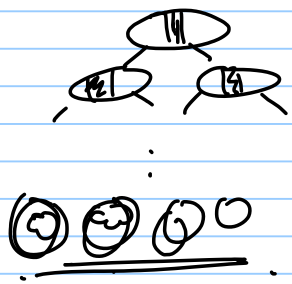


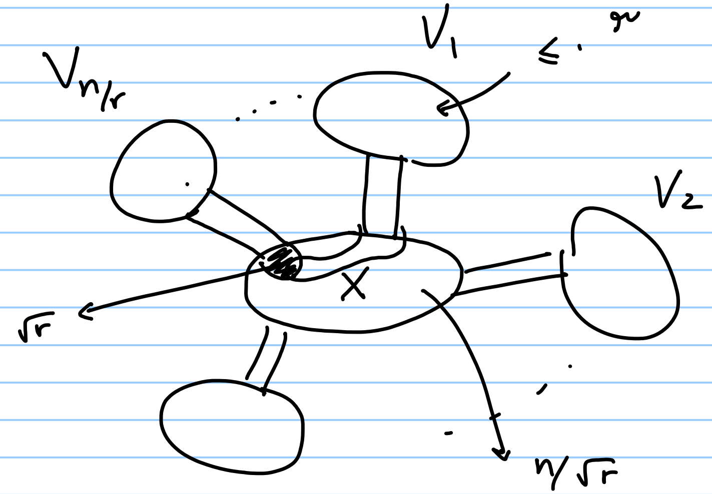
Theorem [Fredrickson '86] (r-divison) For any parameter $r > 0$, and any planar graph $G=(V,E)$ $\exists$ constants $c_1, c_2, c_3 > 0$; and a partition: $V_1, \dots, V_{n/r}, X$ of V, such that:

(i) $|V_i| \leq c_1 r$

(ii) $|X| \leq c_2 n/\sqrt{r}$

(iii) $|V_i \cap X| \leq c_3 \sqrt{r}$

(iv) $(V_i \setminus X) \cap (V_j \setminus X) = \emptyset \ \forall i \neq j$.

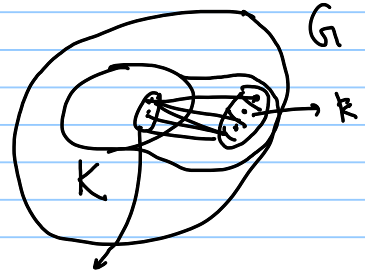


Note Title 13-06-2019

Local search : MIS in planar graphs.

LS(k)

1. Start with any IS K.
2. While possible
3. Swap k vertices from $V \setminus K$ & remove their neighbors in K if it improves the size of the IS.



G $K' = (K \cup S_k) \setminus N(S_k \cap K)$

For $k = O(1/\varepsilon^2)$, LS(k) is a $(1-\varepsilon)$-approx.

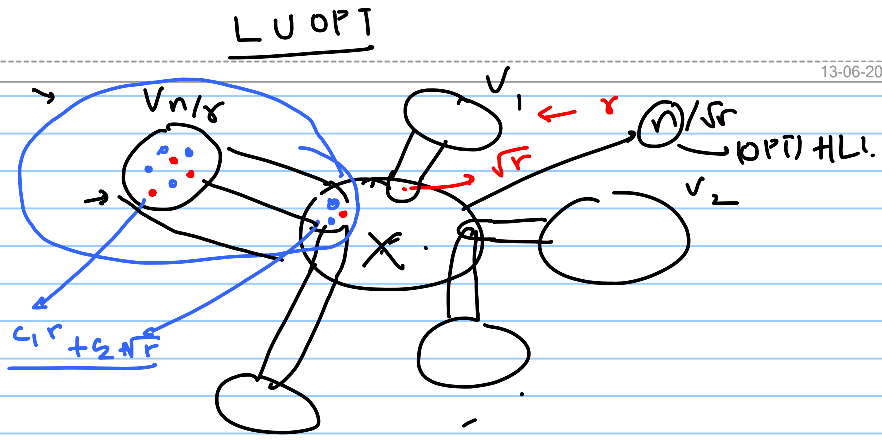
Theorem: LS(k) is a $(1-\varepsilon)$-approx. for MIS in planar graphs.

