Range searching

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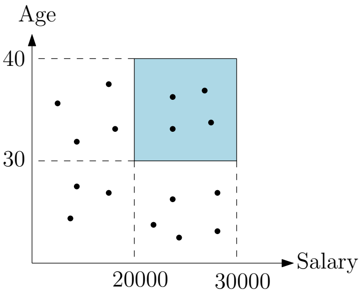
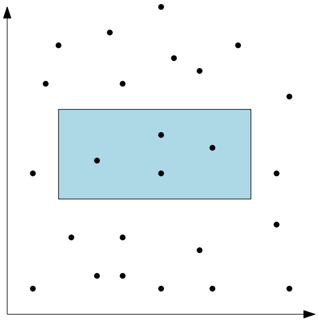
[1]Slide ideas borrowed from Marc van Kreveld and Subhash Suri

Range query

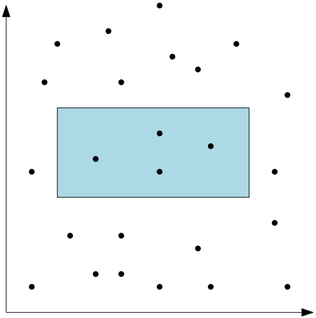


- A range query is a common database operation that retrieves all records where some value is between an upper and lower boundary.
- Range query: Asks for the objects whose coordinates lie in a specified query range (interval)

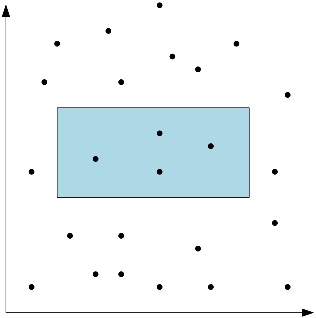
Range searching



- Range Searching: Process a set of given data points efficiently such that given a range window set of points inside the range can e reported "QUICKLY".
- Time-Space tradeoff: the more we preprocess and store, the faster we can solve a query.
- A (search) data structure has a storage requirement, a query time, and a construction time (and an update time)



- Construction time $O(1)$: query time??
- Objective is sub linear query time.



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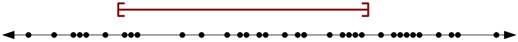
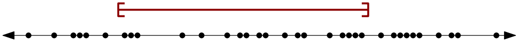
1D range query problem



- 1D range query problem: Preprocess a set of n points on the real line such that the ones inside a 1D query range (interval) can be reported fast.
- The points $p_1 \ldots p_n$ are known beforehand, the query $[x, y]$ arises at run time.
- A solution to a query problem is a data structure description, a query algorithm, and a construction algorithm.



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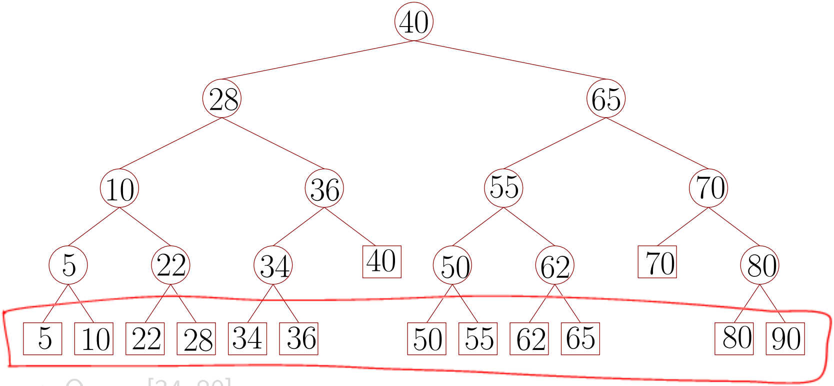
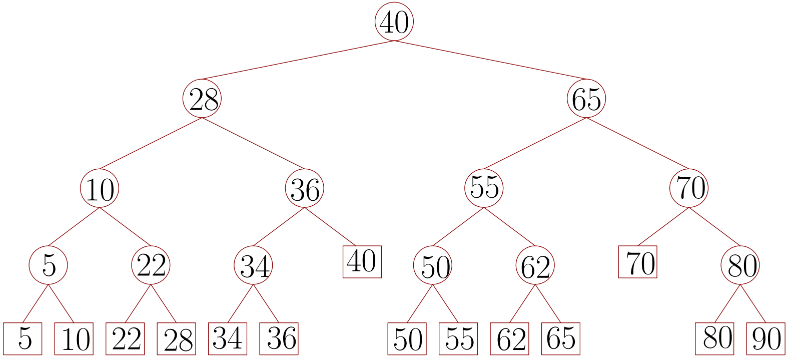
The Data Structure: Balanced binary search trees



- Query $[34, 80]$
- Search path for 34.
- Search path for 80.

