

Analysis of time delay data and clock drift in a network of seismic monitors

Jordi Anguera ¹ Leevi Annala ² Stefan Dimitrijevic ³
Patricia Pauli ⁴ Liisa-Ida Sorsa ⁵ Dimitar Trendafilov ⁶
Christophe Pickard ⁷

¹Autonomous University of Barcelona, Spain ²University of Jyväskylä, Finland

³University of Novi Sad, Serbia ⁴Technical University of Darmstadt, Germany

⁵Tampere University of Technology, Finland

⁶University of Sofia "St. Kliment Ohridski", Bulgaria

⁷University of Grenoble Alpes and Grenoble INP, France

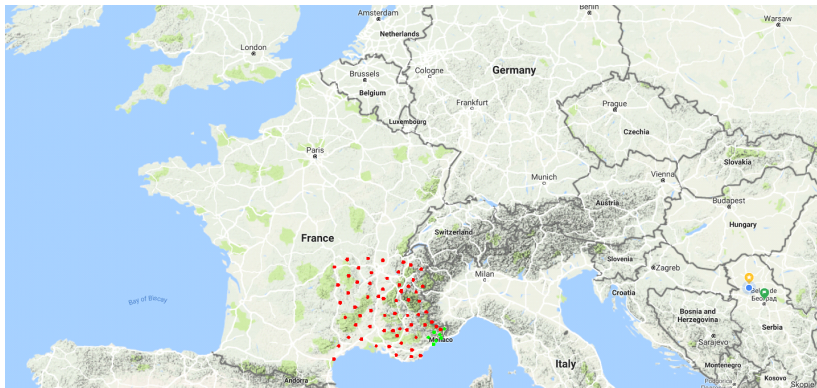
ECMI Modelling Week, 21 July 2018

Problem statement

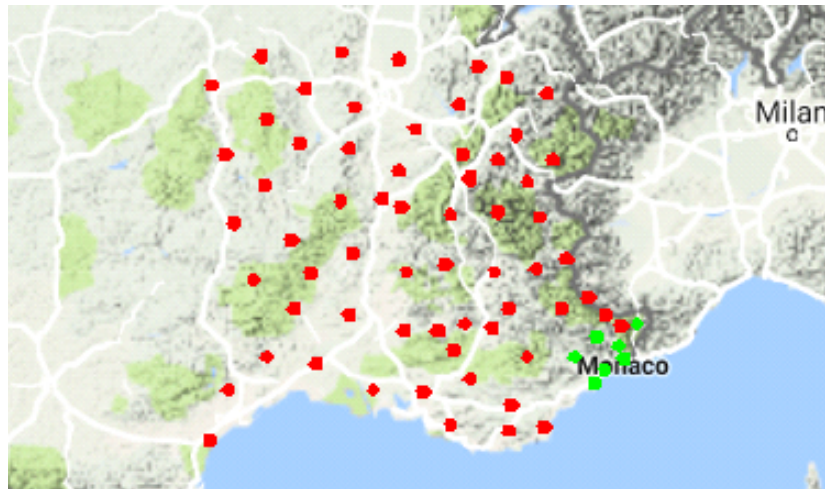
- ▶ Seismic monitoring is used to study the behaviour and composition of the underground floor
- ▶ There is a time drift in the monitors' clocks, causing inaccuracies in analysis
- ▶ Continuous synchronization to the Global Positioning System for accurate timing is not possible
- ▶ A reliable method to correct the time drift of the clocks' is required

The network of seismic monitors

- ▶ The network consists of 73 seismic monitors
- ▶ For initial analysis, 7 stations are selected

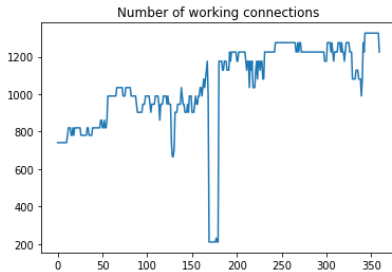
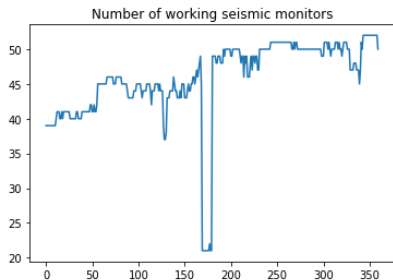


