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Safe Receding Horizon Motion Planning with Infinitesimal Update Interval

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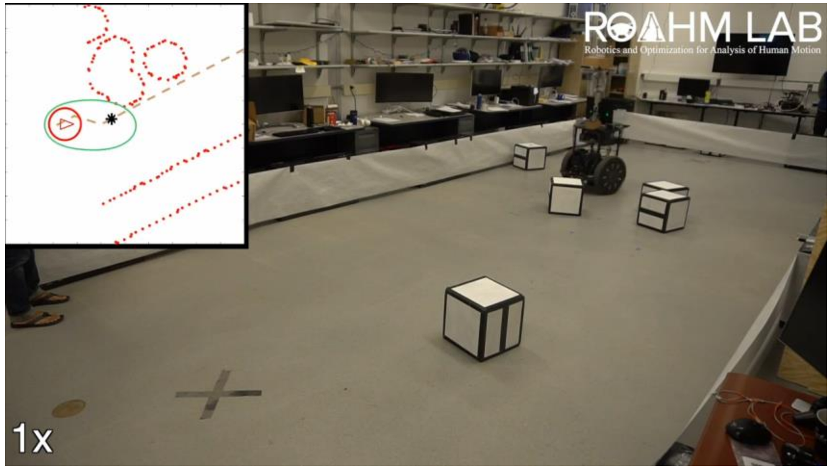
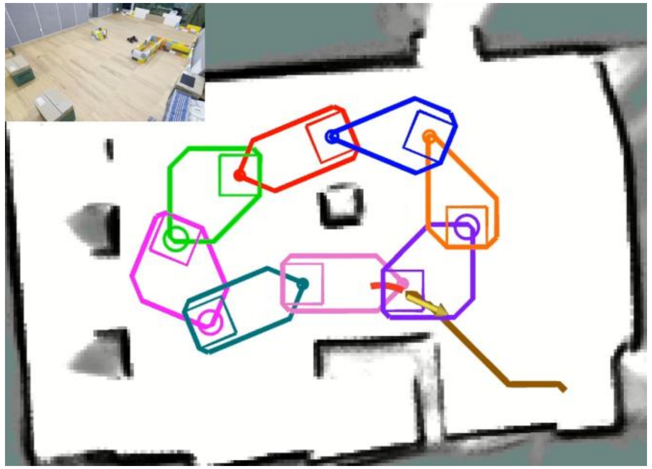
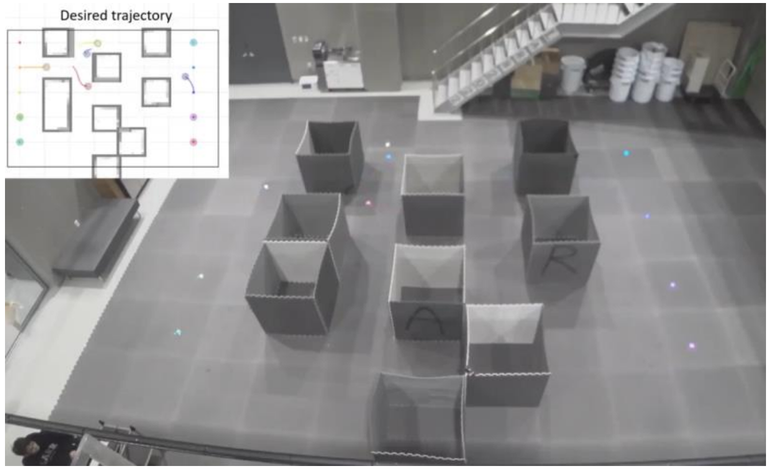
Receding Horizon Motion Planning



Kousik, et al., 2020 [1]



Jang, et al., 2021 [2]



Park, et al., 2023 [3]

[1] Kousik, Shreyas, et al. "Bridging the gap between safety and real-time performance in receding-horizon trajectory design for mobile robots." *The International Journal of Robotics Research* 39.12 (2020): 1419-1469.
[2] Jang, Inkyu, et al. "Robust and recursively feasible real-time trajectory planning in unknown environments." *2021 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2021.
[3] Park, Jungwon, et al. "DLSC: Distributed multi-agent trajectory planning in maze-like dynamic environments using linear safe corridor." *IEEE Transactions on Robotics* 39.5 (2023): 3739-3758.

