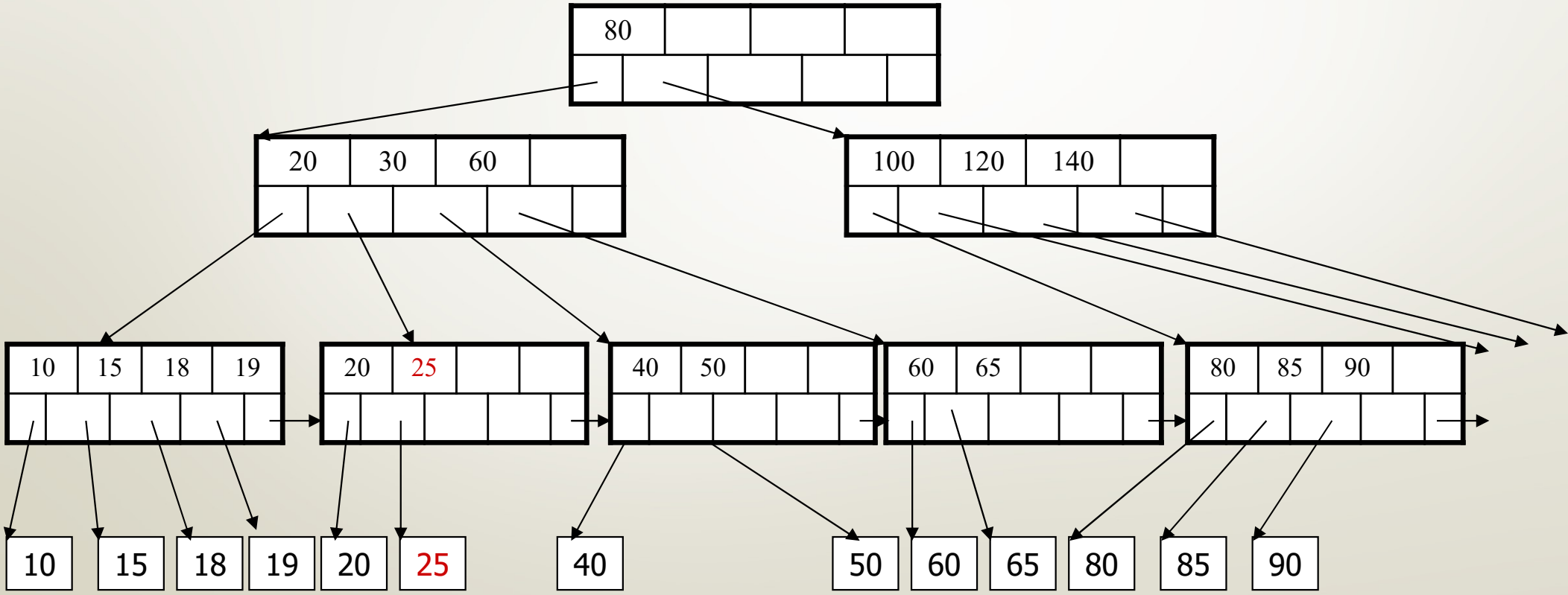
Deletion from a B+ Tree

After deleting 30



Deletion from a B+ Tree

Now delete 25



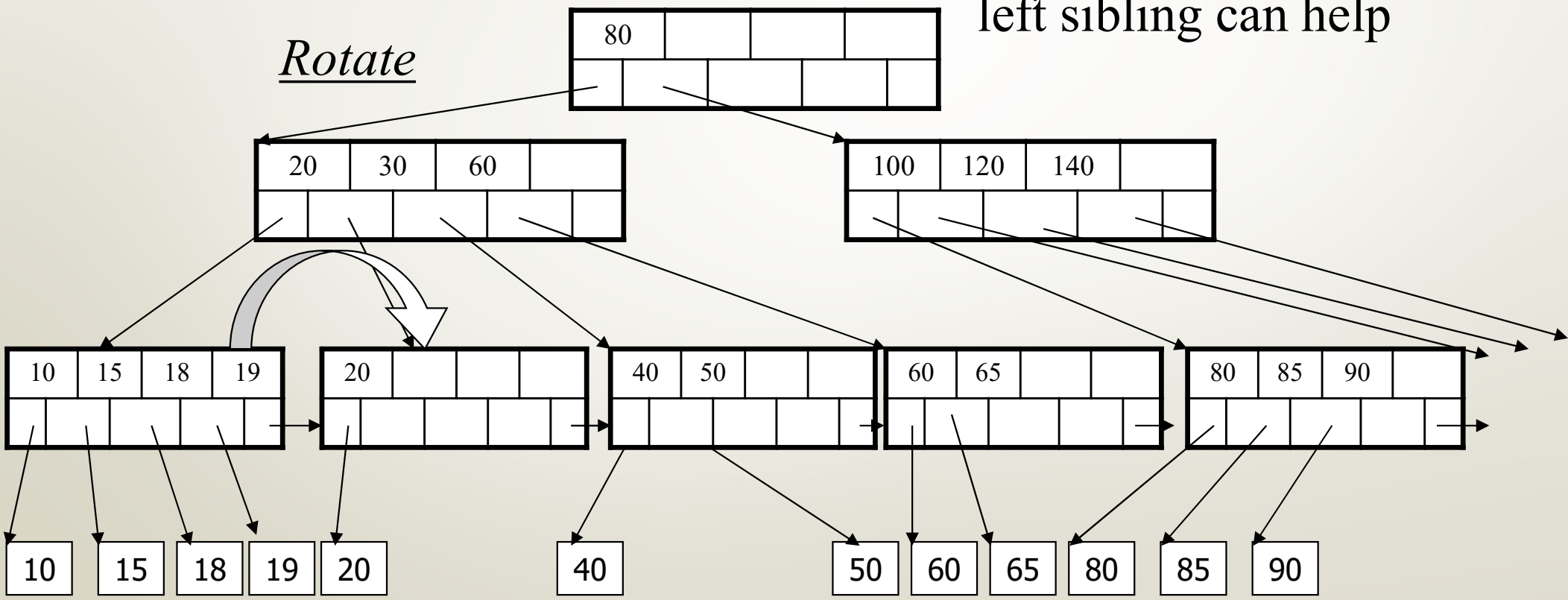
Deletion from a B+ Tree

After deleting 25