Proof:

Let L, OPT:

planar

$G[L \cup OPT]$. $(L \cap OPT = \emptyset)$



- Suppose r is chosen s.t.

  $k > c_1 r + c_2 \sqrt{r}$

- Let OPT be an optimal soln.
- Let L be the soln. returned by LS(k)

Note Title 13-06-2019

<u>Proof</u>. • We can assume: $L \cap OPT = \emptyset$.

• Let $OPT_i = OPT \cap V_i$

$L_i = L \cap V_i$

<u>Obs1</u>. $|L_i| \geq |OPT_i| \quad \forall\, i = 1 \ldots n/r$.

$$k \geq (c_1 + c_2) r$$

Choose $r \leq \frac{k}{c_1 + c_2}$.

$$|OPT| = \sum_{i=1}^{n/r} |OPT_i| + \underbrace{|OPT \cap X|}$$

$$|OPT| \leq \sum |L_i| + |X|$$

$$|OPT| \leq |L| + c_3 \frac{|L \cup OPT|}{\sqrt{r}}.$$

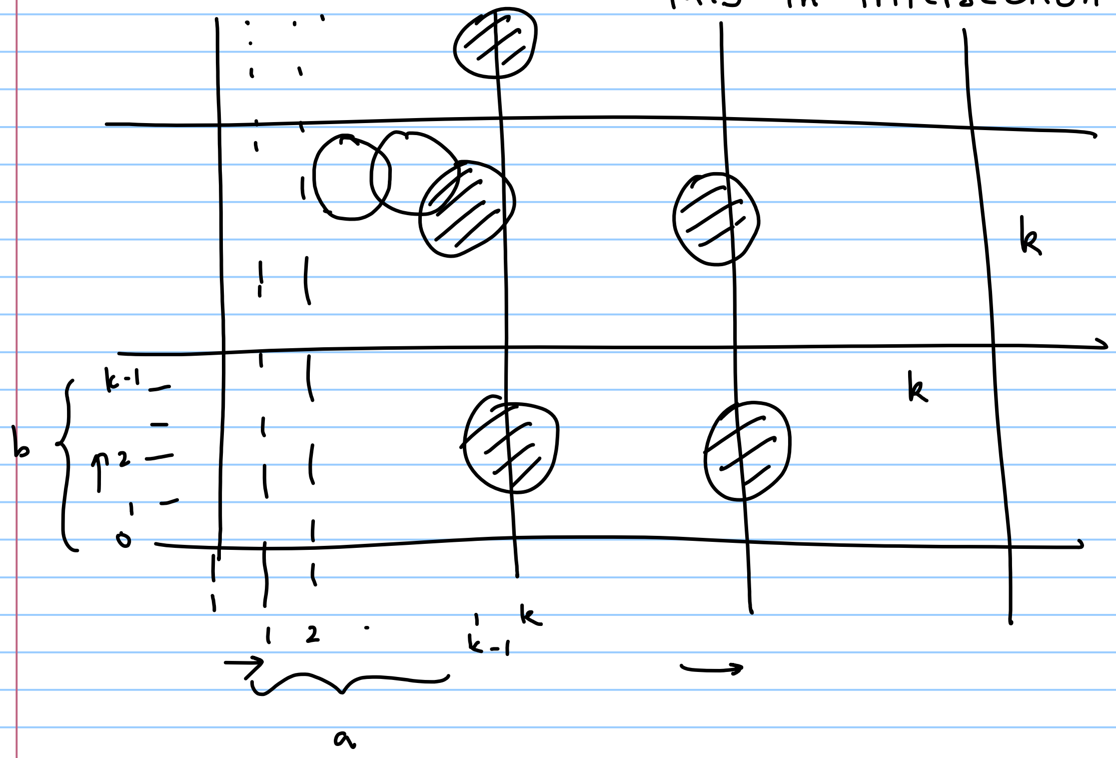
$$|OPT| \leq |L| + \alpha . \frac{|L| + |OPT|}{\sqrt{k}} \qquad k = O\left(\frac{1}{\varepsilon^2}\right)$$

$$|OPT| \left(1 - \frac{\alpha}{\sqrt{k}}\right) \leq |L| \left(1 + \frac{\alpha}{\sqrt{k}}\right).$$

$$|L| \geq |OPT| \frac{(1 - \alpha/\sqrt{k})}{(1 + \alpha/\sqrt{k})} \geq (1 - \varepsilon) |OPT|$$

