VORONOI DIAGRAM

Input: $P = \{p_1, p_2, \dots, p_n\}$ each $p_i \in \mathbb{R}^2$

Assumption: no four or more points are cocircular.

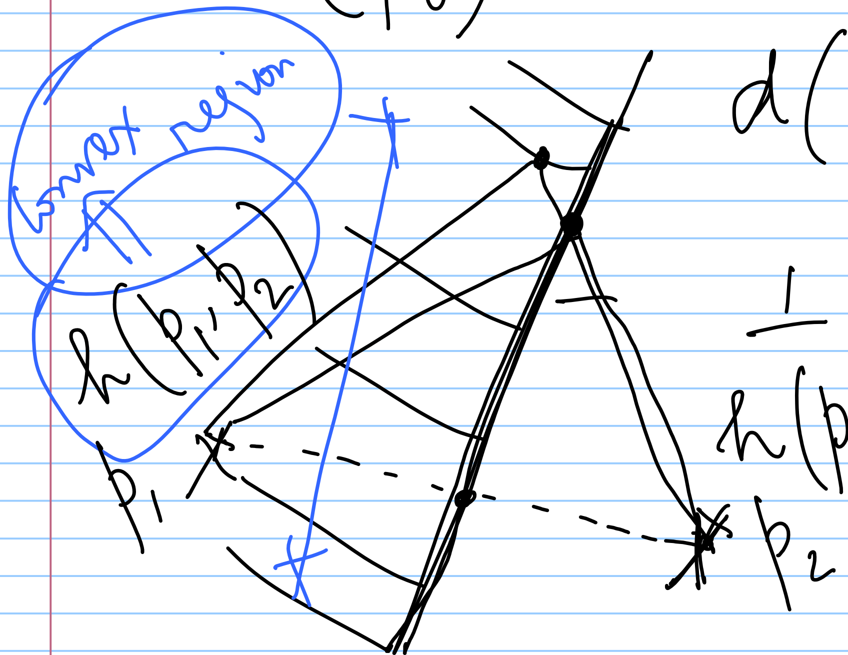
$p = (p_x, p_y) \quad q = (q_x, q_y) \qquad d(p,q) = \sqrt[p]{|p_x - q_x|^p + |p_y - q_y|^p}$

l_2- distance $\quad p = 2$

$P = \{p_1, p_2, \ldots, p_i, \ldots, p_n\}$

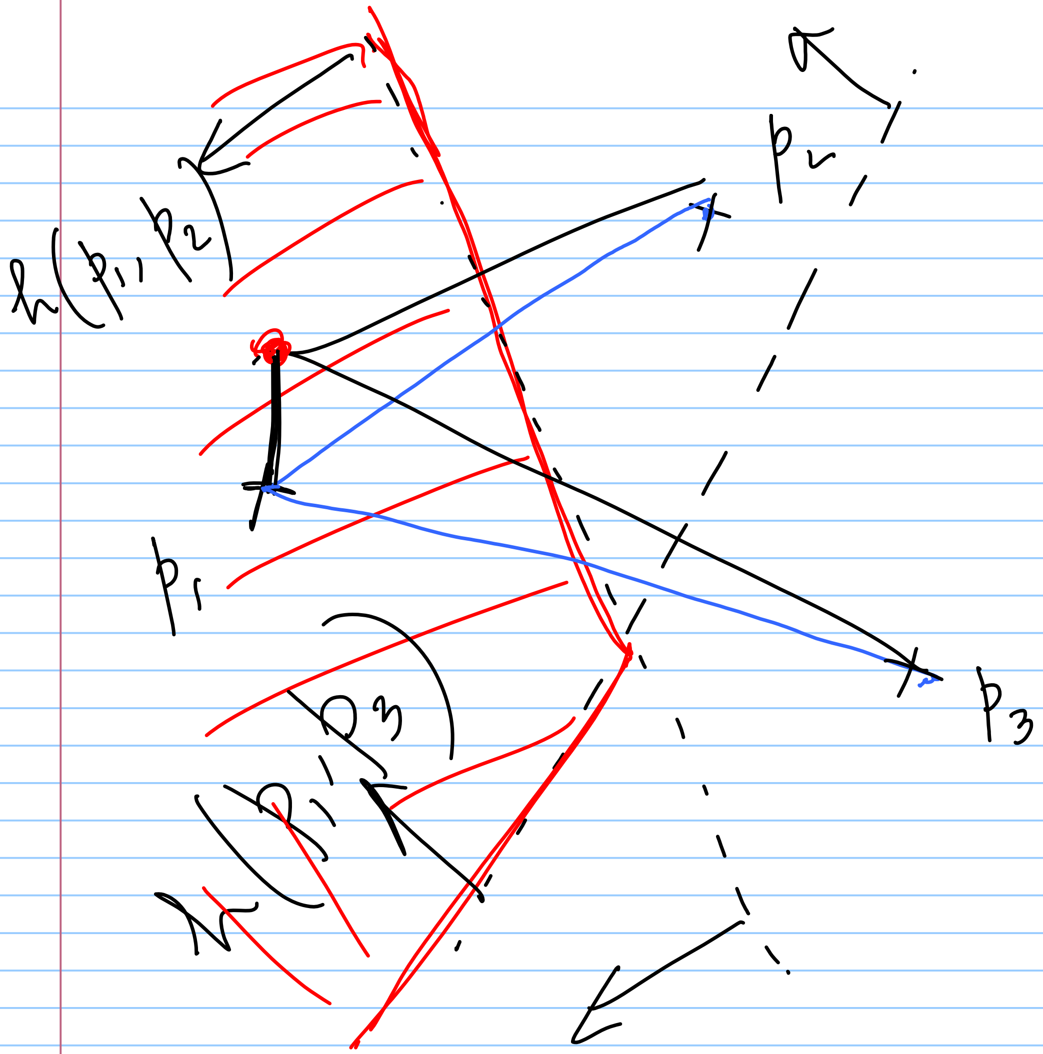
$Vor(p_i)$ = the set of all points $q \in \mathbb{R}^2$, s.t.