Receding Horizon Motion Planning

Common Procedure

1. Initialize $x(0)$ as the robot's initial state.
2. Whenever time t becomes a multiple of τ_{plan}, try to find the next trajectory.
    2-1. If the search succeeds, switch to the next trajectory.
    2-2. If it fails, stay on the current trajectory.
3. Repeat until the task is accomplished.

Optimal Safe Receding Horizon Motion Planning

For a control-affine dynamical system

$\dot{x} = f(x) + g(x) \cdot u$

with safety constraints

$d(x) \geq 0,$

Problem 1

$$\min_{\mathrm{x},\mathrm{u}} \quad \mathcal{J}[\mathrm{x},\mathrm{u}]$$

s.t.	$\mathrm{x}(0) = x(t)$	(initial condition)
	$\partial_\tau \mathrm{x}(\tau) = f\big(\mathrm{x}(\tau)\big) + g\big(\mathrm{x}(\tau)\big) \cdot \mathrm{u}(\tau)$	(dynamic feasibility)
	$d\big(\mathrm{x}(\tau)\big) \geq 0$	(safety)
	$\partial_\tau \mathrm{x}(T) = 0$	(final stop)
	$\mathrm{u}(\tau) \in U$	(input feasibility)

x, u : receding horizon state and input trajectories
U : set of all feasible inputs

Optimal Safe Receding Horizon Motion Planning

Problem 1

$$\min_{\mathrm{x},\mathrm{u}} \quad \mathcal{J}[\mathrm{x},\mathrm{u}]$$

$$\begin{aligned} \text{s.t.} \quad & \mathrm{x}(0) = x(t) \\ & \partial_\tau \mathrm{x}(\tau) = f\big(\mathrm{x}(\tau)\big) + g\big(\mathrm{x}(\tau)\big) \cdot \mathrm{u}(\tau) \\ & d\big(\mathrm{x}(\tau)\big) \geq 0 \\ & \partial_\tau \mathrm{x}(T) = 0 \\ & \mathrm{u}(\tau) \in U \end{aligned}$$

Strengths

1. High expressivity and optimality
2. Recursive feasibility (due to final stop cond.)

Limitations

1. Infinite-dimensional, direct optimization intractable
2. Extremely heavy computation

Parametrized Trajectory Optimization

Problem 1

$$\min_{\mathrm{x},\mathrm{u}} \quad \mathcal{J}[\mathrm{x},\mathrm{u}]$$

$$\text{s.t.} \quad \mathrm{x}(0) = x(t)$$

$$d\big(\mathrm{x}(\tau)\big) \geq 0$$

$$\partial_\tau \mathrm{x}(T) = 0$$

$$\mathrm{u}(\tau) \in U$$

Problem 2

$\min_{k}$	$J(t,k)$	(trajectory cost)
s.t.	$\mathrm{x}(k,0) = x(t)$	(initial condition)
	$w(k) \geq 0$	(safety)
	$k \in K \Leftrightarrow \rho(k) \geq 0$	(parameter feasibility)

k	: Trajectory parameter
$\mathrm{x}(k,\tau)$	: Dynamically feasible trajectory with final stop condition
J	: Trajectory cost
$w(\cdot)$	: Possibly multi-dimensional safety indicator for trajectory
K	: Set of all feasible trajectory parameters

Parametrized Trajectory Optimization

Problem 2

$$
\begin{aligned}
\min_{k}. \quad & J(t,k) \\
\text{s.t.} \quad & \mathrm{x}(k,0) = x(t) \\
& w(k) \geq 0 \\
& \rho(k) \geq 0
\end{aligned}
$$

