Analysis of time delay data and clock drift in a network of seismic monitors

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Problem statement

- Seismic monitoring is used to study the behaviour and composition of the underground floor
- There is a time drift in the monitors' clocks, causing inaccuracies in analysis
- Continuous synchronization to the Global Positioning System for accurate timing is not possible
- A reliable method to correct the time drift of the clocks' is required

The network of seismic monitors

- The network consists of 73 seismic monitors
- For initial analysis, 7 stations are selected
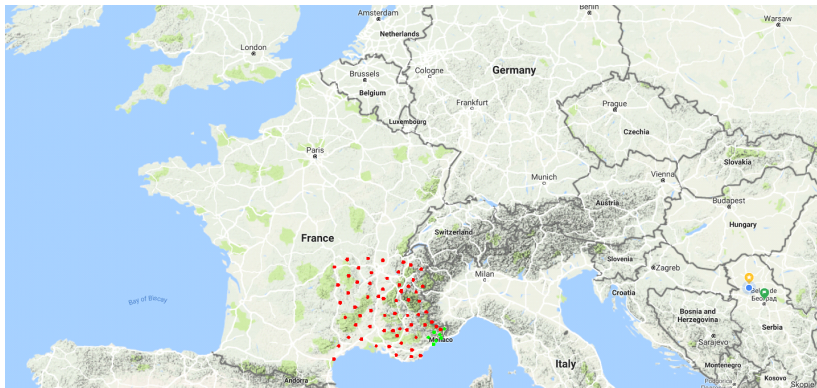
The network of seismic monitors
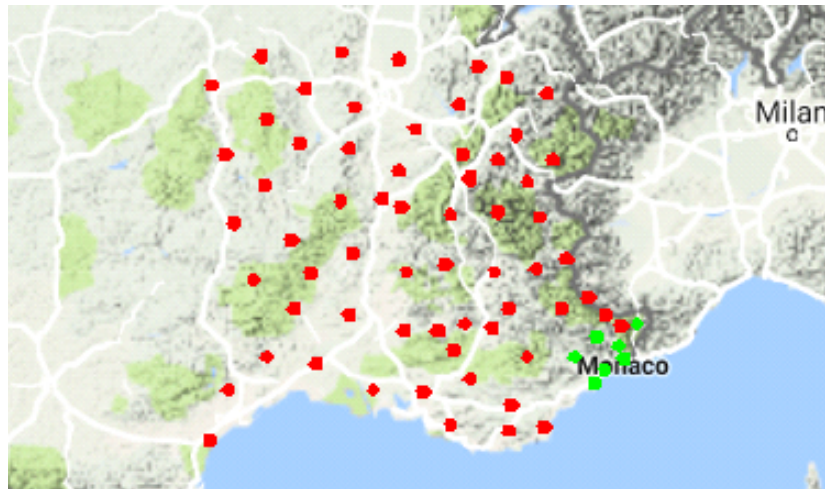
Numbers of working stations and connections to stations during one year

- The maximum of simultaneously working stations is 52 out of 73
- The maximum of connections is 1326
- If two monitors are working simultaneously, they are connected

Number of working seismic monitors

Number of working connections
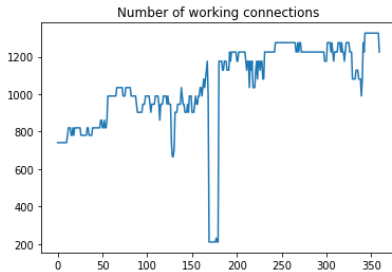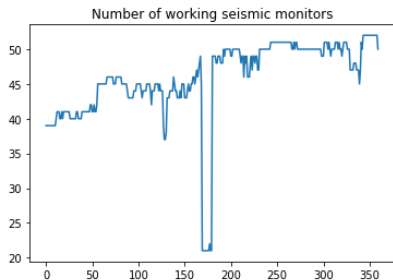
Time-delay signal characteristics

- Data recorded in one second intervals
- Time-delay cross-correlation in computed in one-hour intervals
- Each connection is computed only once
- Tracing time-delays recorded by one station over a year

