Analysis of time delay data and clock drift in a network of seismic monitors

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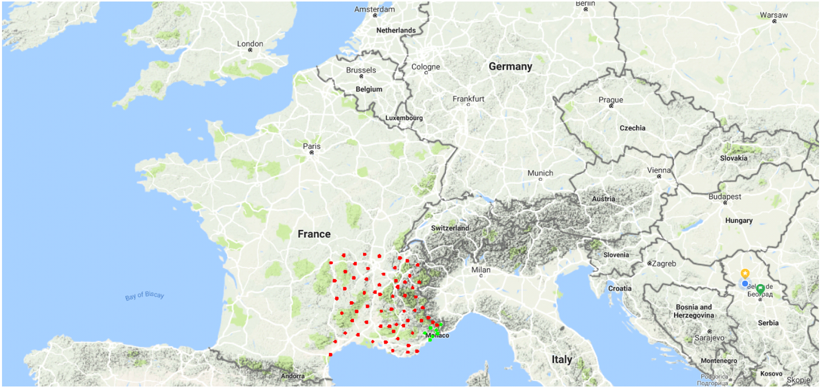
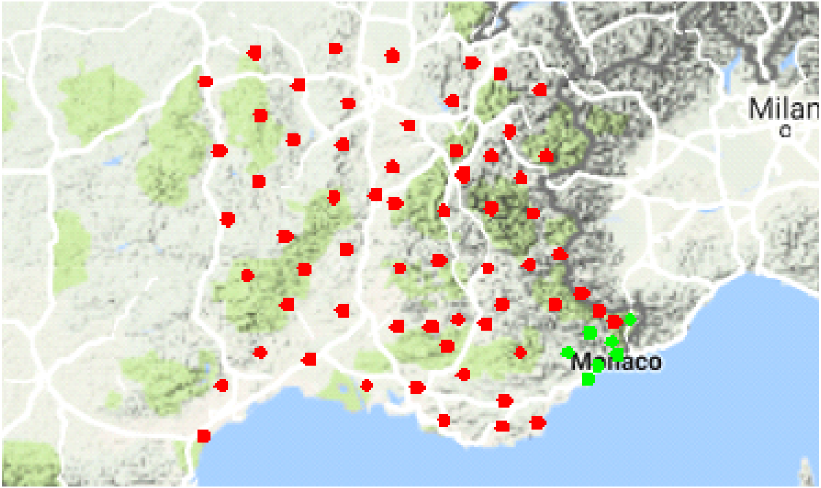
Problem statement

- Seismic monitoring is used to study the behaviour and composition of the underground floor
- Continuous synchronization to the Global Positioning System for accurate timing is not possible
- The instruments' clocks deviate in time causing lack of accuracy in the measurements
- A reliable method to correct for the time deviation is required
- Real data collected from a network of seismic monitors over time is analyzed
- The problem is to discern the time drift of the clock in each monitor from noise and actual data

The network of seismic monitors

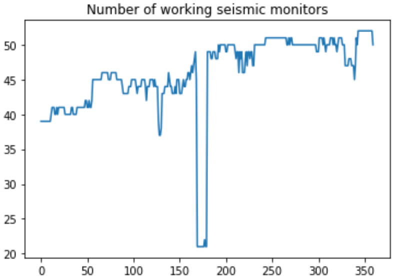


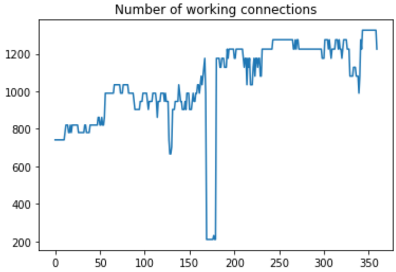
The network of seismic monitors



Numbers of working stations and connections to stations during one year

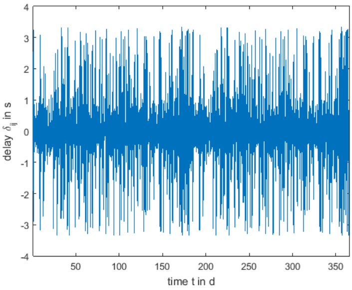
- The network consists of 73 seismic monitors
- Not all monitors work at all times
- If two monitors are working simultaneously, they are connected
- Connections are undirected

