Computational Physics
Exercise 2

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1 Intro

The Ising model consists of a lattice of sites x with spins $s_x \in \{\pm 1\}$ attached. In this exercise we study the 2-dim Ising model. Each spin couples to its neighbouring spins and an external magnetic field. Thus the Ising Hamiltonian of a spin configuration $s = \{s_x : \forall x\}$ is:

$$\mathcal{H}(s) = -J \sum_{\langle s_x, s_y \rangle} s_x s_y - h \sum_x s_x \tag{1}$$

were $\langle s_x, s_y \rangle$ denotes a pair of neighbouring spins. For more information on the Ising model, please refer to the exercise sheet or the relevant literature.

To simulate the 2-dim Ising model we implement a Monte-Carlo simulation using the Metropolis-Hastings algorithm.

1. Generate random initial spin configuration
2. Choose random spin flip
3. Calculate change in action due to spinflip $\Delta S = \beta \left(\mathcal{H}(s') - \mathcal{H}(s') \right)$ (in our case $\beta = 1$)
4. sample random number x between 0 and 1 from uniform distribution
5. if $x < \exp(-\Delta S)$ accept spinflip, else reject
6. repeat steps 2 - 5 until thermalized
7. sweep through the lattice, performing steps 3-5 once on each spin
8. take measurement
9. repeat steps 7 and 8 until enough measurements are taken
10. average measurements

We will be using 1000 thermalization and 2000 measurement steps for the analysis.

2 Question 2

The numerical cost of the calculation of the energy for a given spin configuration scales proportional to Λ if the lattice is larger than 2 in both x and y direction.

3 Question 3

The numerical cost of the calculation of the change in energy for a given spin flip does not depend on Λ if the lattice is larger than 2 in both x and y direction.

4 Question 4

At $h = 0$ there is a phase transition at

$$J_c = \frac{1}{2} \log\left(1 + \sqrt{2}\right) \approx 0.440686793509772 \tag{2}$$

At values of J below J_c the average magnetization is $\langle m \rangle = 0$ while at above J_c the average magnetization is either $+1$ or -1, i.e. $|\langle m \rangle| = 1$. Only in a short interval close to J_c are mean magnetizations other than 0, -1 and $+1$. This is the region of the transition.

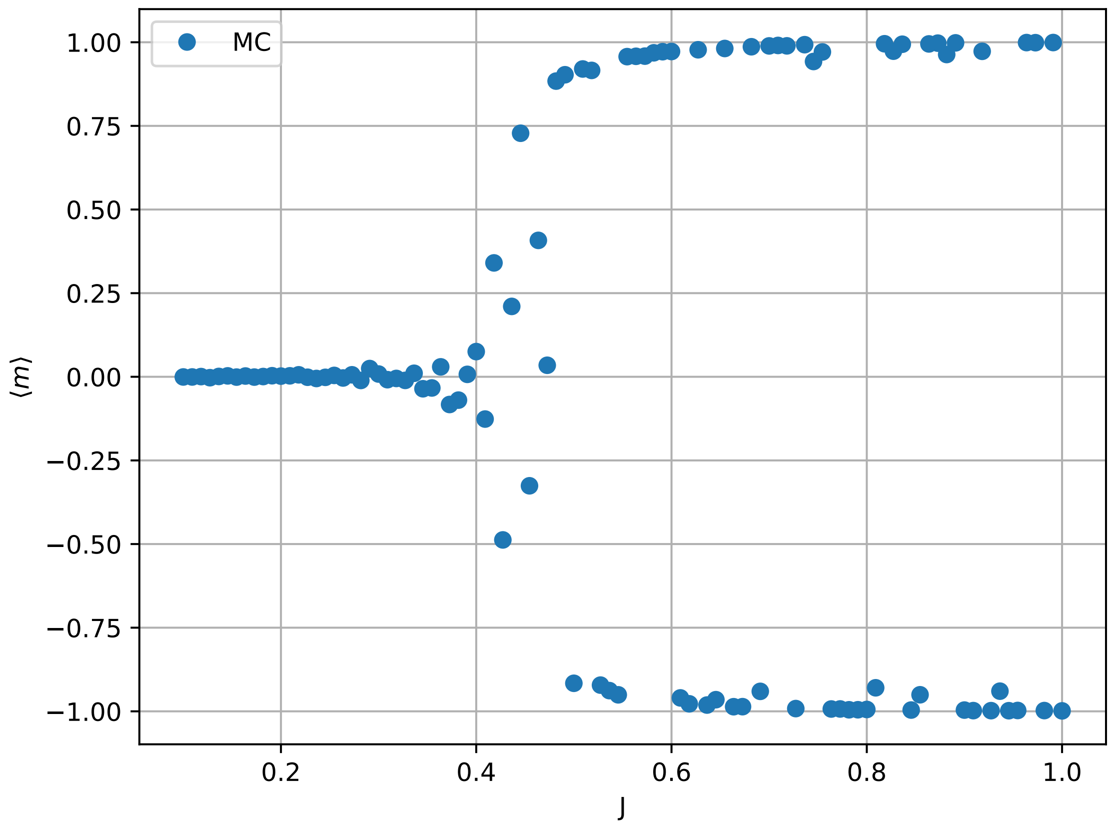

Figure 1: Dependence of the magnetization on J at $h = 0$ and $N = 12$.

