Computational Physics
Exercise 5

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1 Task 1

We were to dumb to solve this within 10 minutes and too lazy to spend more time on an exercise that is only worth 1 point.

2 Task 2

We implemented the Metropolis-Hastings algorithm described on the excercise sheet.

Figures 1 and 2 show our test of the algorithm with $\delta = 2$, $N = 64$ and $\beta = 1$.

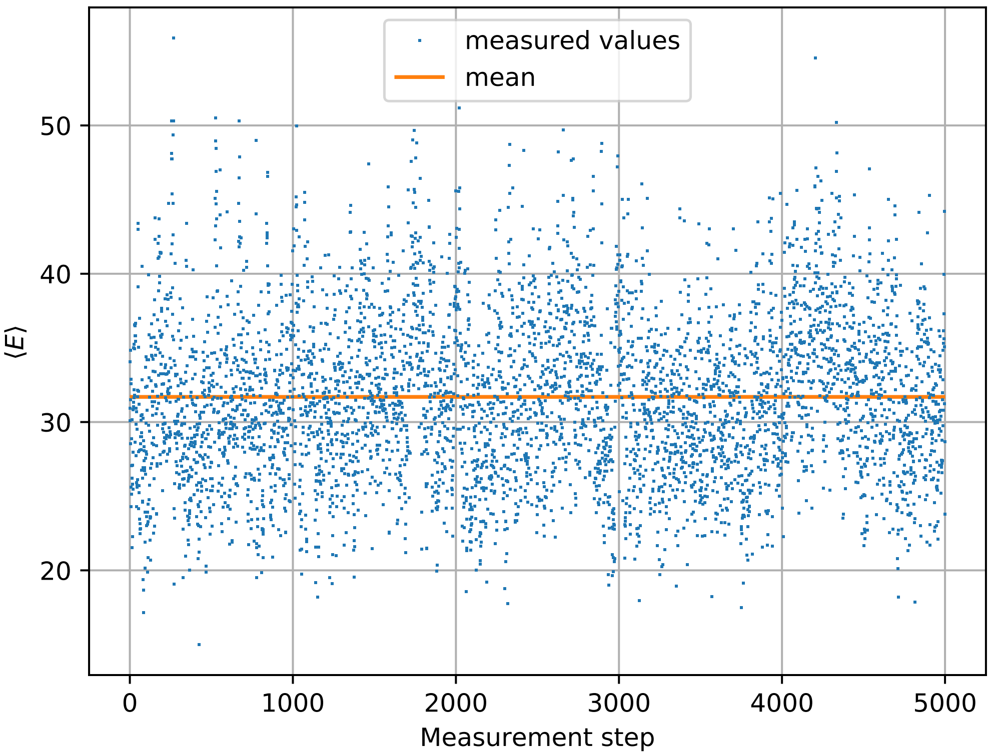

Figure 1: Magnetization m after each Metropolis-Hastings sweep

Since did not solve Task 1 we cannot compare our results to the analytical solution.

