Tutorial 2

ACM Summer School on Geometric Algorithms and Applications 2019

Problem 1.
We will show that given n disjoint disks $\mathcal{D}$ in the plane, there is a rectangle R that intersects $O(\sqrt{n})$ disks, and such that there are at most $2n/3$ disk centers inside, and outside R.
Let R be the smallest $4:3$ ratio rectangle containing at least $2n/3$ disk centers in its interior.

1. Show that there exists a $2:3$ rectangle $R' \subset R$, containing at least $1/3$ disk centers.
2. By scaling and translation, assume that the bottom-left vertex of R' is at the origin, and its top-right vertex is at $(2,3)$. Then, show that each rectangle in the family $\mathcal{F}$ of rectangles, with bottom left $(-\delta, \delta)$, and top-right coordinate $(3-\delta, 4-\delta)$, $\delta \in [0,1]$ contains at least $n/3$ centers inside and outside.
3. Prove that one of these disks contains $O(\sqrt{n})$ rectangles.

Problem 2 (2 points)**.**
Given a tree with maximum degree d, show that there is an edge, whose removal separates the tree into two sub-trees, each of size at least n/d.

Problem 3 (2 points)**.**
Given a set $\mathcal{I}$ of n intervals in $\mathbb{R}$, we want to find a minimum hitting set, i.e., the fewest set of points S so that $I \cap S \neq \emptyset$ for all $I \in \mathcal{I}$.

1. Show that it is sufficient to restrict our attention to a finite set of points in $\mathcal{R}$.
2. Show that a local search algorithm that tries to improve the current solution by swapping two points of the current solution with one point of the optimal is a 2-approximation.
3. Show how to improve the local search algorithm to obtain a $(1+1/k)$-approximation to the optimal.