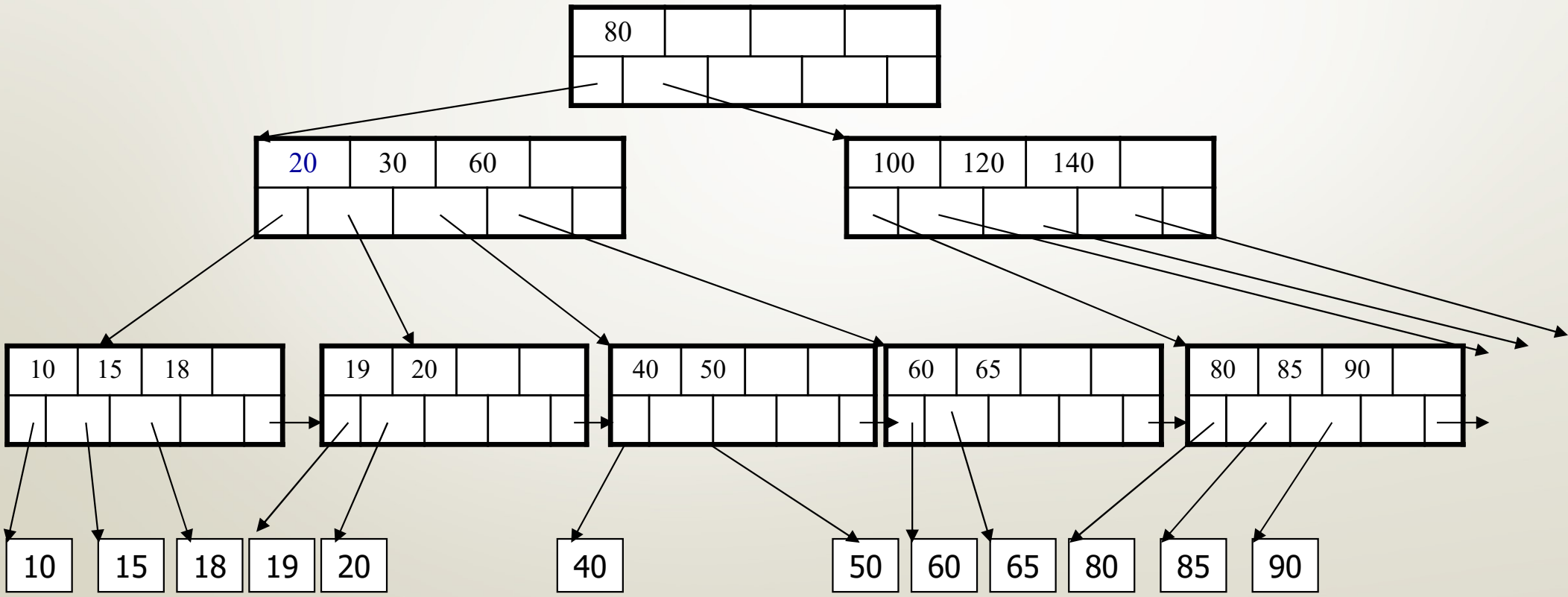
Need to rebalance

Rotate

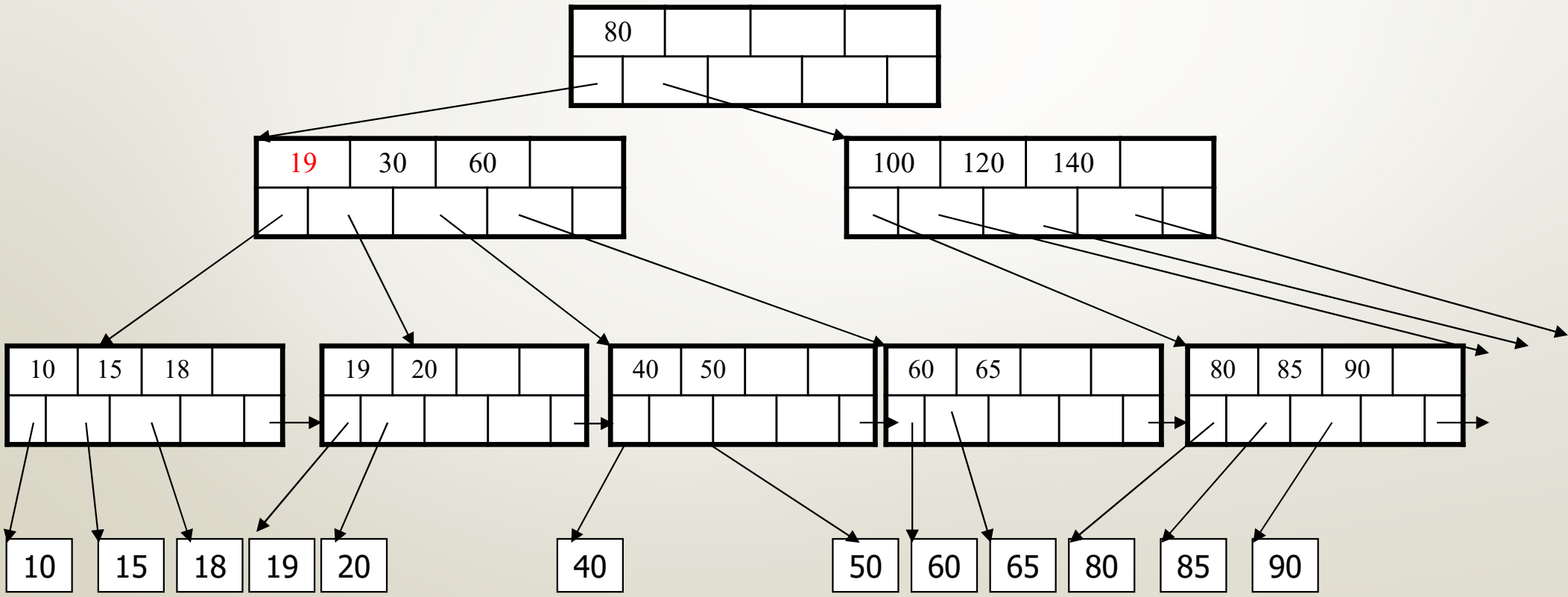
Rotation in general can involve either sibling, but here only the left sibling can help