Delay of single station pair

Delay histogram of single station pair
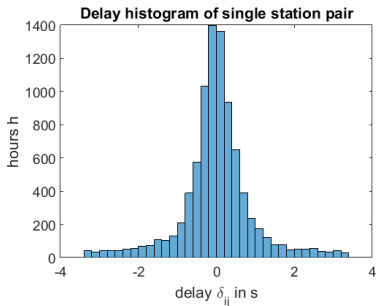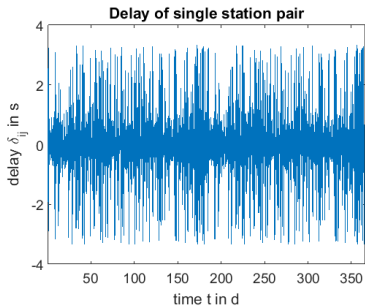
Histograms showing time delays recorded by all active stations over 24 hours
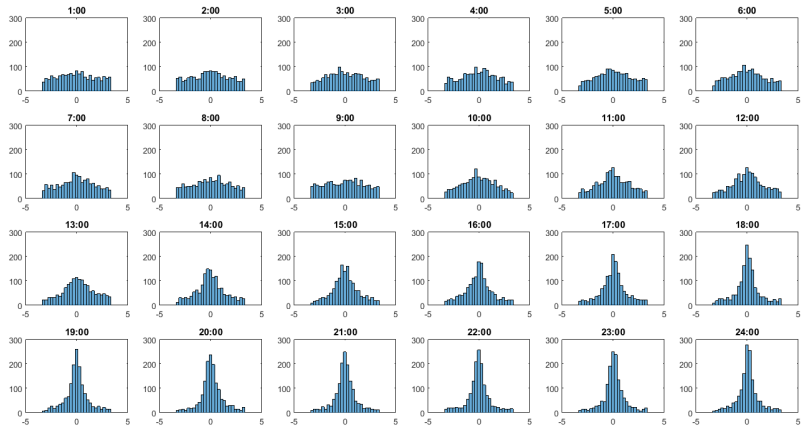
Comparison of time delay of the furthest and the closest station pairs

Comparison of the delay of the furthest and closest station pairs over 10 days
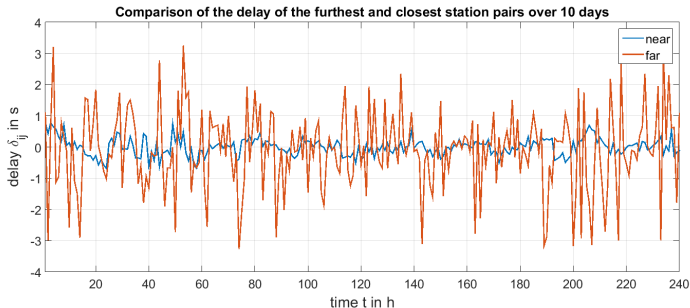
▶ Time delays are smaller between stations which are close to each other

Graph approach to model the data

- The measurements $\hat{\boldsymbol{\delta}}$ of time delays between stations

$$\hat{\boldsymbol{\delta}} = \boldsymbol{\delta} + \boldsymbol{\varepsilon} \tag{1}$$

- A weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \hat{\boldsymbol{\delta}}_t)$
- Values $\hat{\delta}_{ij}$ are normalized to $[0, 1]$
- A normalized distance measure was selected as graph metrics K_m

$$K_m = f_m(r_i) = \sum_j r_{ij}. \tag{2}$$

frametitleGraph approach to model the data

▶ The measurements $\hat{\boldsymbol{\delta}}$ of time delays between stations

$$\hat{\boldsymbol{\delta}} = \boldsymbol{\delta} + \boldsymbol{\varepsilon} \tag{3}$$