The Data Structure: Balanced binary search trees



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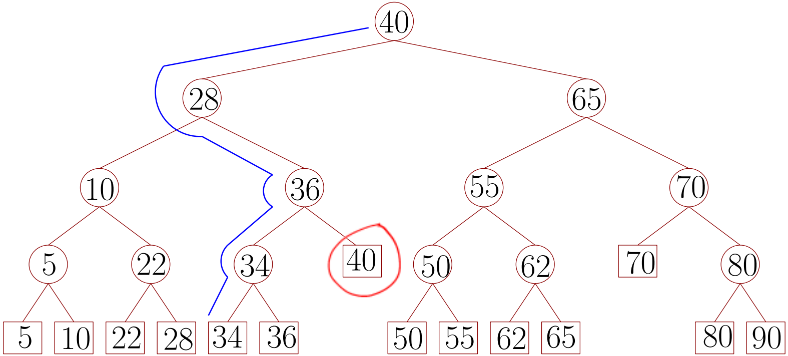

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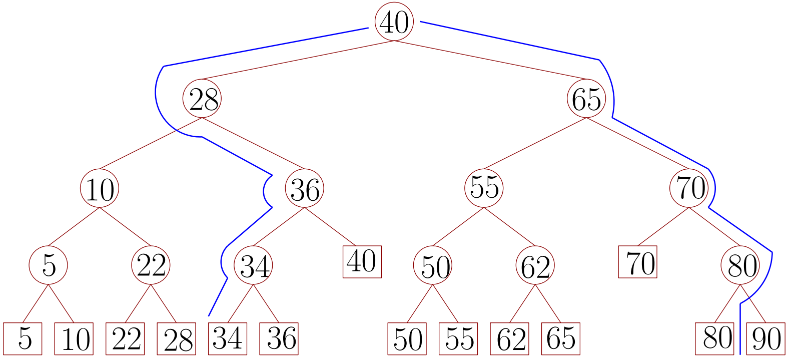

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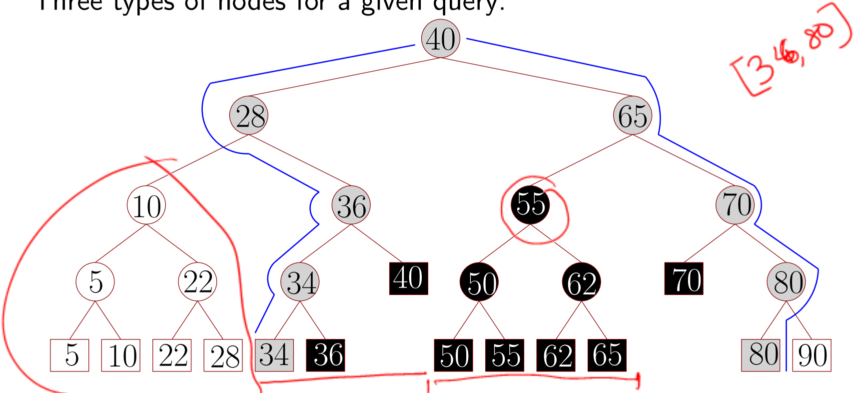
Three types of nodes for a given query:



- **White nodes:** never visited by the query
- **Grey nodes:** visited by the query, unclear if they lead to output
- **Black nodes:** Visited by the query, whole subtree is output

