Note Title 07-06-2019

Point location in a planar subdivision

PSLG : Planar straight line graph

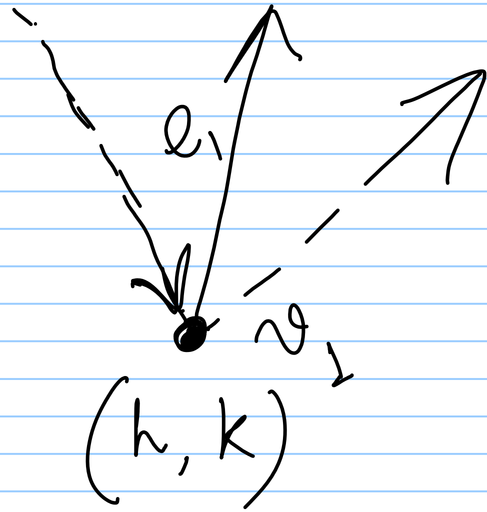
DCEL : Doubly Connected Edge List

Keep records for

1) Vertices

2) Edge

3) Faces

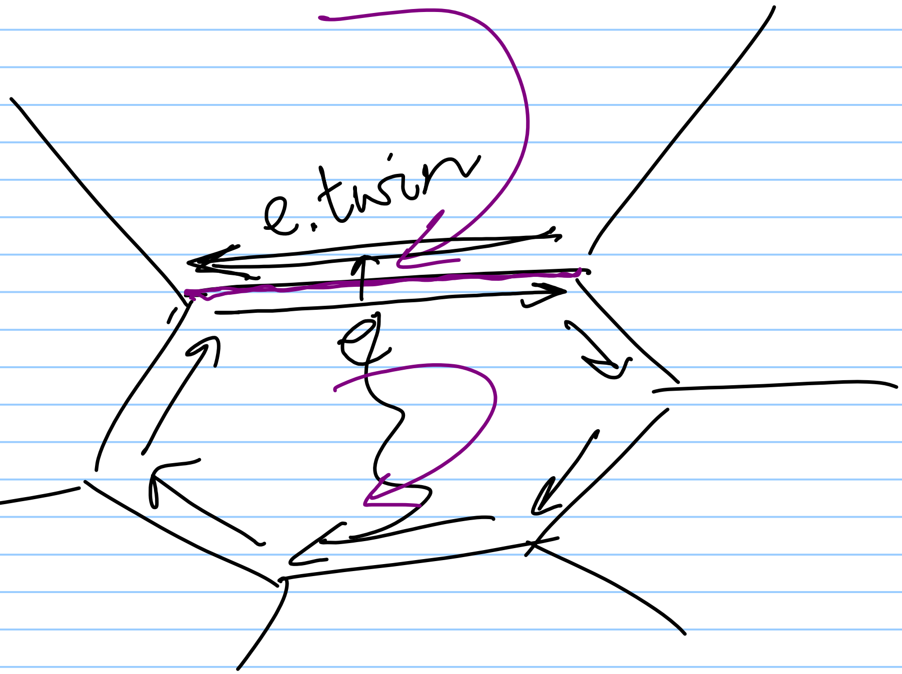


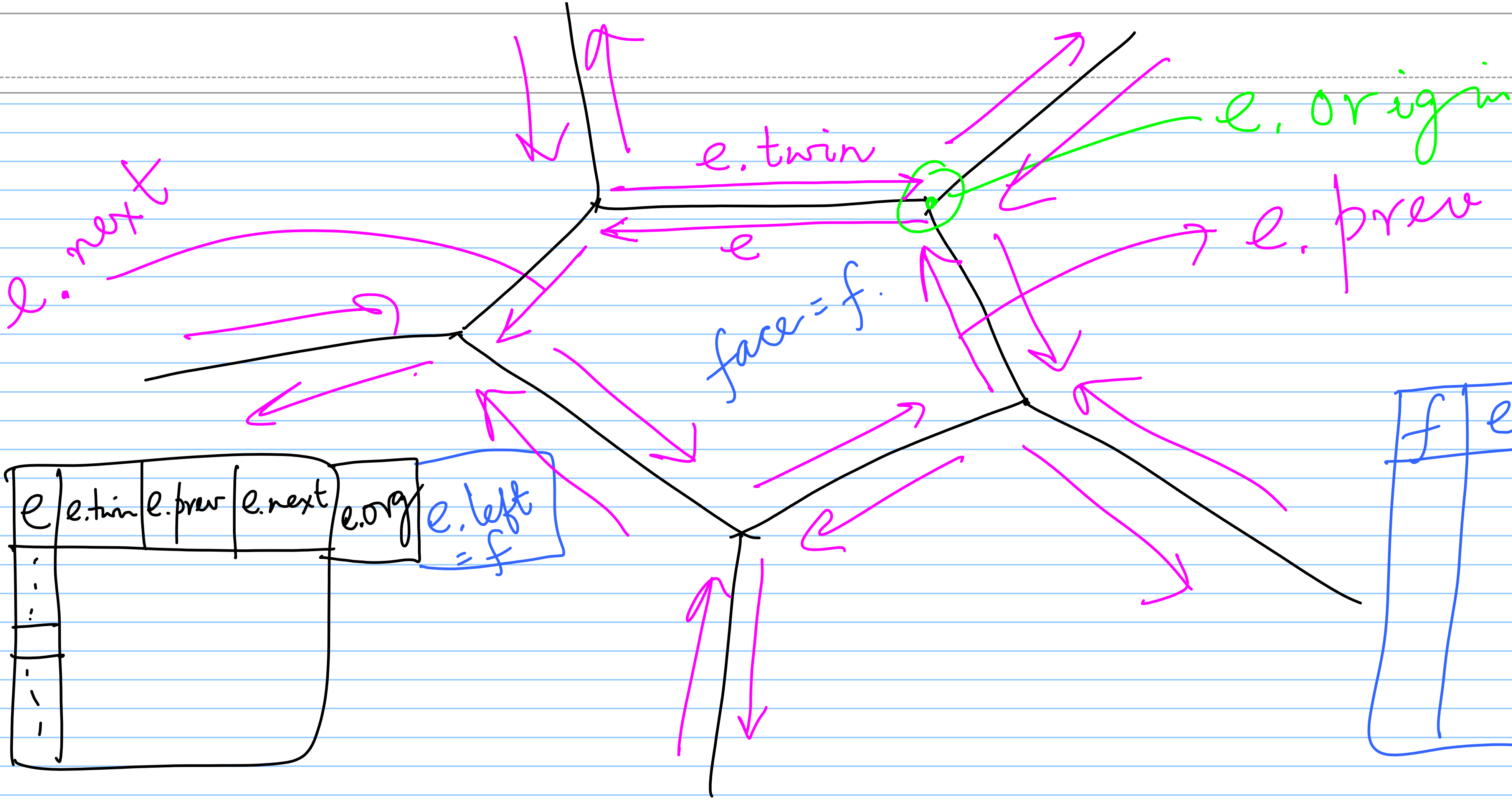
v_1	( )	e_1
v_2	( )	
⋮	⋮	
v_n	⋮	

Vertex Record.

