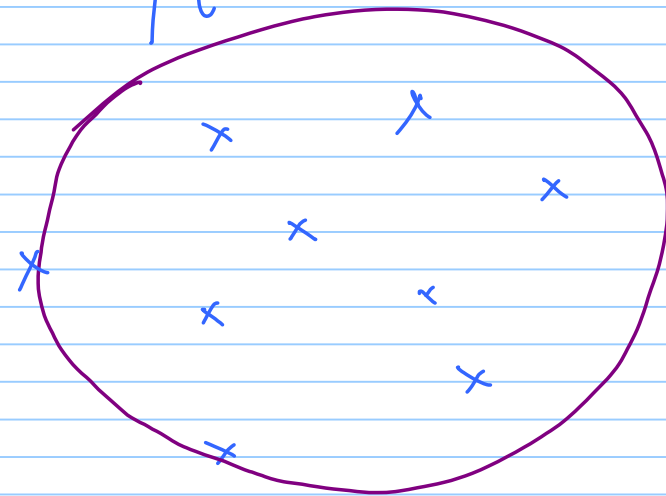
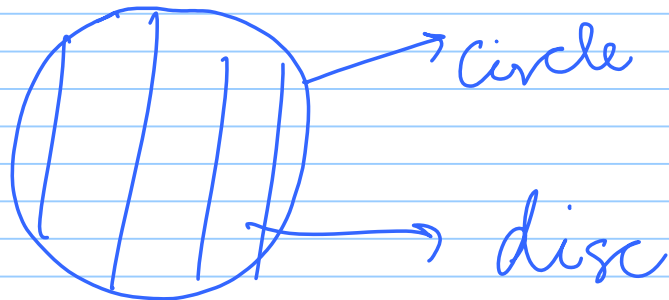


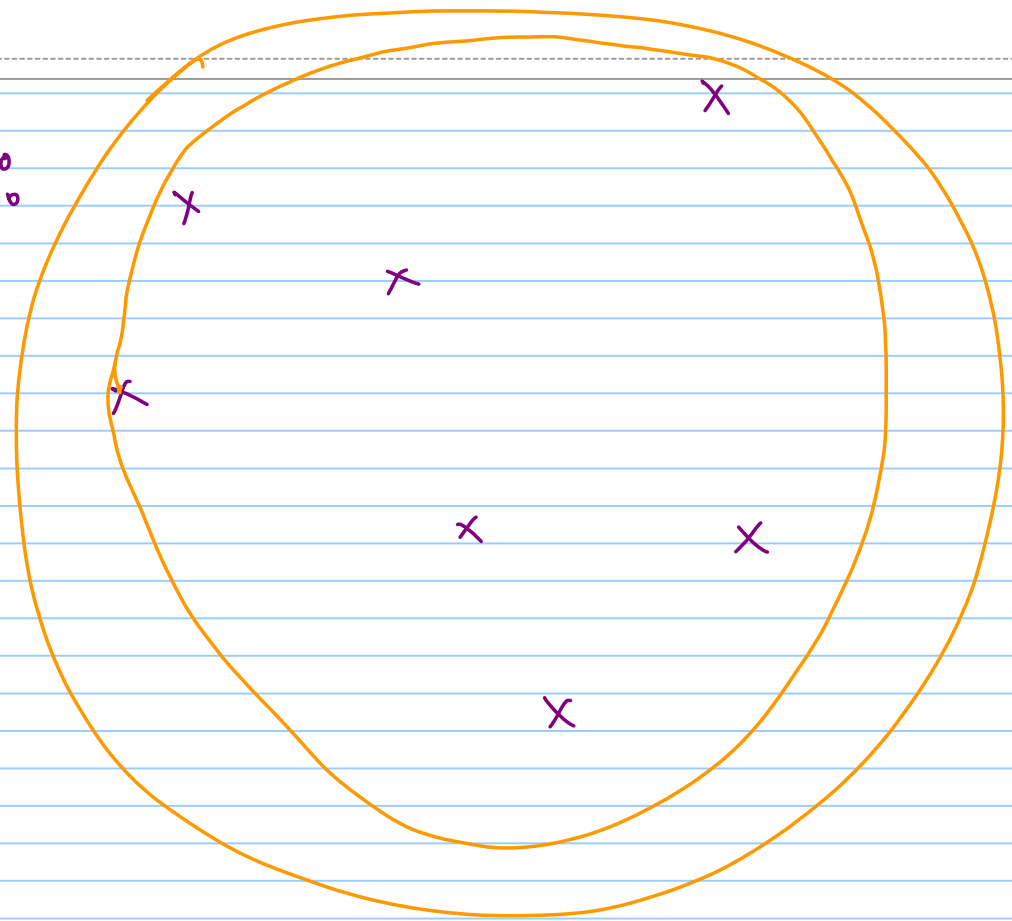
# MINIMUM ENCLOSING DISC

(Smallest) Assumption: No 4 points are cocircular.