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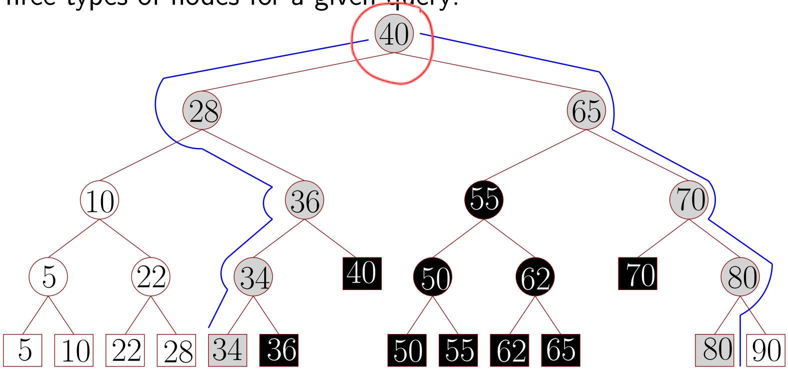

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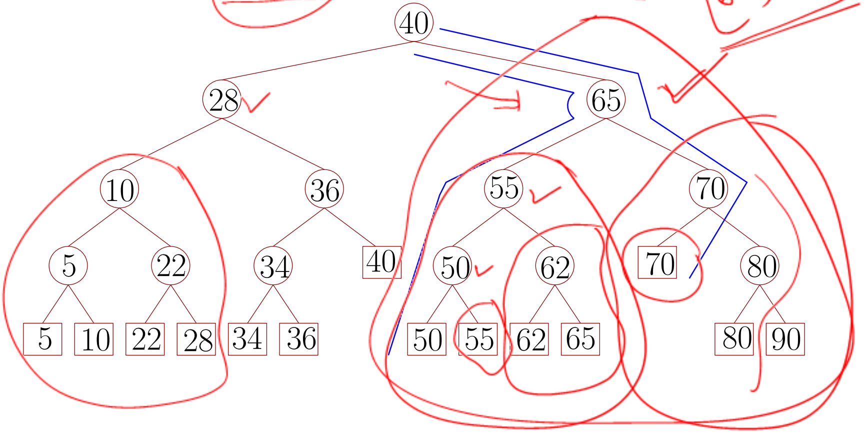
(34, 80)

[64, 70]

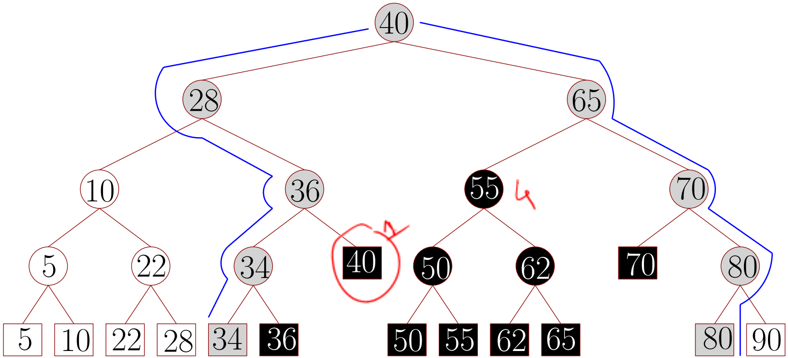
Find a split node [50 – 70]



Algorithm 1 1DRangeQuery($T, [x : y]$)

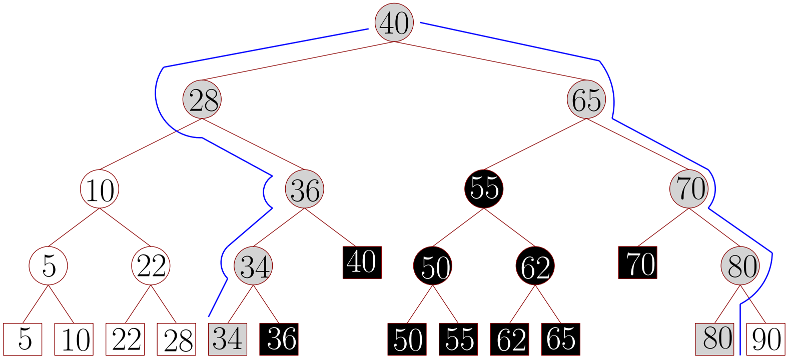
$v_{split} \leftarrow$ FindSplitNode(T, x, y)
if v_{split} is a leaf **then**
    Check if the point in v_{split} must be reported.
else
    $v \leftarrow lc(v_{split})$
    while v v is not a leaf **do**
        if $x \leq value(v)$ **then**
            ReportSubtree($rc(v)$)
            $v \leftarrow lc(v)$
        else
            $v \leftarrow rc(v)$
        end if
    end while
    $v \leftarrow rc(v_{split})$
    Similarly, follow the path to y
end if