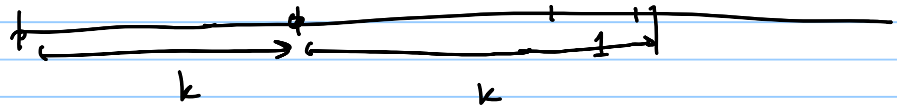
Note Title 13-06-2019

MIS in intersection graph of unit disks. (Hochbaum-Maass '82)

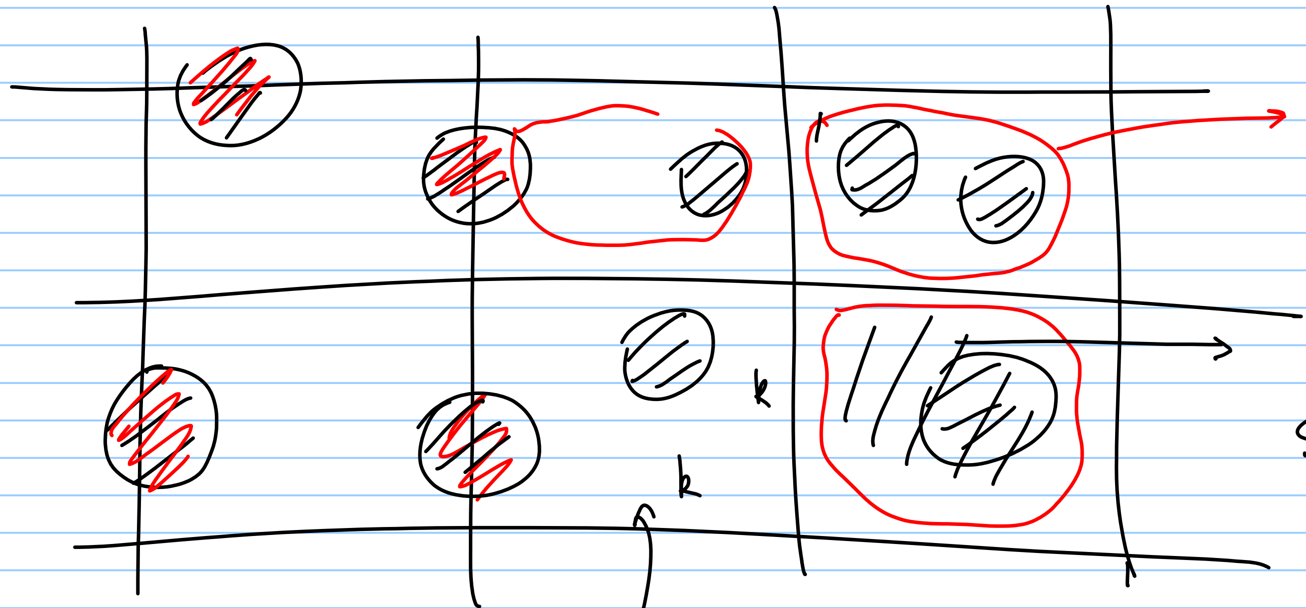


· Fix an optimal soln: OPT

Lemma: $\Pr[$a grid line intersecting a unit disk$] \le 2/k$.



· If we shift randomly; the expected # disks of OPT intersecting a grid line is $\le \frac{2\,OPT}{k}$.