$d(q, p_i) < d(q, p_j) \quad \forall j \neq i$, and $p_j \in P$



$\perp$ bisector of $\overline{p_1 p_2}$

$h(p_2, p_1)$



$\mathrm{Vor}(p_1) = ?$

convex set

$$\underbrace{h(p_1, p_2) \cap h(p_1, p_3)}$$

Ex: $\bigcap$ Convex sets

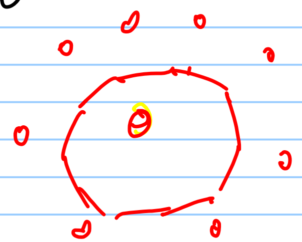
= Convex Set.

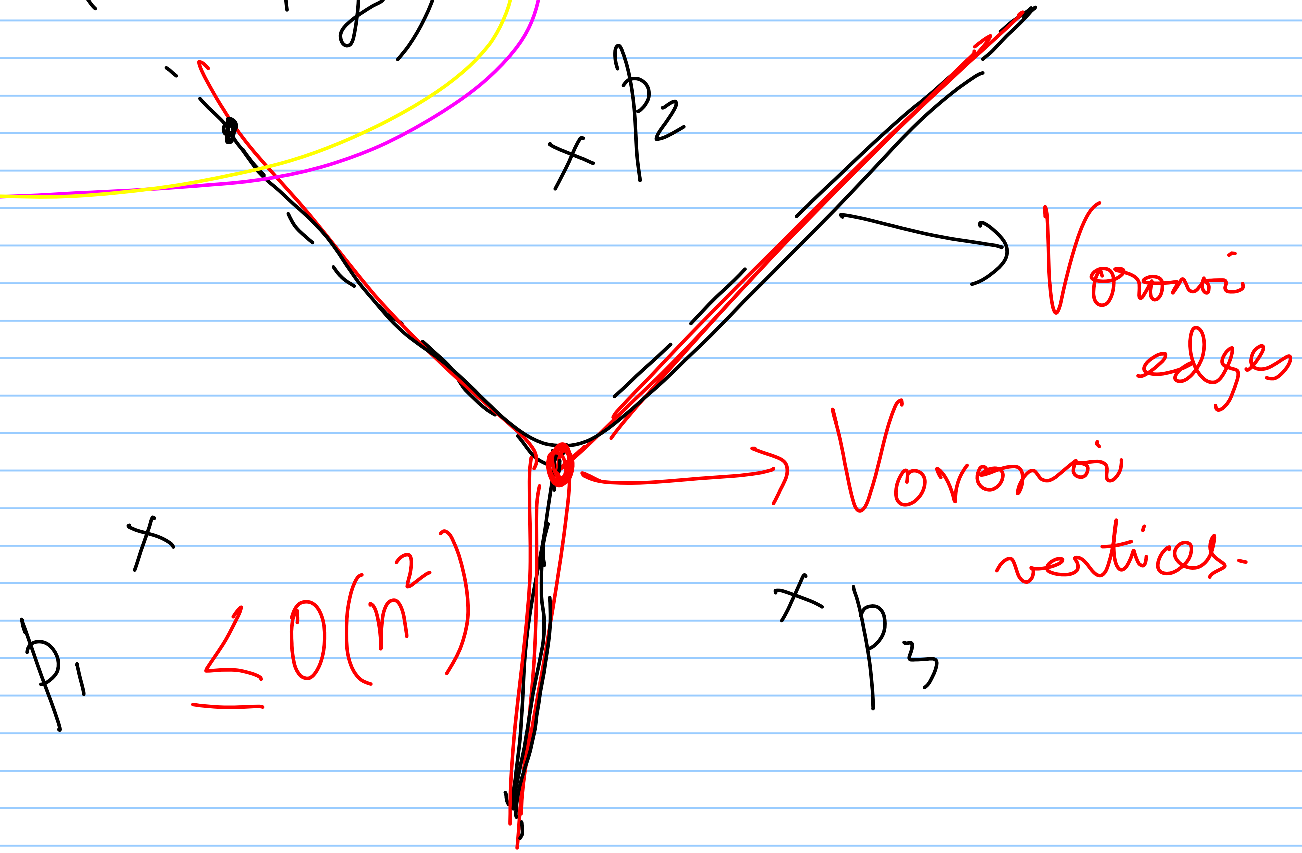
$$Vor(p_i) = \bigcap_{\substack{j \neq i \\ 1 \leq j \leq n}} h(p_i, p_j)$$

