Analysis of time delay data and clock drift in a network of seismic monitors

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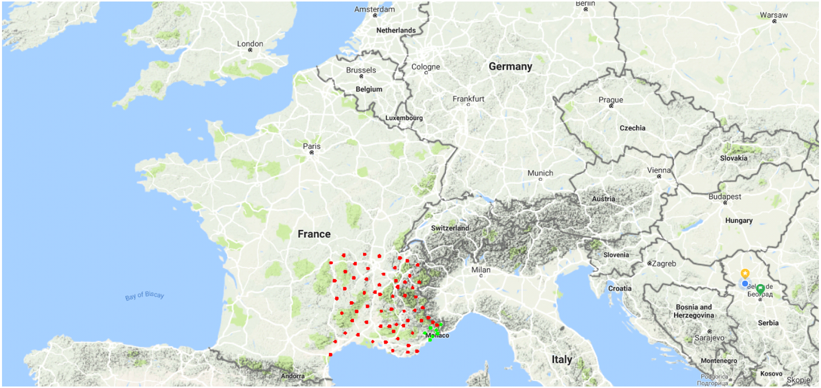
Problem statement

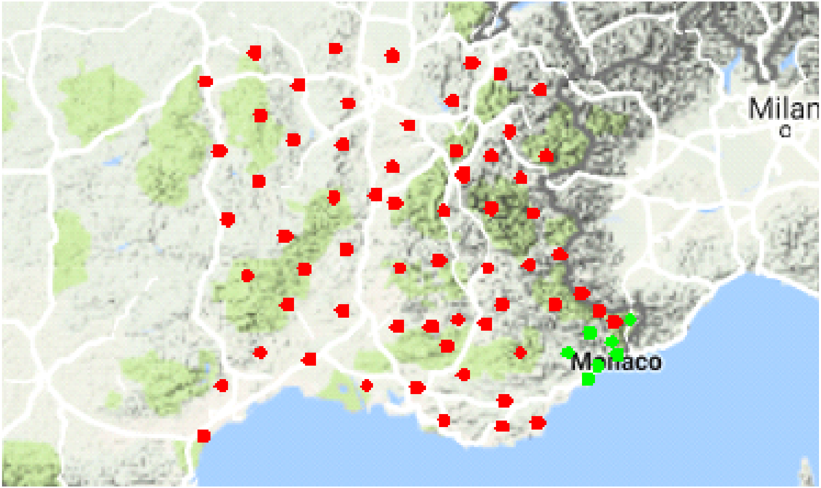
- Seismic monitoring is used to study the behaviour and composition of the underground floor
- There is a time drift in the monitors' clocks, causing inaccuracies in analysis
- Continuous synchronization to the Global Positioning System for accurate timing is not possible
- A reliable method to correct the time drift of the clocks' is required

The network of seismic monitors

- The network consists of 73 seismic monitors
- For initial analysis, 7 stations are selected

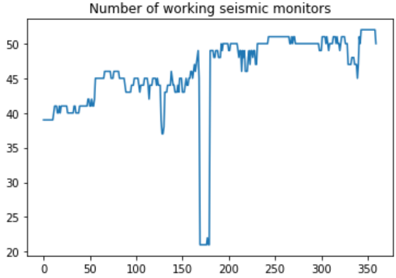


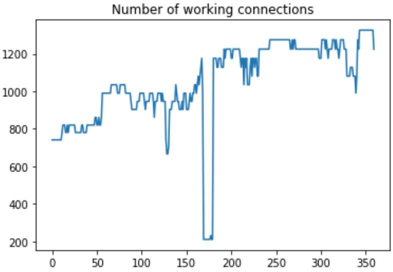
The network of seismic monitors



Numbers of working stations and connections to stations during one year

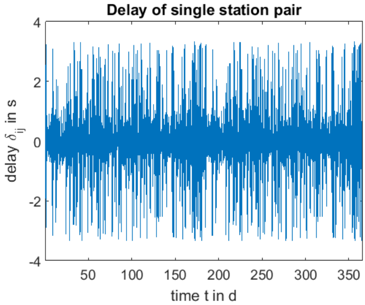
- The maximum of simultaneously working stations is 52 out of 73
- The maximum of connections is 1326
- If two monitors are working simultaneously, they are connected

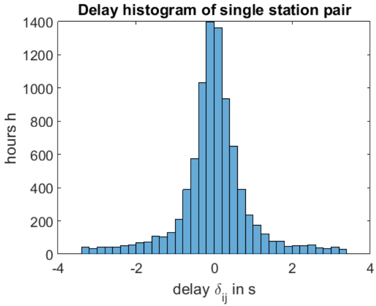




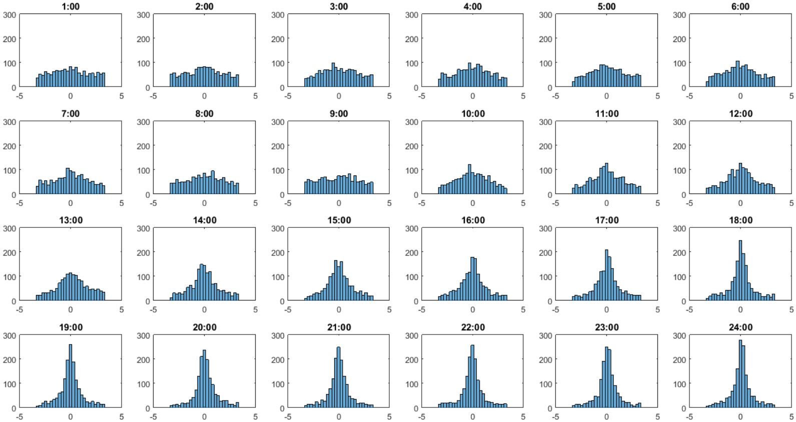
Time-delay signal characteristics

- Data recorded in one second intervals
- Time-delay cross-correlation in computed in one-hour intervals
- Each connection is computed only once
- Tracing time-delays recorded by one station over a year