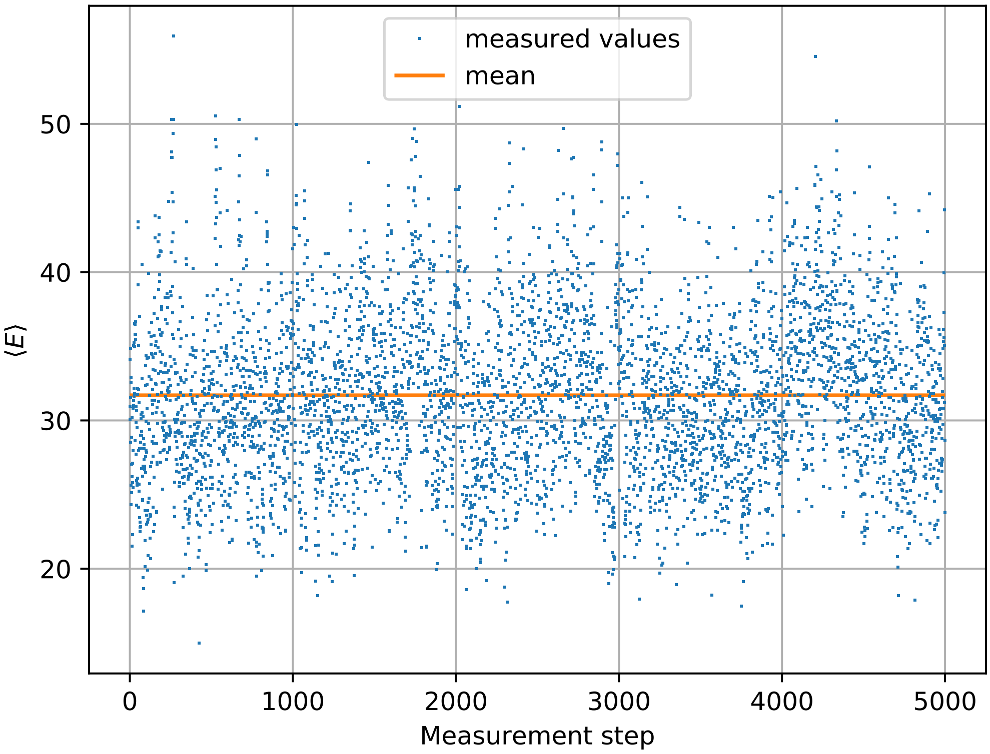

Figure 2: Energy E after each Metropolis-Hastings sweep

3 Task 3

$$\phi_i^{(2a)} = \frac{1}{2}\left(\phi_{2i}^{(a)} + \frac{1}{2}\left(\phi_{2i+1}^{(a)} + \phi_{2i-1}^{(a)}\right)\right) \tag{1}$$

But since we were tasked to set $\phi^{(a)} = 0$, we don't have any external field at any level. We may have misinterpreted the exercise.

Task 3

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$$H_a(u^a) = H_a(\tilde{u}^a) + H_{2a}(u^{2a})$$

$$= \frac{1}{a}\sum_{i=1}^{N}(\tilde{u}_i^a - \tilde{u}_{i-1}^a)^2 + a\sum_{i=1}^{N-1}\phi_i^a\tilde{u}_i^a + \frac{1}{2a}\sum_{i=1}^{N/2}(u_i^{2a} - u_{i-1}^{2a})^2 + 2a\sum_{i=1}^{N/2-1}\phi_i^{2a}u_i^{2a}$$

$$= \frac{1}{a}\sum_{i=1}^{N}(u_i^a - u_{i-1}^a)^2 + a\sum_{i=1}^{N-1}\phi_i^a u_i^a$$

$$= \frac{1}{a}\sum_{i=1}^{N}\left(\tilde{u}_i^a + I_{2a}^a u_i^{2a} - (\tilde{u}_{i-1}^a + I_{2a}^a u_{i-1}^{2a})\right)^2 + a\sum_{i=1}^{N-1}\phi_i^a(\tilde{u}_i^a + I_{2a}^a u_i^{2a})$$

$$\Longrightarrow a\sum_{i=1}^{N-1}\phi_i^a\tilde{u}_i^a + 2a\sum_{i=1}^{N/2-1}\phi_i^{2a}u_i^{2a} \overset{!}{=} a\sum_{i=1}^{N-1}\phi_i^a\left(\tilde{u}_i^a + (I_{2a}^a u^{2a})_i\right)$$

$$\Longrightarrow \cancel{\sum_{i=1}^{N-1}\phi_i^a\tilde{u}_i^a} + 2\sum_{i=1}^{N/2-1}\phi_i^{2a}u_i^{2a} = \cancel{\sum_{i=1}^{N-1}\phi_i^a\tilde{u}_i^a} + \sum_{i=1}^{N-1}\phi_i^a(I_{2a}^a u^{2a})_i$$

$$\Longrightarrow 2\sum_{i=1}^{N/2-1}\phi_i^{2a}u_i^{2a} = \sum_{i=1}^{N-1}\phi_i^a(I_{2a}^a u^{2a})_i$$

$$= \sum_{i=1}^{N-1}\phi_i^a\begin{cases} u_{i/2}^{2a} & , i \text{ even} \\ (u_{(i-1)/2}^{2a} + u_{(i+1)/2}^{2a})/2 & , i \text{ odd}\end{cases}$$

$$= \sum_{i=1}^{N/2-1}\phi_{2i}^a u_i^{2a} + \frac{1}{2}\sum_{i=0}^{N/2-1}\phi_{2i+1}^a\left(u^{2a}_{\underbrace{(2i-1+1)/2}_{i}} + u^{2a}_{\underbrace{(2i+1+1)/2}_{i+1}}\right)$$

$$= \sum_{i=1}^{N/2-1}\phi_{2i}^a u_i^{2a} + \frac{1}{2}\sum_{i=0}^{N/2-1}\phi_{2i+1}^a u_i^{2a} + \frac{1}{2}\sum_{i=1}^{N/2}\phi_{2i-1}^a u_i^{2a}$$

$$\left(u_0^{2a} = u_{N/2}^{2a} = 0\right)$$

$$= \sum_{i=1}^{N/2-1}\left(\phi_{2i}^a + \frac{1}{2}(\phi_{2i+1}^a + \phi_{2i-1}^a)\right)u_i^{2a}$$

$$\Longrightarrow \phi_i^{2a} = \frac{1}{2}\left(\phi_{2i}^a + \frac{1}{2}(\phi_{2i+1}^a + \phi_{2i-1}^a)\right)$$