$P = \{p_1, \ldots, p_i, \ldots, p_n\}$

$\Downarrow$

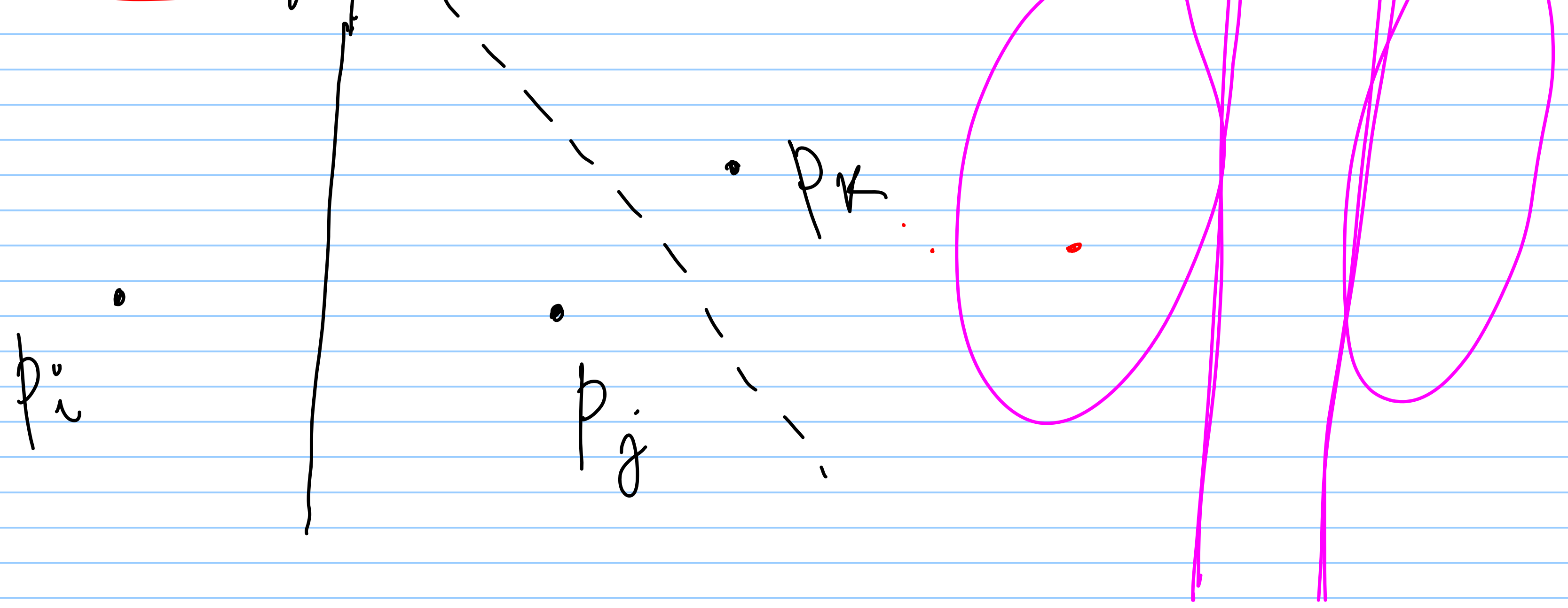
Convex region

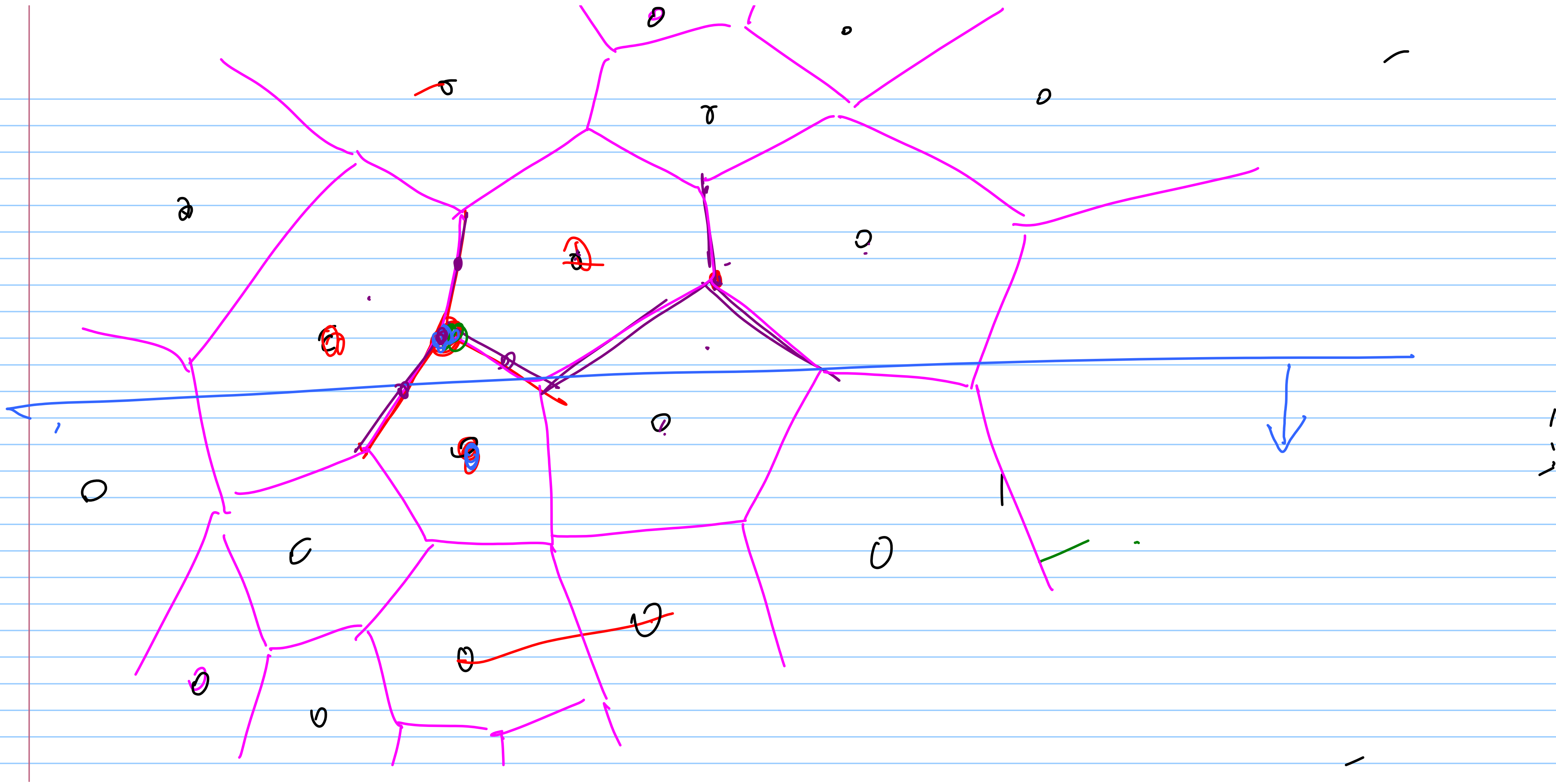




$|P| \geq 3$