Runtime



- White nodes: never visited by the query; no time spent
- Grey nodes: visited by the query, unclear if they lead to output; time determines dependency on n
- Black nodes: visited by the query, whole subtree is output; time determines dependency on k, the output size

Runtime



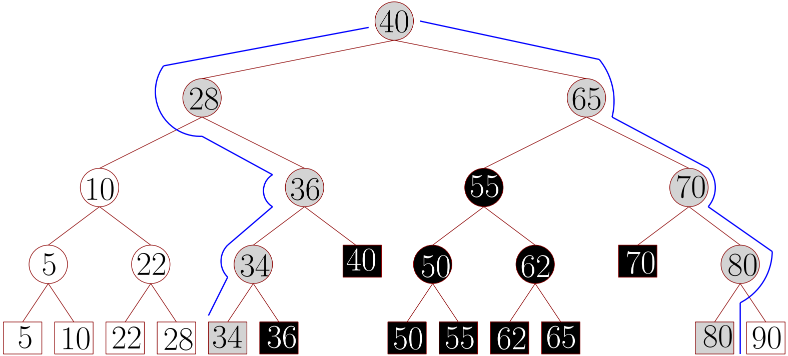
- Grey nodes: they occur on only two paths in the tree, and since the tree is balanced, its depth is $O(\log n)$
- Black nodes: Charged on output

The time spent at each node is $O(1) \Rightarrow O(\log n + k)$ query time

Storage requirement and preprocessing

- A (balanced) binary search tree storing n points uses $O(n)$ storage
- A balanced binary search tree storing n points can be built in $O(n)$ time after sorting, so in $O(n \log n)$ time overall (or by repeated insertion in $O(n \log n)$ time)

A 1-dimensional range tree for range counting queries



Theorem

A set of n points on the real line can be preprocessed in $O(n\log n)$ time into a data structure of $O(n)$ size so that any range counting queries can be answered in $O(\log n)$ time

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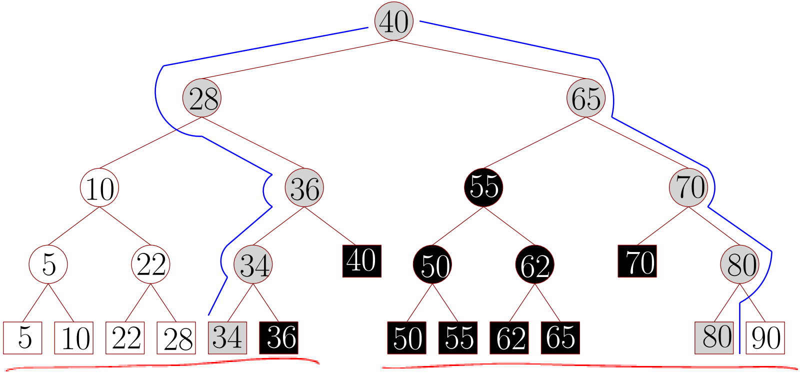

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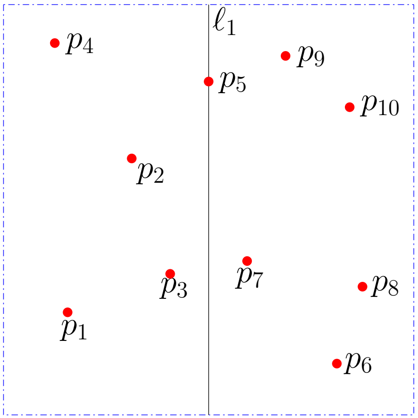
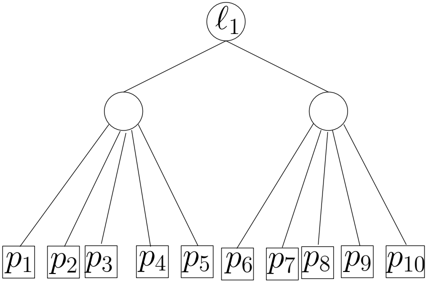
Kd-trees, the idea:

- Split the point set alternatingly by x-coordinate and by y-coordinate
- Split by x-coordinate: split by a vertical line that has half the points left and half right
- Split by y-coordinate: split by a horizontal line that has half the points below and half above

Kd-tree Construction





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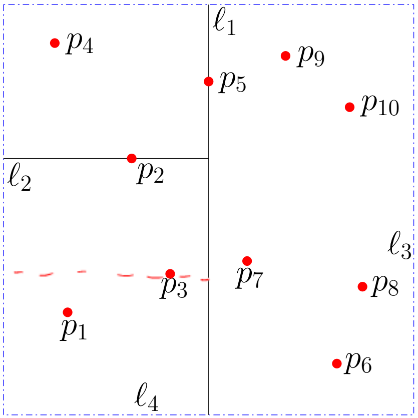

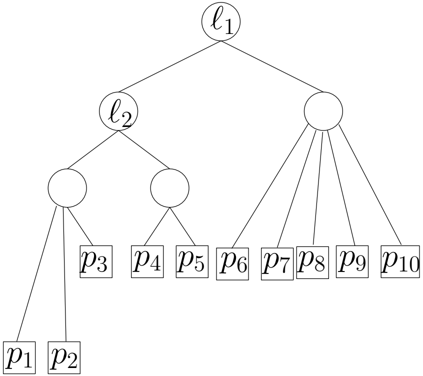

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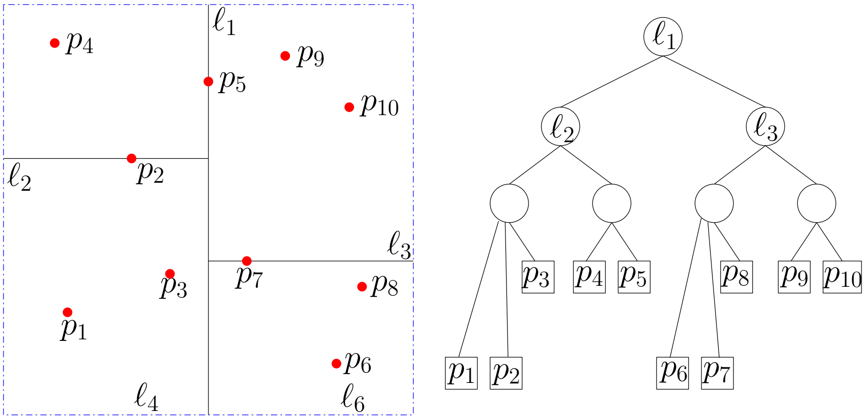

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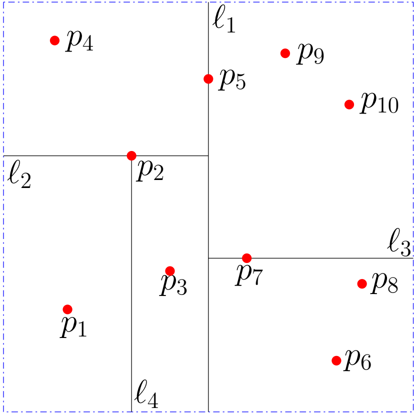

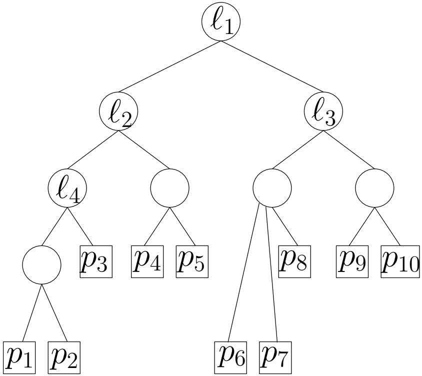

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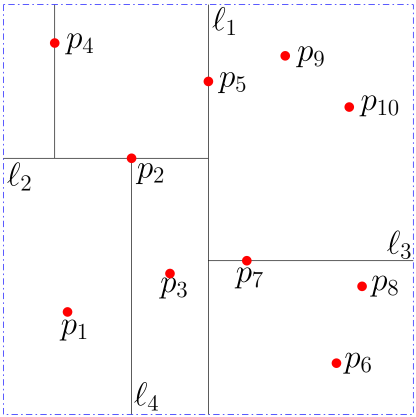