$\exists$ a soln. of size $\geq (1-\frac{2}{k})$OPT

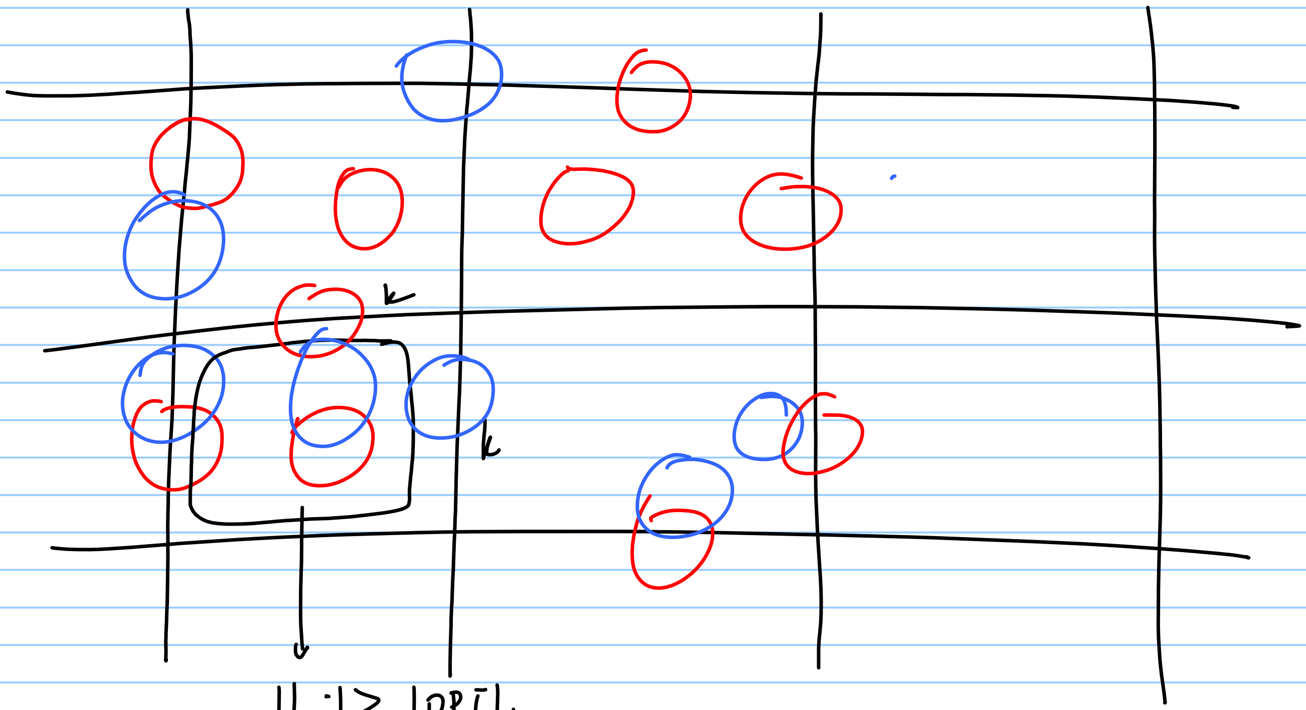
Enumerate all possible sets of size $O(k^2)$: $\binom{n}{k^2} = O(n^{k^2})$.

Packing argument : A $k \times k$ grid has area k^2; a unit disk occupies π

$\Rightarrow$ # of pairwise disjoint disks $\leq O(k^2)$

$k = 1/\varepsilon$

$LS(k^2)$



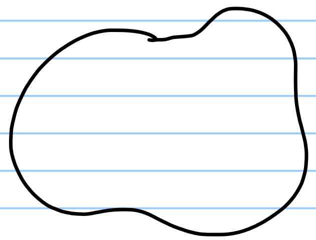
$|L_i| \geq |OPT|_i$

: OPT

: Local.

intersection graph of $OPT \cup Local$.