Voronoi diagram: Voronoi diagram of a pt. set P is a "connected" structure with Voronoi edges being either half lines or line segments.





Euler's formula: planar connected graph: $n_{Vor} \le ?f(n)$ $e_{Vor} \le ?f(n)$

n: vertices e: edges f: faces.

$$n - e + f = 2$$

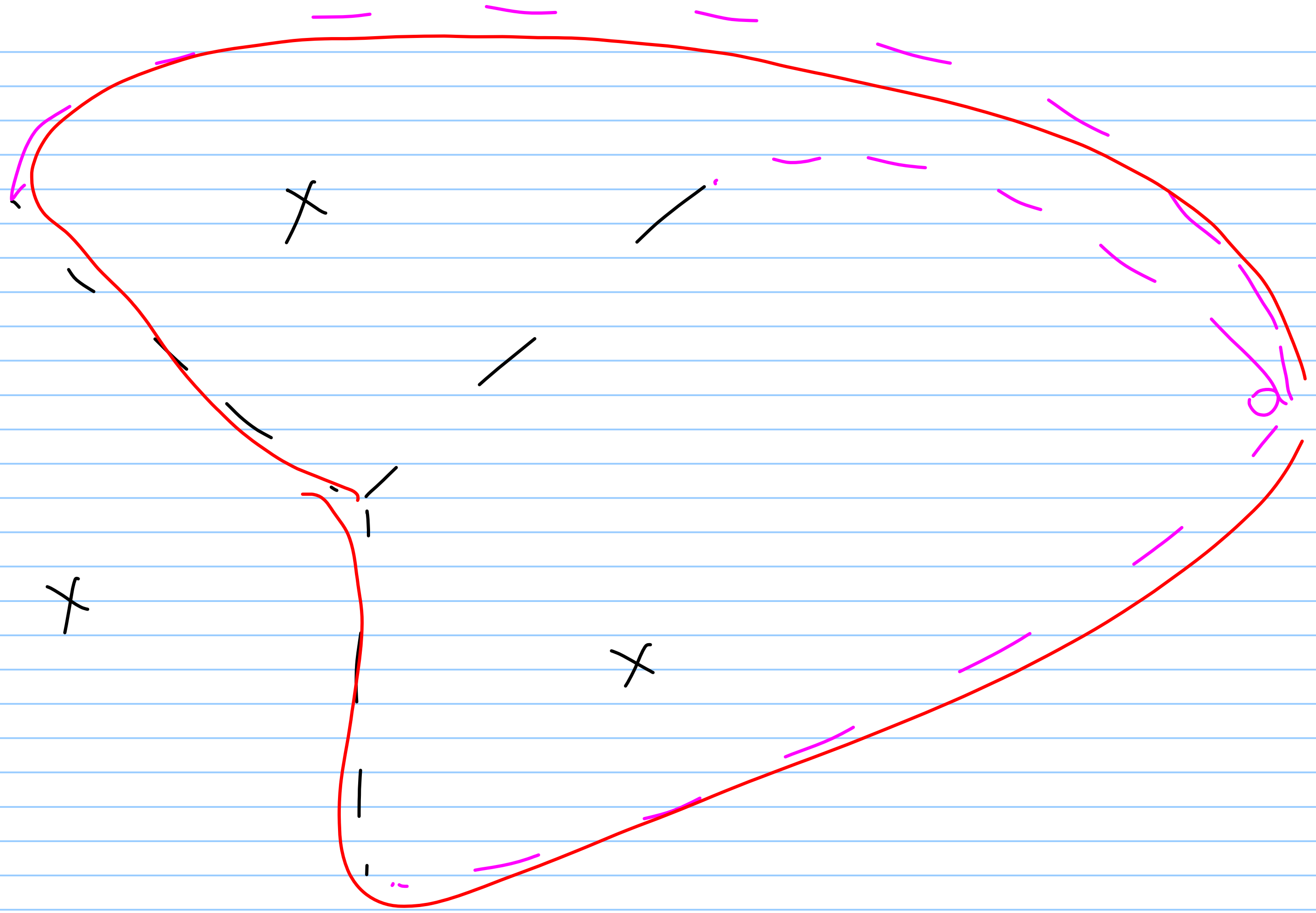
$\#$ Voronoi vertices $= n_{Vor}$ $\#$ Vor edges $= e_{Vor}$ $\#$ Vor faces $= f_{Vor} = n$