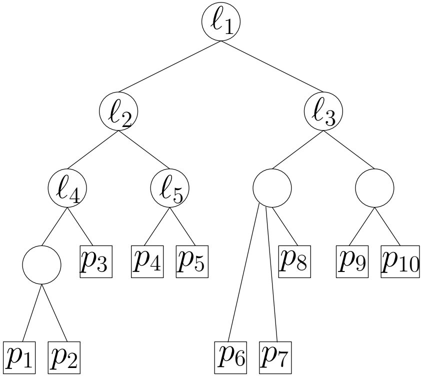

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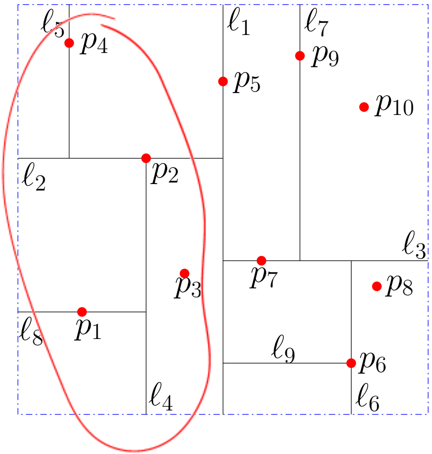
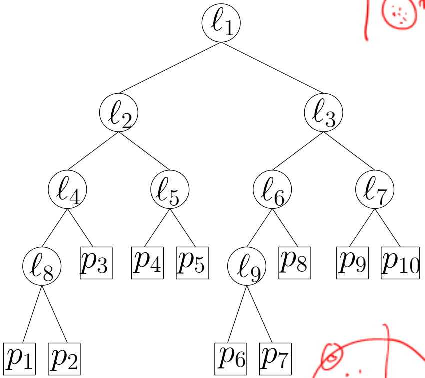
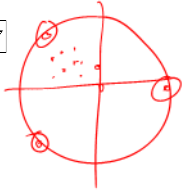






Algorithm

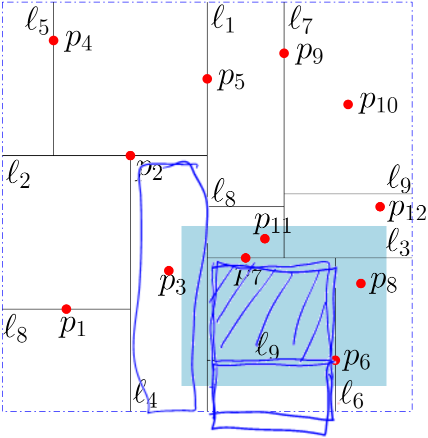
Algorithm 2 BuildKdTree(P, *depth*)

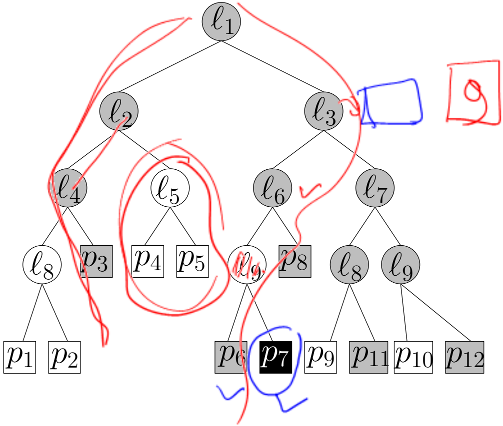
if P contains only one point **then**
  return a leaf storing this point
else if depth is even **then**
  Split P with a vertical line ℓ through the median x-coordinate into P_1 (left of ℓ) and P_2 (right of ℓ)
else
  Split P with a horizontal line ℓ through the median x-coordinate into P_1 (below ℓ) and P_2 (above ℓ)
end if
$left \leftarrow BuildKdTree(P_1, depth + 1)$
$right \leftarrow BuildKdTree(P_2, depth + 1)$
Create a node v storing ℓ, make *left* left the left child of v, and make *right* right the right child of v.
return(v)

Complexity

- The median of a set of n values can be computed in $O(n)$ time
- Let $T(n)$ be the time needed to build a kd-tree on n points