Strengths

1. Online computation doable
2. Safety guaranteed through $w(\cdot)$

Limitations

1. Still heavy computation for high-frequency updates
2. Recursive feasibility not guaranteed, due to the initial condition

Note on the Update Interval τ_{plan}

It **denotes how promptly the system can respond** to possibly changing environmental conditions

➔ It is favorable to keep τ_{plan} as short as possible

It **serves as a time budget** to complete searching for the next trajectory

➔ τ_{plan} cannot be shortened beyond the computation power

What happens when we consider infinitesimal update interval (i.e., $\tau_{\text{plan}} \searrow 0$)?

Parametrized Trajectory Optimization in a Recursively Feasible Form

Problem 2

$$\min_{k} \quad J(t,k)$$

$$\text{s.t.} \quad \mathrm{x}(k,0) = x(t)$$

$$w(k) \geq 0$$

$$\rho(k) \geq 0$$

Problem 3

$$\min_{k,\tau} \quad J(t,k) + \lambda\tau \qquad \text{(augmented cost)}$$

$$\text{s.t.} \quad \mathrm{x}(k,\tau) = x(t) \qquad \text{(initial condition)}$$

$$w(k) \geq 0 \qquad \text{(safety)}$$

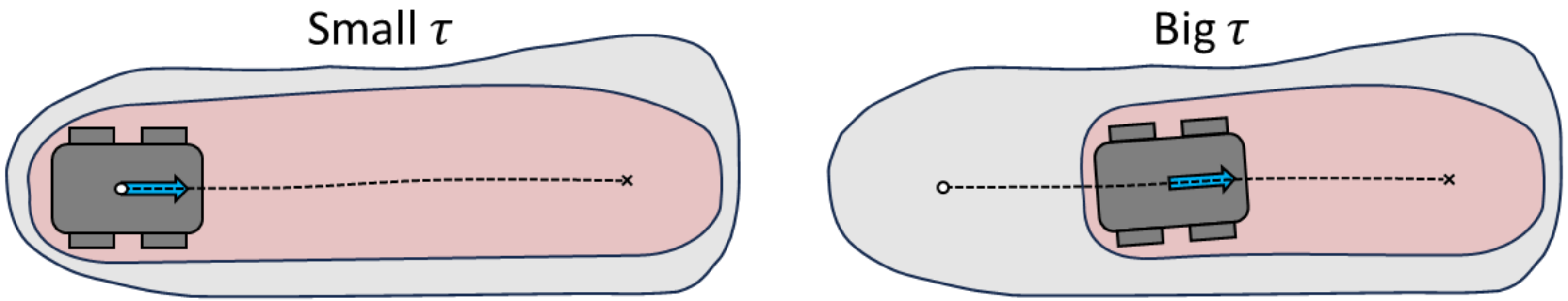
$$\rho(k) \geq 0 \qquad \text{(parameter feasibility)}$$

$$\tau \geq 0 \qquad \text{(starting time feasibility)}$$

λ : A positive constant
τ : The indicator at which time point on the parametrized trajectory the robot will start

Parametrized Trajectory Optimization in a Recursively Feasible Form

Why add auxiliary term $\lambda\tau$?



Receding horizon trajectory $\mathrm{x}(k,\cdot)$

Occupancy of $\mathrm{x}(k,\cdot)$ (i.e., $w(k) \leq 0$)

The actual space needed for safety (i.e., $\min_{\tau' \in [\tau,\infty)} d\big(\mathrm{x}(k,\tau')\big) \leq 0$)

➔ We want to keep τ small to avoid excessive conservatism (waste of space)

Parametrized Trajectory Optimization in a Recursively Feasible Form

Problem 3

$$\min_{k,\tau} \quad J(t,k) + \lambda\tau$$

$$\text{s.t.} \quad \mathrm{x}(k,\tau) = x(t)$$

$$w(k) \geq 0$$

$$\rho(k) \geq 0$$

$$\tau \geq 0$$

Strength

Recursively feasible

Limitation

Still heavy computation for high-frequency updates

Taking the Limit $\tau_{\text{plan}} \searrow 0$

With infinitesimal τ_{plan}, the trajectory parameters k and τ are controlled through $\dot{k}$ and $\dot{\tau}$.