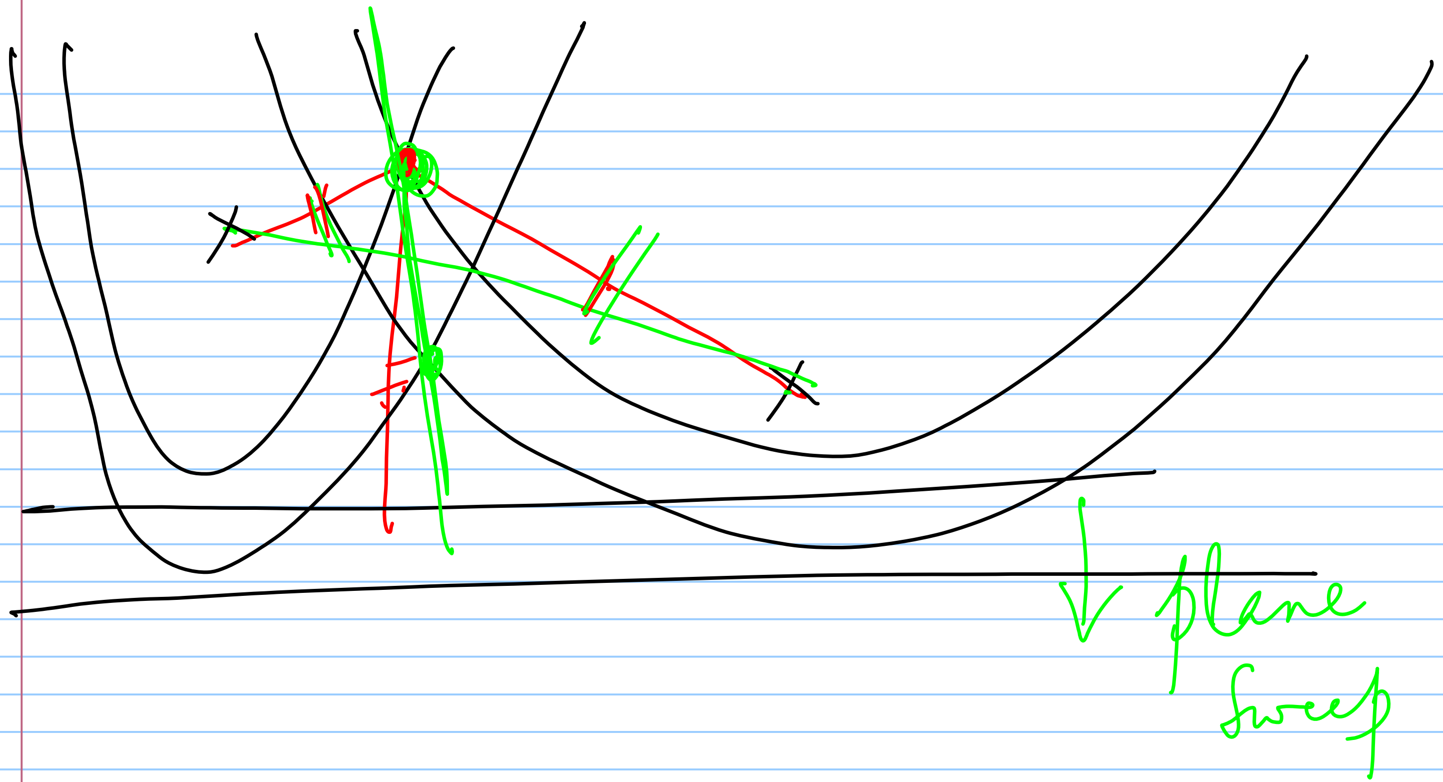
$$(n_{Vor}+1) - e_{Vor} + n = 2 \quad \cdots (1)$$