The network of seismic monitors



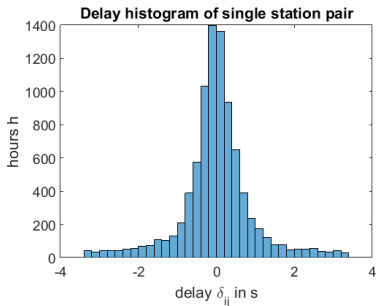
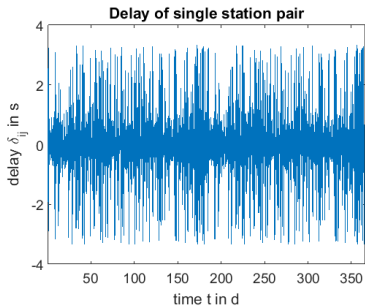
Numbers of working stations and connections to stations during one year

- ▶ The maximum of simultaneously working stations is 52 out of 73
- ▶ The maximum of connections is 1326
- ▶ If two monitors are working simultaneously, they are connected

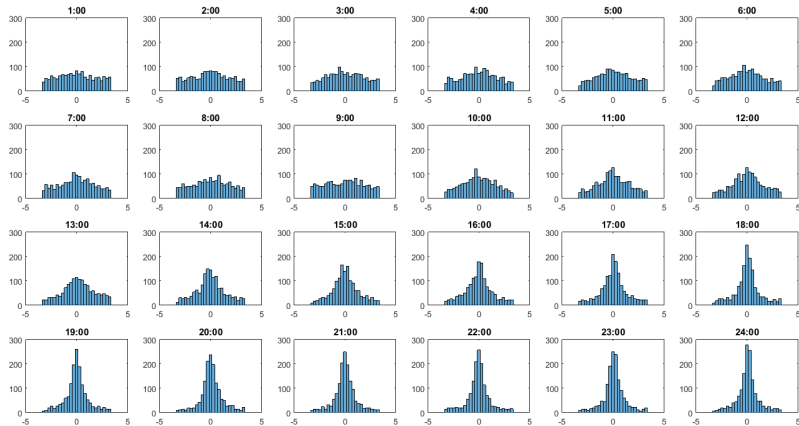


Time-delay signal characteristics

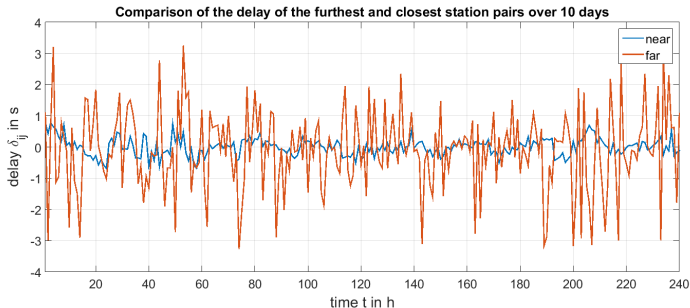
- ▶ Data recorded in one second intervals
- ▶ Time-delay cross-correlation is computed in one-hour intervals
- ▶ Each connection is computed only once
- ▶ Tracing time-delays recorded by one station over a year



Histograms showing time delays recorded by all active stations over 24 hours



Comparison of time delay of the furthest and the closest station pairs



- Time delays are smaller between stations which are close to each other

Graph approach to model the data

- ▶ Measured time delays $\hat{\delta}$ between stations:

$$\hat{\delta} = \delta(t) + \varepsilon(t) \quad (1)$$

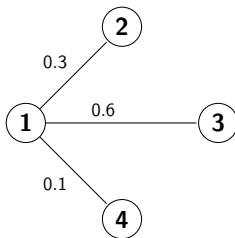
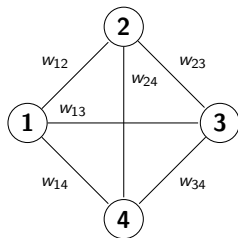
- ▶ Graphs $\mathcal{G}_{\delta} = (\mathcal{V}, \mathcal{E}, \hat{\delta})$, $\mathcal{G}_r = (\mathcal{V}, \mathcal{E}, \mathbf{W})$