Problem 3

$$\begin{aligned}
\min_{k,\tau}.\quad & J(t,k) + \lambda\tau \\
\text{s.t.}\quad & \mathrm{x}(k,\tau) = x(t) \\
& w(k) \geq 0 \\
& \rho(k) \geq 0 \\
& \tau \geq 0
\end{aligned}$$

Problem 4

$$\begin{aligned}
\min_{u,\dot{k},\dot{\tau}}.\quad & \partial_k J(t,k)\cdot\dot{k} + \lambda\dot{\tau} + \frac{1}{2}\mu_k\left\|\dot{k}\right\|^2 + \frac{1}{2}\mu_\tau\dot{\tau}^2 \\
\text{s.t.}\quad & \partial_k \mathrm{x}(k,\tau)\cdot\dot{k} + \partial_\tau \mathrm{x}(k,\tau) = f\big(x(t)\big) + g\big(x(t)\big)\cdot u \\
& \partial_k w(k)\cdot\dot{k} + \alpha\big(w(k)\big) \geq 0 \\
& \partial_k \rho(k)\cdot\dot{k} + \gamma\big(\rho(k)\big) \geq 0 \\
& \dot{\tau} + \sigma(\tau) \geq 0 \\
& u \in U
\end{aligned}$$

Taking the Limit $\tau_{\text{plan}} \searrow 0$

The cost term

$$\min_{k,\tau} \quad J(t,k) + \lambda\tau \quad \Rightarrow \quad \min_{u,\dot{k},\dot{\tau}} \quad \partial_k J(t,k)\cdot\dot{k} + \lambda\dot{\tau} + \frac{1}{2}\mu_k\left\|\dot{k}\right\|^2 + \frac{1}{2}\mu_\tau\dot{\tau}^2$$

$\frac{d}{dt}J(t,k) = \partial_t J(t,k) + \partial_k J(t,k)\cdot\dot{k}$: $\partial_t J$ term is omitted, because it does not depend on $u,\dot{k},\dot{\tau}$

$\frac{1}{2}\mu_k\left\|\dot{k}\right\|^2 + \frac{1}{2}\mu_\tau\dot{\tau}^2$: The quadratic regularization term is added for numerical stability

Taking the Limit $\tau_{\text{plan}} \searrow 0$

Equality Constraint

$$\mathrm{x}(k, \tau) = x(t) \quad \Rightarrow \quad \partial_k \mathrm{x}(k, \tau) \cdot \dot{k} + \partial_\tau \mathrm{x}(k, \tau) = f\big(x(t)\big) + g\big(x(t)\big) \cdot u$$

Take the time derivative on both sides, and the equality is satisfied *indirectly*.

Note: The input u shows up here!

Taking the Limit $\tau_{\text{plan}} \searrow 0$

Inequality Constraint

$$w(k) \geq 0$$
$$\rho(k) \geq 0$$
$$\tau \geq 0$$

$$\Rightarrow$$

$$\partial_k w(k) \cdot \dot{k} + \alpha\big(w(k)\big) \geq 0$$
$$\partial_k \rho(k) \cdot \dot{k} + \gamma\big(\rho(k)\big) \geq 0$$
$$\dot{\tau} + \sigma(\tau) \geq 0$$

The nonnegativity constraints are replaced by *CBF-QP-style* constraints [1].

The functions α, γ, and σ are class K functions.

[1] Ames, Aaron D., et al. "Control barrier functions: Theory and applications." *2019 18th European control conference (ECC)*. IEEE, 2019.