$$P = \{p_1, p_2, \dots, p_n\} \quad p_i \in \mathbb{R}^2$$

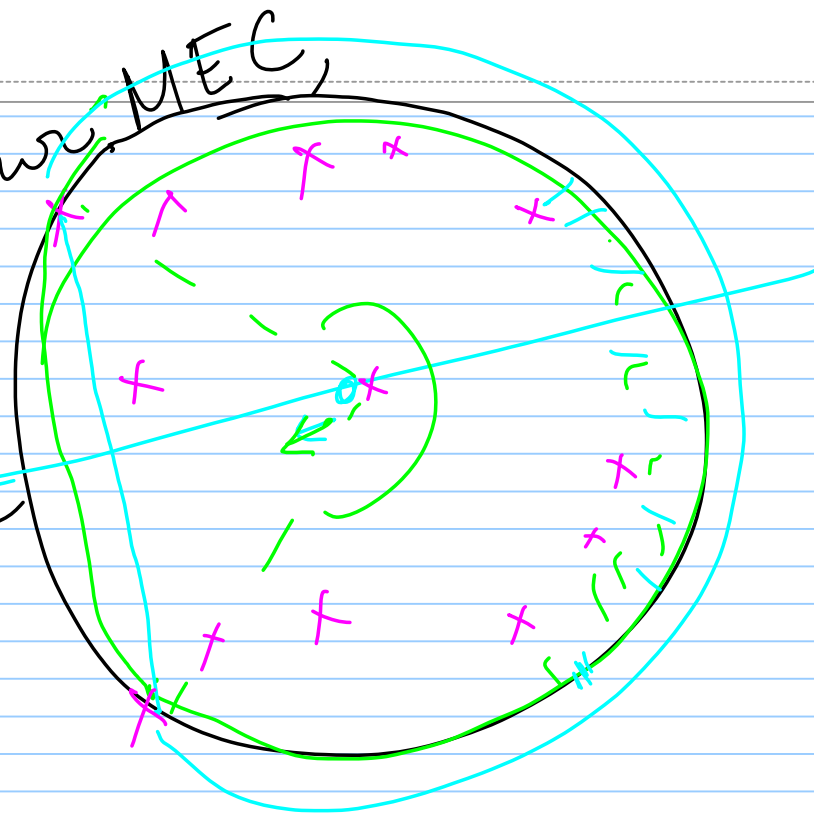


Obs<sup>n</sup> 1 :