Deletion from a B+ Tree

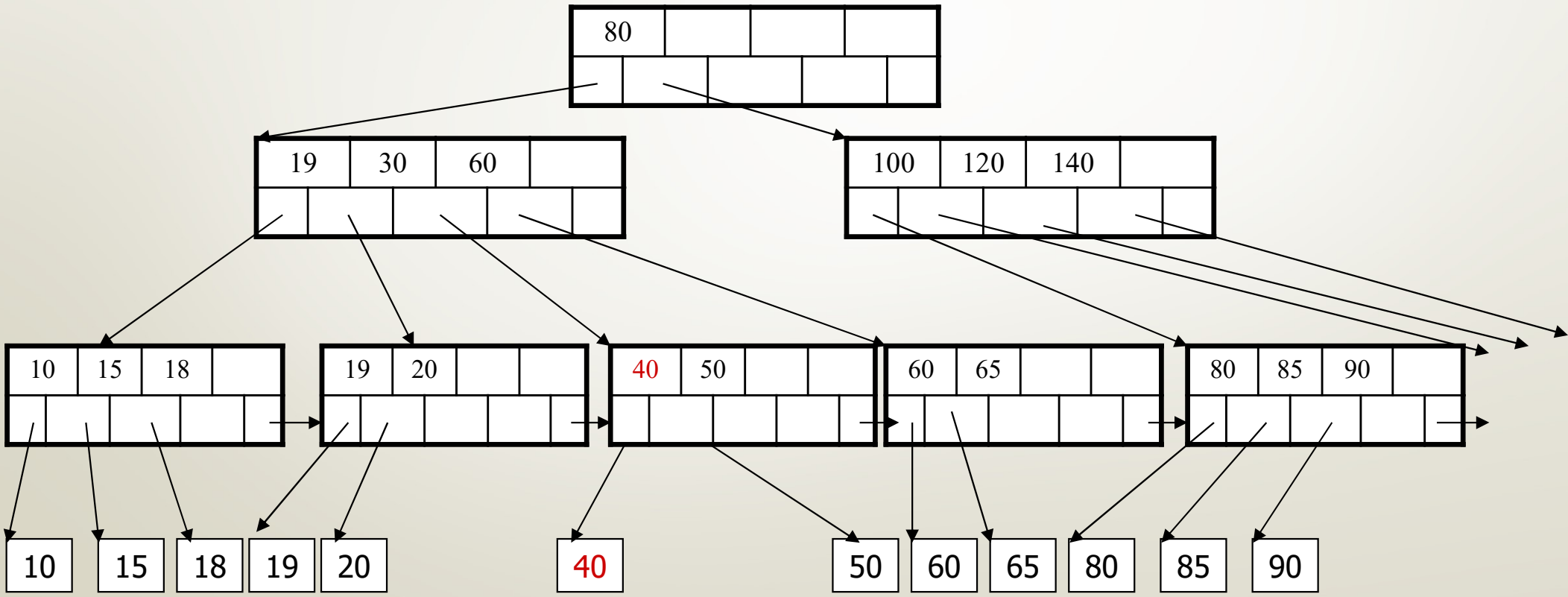


Deletion from a B+ Tree



Deletion from a B+ Tree

Now delete 40

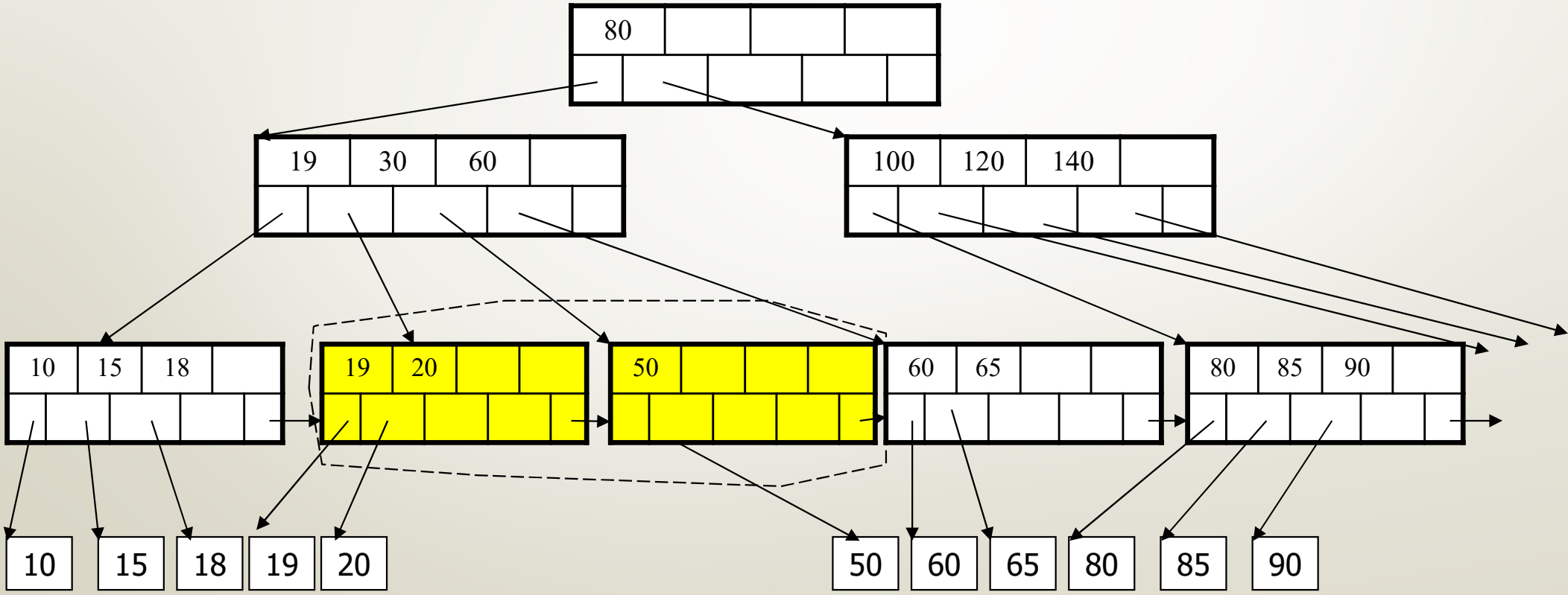


Deletion from a B+ Tree