Edge Record

e	~~e~~ e.twin





e	e.twin	e.prev	e.next	e.org	e.left = f
⋮					
⋮					

f	e

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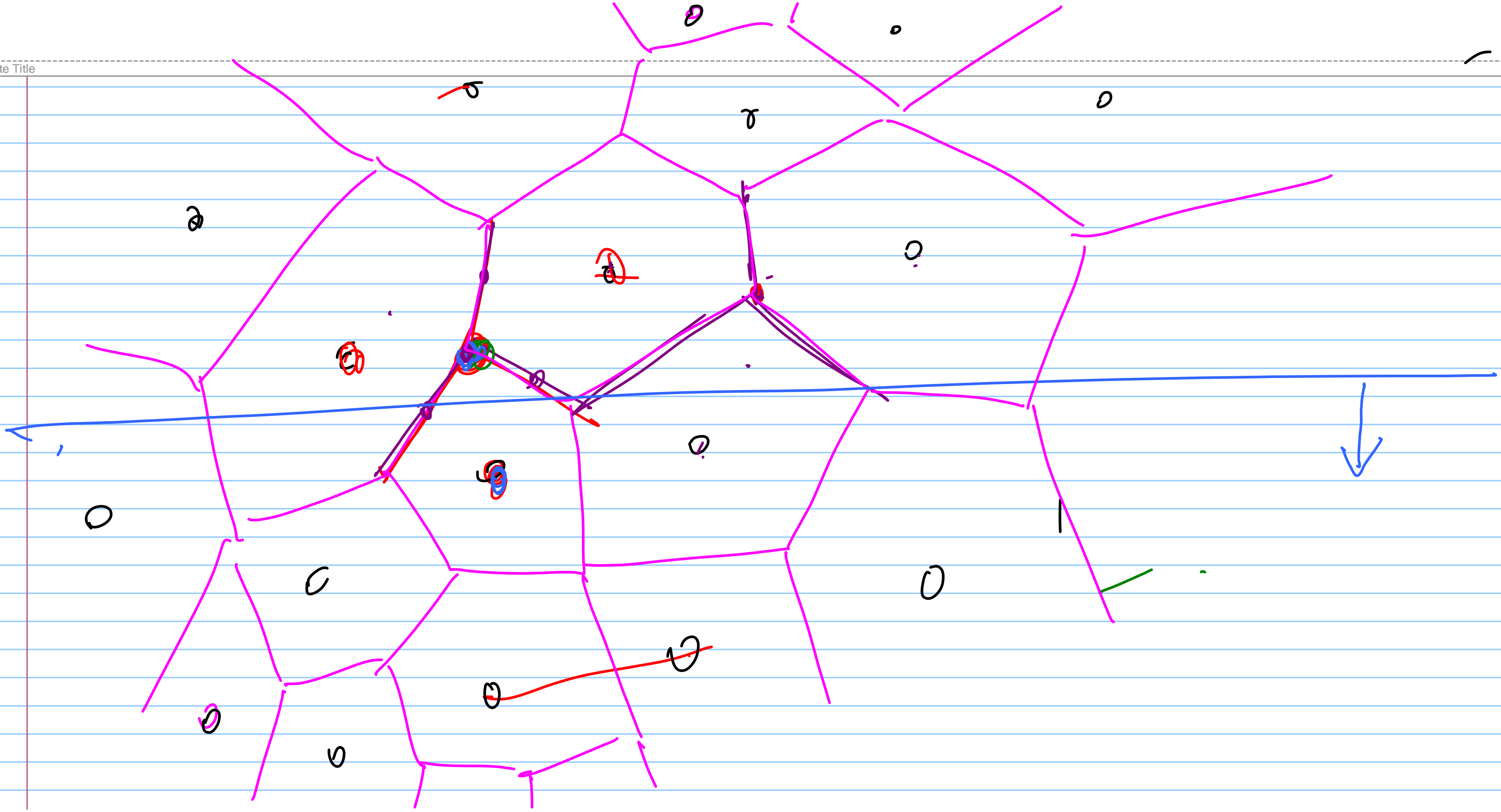