The safety filter

Problem 4

$\min_{u,\dot{k},\dot{\tau}}$	$\partial_k J(t,k)\cdot\dot{k}+\lambda\dot{\tau}+\frac{1}{2}\mu_k\left\lVert\dot{k}\right\rVert^2+\frac{1}{2}\mu_\tau\dot{\tau}^2$	(trajectory cost)
s.t.	$\partial_k \mathrm{x}(k,\tau)\cdot\dot{k}+\partial_\tau \mathrm{x}(k,\tau)=f\big(x(t)\big)+g\big(x(t)\big)\cdot u$	(initial condition)
	$\partial_k w(k)\cdot\dot{k}+\alpha\big(w(k)\big)\geq 0$	(safety)
	$\partial_k \rho(k)\cdot\dot{k}+\gamma\big(\rho(k)\big)\geq 0$	(parameter feasibility)
	$\dot{\tau}+\sigma(\tau)\geq 0$	(starting time feasibility)
	$u\in U$	(input feasibility)

Hardware Experiment

Dynamics (Turtlebot 3 [1])

$x \in X = SE(2) \times \mathbb{R}^2$
(Second-order unicycle dynamics)

$$\dot{x} = \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{\theta} \\ \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ \alpha \end{bmatrix}$$

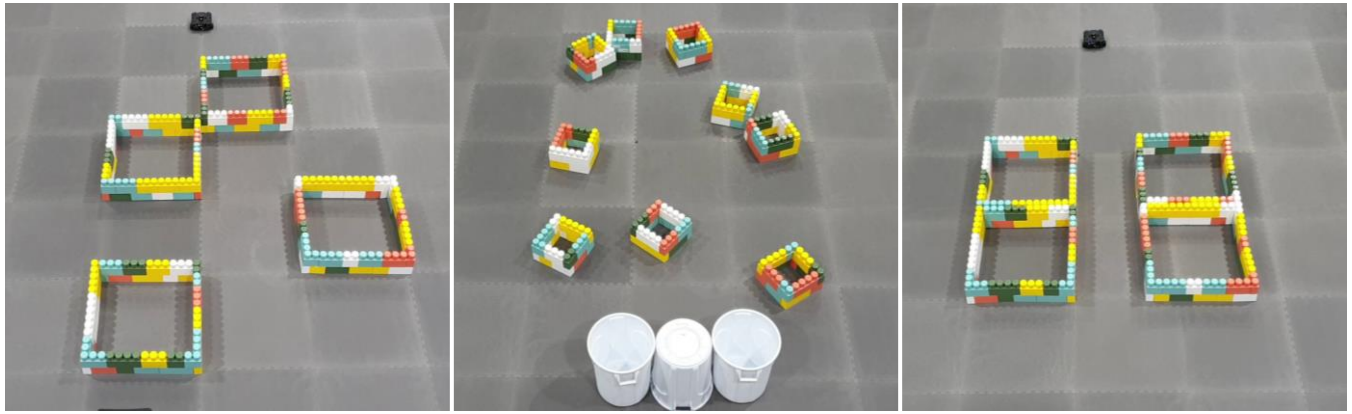
$u = [a, \alpha]^\top, \qquad |a| \leq a_{\max}, |\alpha| \leq \alpha_{\max}$

[1] Turtlebot 3,
`https://emanual.robotis.com/docs/en/platform/turtlebot3/overview/`

Safety Requirements

1. Collision avoidance w.r.t. LiDAR data points
2. Linear speed limit $|v| \leq v_{\max}$
3. Angular speed limit $|\omega| \leq \omega_{\max}$

