

Range Query KD Tree  $O(\sqrt{n} + k)$

$$P = \{p_1, p_2, \dots, p_n\} \rightarrow \text{preprocess } P$$

s.t. one can report the points  $p_i \in P$  that lie inside the query rect.  $Q$

Reporting query -  $O(\sqrt{n} + \overleftarrow{K})$

Counting query -  $O(\sqrt{n}) \rightarrow \underline{Ex}$

$$O\left(n^{1-\frac{1}{d}} + K\right) \quad T(n) \leq 2T\left(\frac{n}{2}\right) + \dots$$

$\longleftarrow d$

kd-tree

$$O(\sqrt{n} + k)$$



Range tree

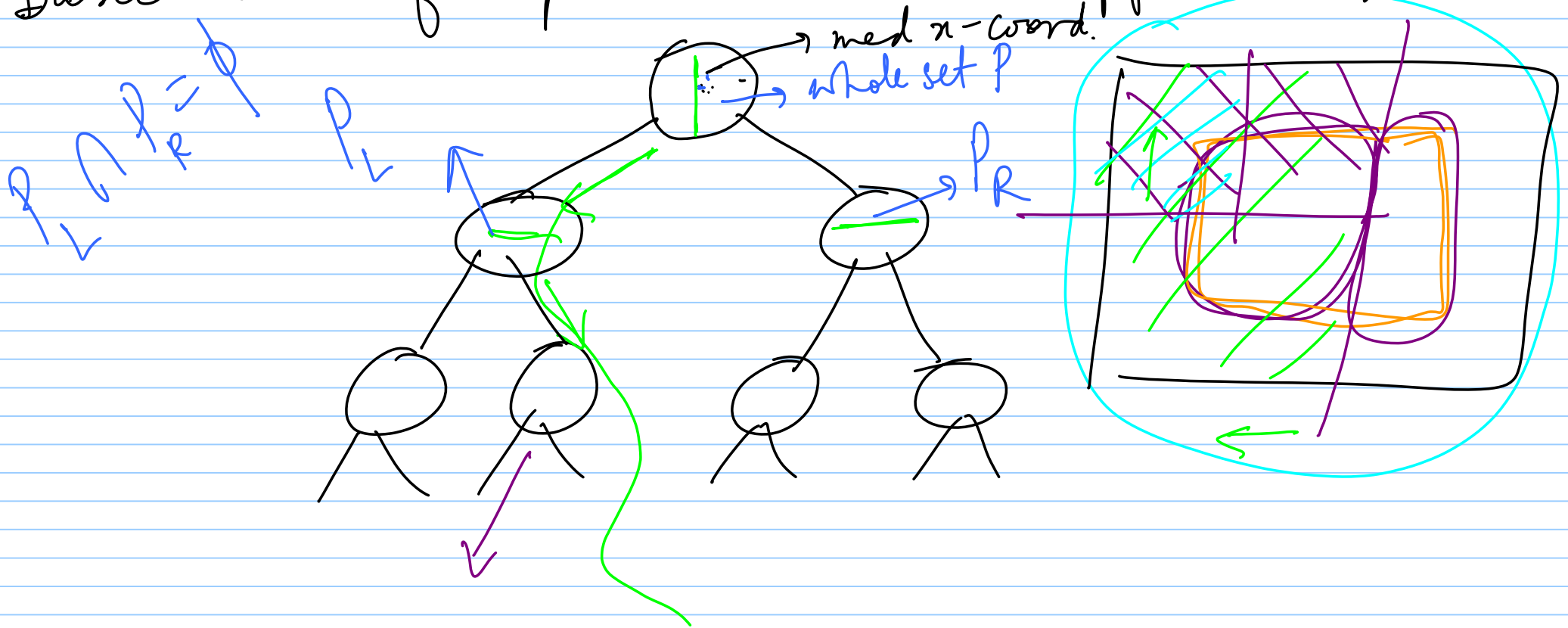
$$O(\log^2 n + k)$$

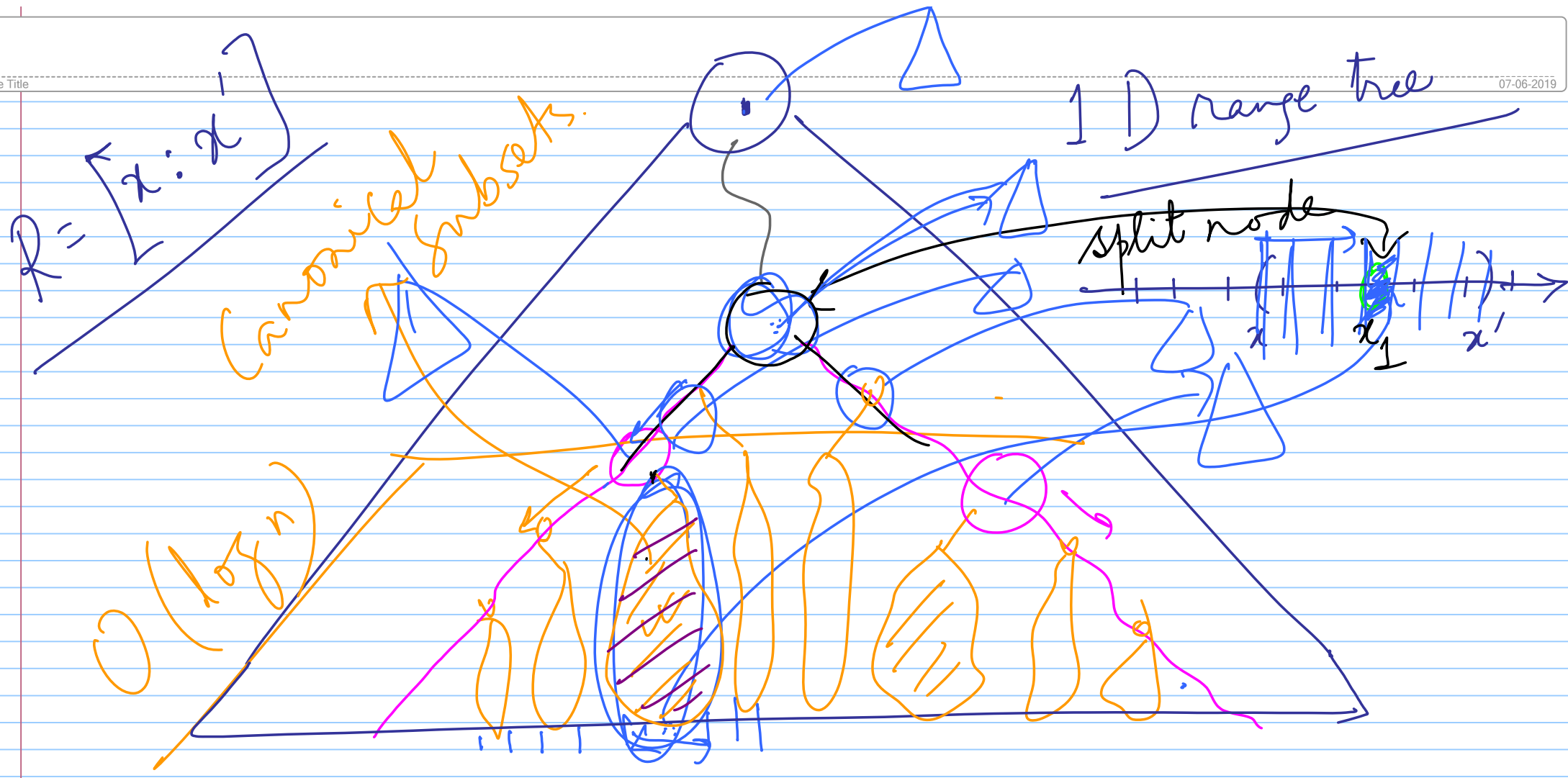
Layered range  
tree

$$O(\log n + k)$$

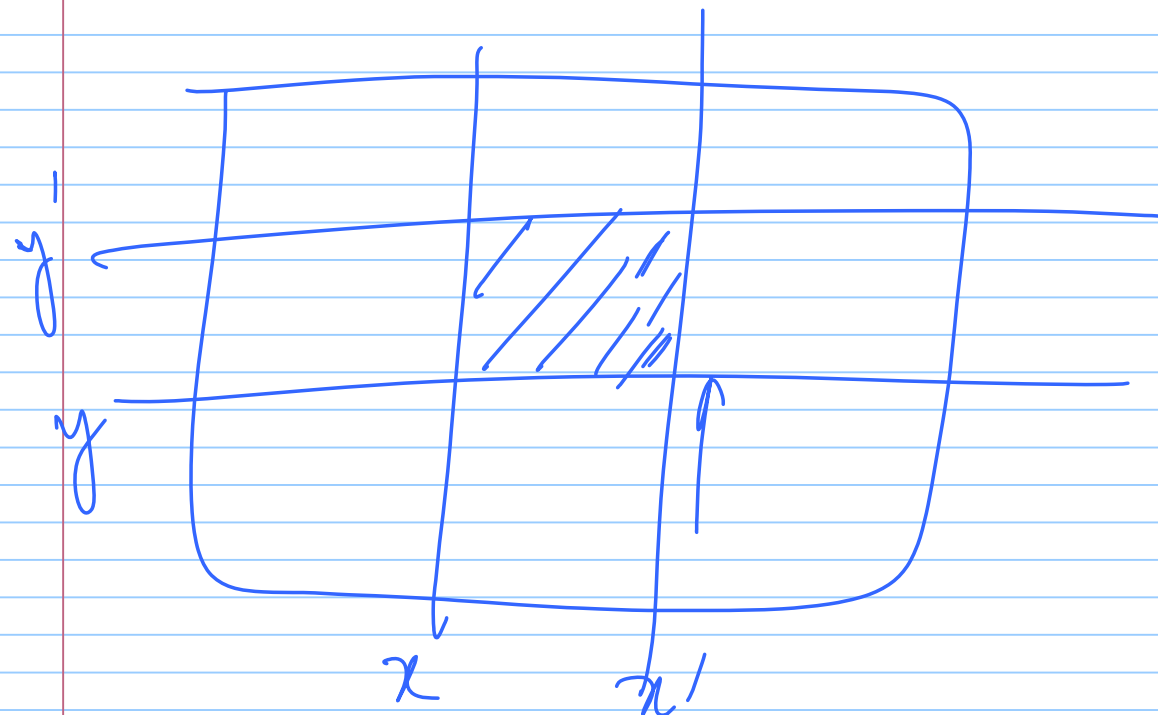
Fractional  
Cascading

Basic idea of any data str. that supports range queries:

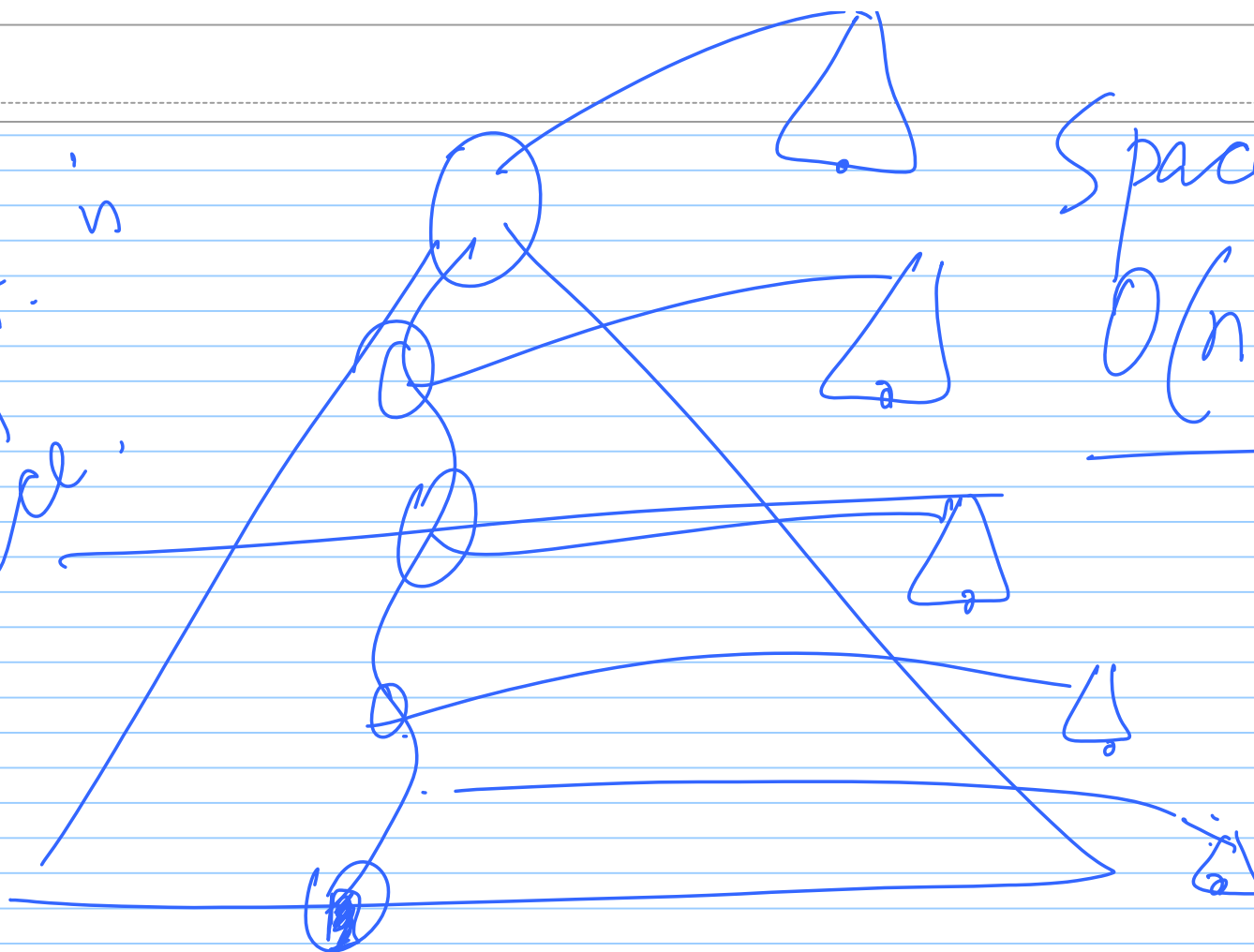




$$R = [x:x'] \times [y:y']$$



per level a pt.  $n$   
Stored once.



Space:  
 $O(n \log n)$

$$\sum_{\log n}^{\log n} O(\log n + R_v) \approx O(\log^2 n + k)$$

Space:  $O(n \log n)$

Query time:  $O(\log^2 n)$

Ex:

Generalize this to  $\mathbb{R}^d$  // use a recurrence



$$O(\log^2 n + k) \rightarrow O(\log n + k)$$

$$\sum_{v \in \log n} O(\log n + K_v) \rightarrow \sum_{\underline{\underline{v \in \log n}}} \underline{O(1 + K_v)}$$

$$O(\log^d n) \rightarrow O(\log^{d-1} n)$$