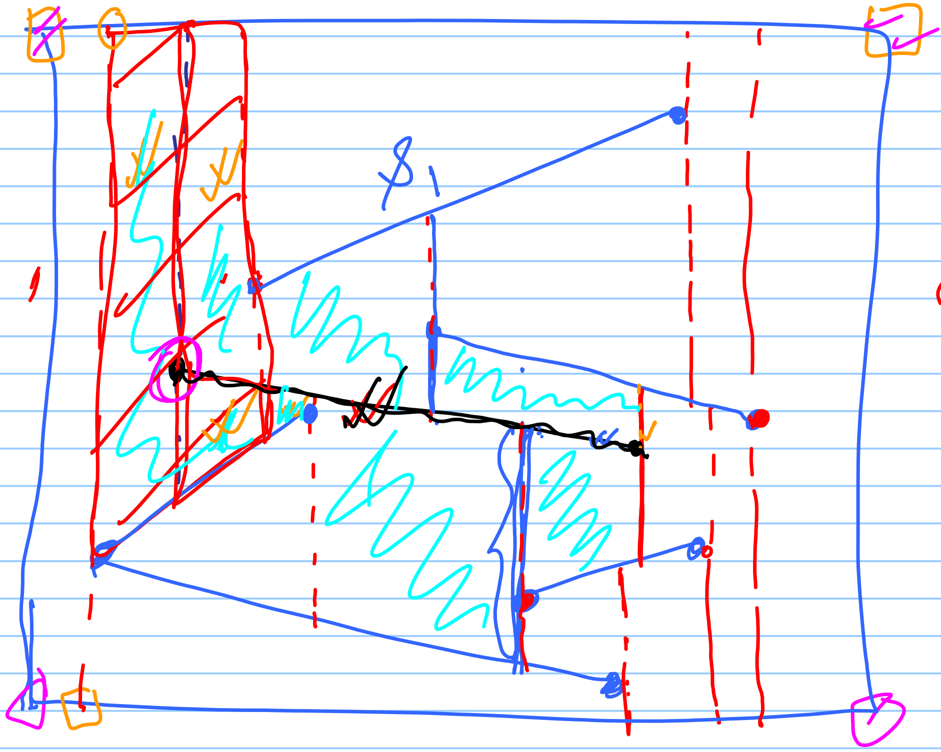
Trapezoidal decomposition

$6n+4$

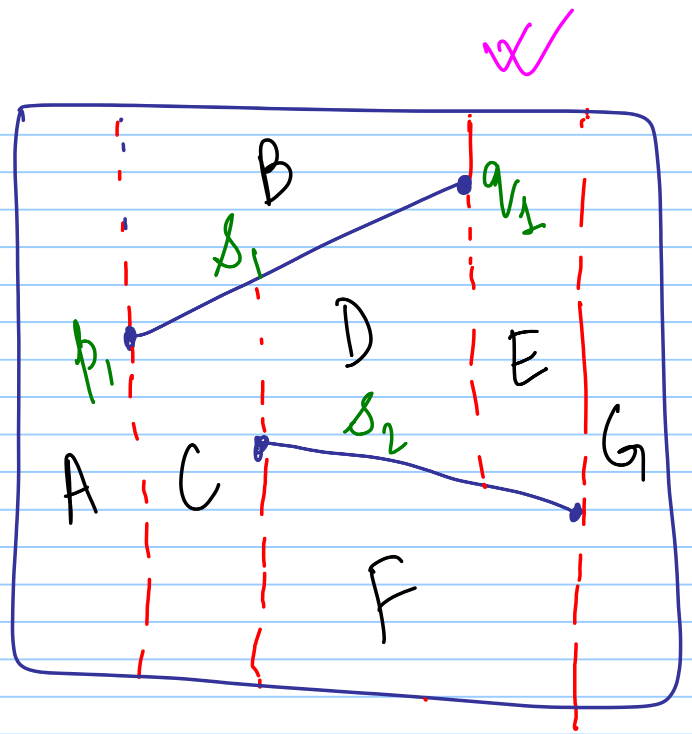
Lemma: # vertices $\leq$

trapezoids $\leq$?