$$2e_{Vor} \geq 3(n_{Vor}+1) \quad \cdots (2)$$

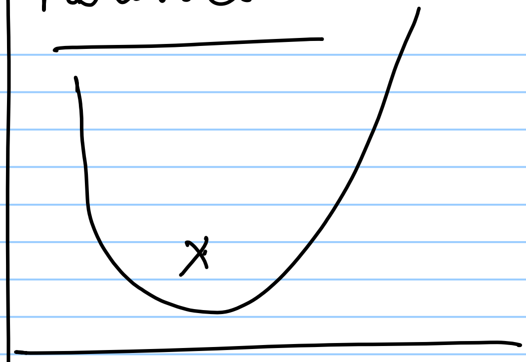
$n_{Vor} \le 2n-5$

$e_{Vor} \le 3n-6$





Parabola



ℓ_2

ℓ_1, ℓ_∞

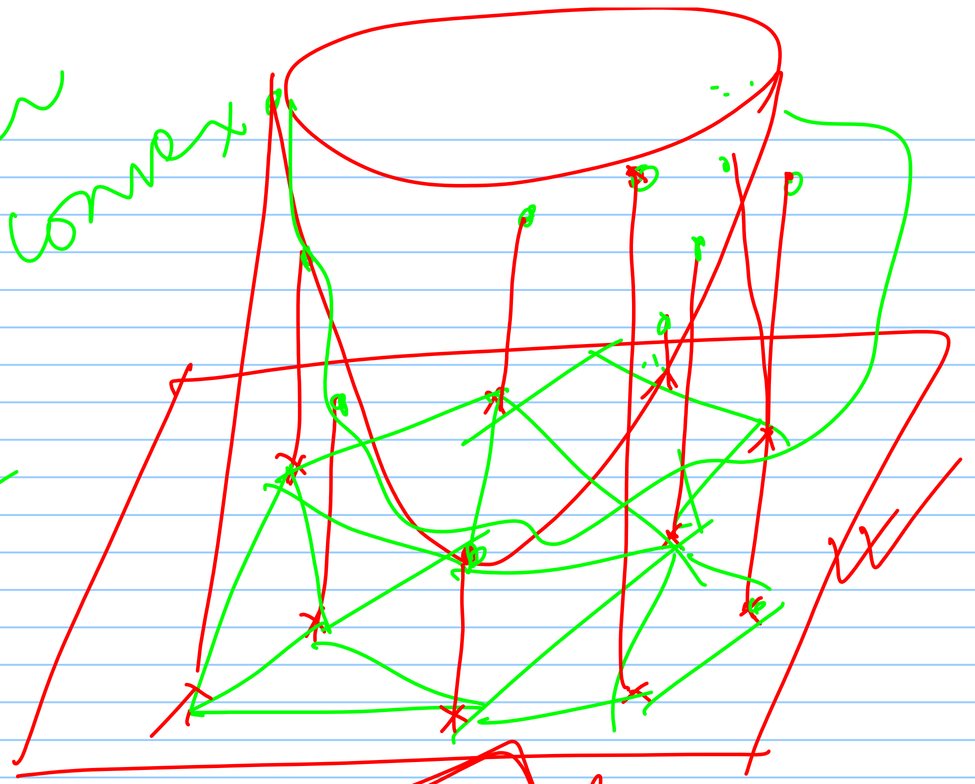

Site event
circle events.
beach line
sweep line
O(n log n)

Convex Hull, Voronoi diagrams

& Delaunay triangulation,

distances.

Project the lower envelope of the convex hull onto $\mathbb{R}^2$



$z = x^2 + y^2$

$(x,y) \longmapsto (x, y, x^2+y^2)$

lower envelope of the convex hull

$p, q, r, s \longleftrightarrow \boxed{p_0, q_0, r_0, s_0}$

$z = -1$