Tutorial 1

ACM Summer School on Geometric Algorithms and Applications 2019

Problem 1.

In the Set Cover problem, we are given a set system $(X, \mathcal{R})$, where $|X| = n, |\mathcal{R}| = m$, and $\cup_{i=1}^{m} R_i = X$. The goal is to find a subset $\mathcal{S} \subseteq \mathcal{R}$ of minimum cardinality such that $\cup_{S \in \mathcal{S}} S = X$. Let us now look at a slightly different problem, called Max-Coverage. The input is the same as the Set Cover problem. We are additionally given a parameter $k \in \{1, \ldots, m\}$. We want to pick a collection $\mathcal{S}$ of at most k subsets from $\mathcal{R}$, such that we maximize $|\cup_{S \in \mathcal{S}} S|$, i.e., we want to cover as many points of X as possible using at most k sets of $\mathcal{R}$. Write an integer programming formulation of the problem, and relax it to obtain a linear program.

Problem 2.
Using the Set Cover LP, show that there is a collection of sets, of total size $O(OPT)$, that covers at least $1/2$ the elements. Therefore, the additional $\log n$ sets we use is essentially to cover the remaining half of the elements.

Problem 3 (2 points)**.**
Give an (infinite family) of examples, for each $n \in \{3, 4, \ldots\}$ for which the ratio OPT_{LP}/OPT_{IP} for Vertex Cover approaches $1/2$ as $n \to \infty$. In other words, starting with the linear programming relaxation we used for Vertex Cover, we can not hope to improve the approximation factor of 2.

Problem 4 (2 points)**.**

1. Show that the Vertex Cover problem is a special case of the Set Cover problem, where each element appears in exactly 2 sets.
2. Given a Set System where each element appears in at most 3 sets, devise an LP-rounding based 3-approximation algorithm.
3. The Vertex Cover LP has a special property called $1/2$-integrality: Any optimal solution to the LP-relaxation for Vertex Cover has the property that all the variable values are from $\{0, 1, 1/2\}$. Suppose each element appeared in exactly 3 sets, is it true that in any optimal solution to the LP satisfies the property that its values lie in $\{0, 1/3, 2/3, 1\}$?