$3n+1$

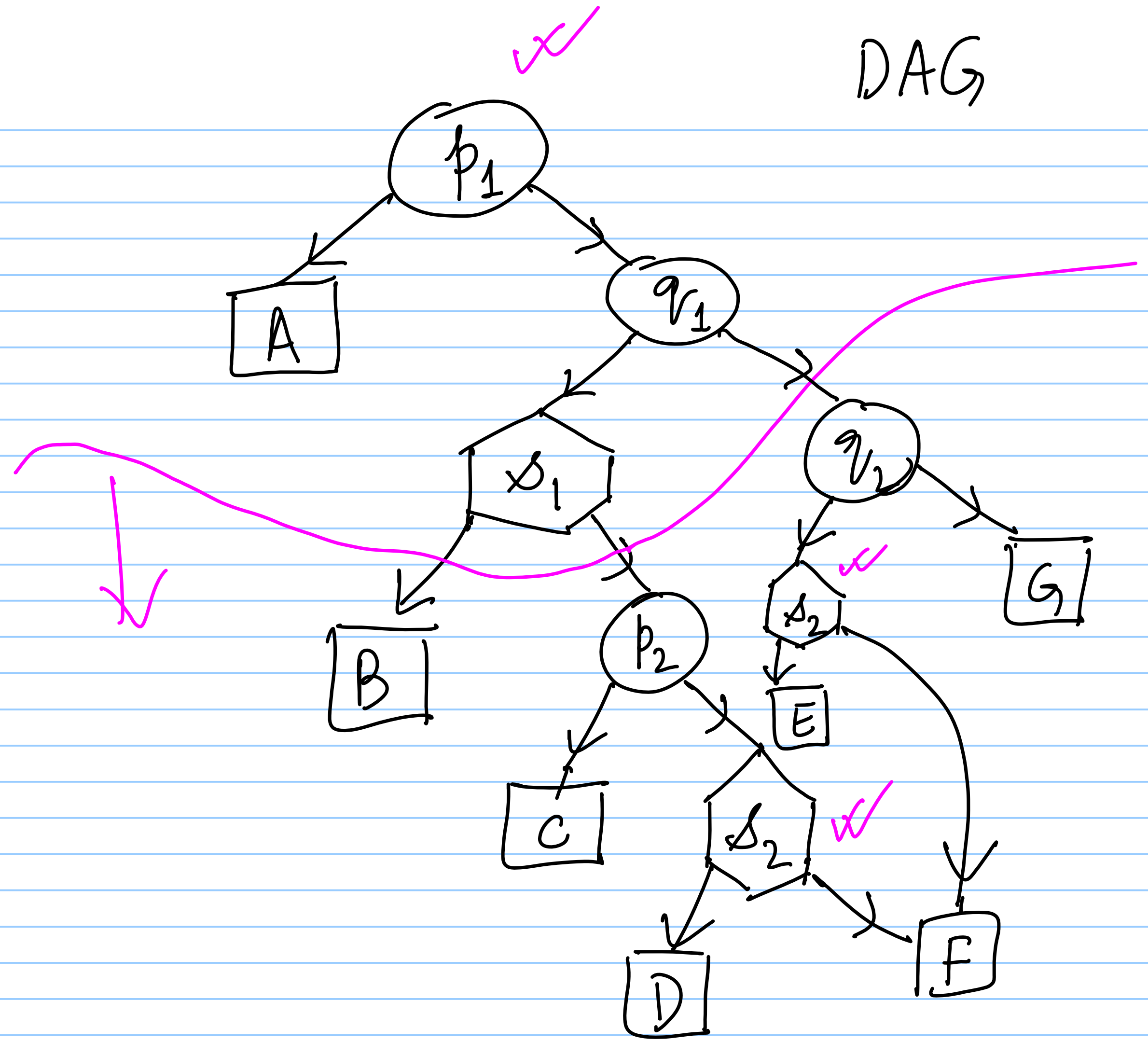


~~O(n)~~



$$|T| = O(n)$$

DAG



Claim: Ignoring the time spent to locate the left end point of a segment, time that it takes to insert the i^{th} segment & update the trapezoidal map is $O(k_i)$ where

k_i : # newly created trapezoids.

Proof (idea): For each bullet path crossed, trim the bullet and create a new face.

$E[k_i] = ?\ O(1)$

Lemma: Consider the RIC of a trapezoidal map, and let

k_i : # new trapezoids created when the i^{th} segment is added.

Then $E[k_i] = O(1)$, where the expectation is taken over all permutations of the segments

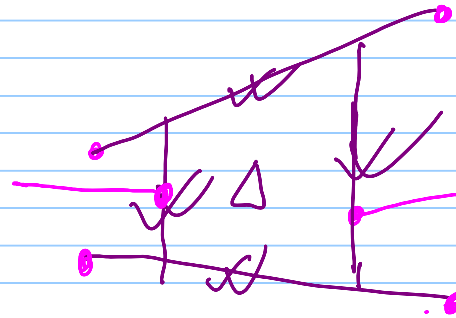
Proof: Let $\mathcal{T}_i$: trapezoidal map after i insertions. $|\mathcal{T}_i| = O(i)$

Let's say that a trapezoid Δ depends on a segment s, if s would have caused Δ to be created, had s been added last.

$$\delta(\Delta, s) = \begin{cases} 1 & \text{if } \Delta \text{ depends on } s \\ 0 & \text{o/w.} \end{cases} \qquad \Big\| \qquad S_i = \{s_1, \dots, s_i\}$$

$$E[R_i] = \frac{1}{i} \boxed{\sum_{s \in S_i}} \boxed{\sum_{\Delta \in T_i} \delta(\Delta, s)}$$

$$= \frac{1}{i} \sum_{\Delta \in T_i} \boxed{\sum_{s \in S_i} \delta(\Delta, s)} \leq 4$$