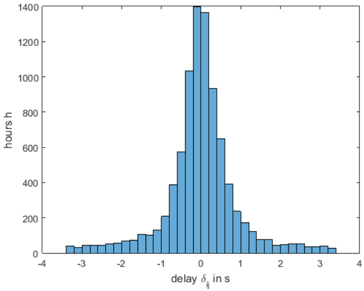




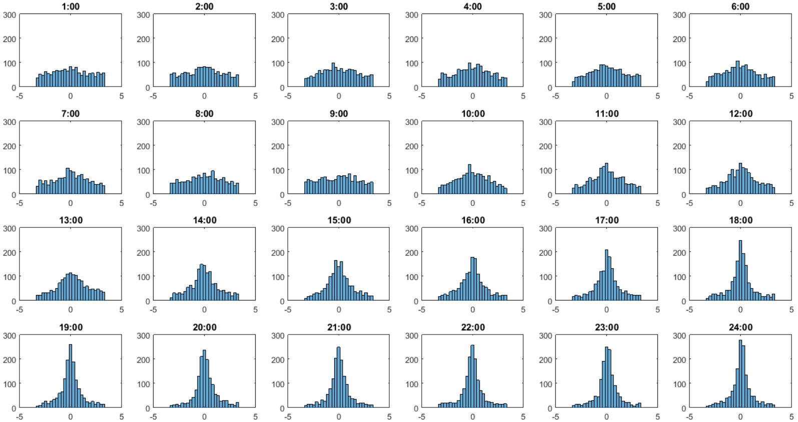
Time-delay signal characteristics

- Data recorded in one second intervals
- Time-delay cross-correlation in one-hour intervals
- Tracing time-delays recorded by one station over a year
- Time delays are computed in both directions (from A $\rightarrow$ B and B $\rightarrow$ A) and are opposite numbers

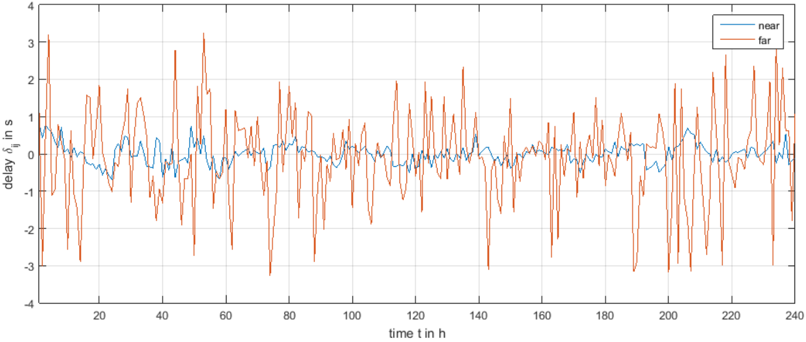




Histograms showing time delays recorded by all active stations over 24 hours



Comparison of time delay of the furthest and the closest station pairs



- Time delays are smaller between stations which are close to each other
- The non-linear trace of the data is thought to be caused by the station's non-ideal clock (clock drift)

Clock drift and errors in the data

- The measurements $\hat{\boldsymbol{\delta}}$ of time delays between stations

$$\hat{\boldsymbol{\delta}} = \boldsymbol{\delta} + \boldsymbol{\varepsilon}. \tag{1}$$