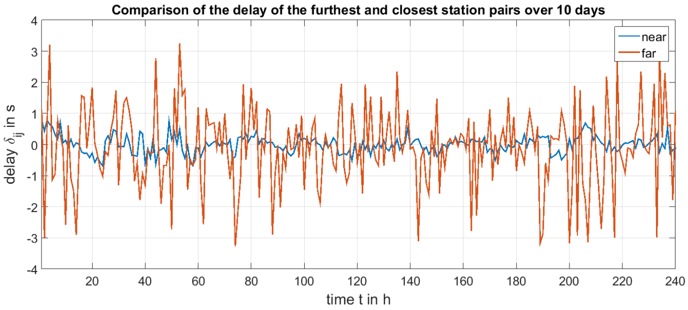




Histograms showing time delays recorded by all active stations over 24 hours



Comparison of time delay of the furthest and the closest station pairs



- Time delays are smaller between stations which are close to each other

Graph approach to model the data

- The measurements $\hat{\boldsymbol{\delta}}$ of time delays between stations

$$\hat{\boldsymbol{\delta}} = \boldsymbol{\delta} + \boldsymbol{\varepsilon} \quad (1)$$

- A weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \hat{\boldsymbol{\delta}}_t)$
- Values $\hat{\delta}_{ij}$ are normalized to $[0, 1]$
- A normalized distance measure was selected as graph metrics K_m

$$K_m = f_m(r_i) = \sum_j r_{ij}. \quad (2)$$

Signal denoising

- We specify a cost function $c(\hat{\boldsymbol{\delta}})$ measuring the deviation from the observed weight matrix's metrics $f_m(\hat{\boldsymbol{\delta}})$ to the estimates of the distance metrics K_m

$$c(\hat{\boldsymbol{\delta}}) = \sum_m e_m^2(\hat{\boldsymbol{\delta}}) = \sum_m (f_m(\hat{\boldsymbol{\delta}}) - K_m)^2 \tag{3}$$

- Error is minimized with gradient descent updates on $\hat{\boldsymbol{\delta}}$:

$$\hat{\boldsymbol{\delta}}^{(t+1)} = \hat{\boldsymbol{\delta}}^t - \mu \sum_m e_m(\hat{\boldsymbol{\delta}}^t) \frac{df_m(\hat{\boldsymbol{\delta}}^t)}{d\hat{\boldsymbol{\delta}}^t}, \tag{4}$$

Exctracting clock drift from noise

- The noise ε_{ij} between each pair of two stations is modelled with
$$\varepsilon_{ij} = \Delta_i - \Delta_j + e_{ij}, \tag{5}$$
in which Δ_i and Δ_j are the clock drifts of stations i, j
e_{ij} accounts for all other noise in the system

Solving individual clock drifts (1/2)

- We have overdetermined inverse problem of the form

$$Gm = s \tag{6}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & \dots & 0 \\ 1 & 0 & -1 & 0 & \dots & 0 \\ 1 & 0 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & 0 & \dots & -1 \\ 0 & 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & 0 & -1 & \dots & 0 \\ 0 & 1 & 0 & 0 & \ddots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \vdots \\ \Delta_k \\ \vdots \\ \Delta_{n-1} \\ \Delta_n \end{bmatrix} = \begin{bmatrix} \Delta_1 - \Delta_2 \\ \Delta_1 - \Delta_3 \\ \Delta_1 - \Delta_4 \\ \vdots \\ \Delta_1 - \Delta_n \\ \Delta_2 - \Delta_3 \\ \Delta_2 - \Delta_4 \\ \vdots \\ \Delta_2 - \Delta_k \\ \vdots \\ \Delta_{n-1} - \Delta_n \end{bmatrix} = \begin{bmatrix} \Delta_{12} \\ \Delta_{13} \\ \Delta_{14} \\ \vdots \\ \Delta_{1n} \\ \Delta_{23} \\ \Delta_{24} \\ \vdots \\ \Delta_{2k} \\ \vdots \\ \Delta_{(n-1)n} \end{bmatrix}$$

- Matrix G rank is $n - 1$

Solving individual clock drifts 1/2

- Ordinary least squares regression

$$\|\mathbf{Gm} - \mathbf{s}\|_2^2. \tag{7}$$

- Tikhonov regularization is employed

$$\|\mathbf{Gm} - \mathbf{s}\|_2^2 + \|\boldsymbol{\Gamma}\mathbf{m}\|_2^2, \tag{8}$$

  for some suitable Tikhonov matrix $\boldsymbol{\Gamma} = \alpha\mathbf{I}$

- The explicit solution is hence given by

$$\hat{\mathbf{s}} = (\mathbf{G}^T\mathbf{G} + \boldsymbol{\Gamma}^T\boldsymbol{\Gamma})^{-1}\mathbf{G}^T\mathbf{m}. \tag{9}$$

Results

Conclusion