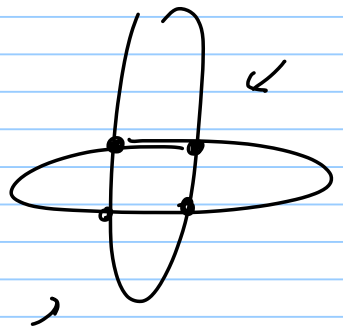
Independent Set in pseudodisks

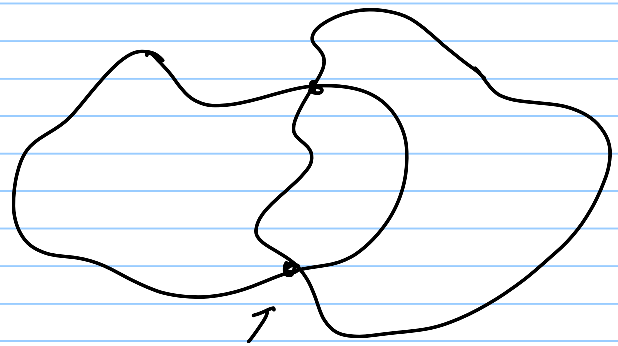


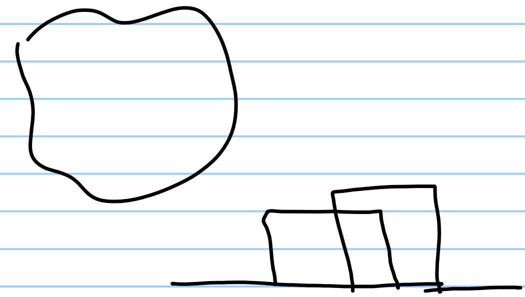
- Each region is def. by a cont. closed curve.

A collection of regions R is said to be a set of pseudodisks if

$\forall A, B \in R$







- either their boundaries intersect twice, or not at all.

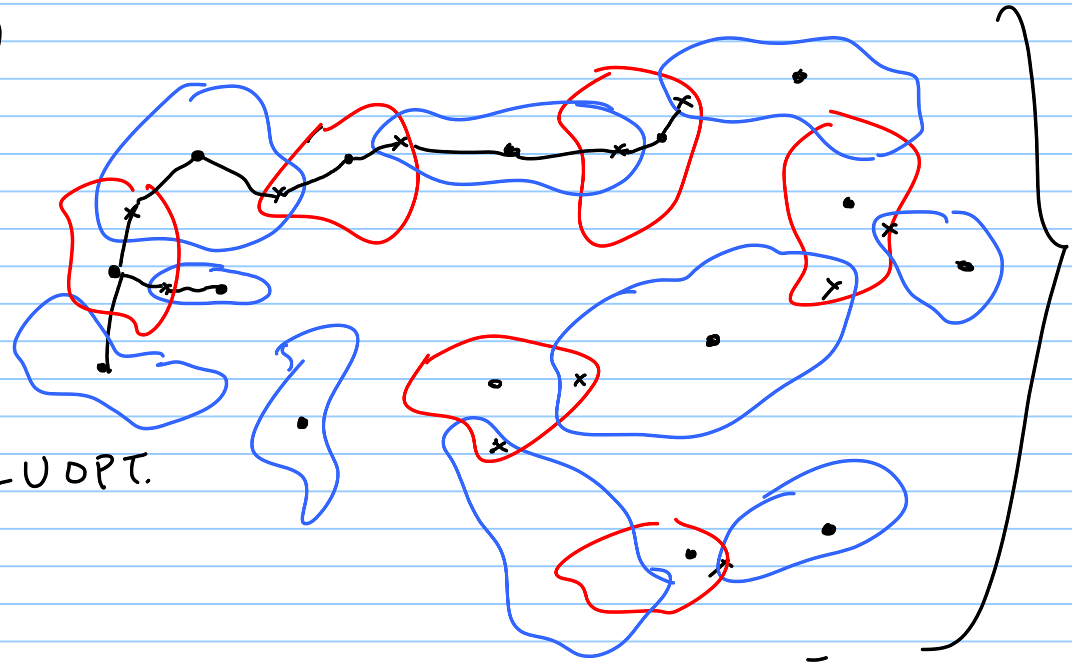
Cannot pierce

Har-Peled & Chan '09.

Alg: LS(k)

Pf. Let L be a LS soln.

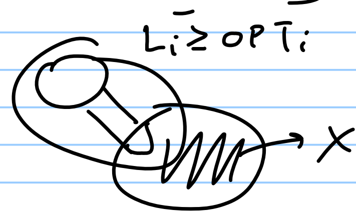
OPT: an optimal soln.