- The error ε_{ij} is error in time delay between stations i and j

$$\varepsilon_{ij} = |\Delta_i - \Delta_j| + e_{ij}, \tag{2}$$

Δ_i is the clock drift of station i
e_{ij} includes other measurement errors

Graph approach to model the data

- A weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \hat{\boldsymbol{\delta}}_t)$
- Weighted adjacency matrix of time delays $\hat{\boldsymbol{\delta}}_t$) is symmetric with $\delta_{ii} = 0$
- Values $\hat{\delta}_{ij}$ are normalized to $[0, 1]$
- A normalized (to $[0, 1]$) distance measure was selected as graph metrics K_m

$$K_m = f_m(r_i) = \sum_j r_{ij}. \tag{3}$$

Signal denoising

- We specify a cost function $c(\hat{\boldsymbol{\delta}})$ measuring the deviation from the observed weight matrix's metrics $f_m(\hat{\boldsymbol{\delta}})$ to the estimates of the distance metrics K_m

$$c(\hat{\boldsymbol{\delta}}) = \sum_m e_m^2(\hat{\boldsymbol{\delta}}) = \sum_m (f_m(\hat{\boldsymbol{\delta}}) - K_m)^2 \tag{4}$$

- Error is minimized with gradient descent updates on $\hat{\boldsymbol{\delta}}$:

$$\hat{\boldsymbol{\delta}}^{(t+1)} = \hat{\boldsymbol{\delta}}^t - \mu \sum_m e_m(\hat{\boldsymbol{\delta}}^t) \frac{df_m(\hat{\boldsymbol{\delta}}^t)}{d\hat{\boldsymbol{\delta}}^t}, \tag{5}$$

Exctracting clock drift from noise

- The noise ε_{ij} between each pair of two stations is modelled with
$$\varepsilon_{ij} = |\Delta_i - \Delta_j| + e_{ij}, \tag{6}$$
in which Δ_i and Δ_j are the clock drifts of stations i, j
e_{ij} accounts for all other noise in the system
- A polynomial curve is fitted to the extracted noise to trace clock drift

Solvin individual clock drifts (1/2)

- We have overdetermined inverse problem of the form

$$Gm = s \tag{7}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & \dots & 0 \\ 1 & 0 & -1 & 0 & \dots & 0 \\ 1 & 0 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & \dots & -1 \\ 0 & 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & 0 & -1 & \dots & 0 \\ 0 & 1 & 0 & 0 & \ddots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \vdots \\ \Delta_k \\ \vdots \\ \Delta_{n-1} \\ \Delta_n \end{bmatrix} = \begin{bmatrix} \Delta_1 - \Delta_2 \\ \Delta_1 - \Delta_3 \\ \Delta_1 - \Delta_4 \\ \vdots \\ \Delta_1 - \Delta_n \\ \Delta_2 - \Delta_3 \\ \Delta_2 - \Delta_4 \\ \vdots \\ \Delta_2 - \Delta_k \\ \vdots \\ \Delta_{n-1} - \Delta_n \end{bmatrix} = \begin{bmatrix} \Delta_{12} \\ \Delta_{13} \\ \Delta_{14} \\ \vdots \\ \Delta_{1n} \\ \Delta_{23} \\ \Delta_{24} \\ \vdots \\ \Delta_{2k} \\ \vdots \\ \Delta_{(n-1)n} \end{bmatrix}$$

- Matrix G rank is $n-1 \rightarrow$ regularization required

Solving individual clock drifts 1/2

- Ordinary least squares regression

$$\|\mathbf{Gm} - \mathbf{s}\|_2^2. \tag{8}$$

- Tikhonov regularization is employed

$$\|\mathbf{Gm} - \mathbf{s}\|_2^2 + \|\mathbf{\Gamma m}\|_2^2, \tag{9}$$

for some suitable Tikhonov matrix $\mathbf{\Gamma} = \alpha \mathbf{I}$

- The explicit solution is hence given by

$$\hat{\mathbf{s}} = (\mathbf{G}^T\mathbf{G} + \mathbf{\Gamma}^T\mathbf{\Gamma})^{-1}\mathbf{G}^T\mathbf{m}. \tag{10}$$

Results

Conclusion