Modeling effect of time delay for large network of seismic monitor

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Abstract

1 Introduction

The purpose of this work is to analyze noise in a large network of seismic monitors and to extract clock drift from the noise.

2 Methods

2.1 Basic variables, model

We have a network of seismic monitors recording ground vibrations (Figure 1). The monitor records compression and decompression as discrete values -1 and 1, respectively, every second. The GPS coordinate position of each monitor in the network is known (Figure !!!TODO: insert map of monitors).

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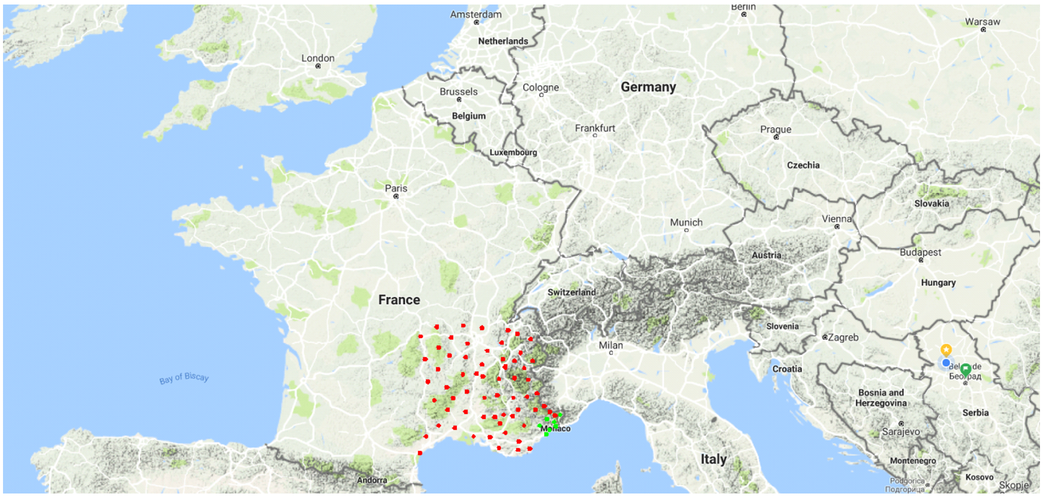

Figure 1: The network of seismic monitors are located in the Southeast France, mostly in the regions of Provence-Alpes-Côte d'Azur and Rhône-Alpes

The network of monitors can be modelled as a complete graph !!!TODO: schematic representation of the graph. The signal of each monitor is cross-correlated with other monitor signals, finally yielding time-delay data between each monitor. The time delay between the monitors comprise information on the real time the signal travels between the monitors, the inaccuracies of the clock (time drift), and other noise affecting the measurement.

Let $\hat{\boldsymbol{\delta}} \in \mathbb{R}^{n \times n}$ be the measured pairwise time delays of the system, $\boldsymbol{\delta}$ the actual pairwise time delays, and $\boldsymbol{\varepsilon}$ an error term including clock drift of the station's clock and other errors

$$\hat{\boldsymbol{\delta}} = \boldsymbol{\delta} + \boldsymbol{\varepsilon}. \tag{1}$$

Each of the monitors are equipped with a clock which runs independent of others. It is synchronized via GPS system once a month and then runs independently. The drift, ε_i in clock i is caused by variation in the clocks oscillator.

The position of each seismic monitor in the network is known. As the monitors are spread across a large area, it is assumed that local tremors are detected by stations that are close by, and therefore the correlations found in the pairwise cross-correlations and time delay data between them have higher likelihood to be linked.

2.2 Computational model

The computational model involves weighted network estimation by the use of topological graph metrics, described in detail in [1].

We have a weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \hat{\delta}_{\mathbf{t}})$ defined by a finite set of nodes $\mathcal{V}$ with $|\mathcal{V}| = n$, a set of edges $\mathcal{E} = \{(v_i, v_j) \in \mathcal{E}\}$, with $|\mathcal{E}| = n^2 - n$ and the weighted adjacency matrix $\hat{\boldsymbol{\delta}}$ with $\delta_{ii} = 0$ for all i. The matrix $\hat{\boldsymbol{\delta}}$ is symmetric and describes the time delays between events in the graph, and is normalized, i.e. $\hat{\delta}_{ij} \in [0, 1]$. The adjacency matrix indicates the strength of connection between nodes. Therefore, the shorter the time delay between nodes, the stronger the connection.

Graph metrics are scalar functions of the weight matrix, $f(\hat{\boldsymbol{\delta}}) : \mathbb{R}^{n^2} \to \mathbb{R}$. We choose graph metrics to:

1. Degree:
$$k_i^{\delta_t} = \sum \delta_{ij} = \mathrm{tr}\{\hat{\boldsymbol{\delta}}\mathbf{R}_i\}, \tag{2}$$
with the degree derivative being:
$$\frac{\partial k_i^{\delta}}{\partial \hat{\boldsymbol{\delta}}} = \mathbf{R_i}^T. \tag{3}$$
Since $\mathbf{R}_i$ is non-zero only for column i it can be computed efficiently as
$$\frac{\partial k_i^{\delta}}{\partial \hat{\boldsymbol{\delta}}_i} = \mathbf{1}_n^T, \tag{4}$$
where $\hat{\boldsymbol{\delta}}_i$ is the ith column of $\hat{\boldsymbol{\delta}}$

2. Centrality of the node: the inverse average distance between every other vertex !!!TODO: equations and derivative to be added

3. Average neighbour degree, resilience:

   The average neighbour degree for node i is given by

References

[1] L. Spyrou and J. Escudero. Weighted network estimation by the use of topological graph metrics. *CoRR*, abs/1705.00892, 2017.