After deleting 40

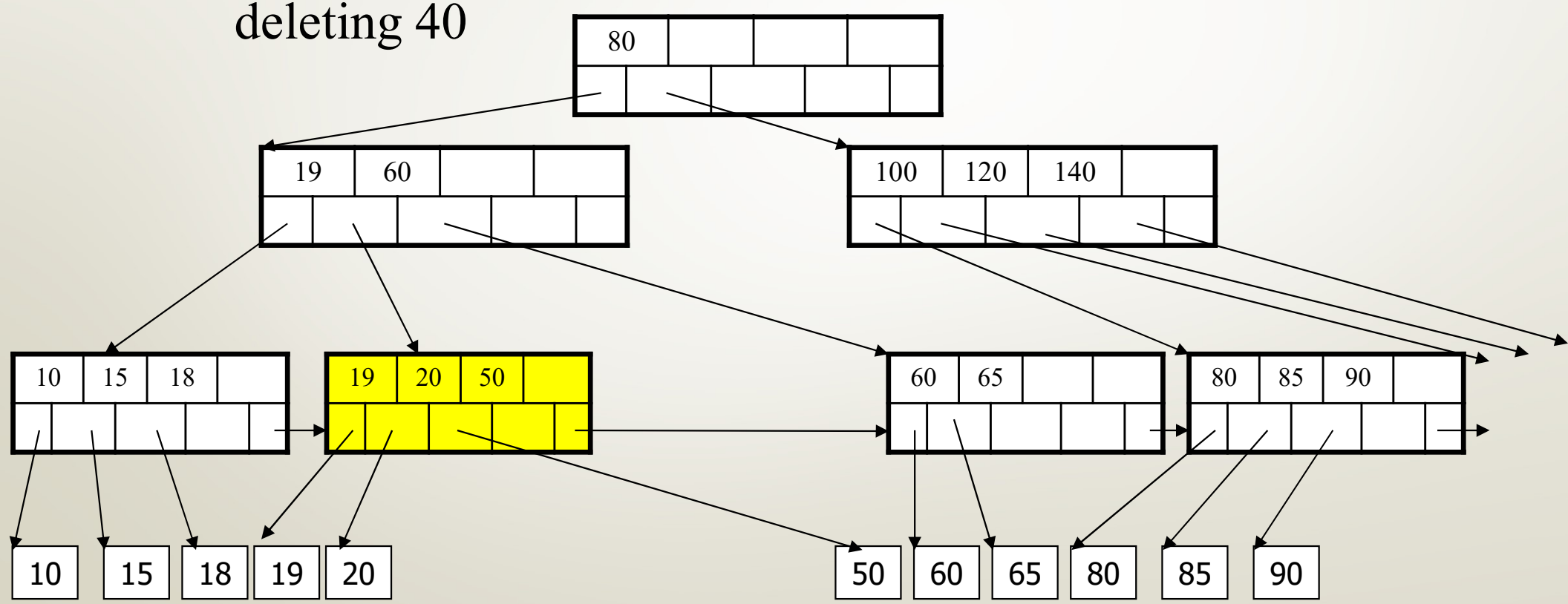
Rotation not possible

Need to *merge* nodes



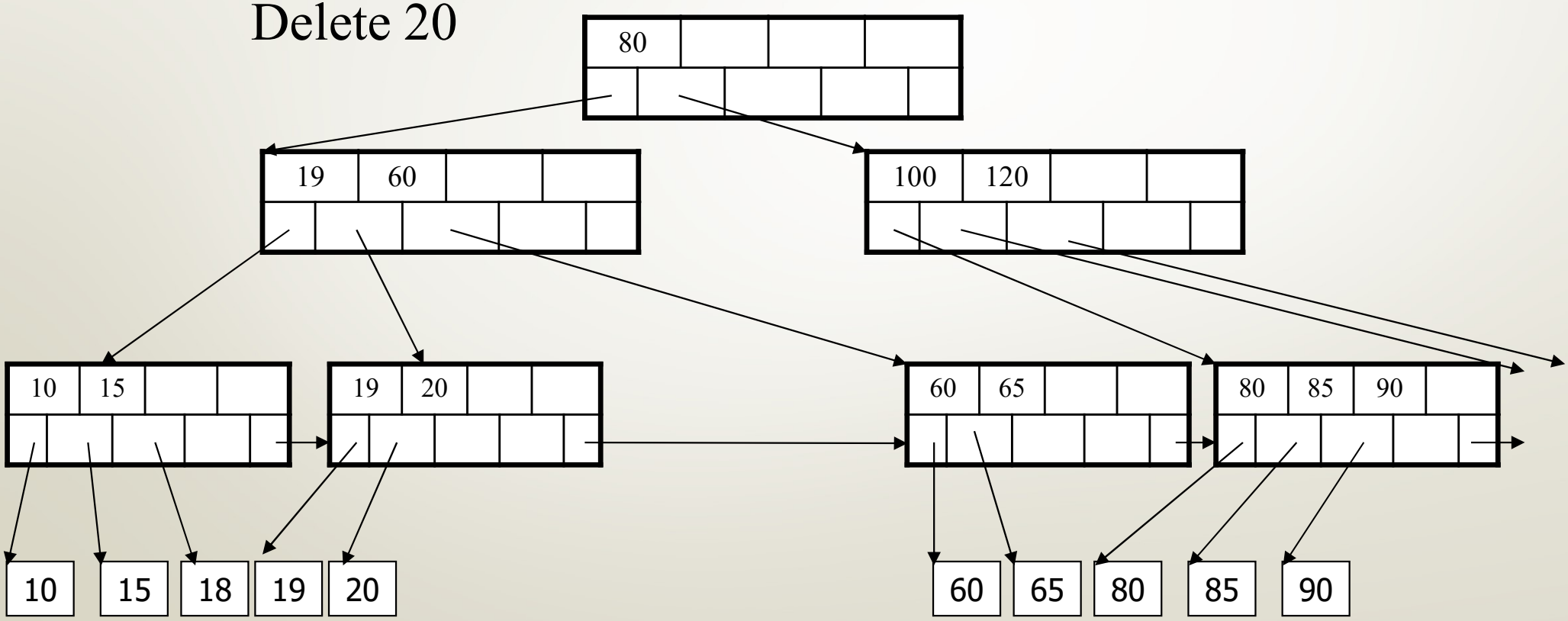
Deletion from a B+ Tree

Tree after deleting 40