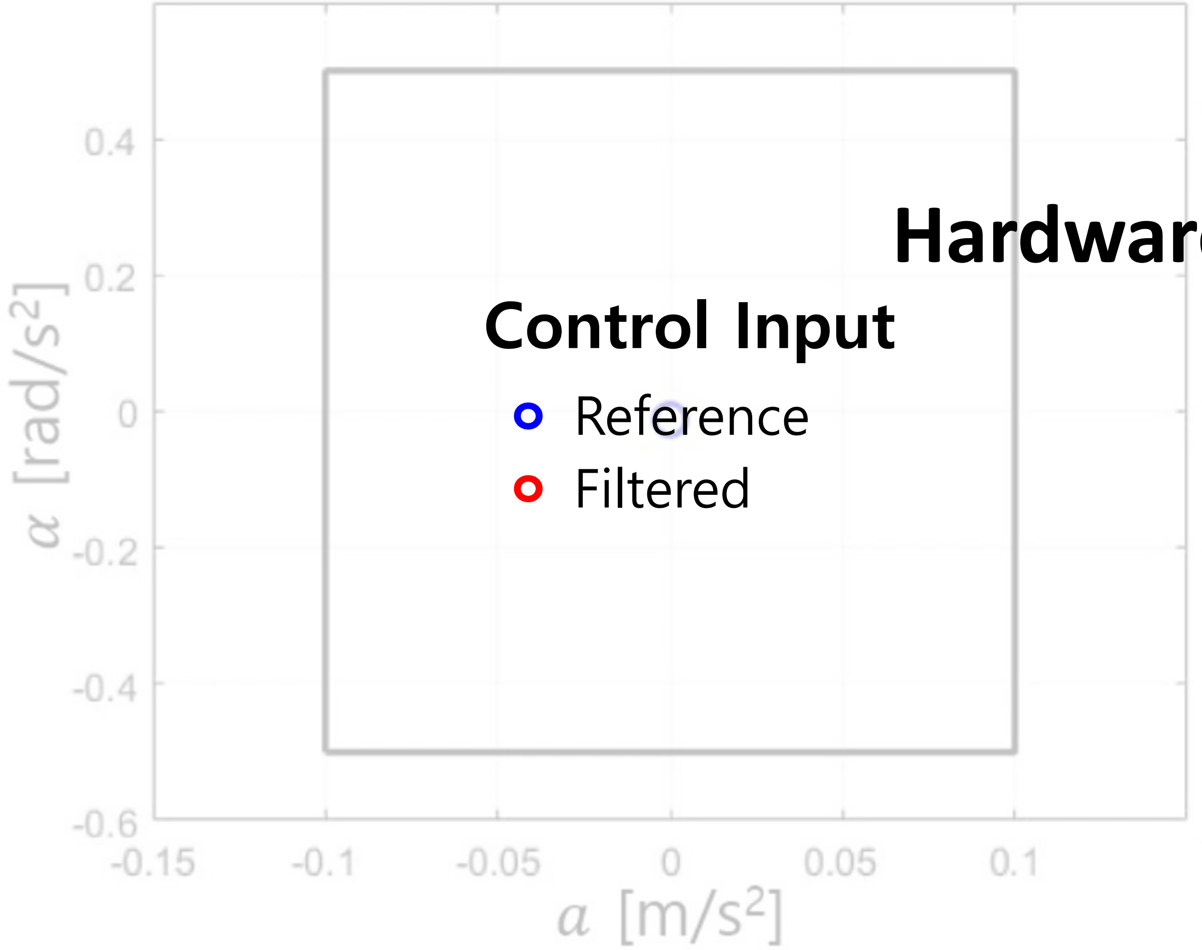
Experiment Environments



Hardware Experiment

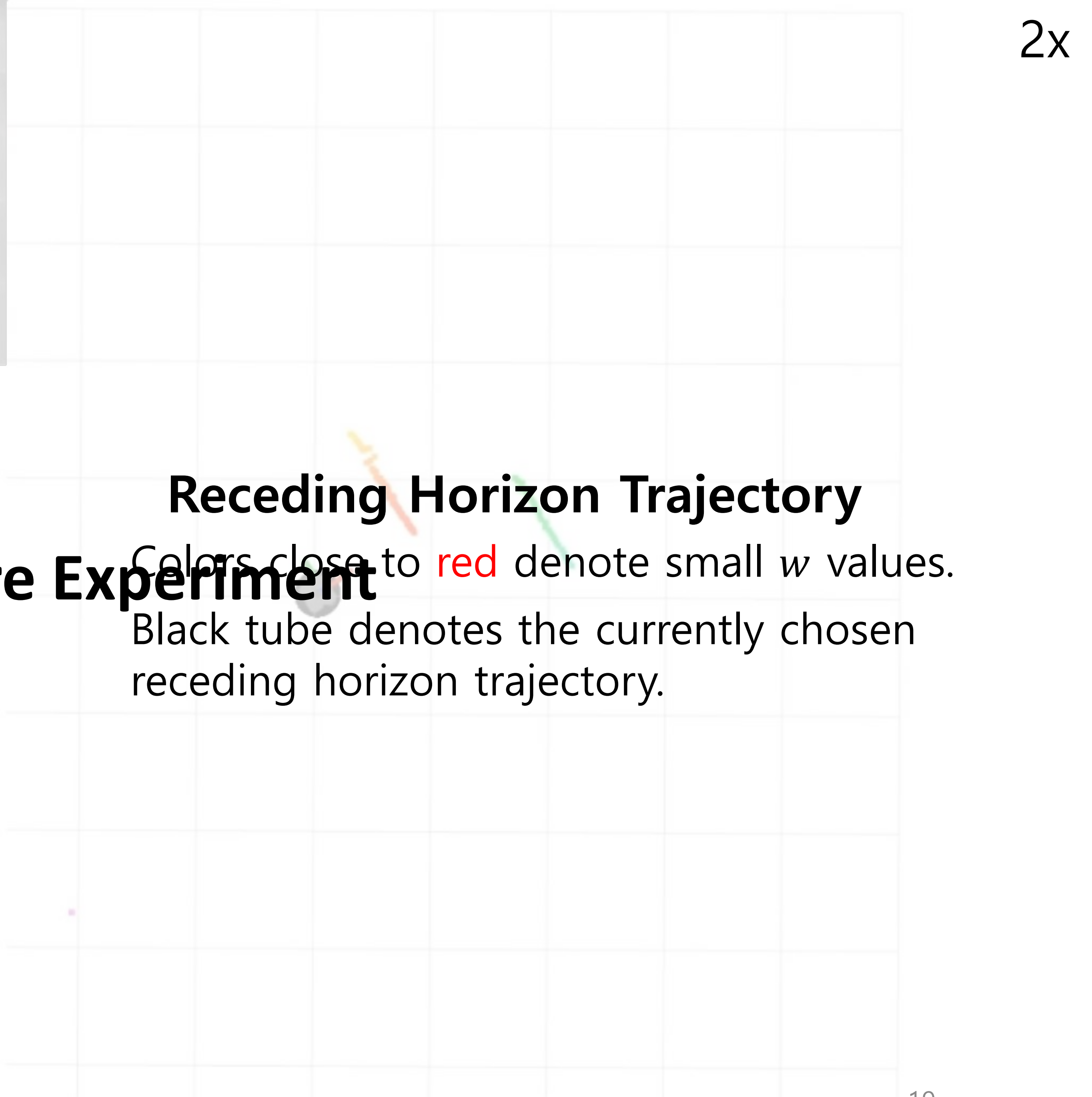
2x

Experiment Video



Control Input

- Reference
- Filtered

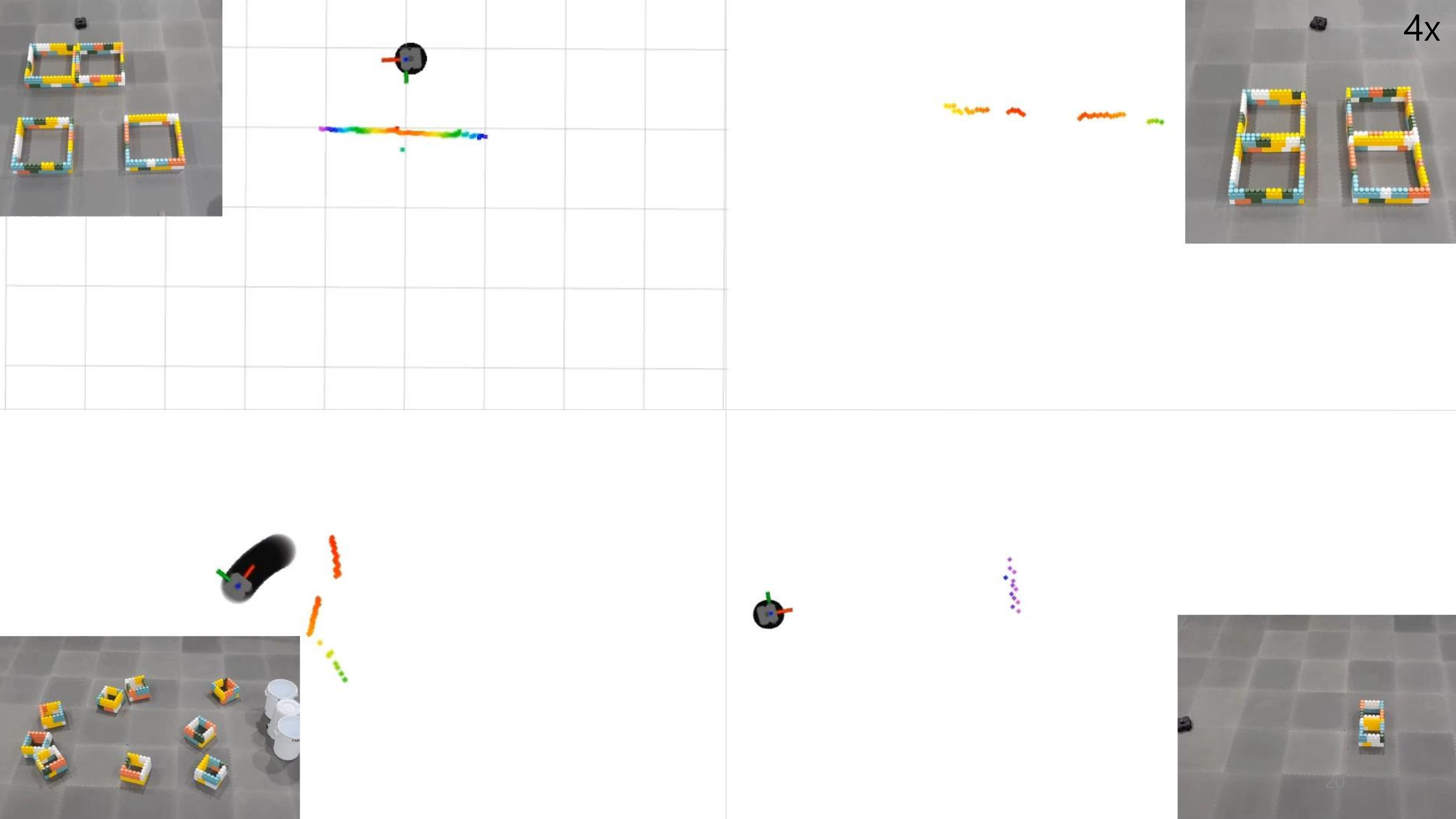


Receding Horizon Trajectory

Colors close to red denote small w values.

Black tube denotes the currently chosen receding horizon trajectory.

Operator instructed to transmit aggressive reference inputs.


4x

Summary

- In this work, we examined the **behavior of safe receding horizon motion planning problems** when the **update interval becomes infinitesimal**.

- With some modifications to the planning problem, infinitesimal update interval yields a **safety filter based on quadratic programming (QP)**.

- The resulting safety filter is **safe**, **recursively feasible**, and can be solved in **real-time**.

- We demonstrated its performance through **real-world experiments**.

Thank you!

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On-Site Presentation

Oral Presentation:

Thursday, May 16, 10:30 – 12:00

Room 301, ThAT33.03 (Integrated Planning and Control Session)

Poster Session:

Thursday, May 16, 13:30 – 15:00