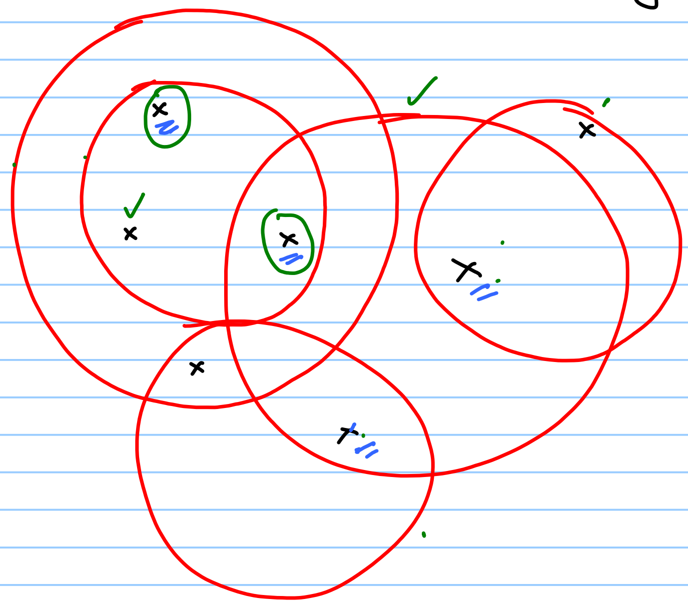
Let G be the intersection graph of $L \cup OPT$.

- **Claim:** G is a planar graph.
- For any set S of $\leq k$ vertices of OPT; $|N(S)|$ in G?

$|N(S)| \geq |S| \quad \forall\, S \subseteq OPT,\ |S| \leq k.$



Hitting set for disks.



$\mathcal{D}$ = Set of disks; V = points

· Find min. # points S :

$\forall\ D \in \mathcal{D};\quad S \cap D \neq \emptyset$

Alg: LS(k)

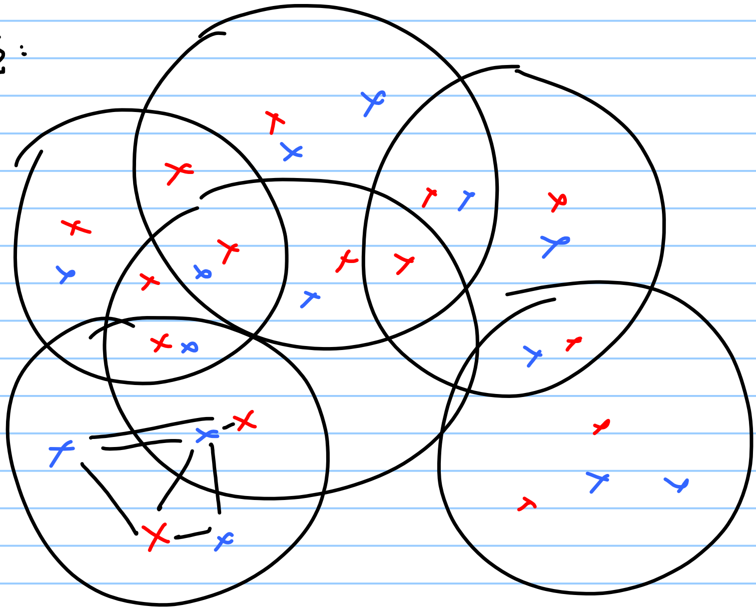
Given the current soln: K

- For each $C \subseteq K$, $|C| \leq k$.
- If we can find a set $N(C) \subseteq V \setminus K$

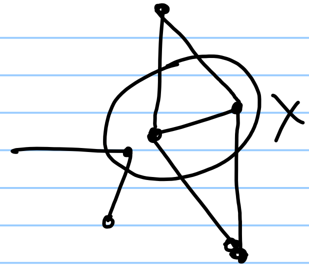
s.t $K' = (K \setminus C) \cup N(C)$ is a hitting set, and

$|K'| < |K|$ then set $K \leftarrow K'$.

Analysis:



Fix an OPT soln. & a LS soln.



$\times$: OPT

$\times$: LS.

- Construct a graph G on $L \cup OPT$.
  s.t. for any set $C \subseteq K$,
  $(K \setminus C) \cup N(C)$ is a hitting set.
- If G is planar ✓

Locality property

$\forall D \in \mathcal{D}$, $\exists$ an edge between a point in $L \cap D$ & $OPT \cap D$. ✓

- Delaunay Triangulation of $OPT \cup LS$

$\rightarrow$ $G[D]$ is connected $\forall D \in \mathcal{D}$.

LS(k).

H-minor free graphs.