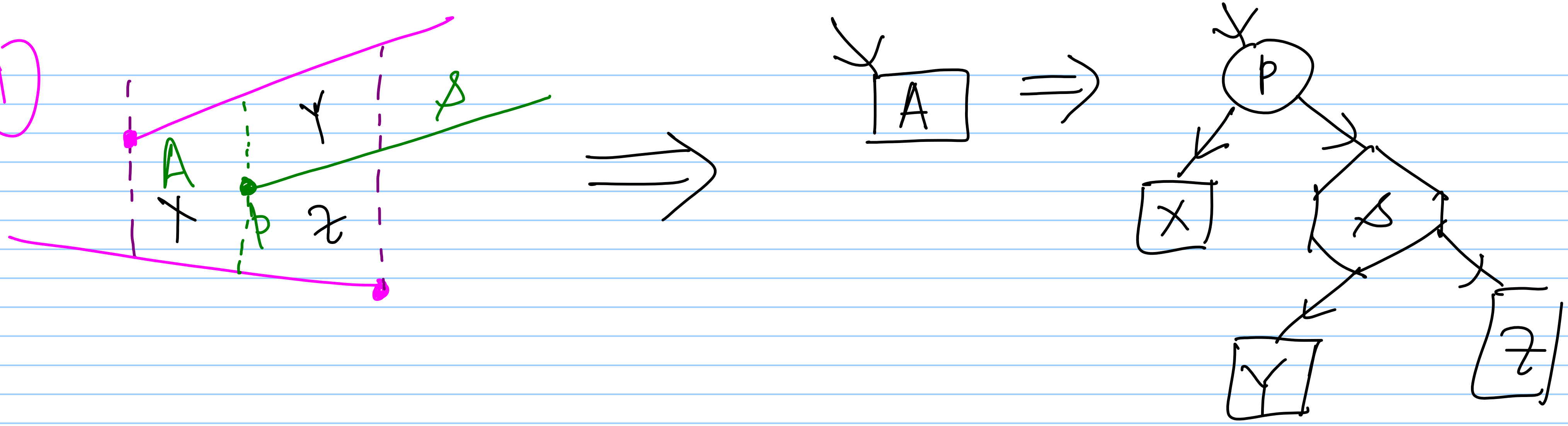


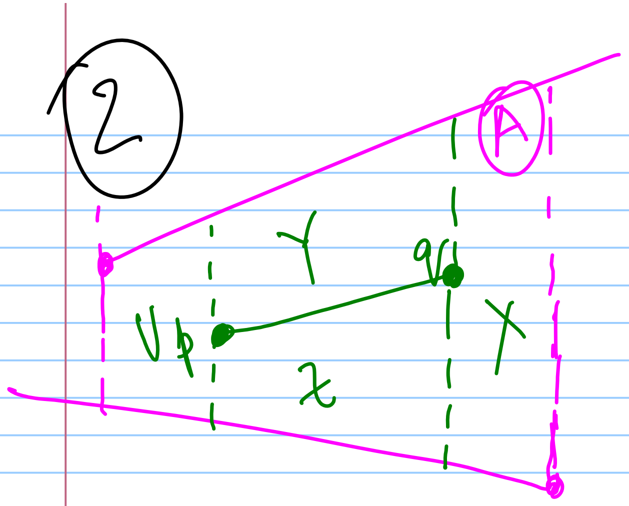
$$= \frac{1}{i} \sum_{\Delta \in \mathcal{T}_i} 4 = \frac{1}{i} 4 \cdot |\mathcal{T}_i| = O(1)$$

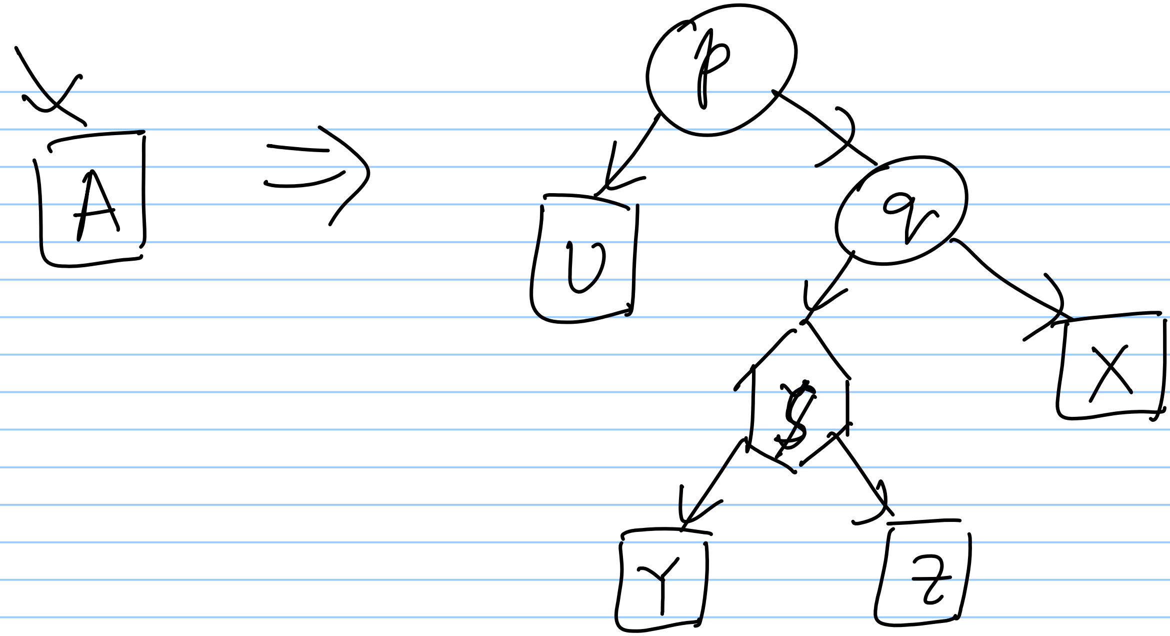
Q: How big can the data structure grow?