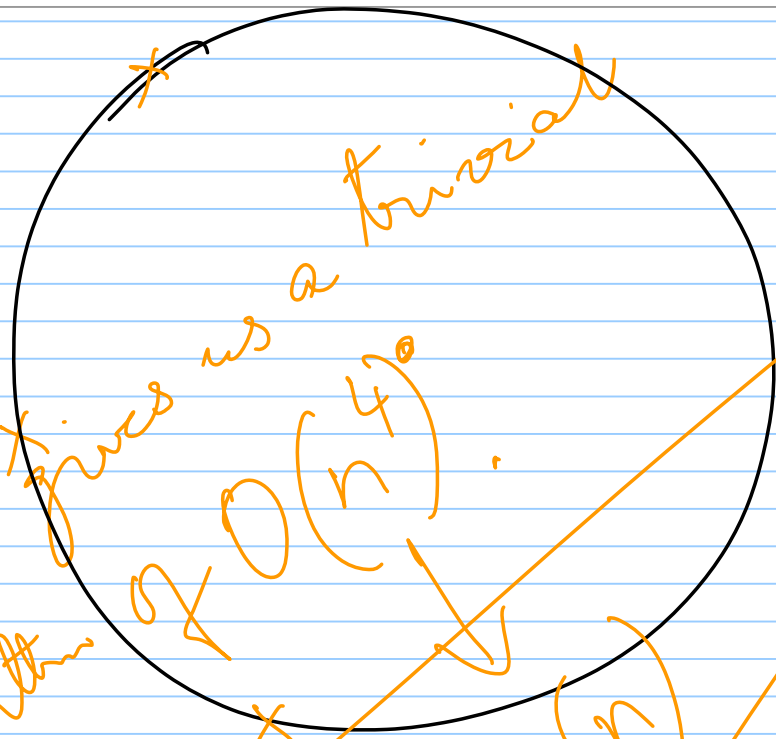


Obs<sup>n</sup> 2

If two pts define our MEC,  
then these two  
points have to be  
diametrically  
opposite.



Obs<sup>n</sup> 3



The above obs<sup>n</sup> gives us a trivial algorithm of  $O(n^4)$ .  
(goal:  $O(n)$ )

Obs<sup>n</sup>

The MEC is either going to be defined by

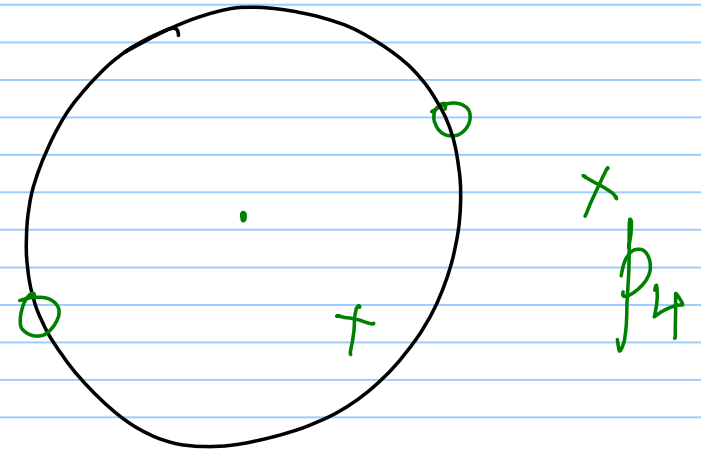
- (i) 2 pts of  $P$  (name them  $p, q$ ) s.t.  $p, q$  are diametrically opposite
- (ii) three points such that the greatest angle / arc subtended is less than  $\pi$ .

Ex: Solve the MEC using farthest neighbor  
Voronoi diagram,  
—  $O(n \log n)$  ??

# Randomized Incremental Construction.

$$\mathcal{P} = \{ \underbrace{p_1, p_2}, p_3, p_4, \dots, p_n \}$$

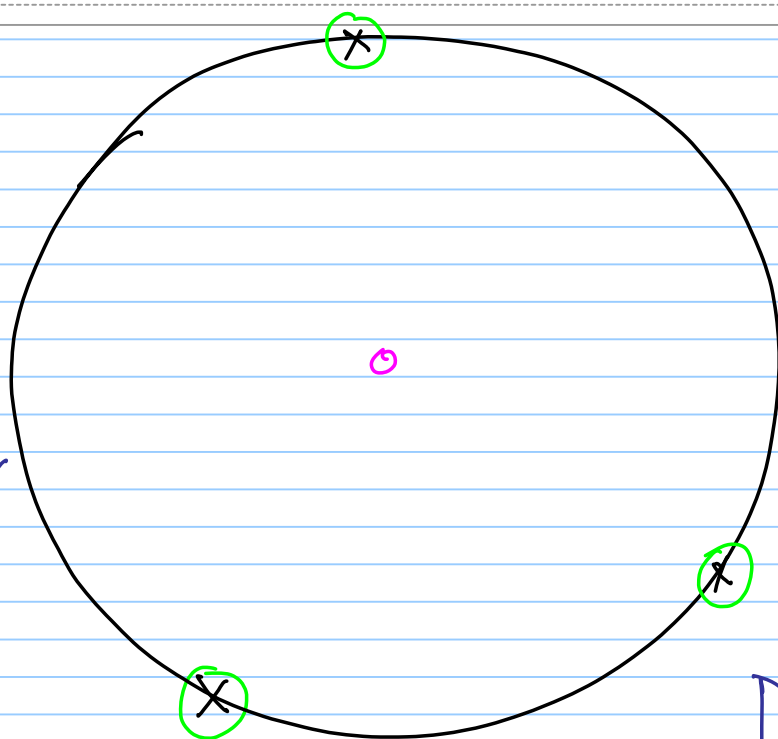
$$\mathcal{P}_i = \{ p_1, p_2, \dots, p_i \}$$



$$p_i \notin D_{i-1}$$

Q: What can we say about  $D_i$ ?  
How is  $D_i$  formed?