Figure 3

4 Task 4

We implemented the multigrid algorithm described on the excercise sheet.

5 Task 5

We test our implementation of the multigrid algorithm with $N = 64$, $\delta = 2$, $\gamma = \{1, 2\}$, $n_{\text{level}} = 3$ and $\nu_{\text{pre}} = \nu_{\text{post}} = \{4, 2, 1\}$. We use 500 thermalization and 5000 measurement steps. The results are shown in figures 4 and 5.

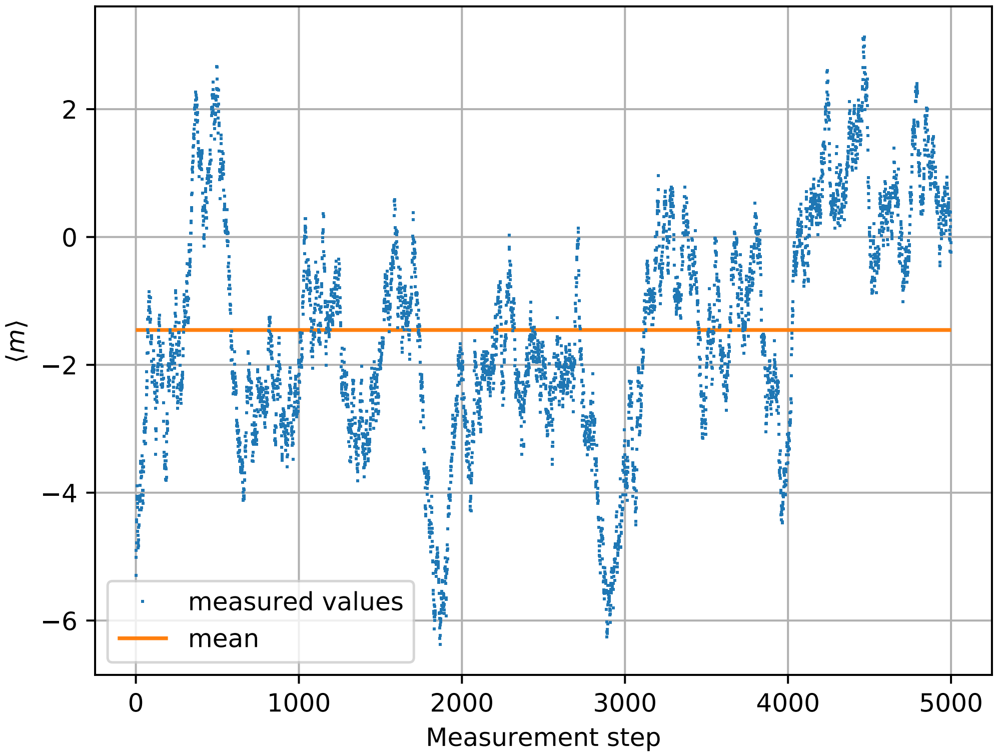

Figure 4: Magnetization m after each Multigrid sweep

We also calculate the normalized autocorrelation of the squared magnetization m^2 for the trajectories determined by the Metropolis-Hastings and Multigrid alogrithms. This is shown in figure 6.

