Analysis of time delay data and clock drift in a network of seismic monitors

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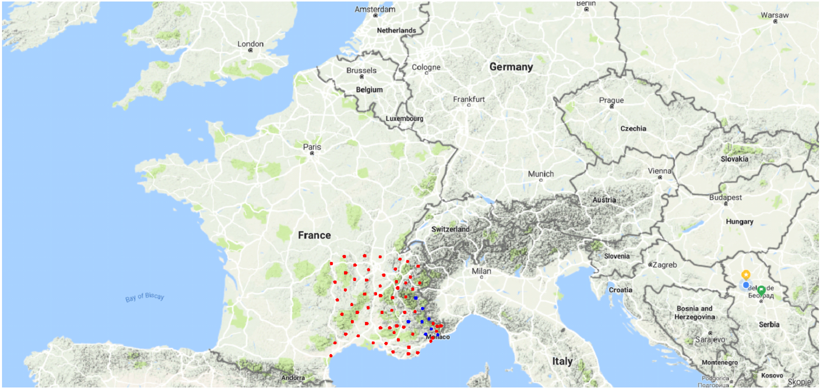
Problem statement

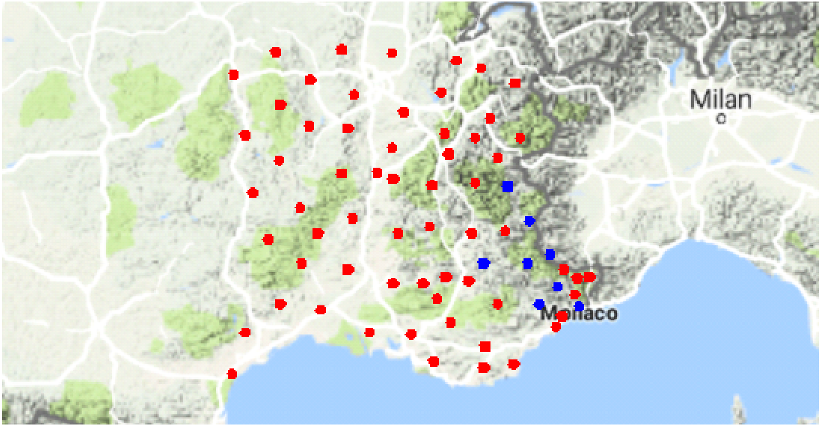
- Seismic monitoring is used to study the behaviour and composition of the underground floor
- There is a time drift in the monitors' clocks, causing inaccuracies in analysis
- Continuous synchronisation to the Global Positioning System for accurate timing is not possible
- A reliable method to correct the time drift of the clocks is required

The network of seismic monitors

- The network consists of 73 seismic monitors
- For initial analysis, 8 stations are selected

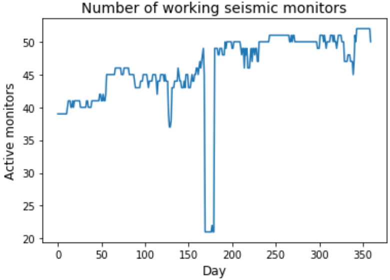


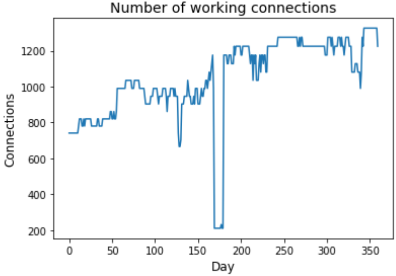
The network of seismic monitors



Numbers of working stations and connections to stations during one year

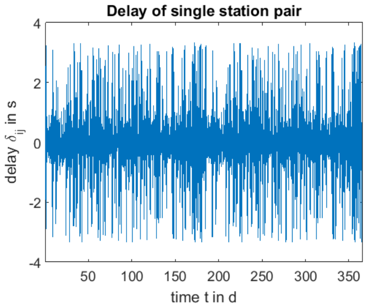
- The maximum number of simultaneously working stations is 52 out of 73
- The maximum number of active connections is 1326
- If two monitors are working simultaneously, they are connected