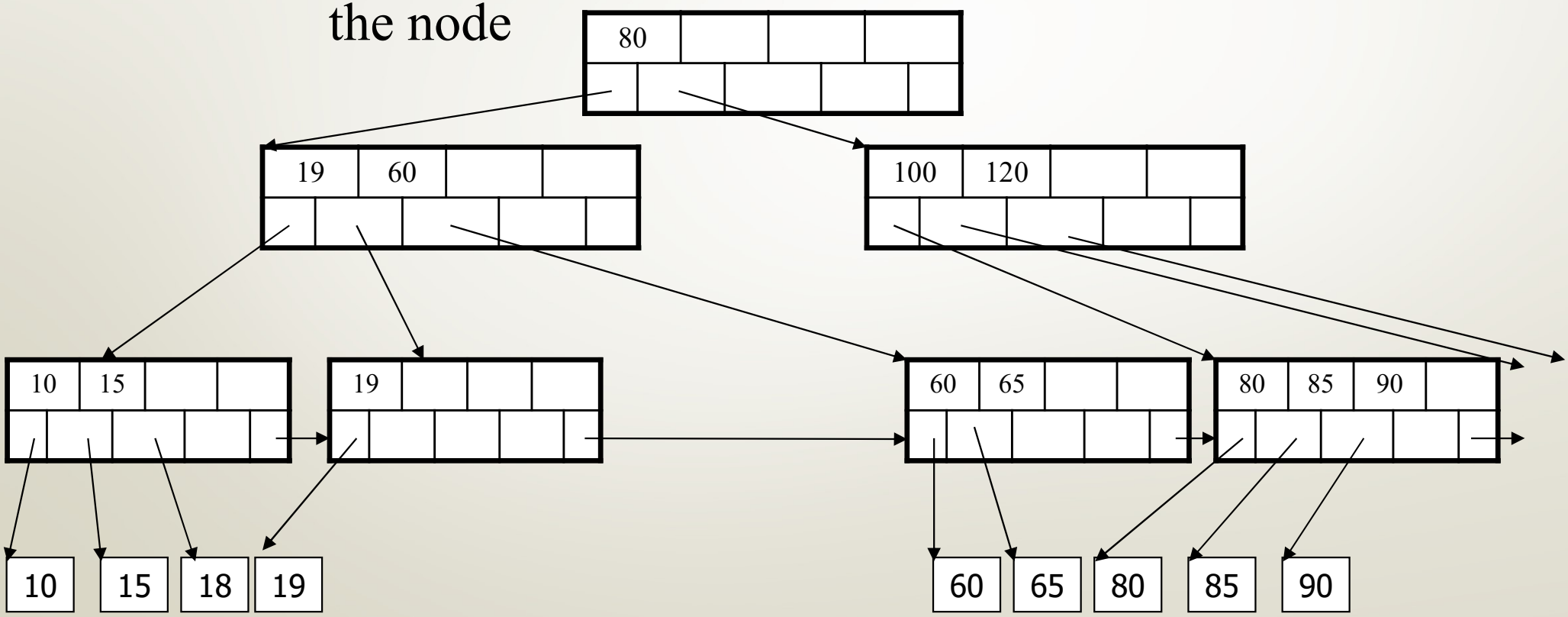
Deletion from a B+ Tree

New tree
Delete 20



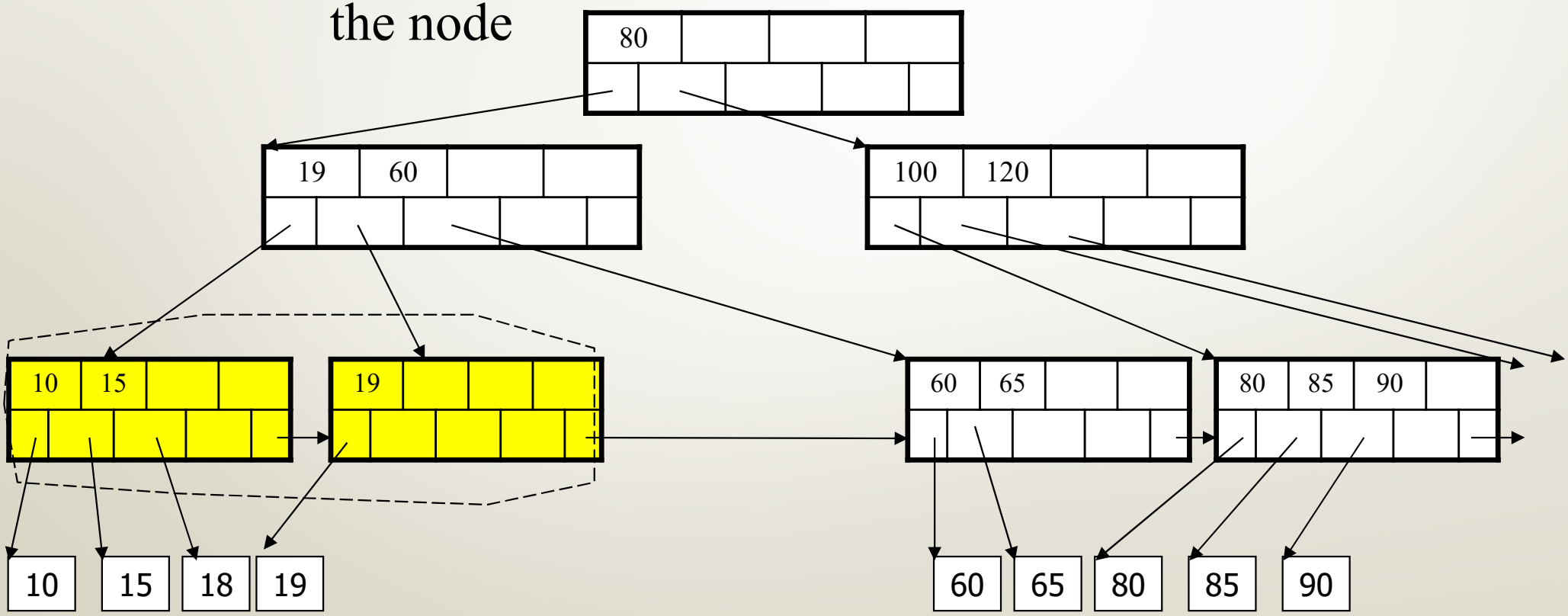
Deletion from a B+ Tree

Rotation is not possible
We have to delete
the node