5 Question 5

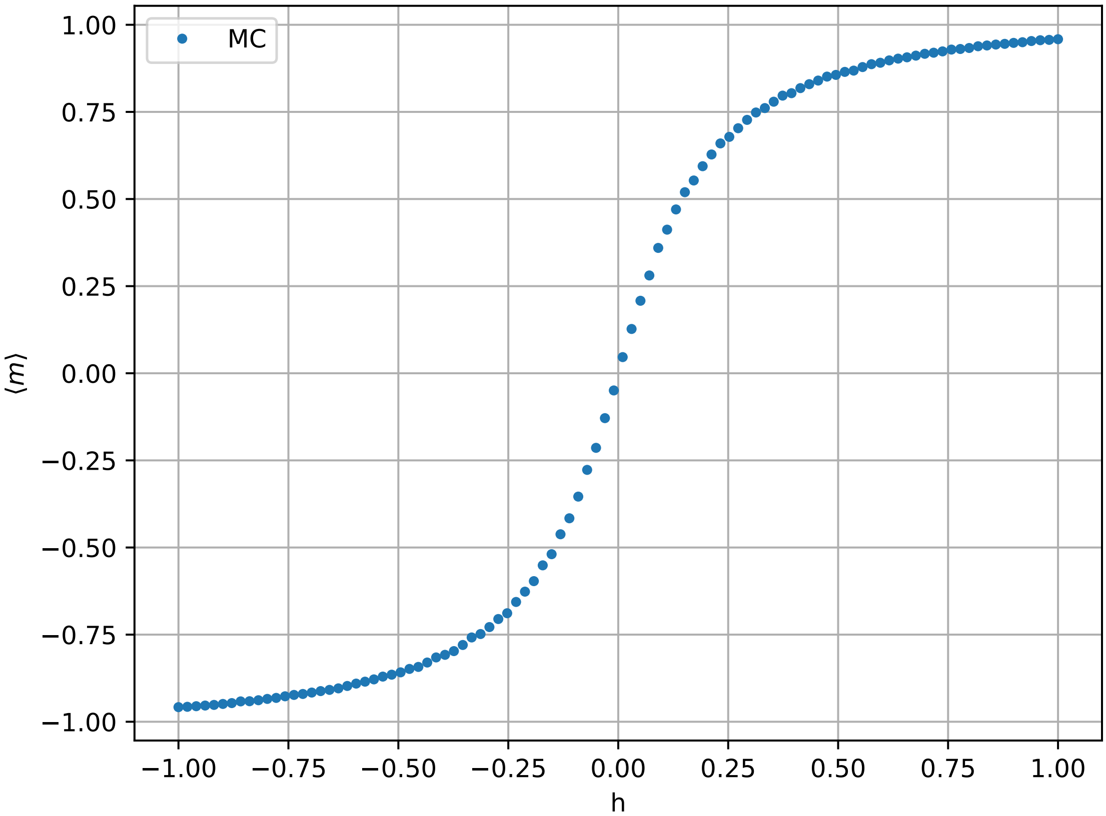

Figure 2: Dependence of the magnetization on h at $N = 12$ and $J = 0.25$.