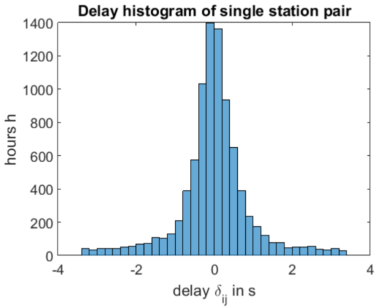




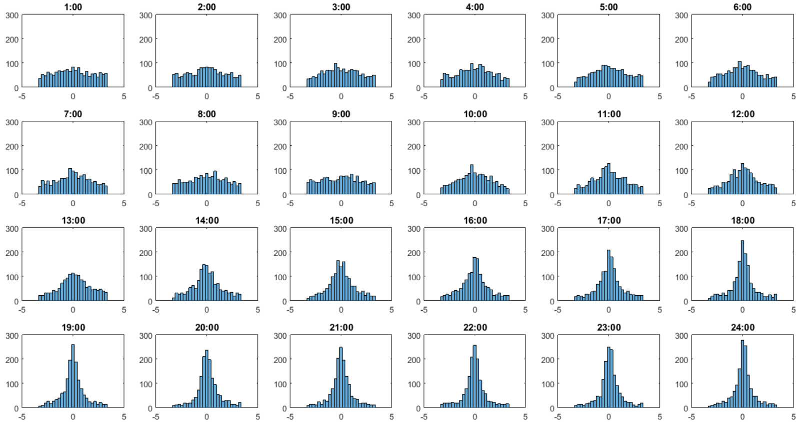
Time-delay signal characteristics

- Data is recorded in one second intervals
- Time-delay cross-correlation is computed in one-hour intervals
- Recording period is one year

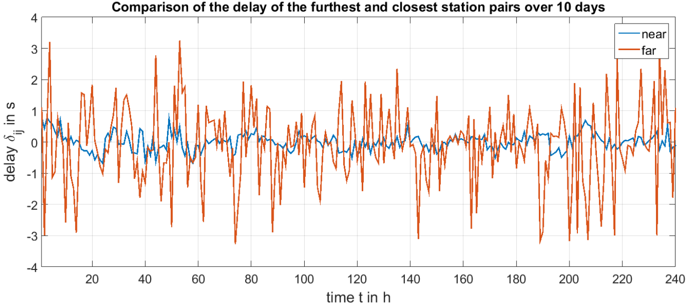




Histograms showing time delays recorded by all active stations over 24 hours



Comparison of time delay of the furthest and the closest station pairs



- Time delays are smaller between stations which are close to each other

Graph approach to model the data

- Measured time delays $\hat{\boldsymbol{\delta}}(t)$ between stations:

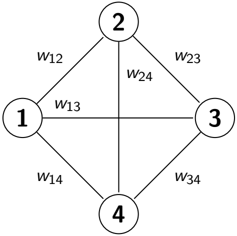
$$\hat{\boldsymbol{\delta}}(t) = \boldsymbol{\delta}(t) + \boldsymbol{\varepsilon}(t) \tag{1}$$

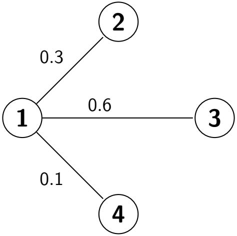
- Graphs: $\mathcal{G}_\delta = (\mathcal{V}, \mathcal{E}, \hat{\boldsymbol{\delta}})$, $\mathcal{G}_r = (\mathcal{V}, \mathcal{E}, \boldsymbol{W})$
- Metric:

$$f_i(\boldsymbol{X}) = K_i \tag{2}$$

with

$$f_i(\hat{\boldsymbol{\delta}}) = \sum_j \hat{\delta}_{ij} \quad \text{and} \quad K_i = f_i(\boldsymbol{W}) = \sum_j w_{ij} \tag{3}$$





normalised weights between $[0, 1]$

Signal denoising and exctracting clock drift

- We specify a cost function $c(\hat{\boldsymbol{\delta}})$ measuring the deviation of the metric:

$$c(\hat{\boldsymbol{\delta}}) = \sum_i e_i^2(\hat{\boldsymbol{\delta}}) = \sum_i (f_i(\hat{\boldsymbol{\delta}}) - K_i)^2 \tag{4}$$

- Error is minimized with gradient descent updates on $\hat{\boldsymbol{\delta}}$:

$$\hat{\boldsymbol{\delta}}^{(k+1)} = \hat{\boldsymbol{\delta}}^k - \mu \sum_i E_i(\hat{\boldsymbol{\delta}}^k) \frac{df_i(\hat{\boldsymbol{\delta}}^k)}{d\hat{\boldsymbol{\delta}}^k} \tag{5}$$

- The noise ε_{ij} between each pair of two stations is caused by clock drifts Δ_i and Δ_j, and other errors E_{ij}:

$$\varepsilon_{ij} = \Delta_i - \Delta_j + e_{ij} \tag{6}$$

Solving individual clock drifts (1/2)