$p_i$

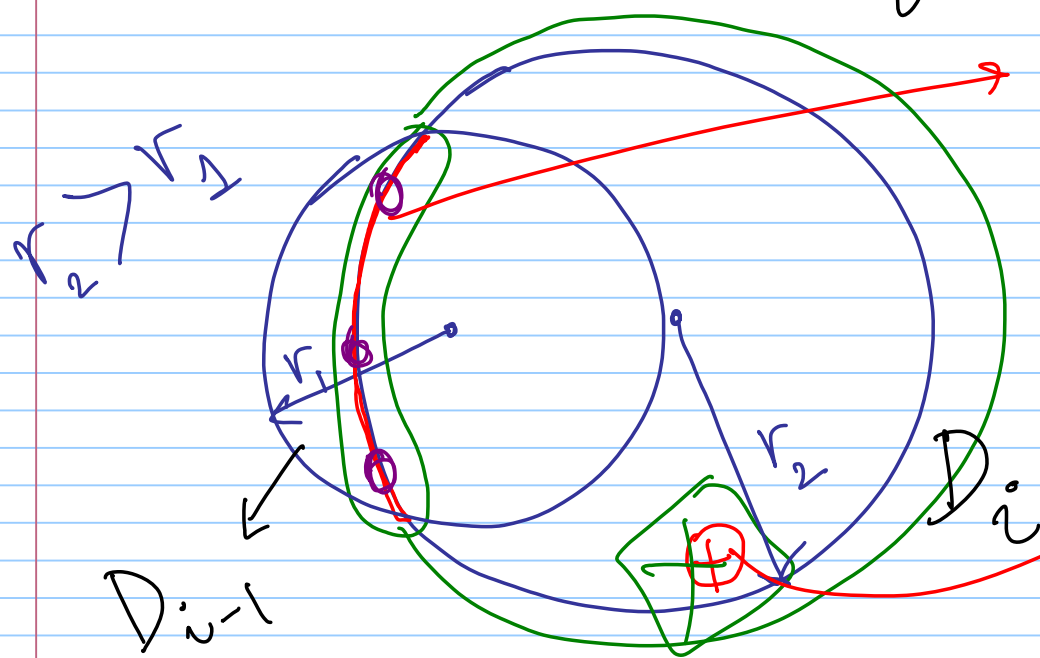


$$\text{MEC}(p_{i-1}) = D_{i-1}$$

$$D_{i-1} : \text{MEC for } p_{i-1}$$

$$D_i : \quad \quad \quad p_i$$

Claim: If  $p_i \notin D_{i-1}$ , then  $p_i$  lies on the boundary of  $D_i$ .



→ this arc subtends an angle less than  $\pi$ .

Assume for a contradiction,  $p_i$  doesn't lie on the boundary of  $D_i$



MinDisk(P)

1. Randomly permute P
2. for (over |P|) {

if  $p_i \in D_{i-1}$ ,  $D_i = D_{i-1}$   
else MinDisk1( $P_i, p_i$ );

}

MinDisk1(P, p)

- {
1. Randomly permute P;
  2. for (
- }

if  $q_i \in D'_{i-1}$ ,  $D'_i = D'_{i-1}$

else MinDisk2(P, p,  $q_i$ );

}

MinDisk1(P, p, q)

{

you have got  
3 precise pts.  
that define  
 $D_i$

$$P_i = \{p_1, p_2, \dots, p_i\}$$

↓  
 $D_i$  is the same

$$O(n) +$$

$$\sum_{i=1}^n \frac{O(1)}{i}$$

$$= O(n)$$

$$\sum_{i=1}^n O(i) \approx O(n^2)$$

Min

$$f(x)$$

$$\sum a_i x_i$$

linear  $\langle a_i \rangle$

s.t. a set of constraints

$$g_i(x) \leq b_i$$

$$x \geq 0$$