▶ A weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \hat{\boldsymbol{\delta}}_t)$
▶ Values $\hat{\delta}_{ij}$ are normalized to $[0, 1]$
▶ A normalized distance measure was selected as graph metrics K_m

$$K_m = f_m(r_i) = \sum_j r_{ij}. \tag{4}$$

blahblah
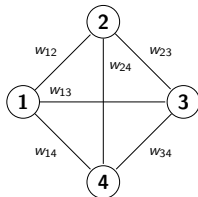
Signal denoising

- We specify a cost function $c(\hat{\boldsymbol{\delta}})$ measuring the deviation from the observed weight matrix's metrics $f_m(\hat{\boldsymbol{\delta}})$ to the estimates of the distance metrics K_m

$$c(\hat{\boldsymbol{\delta}}) = \sum_m e_m^2(\hat{\boldsymbol{\delta}}) = \sum_m (f_m(\hat{\boldsymbol{\delta}}) - K_m)^2 \tag{5}$$

- Error is minimized with gradient descent updates on $\hat{\boldsymbol{\delta}}$:

$$\hat{\boldsymbol{\delta}}^{(t+1)} = \hat{\boldsymbol{\delta}}^t - \mu \sum_m e_m(\hat{\boldsymbol{\delta}}^t) \frac{df_m(\hat{\boldsymbol{\delta}}^t)}{d\hat{\boldsymbol{\delta}}^t}, \tag{6}$$

Exctracting clock drift from noise

- The noise ε_{ij} between each pair of two stations is modelled with

$$\varepsilon_{ij} = \Delta_i - \Delta_j + e_{ij}, \qquad (7)$$

  in which Δ_i and Δ_j are the clock drifts of stations i, j
e_{ij} accounts for all other noise in the system

Solving individual clock drifts (1/2)

▶ We have overdetermined inverse problem of the form

$$Gm = s \tag{8}$$

$$
\begin{bmatrix}
1 & -1 & 0 & 0 & \ldots & 0 \\
1 & 0 & -1 & 0 & \ldots & 0 \\
1 & 0 & 0 & -1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 0 & 0 & 0 & \ldots & -1 \\
0 & 1 & -1 & 0 & \ldots & 0 \\
0 & 1 & 0 & -1 & \ldots & 0 \\
0 & 1 & 0 & 0 & \ddots & 0 \\
\hdotsfor{6} \\
0 & 0 & \ldots & 0 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
\Delta_1 \\
\Delta_2 \\
\Delta_3 \\
\Delta_4 \\
\vdots \\
\Delta_k \\
\vdots \\
\Delta_{n-1} \\
\Delta_n
\end{bmatrix}
=
\begin{bmatrix}
\Delta_1 - \Delta_2 \\
\Delta_1 - \Delta_3 \\
\Delta_1 - \Delta_4 \\
\vdots \\
\Delta_1 - \Delta_n \\
\Delta_2 - \Delta_3 \\
\Delta_2 - \Delta_4 \\
\vdots \\
\Delta_2 - \Delta_k \\
\vdots \\
\Delta_{n-1} - \Delta_n
\end{bmatrix}
=
\begin{bmatrix}
\Delta_{12} \\
\Delta_{13} \\
\Delta_{14} \\
\vdots \\
\Delta_{1n} \\
\Delta_{23} \\
\Delta_{24} \\
\vdots \\
\Delta_{2k} \\
\vdots \\
\Delta_{(n-1)n}
\end{bmatrix}
$$

▶ Matrix G rank is $n - 1$

Solving individual clock drifts 1/2

- Ordinary least squares regression

$$\|\mathbf{Gm} - \mathbf{s}\|_2^2. \tag{9}$$

- Tikhonov regularization is employed

$$\|\mathbf{Gm} - \mathbf{s}\|_2^2 + \|\mathbf{\Gamma m}\|_2^2, \tag{10}$$

for some suitable Tikhonov matrix $\mathbf{\Gamma} = \alpha\mathbf{I}$

- The explicit solution is hence given by

$$\hat{\mathbf{s}} = (\mathbf{G}^T\mathbf{G} + \mathbf{\Gamma}^T\mathbf{\Gamma})^{-1}\mathbf{G}^T\mathbf{m}. \tag{11}$$

Results

Conclusion