$$\longrightarrow O(n)$$

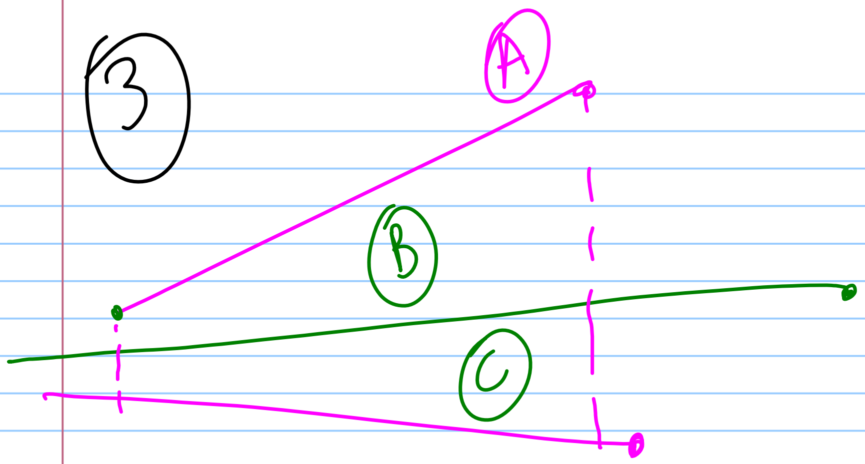
1

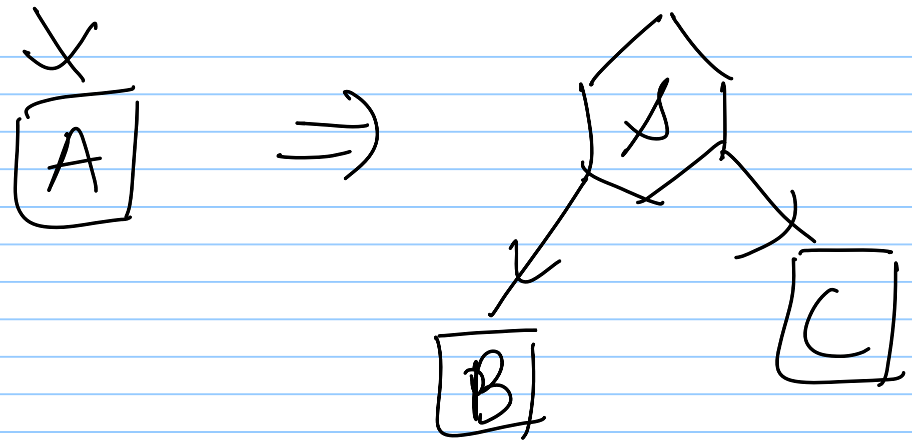



2
A
Y
q
p
x
Y


A
p
q
U
X
Y
Z

3





Query Analysis: Consider a query point q that has been fixed by an adversary w/o the knowledge of the random order in which we insert the lines.

P_i : prob that the trapezoid that contains q changes as a result of the i^{th} insertion.

$\boxed{\text{Expected length}}$ of q's search path in the final structure is at most $\displaystyle\sum_{i=1}^{n} 3P_i \leq 3\sum_{i=1}^{n} \frac{4}{i}$

$$P_i = ?\ \frac{4}{i}$$

$$\approx 12 \log_2 n$$

Argue using backward analysis

CAVEAT !!

$$\Pr\left(\left|X(q) - E[X(q)]\right|\right) \leq ?$$