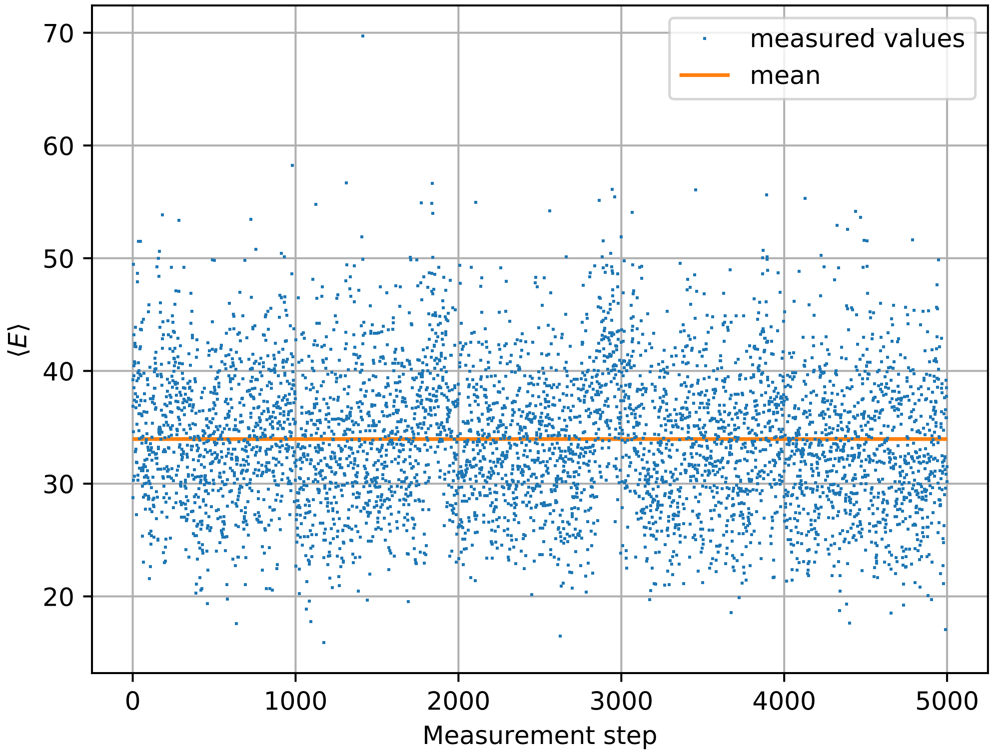

Figure 5: Energy E after each Multigrid sweep

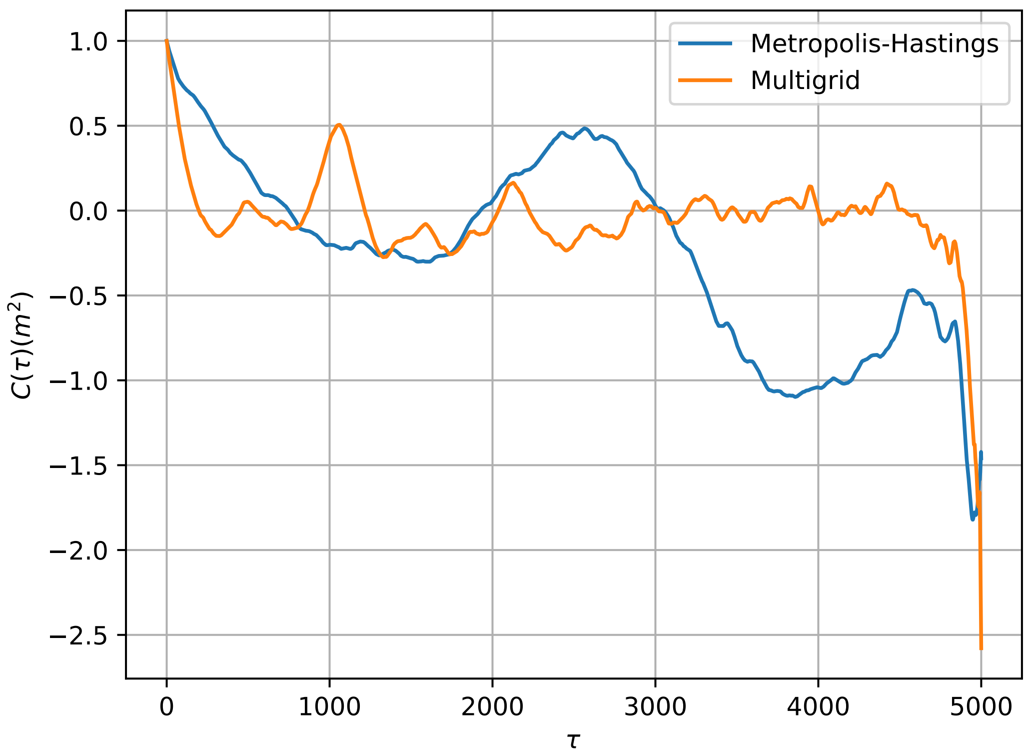

Figure 6: Autocorrelation of the squared magnetization m^2 determined by the Multigrid and Metropolis-Hastings algorithms