- We have overdetermined inverse problem of the form

$$\mathbf{Gs} = \mathbf{m} \tag{7}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & \dots & 0 \\ 1 & 0 & -1 & 0 & \dots & 0 \\ 1 & 0 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & \dots & -1 \\ 0 & 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & 0 & -1 & \dots & 0 \\ 0 & 1 & 0 & 0 & \ddots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \vdots \\ \Delta_k \\ \vdots \\ \Delta_{n-1} \\ \Delta_n \end{bmatrix} = \begin{bmatrix} \Delta_1 - \Delta_2 \\ \Delta_1 - \Delta_3 \\ \Delta_1 - \Delta_4 \\ \vdots \\ \Delta_1 - \Delta_n \\ \Delta_2 - \Delta_3 \\ \Delta_2 - \Delta_4 \\ \vdots \\ \Delta_2 - \Delta_k \\ \vdots \\ \Delta_{n-1} - \Delta_n \end{bmatrix} = \begin{bmatrix} \Delta_{12} \\ \Delta_{13} \\ \Delta_{14} \\ \vdots \\ \Delta_{1n} \\ \Delta_{23} \\ \Delta_{24} \\ \vdots \\ \Delta_{2k} \\ \vdots \\ \Delta_{(n-1)n} \end{bmatrix}$$

- The rank of matrix **G** is $n - 1$

Solving individual clock drifts (2/2)

- Ordinary least squares regression

$$\|\mathbf{Gs} - \mathbf{m}\|_2^2. \tag{8}$$

- Tikhonov regularization is employed

$$\|\mathbf{Gs} - \mathbf{m}\|_2^2 + \|\boldsymbol{\Gamma}\mathbf{s}\|_2^2, \tag{9}$$

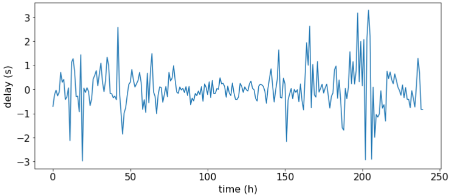
  for some suitable Tikhonov matrix $\boldsymbol{\Gamma} = \alpha\mathbf{I}$

- The explicit solution is hence given by

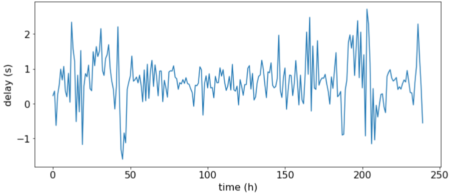
$$\hat{\mathbf{s}} = (\mathbf{G}^T\mathbf{G} + \boldsymbol{\Gamma}^T\boldsymbol{\Gamma})^{-1}\mathbf{G}^T\mathbf{m}. \tag{10}$$

Signal - denoised signal - noise

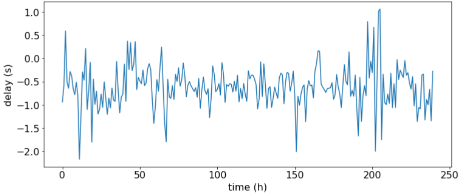
Delay signal $\hat{\delta}$ of one connection for 10 days



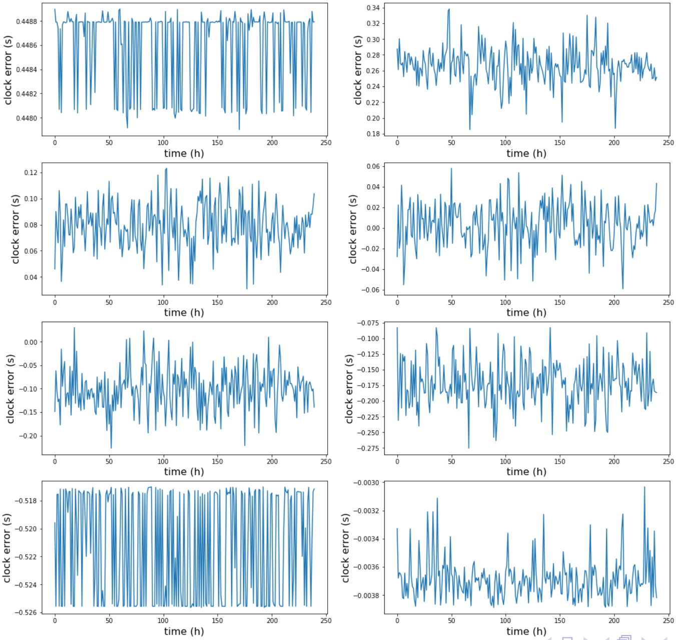
Denoised signal δ of the connection



Signal noise ε



Clock drifts



Conclusion

- A graph method to recover clock drift in large time-delay network was developed
- Clock drifts for the three most influential monitors and five others close-by were computed
- The drift varies over time
- Clock drift may account for up to 1 % inaccuracy in timing at each time point

Future work:

- A synthetic dataset to verify the effectiveness of the method
- Analyse the complete network of monitors
- Develop and test other metrics
- Carry out sensitivity analysis