$T(1) = O(1)$
$T(n) = 2T(n/2) + O(n)$
A kd-tree can be built in $O(n \log n)$ time


ℓ_5
p_4
ℓ_1
ℓ_7
p_9
p_5
p_{10}
ℓ_2
p_2
ℓ_8
ℓ_9
p_{12}
p_{11}
ℓ_3
p_7
p_3
p_8
ℓ_8
p_1
ℓ_9
p_6
ℓ_4
ℓ_6


ℓ_1
ℓ_2
ℓ_3
9
ℓ_4
ℓ_5
ℓ_6
ℓ_7
ℓ_8
p_3
p_4
p_5
ℓ_9
p_8
ℓ_8
ℓ_9
p_1
p_2
p_6
p_7
p_9
p_{11}
p_{10}
p_{12}

Complexity Analysis

White, grey, and black nodes with respect to region(v):

- White node v: R does not intersect region(v)
- Grey node v: R intersects region(v), but region(v) $\not\subseteq R$
- Black node v: region(v)$\subseteq R$

Complexity Analysis

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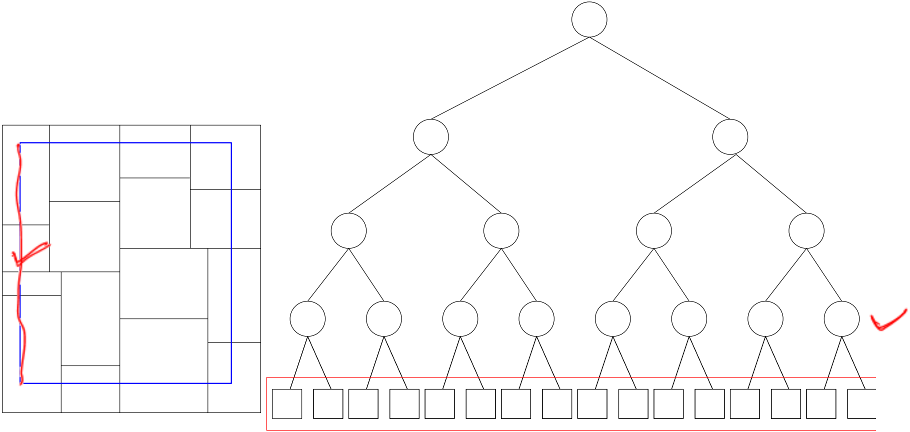
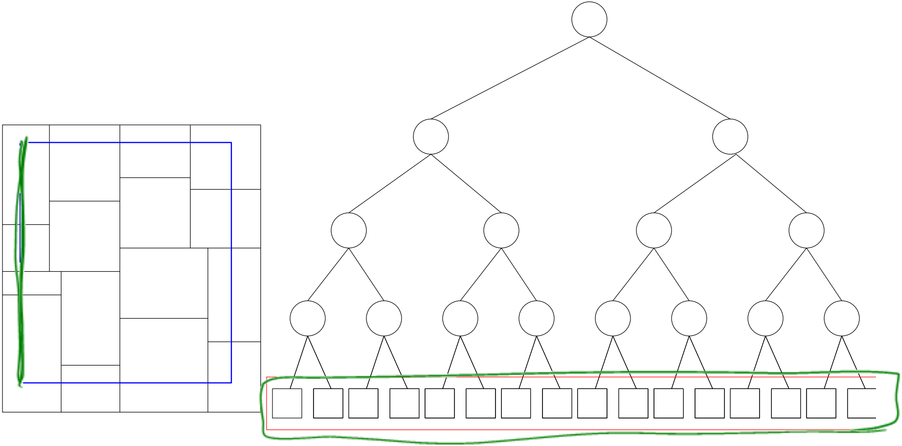
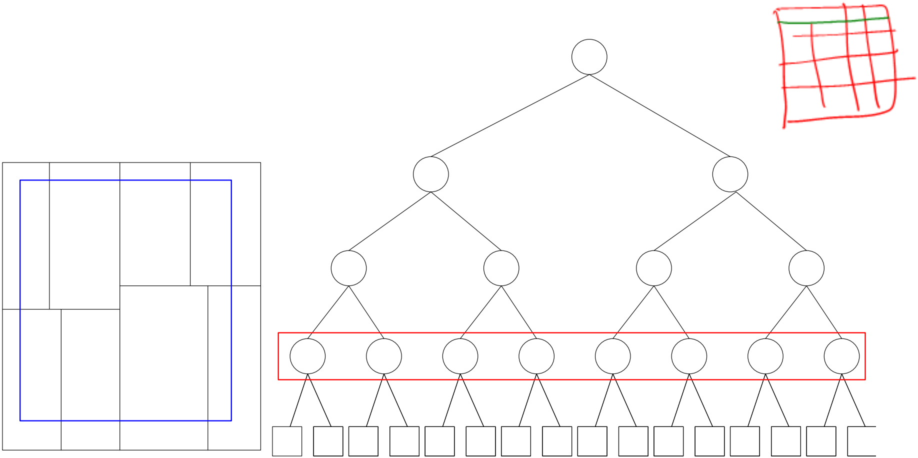