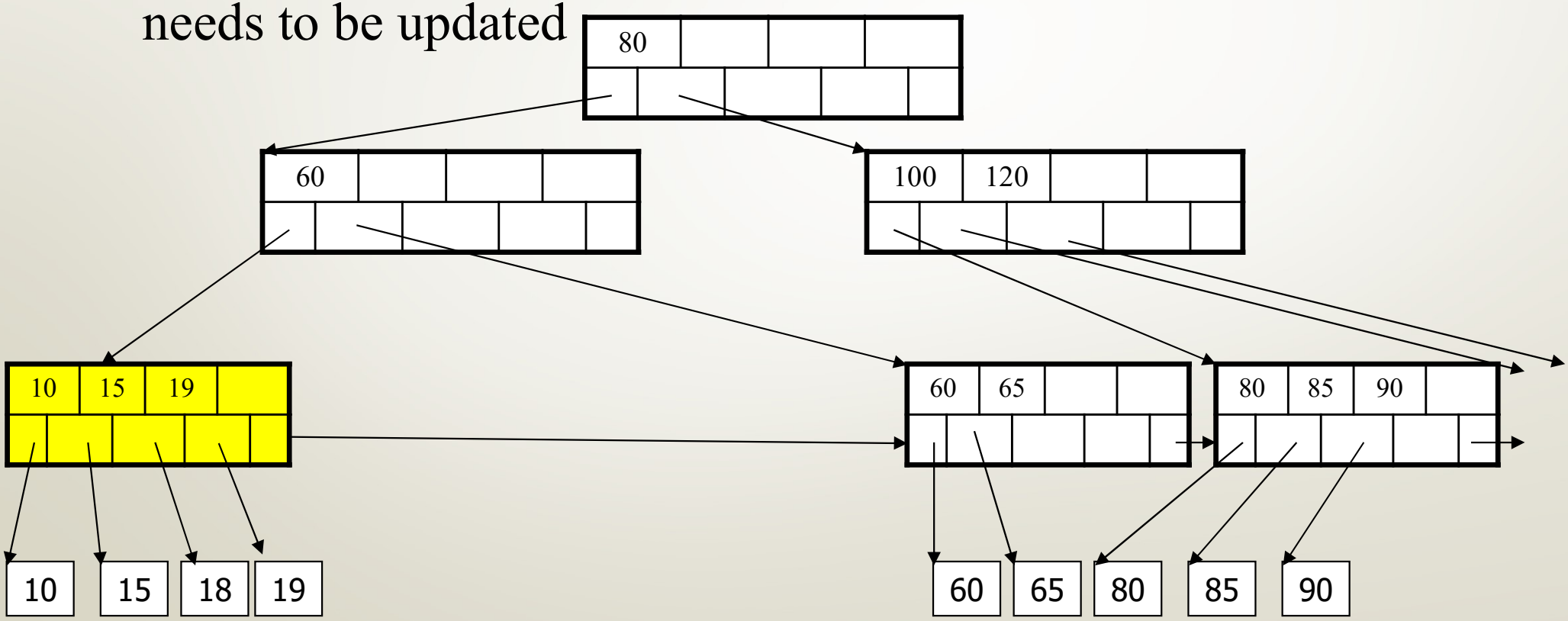
Deletion from a B+ Tree

Rotation is not possible
We have to delete
the node



Deletion from a B+ Tree

The parent node needs to be updated



Deletion from a B+ Tree