- ▶ Metric:

$$f_i(\hat{\delta}) = K_i \quad (2)$$

with

$$f_i(\hat{\delta}) = \sum_j \delta_{ij} \quad \text{and} \quad K_i = f_i(\mathbf{W}) = \sum_j w_{ij} \quad (3)$$



text here

Signal denoising

- ▶ We specify a cost function $c(\hat{\delta})$ measuring the deviation of the metric:

$$c(\hat{\delta}) = \sum_i e_i^2(\hat{\delta}) = \sum_i (f_i(\hat{\delta}) - K_i)^2 \quad (4)$$

- ▶ Error is minimized with gradient descent updates on $\hat{\delta}$:

$$\hat{\delta}^{(t+1)} = \hat{\delta}^t - \mu \sum_i e_i(\hat{\delta}^t) \frac{df_i(\hat{\delta}^t)}{d\hat{\delta}^t} \quad (5)$$

Extracting clock drift from noise

- ▶ The noise ε_{ij} between each pair of two stations is modelled with

$$\varepsilon_{ij} = \Delta_i - \Delta_j + e_{ij}, \quad (6)$$

in which Δ_i and Δ_j are the clock drifts of stations i, j
 e_{ij} accounts for all other noise in the system

Solving individual clock drifts (1/2)

- We have overdetermined inverse problem of the form

$$\mathbf{G}s = \mathbf{m} \quad (7)$$

$$\begin{bmatrix}
 1 & -1 & 0 & 0 & \dots & 0 \\
 1 & 0 & -1 & 0 & \dots & 0 \\
 1 & 0 & 0 & -1 & \dots & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
 1 & 0 & 0 & 0 & \dots & -1 \\
 0 & 1 & -1 & 0 & \dots & 0 \\
 0 & 1 & 0 & -1 & \dots & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
 0 & 1 & 0 & 0 & \dots & 0 \\
 \dots & \dots & \dots & \dots & \dots & \dots \\
 0 & 0 & \dots & 0 & 1 & -1
 \end{bmatrix}
 \begin{bmatrix}
 \Delta_1 \\
 \Delta_2 \\
 \Delta_3 \\
 \Delta_4 \\
 \vdots \\
 \Delta_k \\
 \vdots \\
 \Delta_{n-1} \\
 \Delta_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 \Delta_1 - \Delta_2 \\
 \Delta_1 - \Delta_3 \\
 \Delta_1 - \Delta_4 \\
 \vdots \\
 \Delta_1 - \Delta_n \\
 \Delta_2 - \Delta_3 \\
 \Delta_2 - \Delta_4 \\
 \vdots \\
 \Delta_2 - \Delta_k \\
 \vdots \\
 \Delta_{n-1} - \Delta_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 \Delta_{12} \\
 \Delta_{13} \\
 \Delta_{14} \\
 \vdots \\
 \Delta_{1n} \\
 \Delta_{23} \\
 \Delta_{24} \\
 \vdots \\
 \Delta_{2k} \\
 \vdots \\
 \Delta_{(n-1)n}
 \end{bmatrix}$$

- Matrix \mathbf{G} 's rank is $n - 1$

Solving individual clock drifts (2/2)

- ▶ Ordinary least squares regression

$$\|\mathbf{G}\mathbf{s} - \mathbf{m}\|_2^2. \quad (8)$$

- ▶ Tikhonov regularization is employed

$$\|\mathbf{G}\mathbf{s} - \mathbf{m}\|_2^2 + \|\mathbf{\Gamma}\mathbf{s}\|_2^2, \quad (9)$$

for some suitable Tikhonov matrix $\mathbf{\Gamma} = \alpha \mathbf{I}$

- ▶ The explicit solution is hence given by

$$\hat{\mathbf{s}} = (\mathbf{G}^T \mathbf{G} + \mathbf{\Gamma}^T \mathbf{\Gamma})^{-1} \mathbf{G}^T \mathbf{m}. \quad (10)$$

Results

Conclusion