6 Question 6

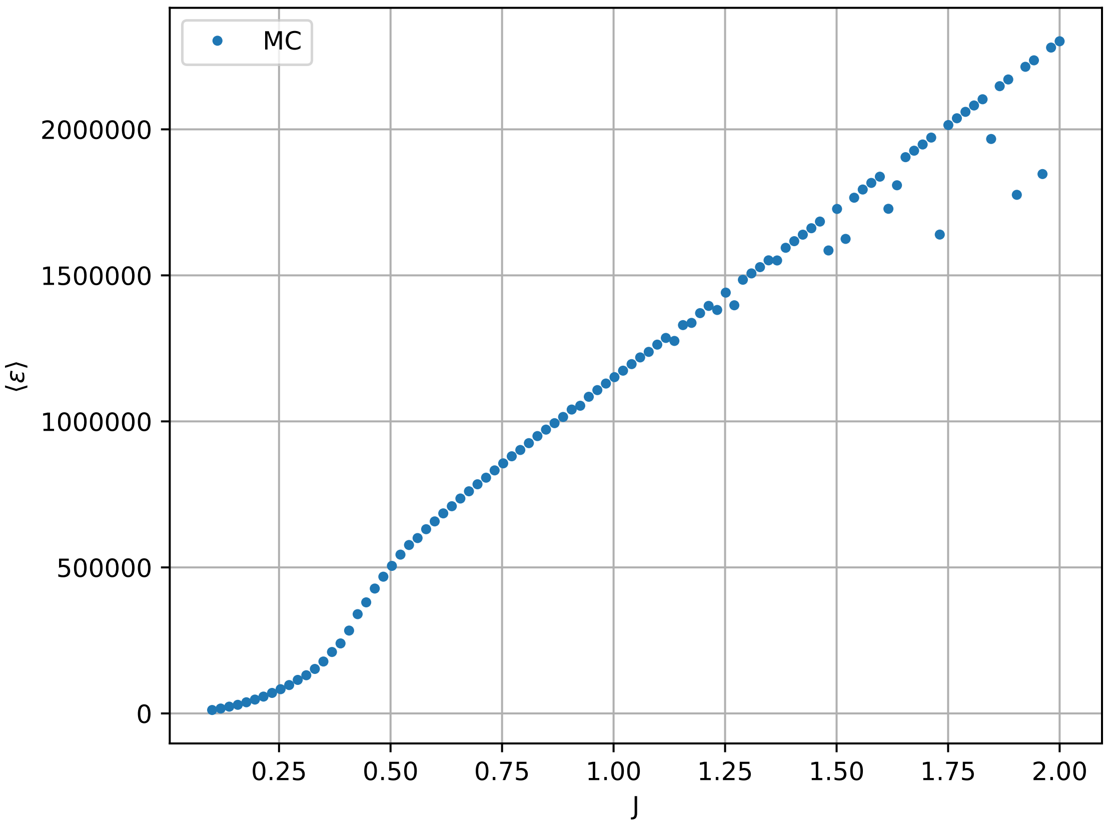

Figure 3: Dependence of the magnetization on J at $h = 0$ and $N = 12$.

7 Question 7

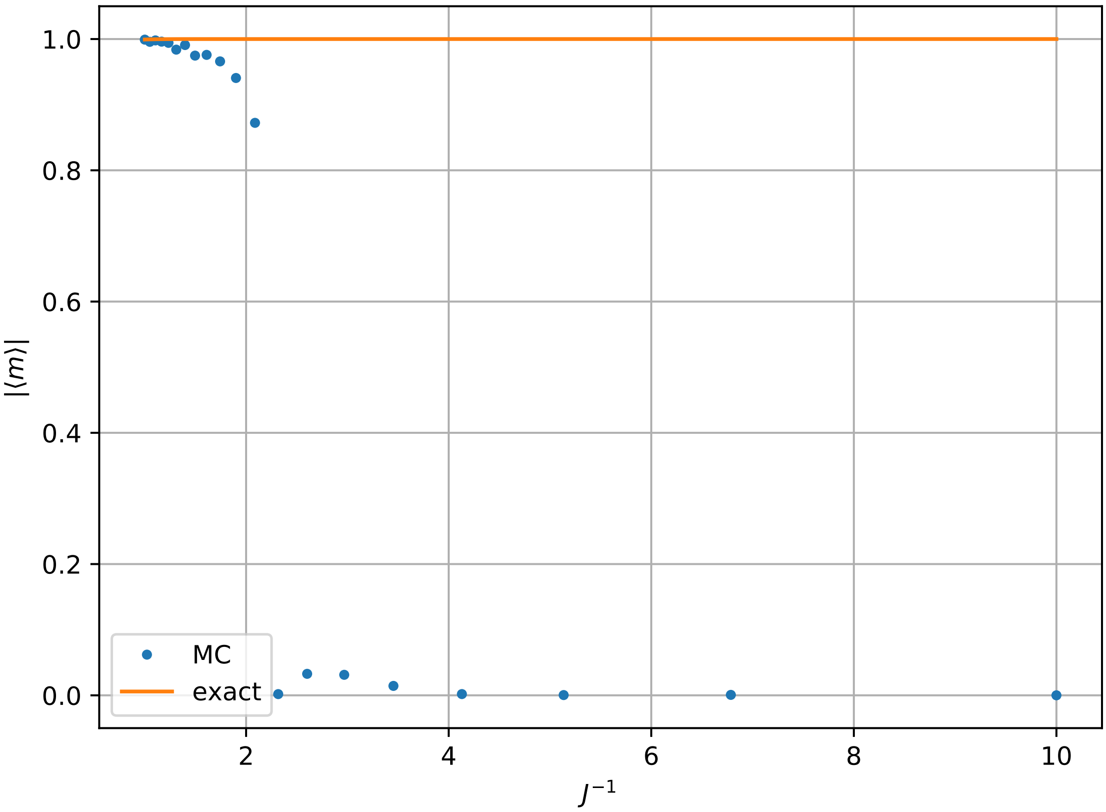

Figure 4: Dependence of the absolute value of the magnetization on and J^{-1} at $h = 0$ and $N = 12$