Rotation is not possible
We have to delete
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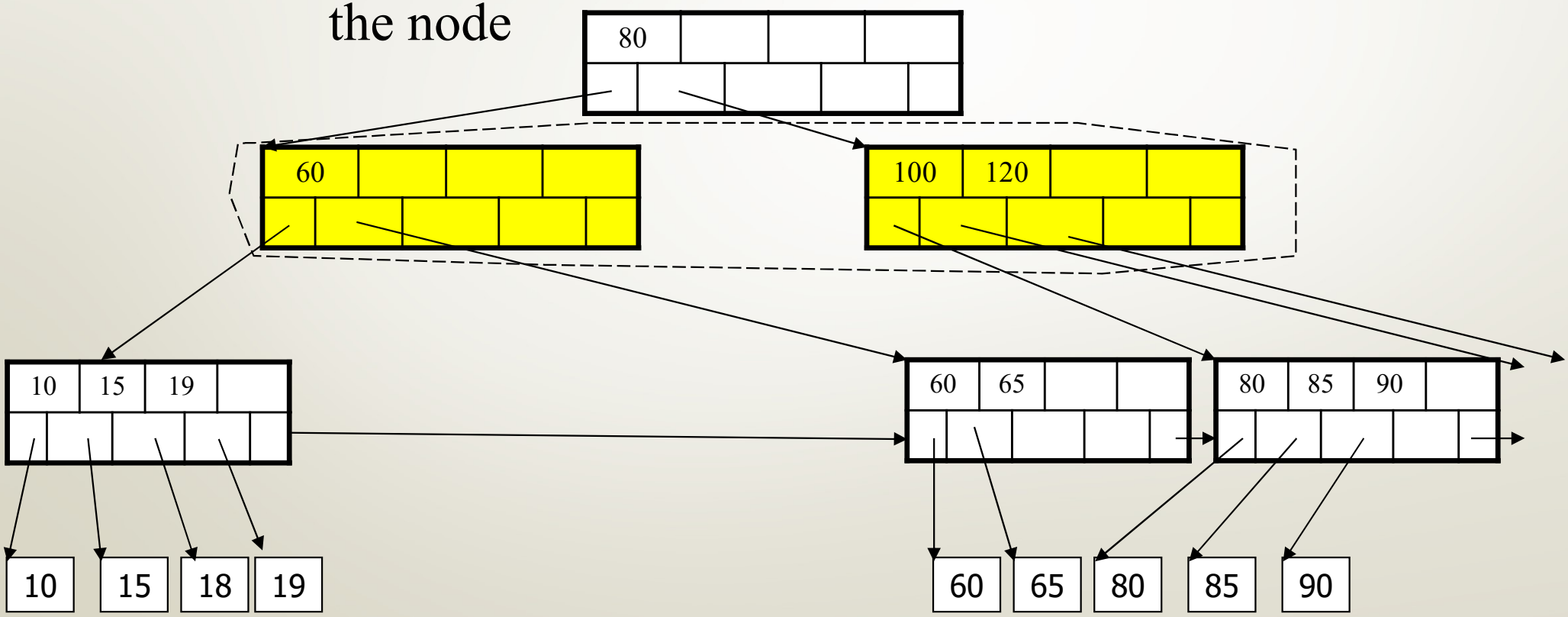
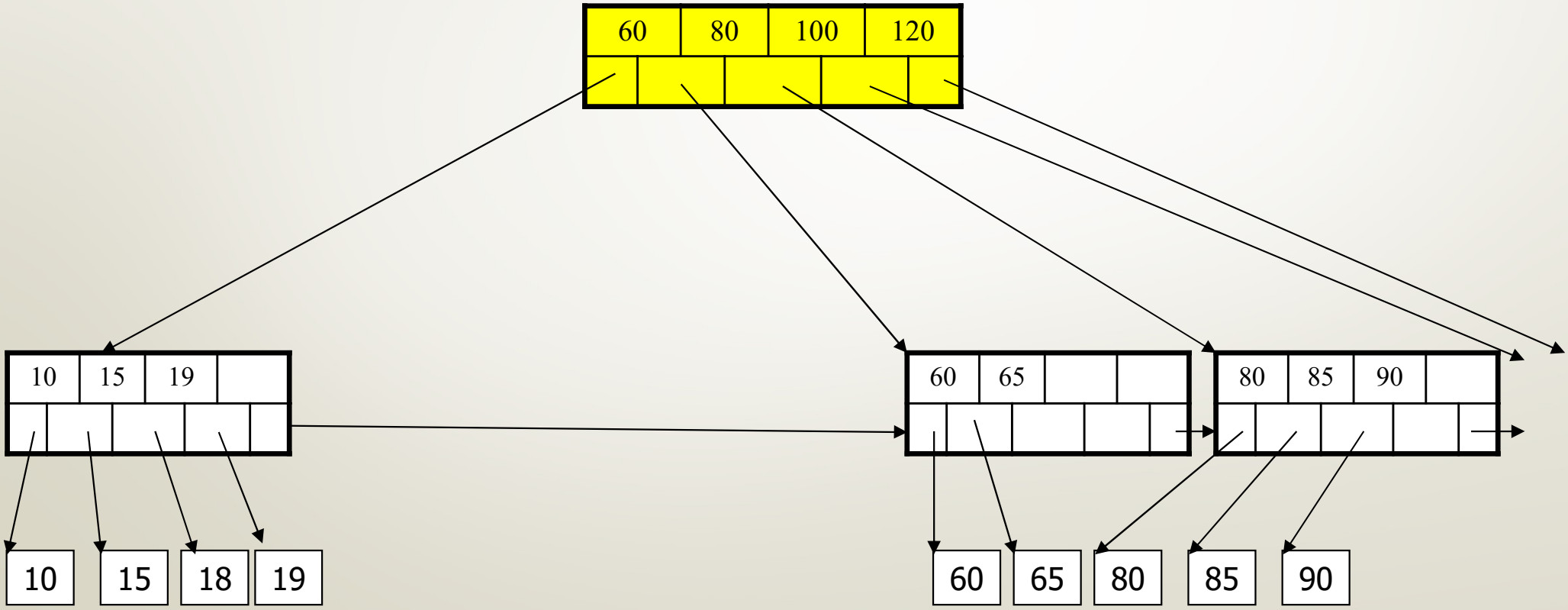


Deletion from a B+ Tree

Final tree



Advantages of B+Trees

- Balanced → Uniform space utilization
  - Predictable organization
  - Predictable time (logarithmic); unbalanced can be linear in worst case
- Good for range queries

Can we do better?