- How many grey nodes can be there among the leaf nodes.
- How many regions can be intersected by a axis parallel straight line.



- At max $O(\sqrt(n))$
- In the previous level $O(\sqrt(n/2))$

Complexity Analysis



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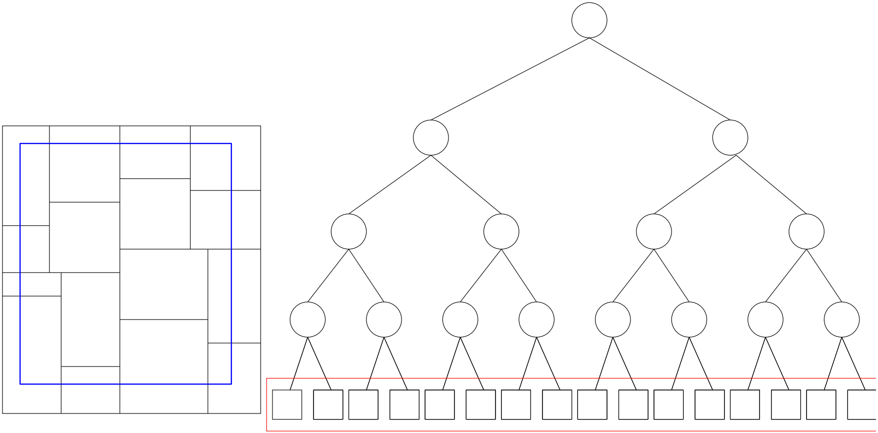

- Total no of Gray cells are $\sqrt{n}(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{8}} \dots)$
- $O(\sqrt{(n)})$

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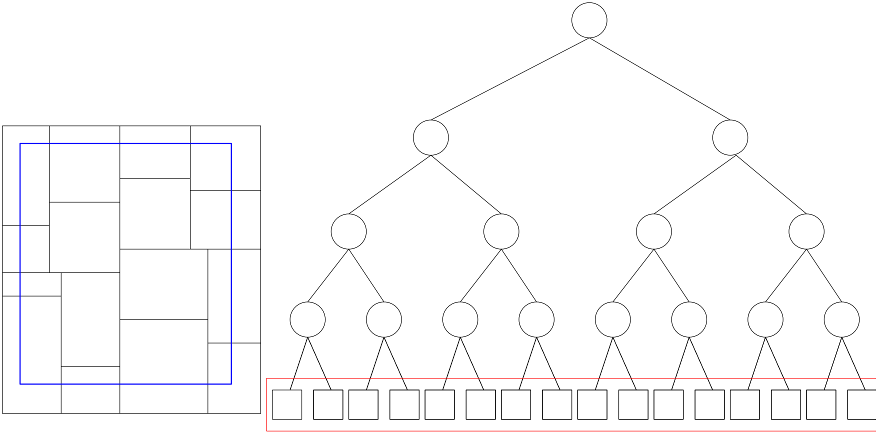

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Higher dimensions

- A 3-dimensional kd-tree alternates splits on x, y, and z coordinate
- A 3D range query is performed with a box

Theorem

A set of n points in d-space can be preprocessed in $O(n \log n)$ time into a data structure of $O(n)$ size so that any d-dimensional range query can be answered in $O(n^{1-1/d} + k)$ time, where k is the number of answers reported.

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